

## Magnetic experiments on board the LTP

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### Approval List

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## Acronyms

ASD	Astrium GmbH, Astrium Deutschland/Germany
ASU	Astrium Ltd., Astrium UK
CI	Configuration Item
DC	Direct Current
DDS	Data Management and Diagnostics Sub-System
DFACS	Drag Free and Attitude Control System
DMU	Data Management Unit
DWS	Differential Wavefront Sensing
EH	Electrode Housing
EM	Engineering Model
EMP	Experiment Master Plan
ESA	European Space Agency
ESAC	European Space Astronomy Centre (near Madrid)
GW	Gravitational Wave
ICD	Interface Control Document
IEEC	Institut d'Estudis Espacials de Catalunya
LISA	Laser Interferometer Space Antenna
LPF	LISA Pathfinder (formerly SMART-2)
LTP	LISA Technology Package
LTPA	LTP Architect
LTPDA	LTP Data Analysis
MBW	Measurement Bandwidth
NASA	National Aeronautics and Space Administration
NTE	Nuevas Tecnologías Espaciales, S.A. (Lliçà d'Amunt, Spain)
OMS	Optical Metrology Subsystem
SNR	Signal to Noise Ratio
TM	Telemetry / Test mass
UTN	University of Trento
VE	Vacuum Enclosure

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## 1 Applicable documents

Ref.	Title	Doc Number	Issue	Date
AD1	Science Requirements and Top-level Architecture Definition for the LISA Technology Package (LTP) on Board LISA Pathfinder (SMART-2)	LTPA-UTN-ScRD	3.1	30/Jun/2005
AD2	DDS Subsystem Specification	S2-ASD-RS-3004	4.4	02/May/2007

## 2 Reference documents

Ref.	Title	Doc Number	Issue	Date
RD1	DFACS User Manual	S2-ASD-MA-2004	1.0	22/May/2008
RD2	Measurement of LTP dynamical coefficients by system identification	S2-UTN-TN-3045	1.1	05/Jun/2007
RD3	Diagnostic elements-DAUs interface configuration	S2-IEC-TN-3023	1.3	03/Dec/07
RD4	DDS Design Specification	S2-NTE-DS-3001	1.3	29/Sep/2006
RD5	J Schoukens and R Pintelon, <i>Identification of Linear Systems</i>	—	—	Pergamon Press, 1991
RD6	A Schnabel, L Trougnou and E Balaguer, <i>LTP EM TM Remnant Magnetic Moment Measurements</i>	S2-EST-TR-3002	2.0	2/Feb/2007
RD7	M Nofrarias and J Sanjuán, <i>Thermal experiments on board LPF</i>	S2-IEC-TN-3042	0.4	12/Sep/2008
RD8	A Schleicher, N Brandt and T Ziegler, <i>DFACS External ICD</i>	S2-ASD-ICD-2011	1.4	15/Sep/2008
RD9	J Fauste, M Armano, <i>LTP Investigations Description Document</i>	S2-ESAC-TN-5005	1.0	20/Oct/2008



## 3 Introduction

### 3.1 Scope

This note is released in support of the Experimental Master Plan [RD1] and describes the experiments that will be performed on board the LTP related to magnetic services on board.

### 3.2 Acronyms

ASD	Astrium GmbH, Astrium Deutschland/Germany
ASU	Astrium Ltd., Astrium UK
CI	Configuration Item
DC	Direct Current
DDS	Data Management and Diagnostics Sub-System
DFACS	Drag Free and Attitude Control System
DMU	Data Management Unit
DWS	Differential Wavefront Sensing
EH	Electrode Housing
EM	Engineering Model
EMP	Experiment Master Plan
ESA	European Space Agency
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GW	Gravitational Wave
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LISA	Laser Interferometer Space Antenna
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LTP	LISA Technology Package
LTPA	LTP Architect
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MBW	Measurement Bandwidth
NASA	National Aeronautics and Space Administration
NTE	Nuevas Tecnologías Espaciales, S.A. (Lliçà d'Amunt, Spain)
OMS	Optical Metrology Subsystem
SNR	Signal to Noise Ratio
TM	Telemetry / Test mass
UTN	University of Trento
VE	Vacuum Enclosure

## 4 Experiment objectives and global overview

The LTP is endowed with two magnetic coils, whose common axis is the line joining the two proof masses, and placed in the outer sides of each of the vacuum enclosures. By feeding currents of various frequencies in the MBW to these coils one generates controlled magnetic fields, which are sufficient to exert forces on the TMs, due to their remanent magnetic moments and magnetic susceptibility. The resulting accelerations are measured by the LTP interferometer, thereby making possible the estimation of the above magnetic parameters of the TMs. Once these are known, knowledge of the magnetic environmental magnetic field during science operations (with coils off) by means of a set of four high sensitivity magnetometers, that part of the total LTP noise attributable to magnetic fluctuations can be estimated. Layout details are shown in Figures 4.1 and 4.2. The rest of this document concentrates on the analysis of the required signals and resulting SNRs, thereby defining the sequence of experiments which should be arranged for during mission flight —see tables 7.1 towards the end of this document. The analysis of parameter estimation follows the general principles described in more detail in reference [RD7], and will be omitted here.

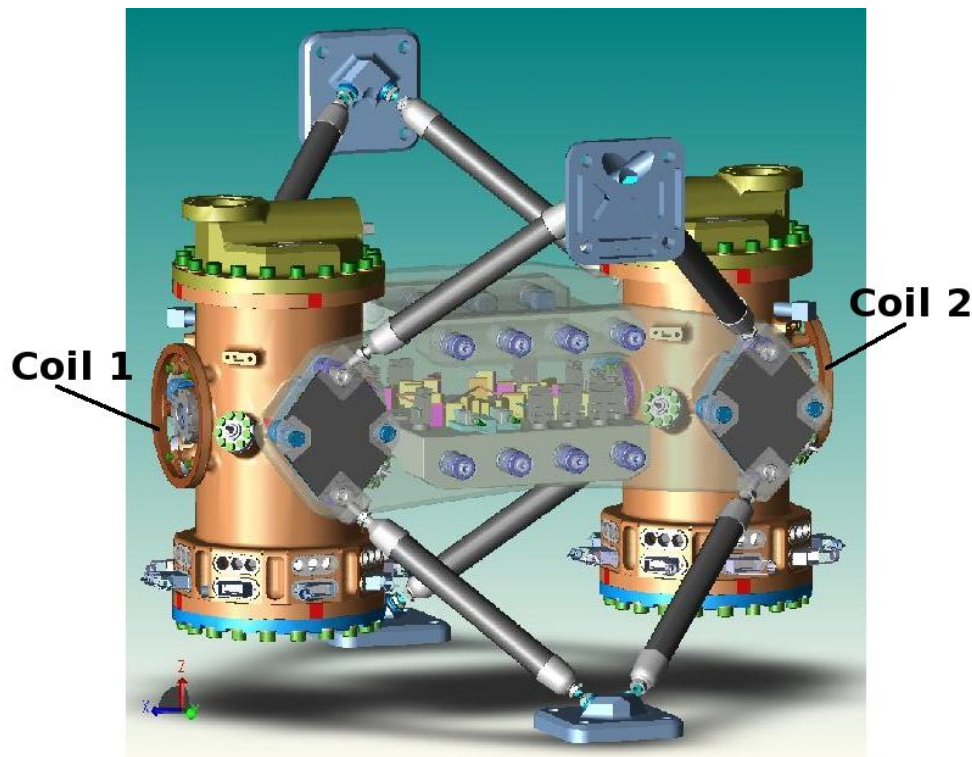


Figure 4.1: Design picture of the LTP. Coils can be seen around the VE flanges in the left and right ends of the drawing.

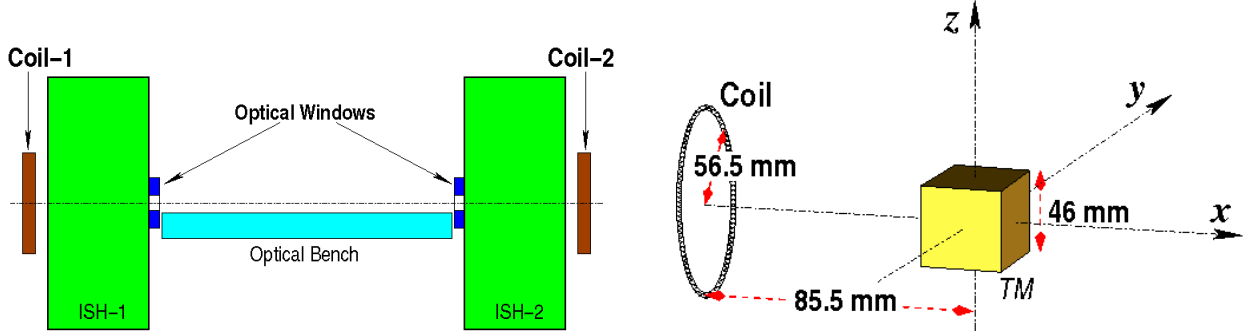


Figure 4.2: Global concept drawing of the coils' positions (left), and geometric data (right), applicable to either TM.

## 5 Theory and signals

The LTP TMs are at all times under the influence of an environmental magnetic field,  $\mathbf{B}_{env}$ , whose DC value is required to be below  $10 \mu\text{T}$ , and whose DC gradient should be below  $5 \mu\text{T/m}$ . If a coil is switched on, it will generate an applied magnetic field,  $\mathbf{B}_{app}$ , which adds linearly to the environmental one and gives rise to a forces and torques on each TM; these are given by

$$F_x = \left\langle \left[ \left( \mathbf{M} + \frac{\chi}{\mu_0} \mathbf{B} \right) \cdot \nabla \right] B_x \right\rangle V \quad (5.1)$$

for the force along the sensitive axis, and

$$\mathbf{N} = \left\langle \mathbf{M} \times \mathbf{B} + \mathbf{r} \times \left[ (\mathbf{M} \cdot \nabla) \mathbf{B} + \frac{\chi}{\mu_0} (\mathbf{B} \cdot \nabla) \mathbf{B} \right] \right\rangle V \quad (5.2)$$

for the torque relative to the TM centre of mass. The meaning of symbols in the above formulas is as follows:

$\mathbf{B} = \mathbf{B}_{env} + \mathbf{B}_{app}$	Total magnetic field in the TM
$\mathbf{M}$	Density of magnetic moment (magnetisation) of the TM
$\mathbf{r}$	Vector distance to the TM centre of mass
$V$	Volume of the TM
$\chi$	Magnetic susceptibility of the TM
$\mu_0$	Vacuum magnetic constant ( $4\pi \times 10^{-7} \text{ m kg s}^{-2} \text{ A}^{-2}$ )
$\langle \dots \rangle$	TM volume average of enclosed quantity

Due to axial symmetry, the magnetic field vector created by the  $N$ -turn coil has only two relevant components: *longitudinal*,  $\mathbf{B}_{app,x}$ , and *transverse*,  $\mathbf{B}_{app,\rho}$ . These are given by rather cumbersome analytic expressions involving elliptic integrals with complicated arguments, which we omit to quote here. On the coil's axis those expressions reproduce the well known formulas

$$B_{app,x}(x, \rho = 0) = \frac{\mu_0}{4\pi} \frac{2\pi a^2 N I}{(a^2 + x^2)^{3/2}} \quad \text{and} \quad B_{app,\rho}(x, \rho = 0) = 0 \quad (5.3)$$

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Table 5.1: On-axis magnetic field and gradient values at three points on the TM for a 1 mA DC current fed to the 2400 turn LTP coil. They are the closest point to the TM, the TM centre and the farthest point. Geometrical data are displayed in figure 4.2, right.

	$x = 62.5$ mm	$x = 85.5$ mm	$x = 108.5$ mm
$B_{\text{app},x}$ [ $\mu\text{T}$ ]	8.05	4.47	2.63
$\partial B_{\text{app},x}/\partial x$ [ $\mu\text{T}/\text{m}$ ]	-212.6	-109.2	-57.2

A useful reference is provided by an estimate of the longitudinal field and its gradient when a DC current of 1 mA is fed to the 2400 turn LTP coil. Values are shown in table 5.1.

The idea of the experiment is to measure linear and angular accelerations inflicted upon the TMs by the coil magnetic fields and gradients, thereby to characterise the magnetic properties of the TMs. Once these are known, it will be possible to determine the magnetic noise, which will relate magnetic field measurements (by suitable magnetometers) to TM accelerations, or indeed *transfer function* between those measurements and OMS data.

The applied magnetic fields will be *periodic* signals at various frequencies inside the LTP MBW. These are obtained by injecting into the coil sinusoidal currents,  $I(t) = I_0 \sin \omega_0 t$ , so that

$$\mathbf{B}_{\text{app}}(\mathbf{x}, t) = \mathbf{B}_0(\mathbf{x}) \sin \omega_0 t \quad (5.4)$$

For example, on the coils axis we would have

$$B_{0,x}(x, \rho = 0) = \frac{\mu_0}{4\pi} \frac{2\pi a^2 N I_0}{(a^2 + x^2)^{3/2}} \quad (5.5)$$

As we see in table 5.1, the gradient of the applied field is much larger than that of the environmental field —the latter being required to be less than  $5 \mu\text{T}/\text{m}$ , see [AD1]. We hence safely assume

$$\nabla B_{\text{env},x} \ll \nabla B_{\text{app},x} \quad \Rightarrow \quad \nabla B_{\text{env},x} + \nabla B_{\text{app},x} \simeq \nabla B_{\text{app},x} \quad (5.6)$$

Replacing equation (5.4) into (5.1) we find there are two groups of terms, as follows:

$$F_x = \left\langle \left( \mathbf{M} + \frac{\chi}{\mu_0} \mathbf{B}_{\text{env}} \right) \cdot \nabla B_{0,x} \right\rangle V \sin \omega_0 t \quad (5.7.a)$$

$$+ \frac{\chi V}{\mu_0} \langle \mathbf{B}_0 \cdot \nabla B_{0,x} \rangle \sin^2 \omega_0 t \quad (5.7.b)$$

Linear acceleration signals are thus seen to come in two separate frequencies, one at the frequency of the applied current and the other at twice that frequency. We also note there is a DC component in the response signal, since  $\sin^2 \omega_0 t = (1 - \cos 2\omega_0 t)/2$ . We consider all these frequency components separately in the ensuing analysis.

A similar response scheme is observed in the angular acceleration response following the applied torques:

$$\mathbf{N} = \left\langle \mathbf{M} \times \mathbf{B}_0 + \mathbf{r} \times [(\mathbf{M} \cdot \nabla) \mathbf{B}_0] + \mathbf{r} \times \frac{\chi}{\mu_0} [(\mathbf{B}_{\text{env}} \cdot \nabla) \mathbf{B}_0] \right\rangle V \sin \omega_0 t + \quad (5.8.a)$$

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$$+ \left\langle \mathbf{r} \times \frac{\chi}{2\mu_0} \nabla (B_0^2) \right\rangle V \sin^2 \omega_0 t \quad (5.8.b)$$

It must be noted that, because of the symmetry of the applied magnetic field the line (5.8.b) vanishes, so that

$$\mathbf{N} = \left\langle \mathbf{M} \times \mathbf{B}_0 + \mathbf{r} \times [(\mathbf{M} \cdot \nabla) \mathbf{B}_0] + \mathbf{r} \times \frac{\chi}{\mu_0} [(\mathbf{B}_{\text{env}} \cdot \nabla) \mathbf{B}_0] \right\rangle V \sin \omega_0 t \quad (5.9)$$

Only the torque's  $y$  and  $z$  components can be measured with the *LTP* interferometer, as  $N_x$  generates rotations around the laser beam direction. Those two components will be used in this experiment for the estimation of  $m_z$  and  $m_y$ , respectively. We come to this further down.

## 6 Signal-to-noise ratios

### 6.1 Linear acceleration measurements

#### 6.1.1 The $2\omega$ signal

Here we refer to TM linear accelerations. The  $2\omega$  signal is given by

$$a_{2\omega}(t) = \frac{\chi V}{2\mu_0 m_{\text{TM}}} \langle \mathbf{B}_0 \cdot \nabla B_{0,x} \rangle \cos 2\omega_0 t \quad (6.1)$$

where  $m_{\text{TM}}$  is the mass of a TM.  $a_{2\omega}$  can be measured with the OMS; all the remaining quantities in equation (6.1) are known from the experiment setup *except* the magnetic susceptibility,  $\chi$ , or

$$a_{2\omega}(t) = \chi A_0 \cos 2\omega_0 t, \quad \text{with} \quad A_0 = \frac{V}{2\mu_0 m_{\text{TM}}} \langle \mathbf{B}_0 \cdot \nabla B_{0,x} \rangle \quad (6.2)$$

Here,  $A_0$  can be accurately calculated. Therefore the problem of estimating  $\chi$  is one of calculating the amplitude of a sinusoidal signal contaminated with noise. Signal-to-noise ratio is given by the usual formula

$$SNR^2 = 4 \int_{\text{MBW}} \frac{|\tilde{a}(\omega)|^2 d\omega}{S_a(\omega) 2\pi} \quad (6.3)$$

where  $S_a(\omega)$  is the linear, one-sided acceleration noise spectral density in the LTP,

$$S_a^{1/2}(\omega) = 3 \times 10^{-14} \left[ 1 + \left( \frac{\omega/2\pi}{3 \text{ mHz}} \right)^2 \right] \text{ m s}^{-2}/\sqrt{\text{Hz}} \quad (6.4)$$

in the MBW, and  $\tilde{a}(\omega)$  is the Fourier transform of (6.2), given by

$$\tilde{a}(\omega) = \chi A_0 T \text{ sinc} [(\omega - 2\omega_0) T/2] \quad (6.5)$$

where  $\text{sinc} \theta \equiv \sin \theta / \theta$ , and  $T$  is the experiment integration time. An estimate of the SNR requires an evaluation of the amplitude  $A_0$ , which is an easy numerical integration exercise, plus the adoption of some nominal value for the susceptibility; the natural choice is the requirement,  $\chi \sim 2.5 \times 10^{-5}$ . With this choice, and integration times which include a few cycles of the signal, the results shown in table 6.1 obtain.

Table 6.1: Amplitude SNR for  $a_{2\omega}$  for various measurement parameters. A nominal value  $\chi = 2.5 \times 10^{-5}$  has been assumed. Signals peak at  $2\omega_0$ .  $T$  is the integration time.

$2\omega_0/2\pi$	$I_0 = 0.5$ mA	$I_0 = 1$ mA	$I_0 = 1.5$ mA	$I_0 = 2$ mA
1 mHz ( $T=12000$ s, 12 cycles)	590	1180	1780	2370
3 mHz ( $T=4000$ s, 12 cycles)	190	380	570	760
5 mHz ( $T=2400$ s, 12 cycles)	80	155	230	310
7 mHz ( $T=5000$ s, 35 cycles)	65	130	200	260

### 6.1.2 The DC component

The term in equation (5.7.b) actually incorporates a DC component as well as the  $2\omega_0$  one, because  $\sin^2 \omega_0 t = (1 - \cos 2\omega_0 t)/2$ . This results in a DC force applied to the TM, which causes it to drift with a constant acceleration given by

$$a_{\text{DC}} = \frac{\chi V}{2\mu_0} \langle \mathbf{B}_0 \cdot \nabla B_{0,x} \rangle \quad (6.6)$$

If there was no control loop in the LTP, the TM would eventually hit the electrodes after  $\sim 65$  ( $1 \text{ mA}/I_0$ ) hours. Nevertheless, the action of the control loops does prevent the crash. Figure 6.1 shows the  $x_1-x_2$  interferometer channel signal when TM-1 is subjected to the oscillating magnetic field of the closest coil, together with the  $x_1$  interferometer channel readout, which gives a measure of the distance between the TM and its Electrode Housing (EH). As shown, the TM offset only varies by a few nano-metres during the experiment.

### 6.1.3 The $1\omega$ signal

The  $1\omega$  TM acceleration signal is given by

$$a_{1\omega} = \frac{V}{m_{\text{TM}}} \left\langle \left( \mathbf{M} + \frac{\chi}{\mu_0} \mathbf{B}_{\text{env}} \right) \cdot \nabla B_{0,x} \right\rangle \sin \omega_0 t \quad (6.7)$$

The environmental field  $\mathbf{B}_{\text{env}}$  is required to have a DC value below  $10 \mu\text{T}$ . The remanent magnetic moment of the TM is required to be of order  $2 \times 10^{-8} \text{ Am}^2$ , which means the magnetisation  $\mathbf{M}$  is approximately  $10^{-4} \text{ A/m}$ . If we adopt these limit nominal values then the two terms in round brackets in equation (6.7) are the same order of magnitude. It however turns out that the actual value of  $\mathbf{B}_{\text{env}}$  is very appreciably smaller than  $10 \mu\text{T}$ , in fact in the order of  $0.1 \mu\text{T}$ , which is nearly two orders of magnitude below the requirement<sup>1</sup>. We can take advantage of this circumstance to introduce a convenient simplification in equation (6.7):

$$a_{1\omega} = \frac{V}{m_{\text{TM}}} \langle \mathbf{M} \cdot \nabla B_{0,x} \rangle \sin \omega_0 t \quad (6.8)$$

<sup>1</sup>This information comes from magnetic cleanliness data advanced by ASU to IEEC on 28 November 2006, thanks are due to Dave Wealthy. Such data did not come from real hardware tests, to be done shortly, but on estimates in terms of equipment specifications.

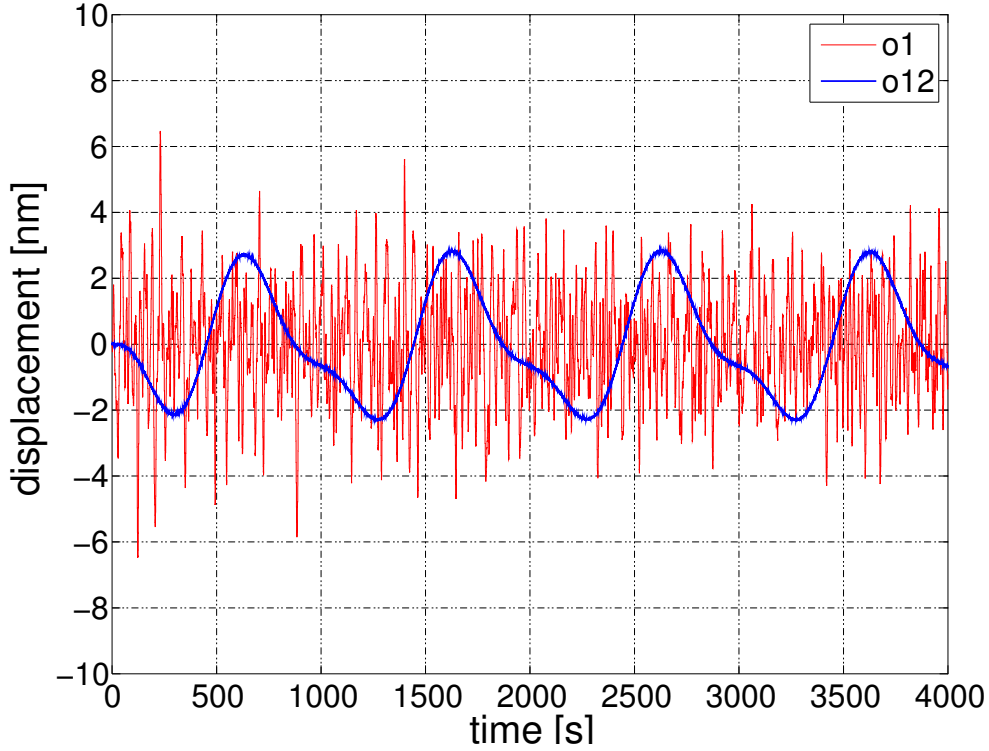


Figure 6.1: Relative distance variations between TM1 and TM2 (blue line) and variations of the distance between TM1 and the EH (red line) when a current of 1 mA at a frequency of 1 Hz is fed to the coil next to TM1. The red line shows that the S/C motion is considerably noisier than that of the TMs.

If the TM magnetisation happens to be *homogeneous* throughout its volume then we have the simplified expression

$$a_{1\omega} = \frac{1}{m_{\text{TM}}} \mathbf{m}_0 \cdot \langle \nabla B_{0,x} \rangle \sin \omega_0 t, \quad \text{for a homogeneous TM} \quad (6.9)$$

where  $\mathbf{m}_0 \equiv (m_x, m_y, m_z)$  is the total remanent magnetic moment of the TM. The above further simplifies to

$$a_{1\omega} = \frac{1}{m_{\text{TM}}} m_x \left\langle \frac{\partial B_{0,x}}{\partial x} \right\rangle \sin \omega_0 t, \quad \text{in a homogeneous TM} \quad (6.10)$$

since the  $y$  and  $z$  components of  $\nabla B_{0,x}$  average to zero due to symmetry of the coil field. This means the only measurable component of  $\mathbf{m}_0$  is  $m_x$ , in case the TM remanent magnetisation is indeed homogeneous. Information on the magnetic remanence of the EM TMs is available in [RD6], which does not specially hint at any detected inhomogeneities, although it indicates they could be present. This seems a realistic possibility, and a procedure to estimate them, hence determine  $\mathbf{m}_0$ , can be set up —see below.

Following the same mathematical procedure described in section 6.2, we can estimate the SNR for the determination of  $m_x$ . The average value of the gradient across the TM is estimated to be  $\langle \partial B_{0,x} / \partial x \rangle \simeq 120 \mu\text{T/m}$  (for a 1 mA current, and scaling linearly), and adopting the nominal value of  $2 \times 10^{-8} \text{ Am}^2$  for  $m_x$ , the SNRs obtained for various configuration parameters are shown in table 6.2.

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Table 6.2: Amplitude SNR for  $a_\omega$  for various measurement parameters. A nominal value  $m_x = 2 \times 10^{-8} \text{ Am}^2$  has been assumed.  $T$  is the integration time.

$\omega_0/2\pi$	$I_0 = 0.5 \text{ mA}$	$I_0 = 1 \text{ mA}$	$I_0 = 1.5 \text{ mA}$	$I_0 = 2 \text{ mA}$
1 mHz ( $T=12000 \text{ s}$ , 12 cycles)	1420	2850	4270	5700
3 mHz ( $T=4000 \text{ s}$ , 12 cycles)	460	900	1370	1825
5 mHz ( $T=2400 \text{ s}$ , 12 cycles)	190	370	560	750
7 mHz ( $T=5000 \text{ s}$ , 35 cycles)	160	320	475	630

## 6.2 Angular acceleration measurements

As stressed above, only the  $1\omega$  frequency appears in the angular motions of the TM in response to torque actuation on the TMs. Considering again the uniform magnetisation approximation, the measurable torque components are the following:

$$N_y = \left\langle B_{0,x} + z \frac{\partial B_{0,x}}{\partial z} - x \frac{\partial B_{0,z}}{\partial z} \right\rangle m_z \sin \omega_0 t \quad (6.11.a)$$

$$N_z = \left\langle -B_{0,x} + x \frac{\partial B_{0,y}}{\partial y} - y \frac{\partial B_{0,x}}{\partial y} \right\rangle m_y \sin \omega_0 t \quad (6.11.b)$$

Because the test mass is (almost) a perfect cube, i.e., all its three moments of inertia are equal, the equations of motion for the angular degrees of freedom  $\eta$  (around the  $y$ -axis) and  $\phi$  (around the  $z$ -axis) are

$$\mathcal{I}\ddot{\eta} = N_y \quad \text{and} \quad \mathcal{I}\ddot{\phi} = N_z \quad (6.12)$$

where  $\mathcal{I}$  is the moment of inertia

$$\mathcal{I} = \frac{1}{6} m_{\text{TM}} a^2 = 6.9 \times 10^{-4} \text{ kg m}^2 \quad (6.13)$$

with  $a = 4.6 \text{ mm}$  the length of the side of the TM. The angular accelerations can be derived from the Differential Wavefront Sensing (DWS) of the interferometer channel, hence the  $m_y$  and  $m_z$  components of the remanent magnetic moment determined with equations (6.11) and (6.12).

The SNRs with which this can be accomplished depend on the noise in the torque components. This can be readily derived from [RD8], pages 23–24:

$$S_{N_y}^{1/2}(\omega) = 1.2 \times 10^{-15} \left[ 1 + \left( \frac{\omega/2\pi}{3 \text{ mHz}} \right)^2 \right] \text{ N m}/\sqrt{\text{Hz}} \quad (6.14.a)$$

$$S_{N_z}^{1/2}(\omega) = 7.9 \times 10^{-16} \left[ 1 + \left( \frac{\omega/2\pi}{3 \text{ mHz}} \right)^2 \right] \text{ N m}/\sqrt{\text{Hz}} \quad (6.14.b)$$

Signal-to-noise ratios for  $m_y$  and  $m_z$  are then evaluated in the usual way:

$$SNR^2[m_y] = 4 \int_{\text{MBW}} \frac{|\tilde{N}_y(\omega)|^2}{S_{N_y}(\omega)} \frac{d\omega}{2\pi}, \quad SNR^2[m_z] = 4 \int_{\text{MBW}} \frac{|\tilde{N}_z(\omega)|^2}{S_{N_z}(\omega)} \frac{d\omega}{2\pi} \quad (6.15)$$

Tables 6.3 and 6.4 show the SNR values available for this experiment.



Table 6.3: Amplitude SNR for  $N_y$  for various measurement parameters. A nominal value  $m_y = 2 \times 10^{-4}$  has been assumed. Signals peak at  $\omega_0$ .  $T$  is the integration time.

$\omega_0/2\pi$	$I_0 = 0.5$ mA	$I_0 = 1$ mA	$I_0 = 1.5$ mA	$I_0 = 2$ mA
1 mHz ( $T=12000$ s, 12 cycles)	3500	7000	10500	14000
3 mHz ( $T=4000$ s, 12 cycles)	1120	2240	3365	4490
5 mHz ( $T=2400$ s, 12 cycles)	460	900	1400	1800
7 mHz ( $T=5000$ s, 35 cycles)	390	780	1200	1600

Table 6.4: Amplitude SNR for  $N_z$  for various measurement parameters. A nominal value  $m_z = 2 \times 10^{-4}$  has been assumed. Signals peak at  $\omega_0$ .  $T$  is the integration time.

$\omega_0/2\pi$	$I_0 = 0.5$ mA	$I_0 = 1$ mA	$I_0 = 1.5$ mA	$I_0 = 2$ mA
1 mHz ( $T=12000$ s, 12 cycles)	5300	10500	15700	20000
3 mHz ( $T=4000$ s, 12 cycles)	1690	3400	5000	6750
5 mHz ( $T=2400$ s, 12 cycles)	700	1400	2000	2800
7 mHz ( $T=5000$ s, 35 cycles)	580	1170	1750	2350

## 7 Runs description

The above measurements are to be implemented by means of runs which need to be specified in the Experiment Master Plan (EMP). In view of the results obtained in the previous sections, table 7.1 indicates which are the selected runs. According to Table 1 of [RD9], this experiment has the following identifiers:

Parent experiment: EMP  
Experiment id.: D  
Acronym: DIAG

As shown, there are basically two groups of runs, one per TM. In each of them the corresponding coil is switched on while the other is kept off. Given the excellent SNRs available, the proposed runs should suffice to ensure good estimates of the TMs magnetic properties.

The total duration of the magnetic experiment, 113 600 seconds, is slightly over one day. There is however some margin for adjustments, if needed.

Table 7.1: Summary of runs with the control coils. The column *Offset* makes reference to a mismatch between the telecommand value of the current to be sent to the coils and the current actually sent. This is to be fixed by means of a look-up table, in preparation at the time of writing. The column *No. exec* refers to the number of executions with the same values of the investigation parameters. The investigation id. is on the column to the right, under *Inv. ID*.

Group	DDS input sine signal parameters						DDS output data channels	Comments
	Frequency	Coil current	Offset	Duration (s)	No. exec	Inv. ID		
TM-1	1 mHz	0.5 mA	0	12000	1	COI1F10C05	12 magnetometer channels	<b>DFACS Mode: M3</b> <b>Coil-1 on, Coil-2 off</b> x <sub>12</sub> data at 2 $\omega$ -> $\chi$ x <sub>12</sub> data at 1 $\omega$ -> m <sub>x</sub> x <sub>1</sub> data at 1 $\omega$ , DWS( $\eta$ ) -> m <sub>z</sub> x <sub>1</sub> data at 1 $\omega$ , DWS( $\phi$ ) -> m <sub>y</sub>
		1 mA	0	12000	1	COI1F10C10		
	3 mHz	1 mA	0	4000	1	COI1F30C10		
		1.5 mA	0	4000	1	COI1F30C15		
	5 mHz	1.5 mA	0	2400	1	COI1F50C15		
		2 mA	0	2400	1	COI1F50C20		
	7 mHz	1.5 mA	0	10000	1	COI1F70C15		
		2 mA	0	10000	1	COI1F70C20		
<b>Total TM-1 group duration (s):</b>				<b>56800</b>				

Group	DDS input sine signal parameters						DDS output data channels	Comments
	Frequency	Coil current	Offset	Duration (s)	No. exec	Inv. ID		
TM-2	1 mHz	0.5 mA	0	12000	1	COI2F10C05	12 magnetometer channels	<b>DFACS Mode: M3</b> <b>Coil-1 on, Coil-2 off</b> x <sub>1</sub> -x <sub>12</sub> data at 2 $\omega$ -> $\chi$ x <sub>1</sub> -x <sub>12</sub> data at 1 $\omega$ -> m <sub>x</sub> x <sub>1</sub> -x <sub>12</sub> data at 1 $\omega$ , DWS( $\eta$ ) -> m <sub>z</sub> x <sub>1</sub> -x <sub>12</sub> data at 1 $\omega$ , DWS( $\phi$ ) -> m <sub>y</sub>
		1 mA	0	12000	1	COI2F10C10		
	3 mHz	1 mA	0	4000	1	COI2F30C10		
		1.5 mA	0	4000	1	COI2F30C15		
	5 mHz	1.5 mA	0	2400	1	COI2F50C15		
		2 mA	0	2400	1	COI2F50C20		
	7 mHz	1.5 mA	0	10000	1	COI2F70C15		
		2 mA	0	10000	1	COI2F70C20		
<b>Total TM-2 group duration (s):</b>				<b>56800</b>				

## 8 Conclusions

The main purpose of the activation of magnetic coils in the LTP is to generate signals which will enable the determination of the magnetic properties of the TMs, more specifically their magnetic susceptibility  $\chi$  and remanent magnetic moment  $\mathbf{m}$ .

Sinusoidal electric current signals are to be fed to the coils, which result in TM linear acceleration responses at two frequencies: the original one, and double this, plus DC. The latter show up as a consequence of non-zero susceptibility, which couples the coil's field to its own gradient.

Electric currents also generate torques which cause the TM to rotate depending on its remanent magnetic moment orientation and magnitude. Angular acceleration response signals only happen at the current's nominal frequency.

Fourier analysis of the linear and angular acceleration interferometer channels thus enables a clear determination of  $\chi$  and  $\mathbf{m}$  by a systematic procedure with considerably high SNR.