Development of a Fault-Tolerant Control with MATLAB and Its Application to the Twin-Rotor System

Marcel Luzar*, Marcin Witczak*
Vicenç Puig, Fatiha Nejjari****

**Institute of Control and Computation Engineering, University of Zielona Góra,*
ul. Podgórna 50, 65-246 Zielona Góra, Poland
(e-mail: {m.luzar,m.witczak}@issi.zgora.pl)
***Automatic Control Department, Campus de Terrassa, Universidad Politécnica de Cataluña,*
Rambla Sant Nebridi 10, 08222 Terrassa, Spain
(e-mail: {vicenc.puig,fatiha.nejjari}@upc.edu)

Abstract: The main objective of this paper is to present the MATLAB block-set that can be used for fault-tolerant control of linear-parameter varying systems. In particular, the introductory part of the paper presents a theoretical background regarding the fault identification and control strategy. Subsequently, it is shown how to implement these theoretical results in Matlab/Simulink. The final part of the paper presents the experimental study regarding the twin-rotor system, which confirms the effectiveness of the developed tool.

Keywords: fault diagnosis, fault-tolerant systems, fault identification, observers, control system

1. INTRODUCTION

Fault-Tolerant Control (FTC) is a relatively new idea that makes possible to develop a control feedback that allows keeping the required system performance in the case of faults. The control strategy can be perceived fault tolerant when there is an adaptation mechanism that changes the control law in the case of faults. Another solution is to use hardware redundancy in sensors and/or actuators.

The main objective of the work is to develop a FTC block-set, based on the strategy proposed by Jai and Hamzaoui [2009]. In particular, the goal is to develop an observer that estimates an unknown state of the system. Subsequently a fault identification system is implemented that is based on the states estimates, and finally the control strategy is implemented.

The class of the systems being investigated is a Linear Parameter Varying (LPV) one. This class of systems is widely used in the literature and practice. The work contains also a comprehensive experimental study regarding a twin-rotor MIMO System, which confirms the effectiveness of the implemented technique.

2. THEORETICAL BACKGROUND

2.1 Linear Systems Case

The main objective of this point is to give a brief description of FTC technique developed by Dziekan, Witczak, Puig and Korbicz [2007]. The considered approach relies on the general idea of a virtual actuator.
Let us consider the following reference model:

$$x_{k+1} = Ax_k + Bu_k, \tag{1}$$

$$y_{k+1} = Cx_{k+1}, \tag{2}$$

where $x_k \in \mathbb{R}^n$ stands up for the reference state, $y_k \in \mathbb{R}^m$ is the reference output, and $u_k \in \mathbb{R}^r$ denotes the nominal control input. Let us also consider a possibly faulty system described by following equations:

$$x_{k+1} = Ax_k + Bu_k + Lf_k, \tag{3}$$

$$y_{k+1} = Cx_{k+1}, \tag{4}$$

where $x_{f,k} \in \mathbb{R}^n$ stands for the system state, $y_{f,k} \in \mathbb{R}^m$ is the system output, $u_{f,k} \in \mathbb{R}^r$ denotes the system input, $f_k \in \mathbb{R}^s$, $(s \leq m)$ is the fault vector, and L stands for its distribution matrix which is assumed to be known.

The main objective is to design a control strategy which can be used for determining the system input $u_{f,k}$ such that:

- the control loop for the system (3)-(4) is stable,
- $x_{f,k+1}$ converges asymptotically to x_{k+1} irrespective of the presence of the fault f_k.

The subsequent part of this section shows the development details of the scheme that is able to settle such a challenging problem.

The crucial idea is to use the following control strategy:

$$u_{f,k} = -S\hat{f}_k + K_1(x_k - x_{f,k}) + u_k \tag{5}$$

where $\hat{f}_k$ is the fault estimate. Note that, due to the separation principle, it is not assumed that $x_{f,k}$ is available, i.e. an estimate $\hat{x}_{f,k}$ can be used instead. Thus, the following problems arise:

- to determine $\hat{f}_k$
- to design K_1 in such a way that the control loop is stable, i.e. the stabilization problem.

The necessary condition for a existence of a solution is:

$$rank(CL) = rank(L) = s. \tag{6}$$

This implies that it is possible to calculate

$$H = (CL)^{+} = [(CL)^T CL]^{-1}(CL)^T$$

By multiplying (4) by H and then substituting (3) it can be shown that:

$$f_k = H(y_{f,k+1} - CAx_{f,k} - CBu_{f,k}). \tag{7}$$

Thus, if $\hat{x}_{f,k}$ is used instead of $x_{f,k}$ then the fault estimate is given as follows:

$$\hat{f}_k = H(y_{f,k+1} - CA\hat{x}_{f,k} - CBu_{f,k}). \tag{8}$$

Unfortunately, the crucial problem with practical implementation of (8) is that it requires $y_{f,k+1}$ and $u_{f,k}$ to calculate $\hat{f}_k$ and hence it cannot be directly used to obtain (5). To settle this problem, it is assumed that there exists a diagonal matrix α_k such that $\hat{f}_k = \alpha_k \hat{f}_{k-1}$ and hence the practical form (5) boils down to

$$u_{f,k} = -S\alpha_k \hat{f}_{k-1} + K_1(x_k - x_{f,k}). \tag{9}$$

Consequently, by substituting (7) into (3) it is possible to show that

$$x_{f,k+1} = \bar{A}x_{f,k} + \bar{B}u_{f,k} + \bar{L}y_{f,k+1} \tag{10}$$

where

$$\bar{A} = (I - LHC)A,\ \bar{B} = (I - LHC)B,\ \bar{L} = LH.$$

Thus, the observer structure, which can be perceived as an unknown input observer is given by

$$\hat{x}_{f,k+1} = \bar{A}\hat{x}_{f,k} + \bar{B}u_{f,k} + \bar{L}y_{f,k+1} + K_2(y_{f,k} - C\hat{x}_{f,k}). \tag{11}$$

The main objective is to summarize the presented results within an integrated framework for the development of fault identification and fault tolerant control scheme. First, let us start with following two crucial assumptions:

- the pair $(\bar{A}, C)$ is detectable,
- the pair (A, B) is stabilizable.

Under these assumptions, it is possible to design the matrices K_1 and K_2 in such a way that the extended error

$$\bar{e}_k = \begin{bmatrix} e_k \\ e_{f,k} \end{bmatrix}, \tag{12}$$

with

$$e_k = x_k - x_{f,k}, \quad e_{f,k} = x_{f,k} - \hat{x}_{f,k},$$

described by

$$\bar{e}_{k+1} = \begin{bmatrix} A - BK_1 & LHCA \\ 0 & \bar{A} - K_2C \end{bmatrix} \bar{e}_k = A_0\bar{e}_k, \tag{13}$$

converges asymptotically to zero.

2.2 Extension to LPV

This section presents an extension of the above-described strategy to Linear-Parameter Varying System that was developed by Montes de Oca, Puig, Witczak and Quevedo [2008].

The main objective is to summarize the presented results within an integrated framework for the development of fault identification and fault tolerant control scheme. First, let us start with following two assumption that are required to apply existing results on LPV gain scheduling control:

- the pair $(\bar{A}(\theta), C(\theta))$ is detectable,
- the pair $(A(\theta), B(\theta))$ is stabilizable,

for all $\theta \in \Theta$.

Moreover, note that the first assumption is typical for unknown input observers. Under these assumption, it is possible to design the matrices $K_{1,j}$ and $K_{2,j}$ in such a way that the extended error:

$$\bar{e}_k = \begin{bmatrix} e_k \\ e_{f,k} \end{bmatrix}$$

described by

$$\bar{e}_{k+1} = \sum_{j=1}^{N} \alpha_k^j \begin{bmatrix} A_j - B_jK_{1,j} & L_jH_jC_jA_j \\ 0 & \bar{A}_j - K_{2,j}C_j \end{bmatrix} \bar{e}_k = \sum_{j=1}^{N} \alpha_k^j A_{0,j}\bar{e}_k \tag{14}$$

converges asymptotically to zero.

The ability and stability of the closed loop system is design using an efficient LMI pole placement designed. For many problems, an exact pole assignment may not be necessary and it suffices to locate the poles of the closed-loop system in a sub region of the unit circle.

Consequently, define a disk region LMI called D included in the unit circle with an affix (-q, 0) and a radius r such that (q + r) < 1. These two scalars q and r are used to determine a specific region included in the unit circle so as to place closed-loop system eigenvalues. The pole placement of the closed-loop system (Rodrigues et al. [2007]):

$$x_{f,k+1} = \sum_{j=1}^{N} \alpha_k^j(\theta)\left[A_j x_{f,k} + B_j u_{f,k} + L_j f_k\right],$$

$$y_{f,k} = \sum_{j=1}^{N} \alpha_k^j(\theta)\left[C_j x_{f,k}\right]$$

for all models $j \in [1, \ldots, N]$ in the LMI region can be expressed as follows:

$$\begin{pmatrix} -rX_j & qX_j + (A_{0,j}X_j)^T \\ qX_j + A_{0,j}X_j & -rX_j \end{pmatrix} < 0 \,. \qquad (15)$$

$A_{0,j}$ is stable if and only if there exists a symmetric matrix such that $X_j = X_j^T > 0$.

Assuming that X_j has the block diagonal form $X_j = diag(X_{1,j}, X_{2,j})$ and defining $A_{0,j}$ of (15) as:

$$A_{0,j} = \begin{pmatrix} A_j - B_j K_{1,j} & L_j H_j C_j A_j \\ 0 & \bar{A}_j^T - C_j^T K_{2,j}^T \end{pmatrix} \,. \qquad (16)$$

It can be observed from the structure of $A_{0,j}$ in (16) that the eigenvalues of matrix $A_{0,j}$ are the union of $A_j - B_j K_{1,j}$ and $\bar{A}_j^T - C_j^T K_{2,j}^T$. This clearly indicates that the design of the state feedback and the observer can be carried out independently (separation principle). Thus, the inequalities are:

$$\begin{bmatrix} -rX_{1,j} & qX_{1,j} + X_{1,j}^T(A_j^T - K_{1,j}^T B_j^T) \\ (q + A_j - B_j K_{1,j})X_{1,j} & -rX_{1,j} \end{bmatrix} < 0, \qquad (17)$$

$$\begin{bmatrix} -rX_{2,j} & qX_{2,j} + X_{2,j}^T(\bar{A}_j^T - K_{2,j}C_j) \\ (q + \bar{A}_j^T - C_j^T K_{2,j}^T)X_{2,j} & -rX_{2,j} \end{bmatrix} < 0. \qquad (18)$$

We should note that expression (17) and (18) are Bilinear Matrix Inequalities (BMIs) which cannot be solved with LMI tools, but substituting $W_{1,j} = K_{1,j}X_{1,j}$ and $W_{2,j} = K_{2,j}^T X_{2,j}$ it is possible to transform into:

$$\begin{bmatrix} -rX_{1,j} & qX_{1,j} + X_{1,j}^T A_j^T - W_{1,j}^T B_j^T) \\ (q + A_j)X_{1,j} - B_j W_{1,j} & -rX_{1,j} \end{bmatrix} < 0, \qquad (19)$$

$$\begin{bmatrix} -rX_{2,j} & qX_{2,j} + X_{2,j}^T \bar{A}_j^T - W_{2,j}C_j) \\ (q + \bar{A}_j^T)X_{2,j} - C_j^T W_{2,j}^T & -rX_{2,j} \end{bmatrix} < 0. \qquad (20)$$

Finally, the design procedure boils down to solving the LMIs (19) and (20), and then determining $K_{1,j} = W_{1,j}(X_{1,j})^{-1}$ and $K_{2,j} = (W_{2,j}(X_{2,j})^{-1})^T$.

3. DESCRIPTION OF THE TWIN-ROTOR SYSTEM

The Twin-Rotor MIMO System (TRMS) is a laboratory set-up developed by Feedback Instruments Limited for control experiments. The system is perceived as an engineering problem owing to its high non-linearity, cross-coupling between its two axes and inaccessibility of some of its states and outputs for measurements. TRMS is available at the laboratories of the Advanced Control Systems in the Automatic Control Department (ESAII) of Technical University of Catalonia, where this work was conducted. The TRMS is shown on Fig. 1.

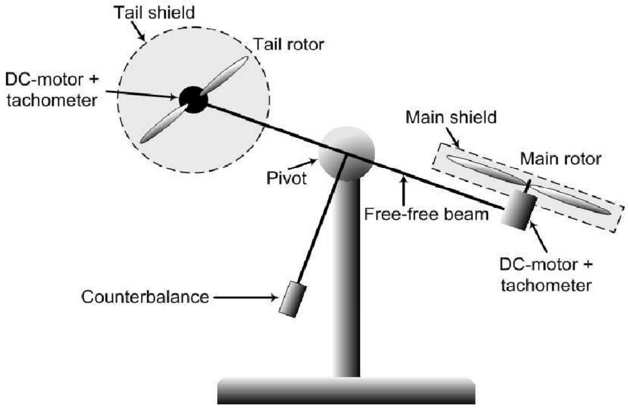

Fig. 1. Aero-dynamical model of the Twin Rotor MIMO System

The block which is a representation of TRMS in Simulink is given with equipment by the producer (Feedback Instruments Limited [1998]) along with comprehensive mathematical description. The mathematical model of TRMS becomes a sets of four non-linear differential equations with two linear differential equations and four non-linear functions. Some of the parameters of this model can be obtained from (13), while some others should be obtained by experiments such as magnitudes of physical propeller, length, mass, inertia, coefficients of friction and impulse force. The system can be defined by the input vector $U = [u_h, u_v]^T$ where u_h is the input voltage of the tail motor and u_v is the input voltage of the main motor. Also can be defined the output vector $Y = [\Omega_h, \alpha_h, \omega_t, \Omega_v, \alpha_v, \omega_m]^T$ where Ω_h is the angular velocity around the vertical axis, α_h is the azimuth angle of beam, ω_t is the rotational velocity of the tail rotor, Ω_v is the angular velocity around the horizontal axis, α_v is the pitch angle of beam and ω_m is the rotational velocity of the main rotor. It should be pointed out that in the remaining part of this paper a reduced output vector is utilized, which is $Y = [\alpha_h, \alpha_v, \omega_m]^T$. Moreover, two kind of faults are considered, namely f1 – main rotor fault, f2 – tail rotor fault. The LPV model was obtained by linearization of the non-linear system around different operating points:
$(u_h = 0, u_v^1 = 0, u_v^2 = 0.05, u_v^3 = 0.1, u_v^4 = 0.15, u_v^5 = 0.2)$.

4. IMPLEMENTATION

The scope of this point is focused on development of the state observer, fault identification and control scheme for the TRMS described in Section II. Thus the design problem is to develop Simulink blocks implementing state observer described by (11) for LPV systems with N=5 is given in Fig. 2, whilst Fig. 3 presents a detailed insight view of the blocks described by the name “Model”. All the parameters being used in the scheme can be easily obtained by solving (20).

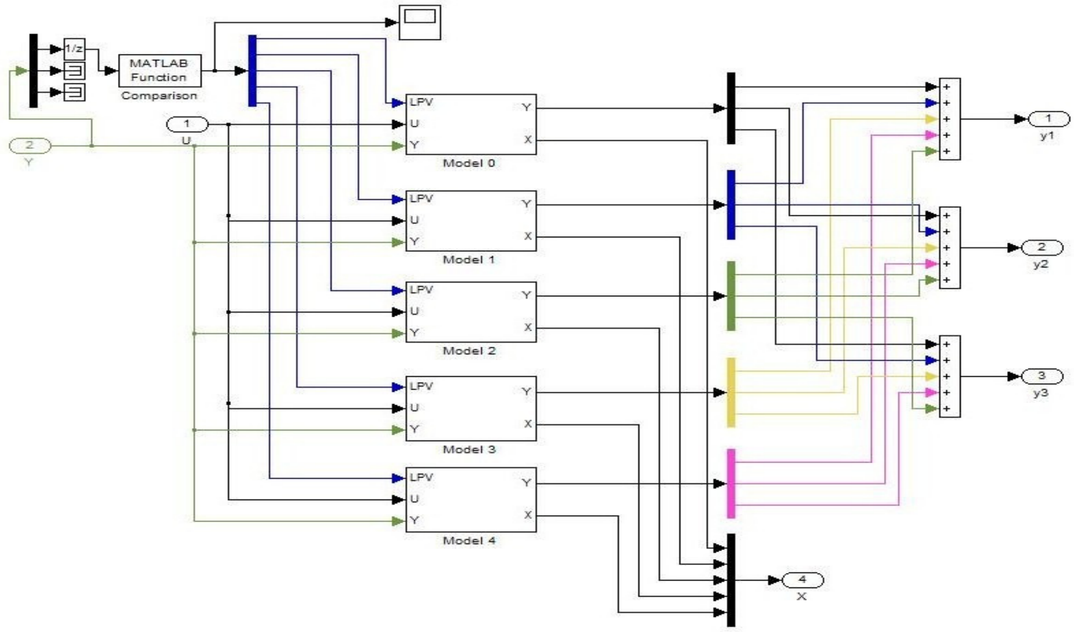

Fig. 2. Observer scheme

The block “Comparison” (cf. Fig. 2) contains the switching rules between the five models. Since the observer is available, it is possible to estimate the faults. Indeed, the fault identification strategy described in the preceding part of this paper is implemented as shown in Fig. 4.

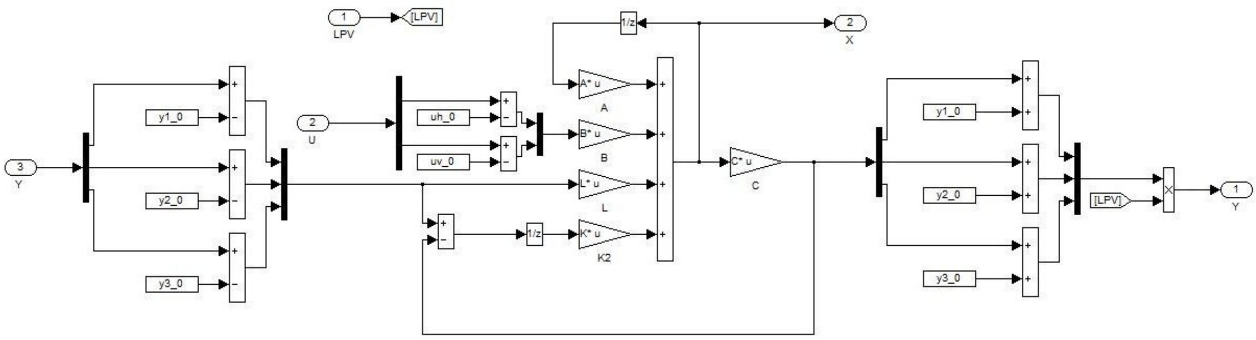

Fig. 3. Observer scheme – a detailed view

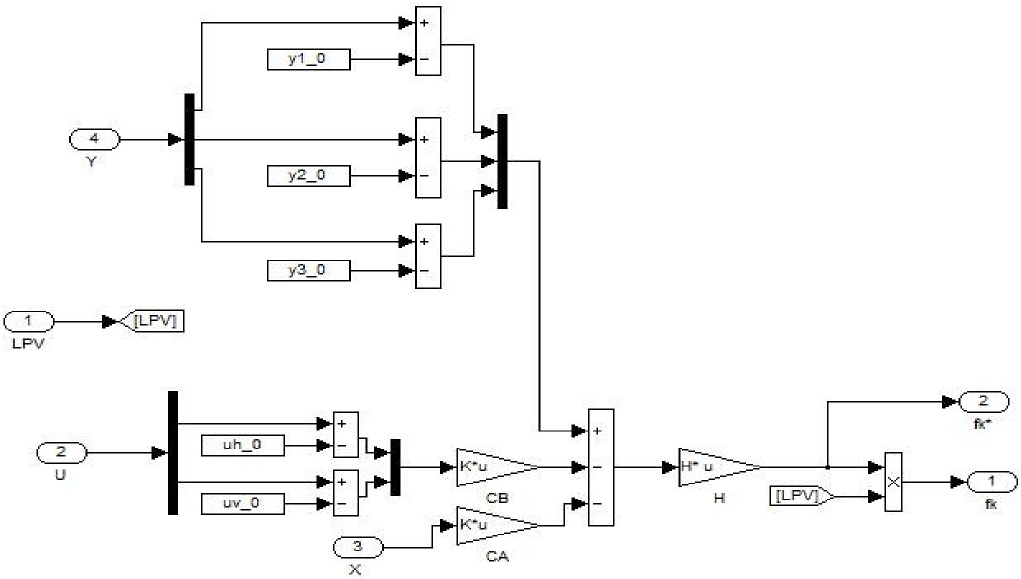

Fig. 4. Fault estimate scheme – a detailed view

Since the state and fault estimates are available, it is possible to develop the control strategy. Fig. 5 presents a complete implementation, while Fig. 6 presents a detailed view of the blocks described by the name "Model".

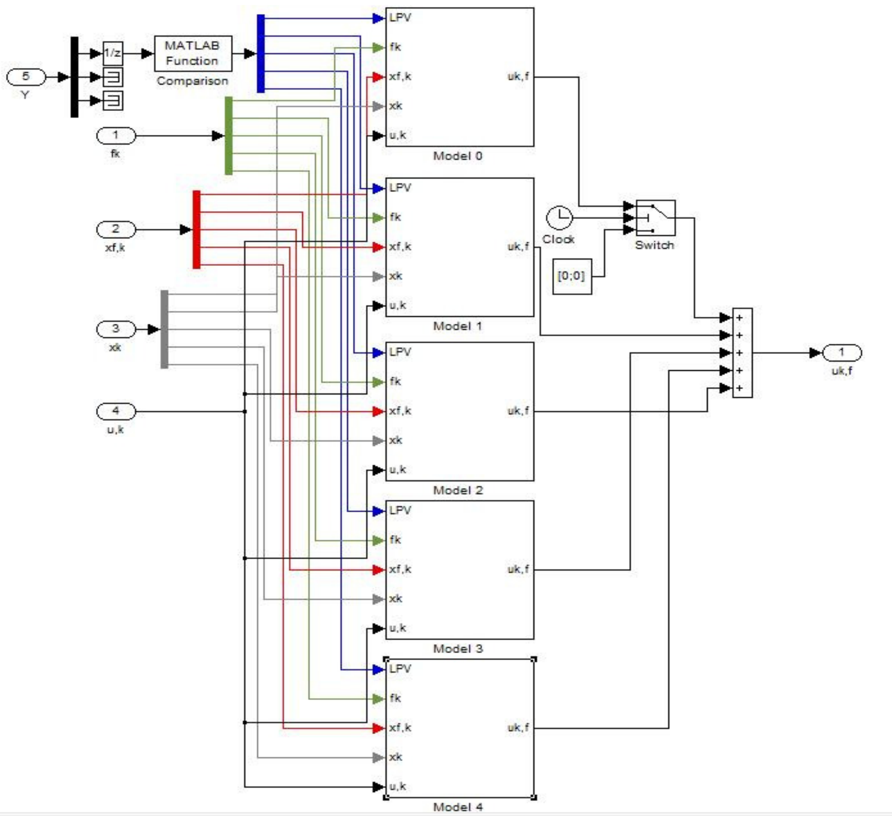

Fig. 5. Control scheme

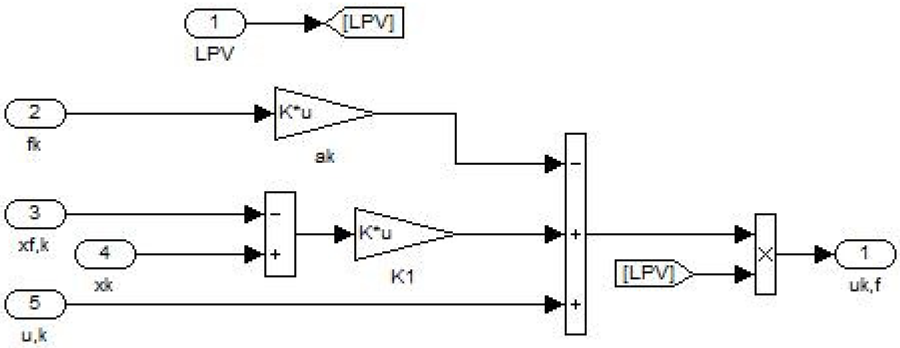

Fig. 6. Strategy control scheme – a detailed view

Finally, Fig. 7. presents a completely connected FTC scheme.

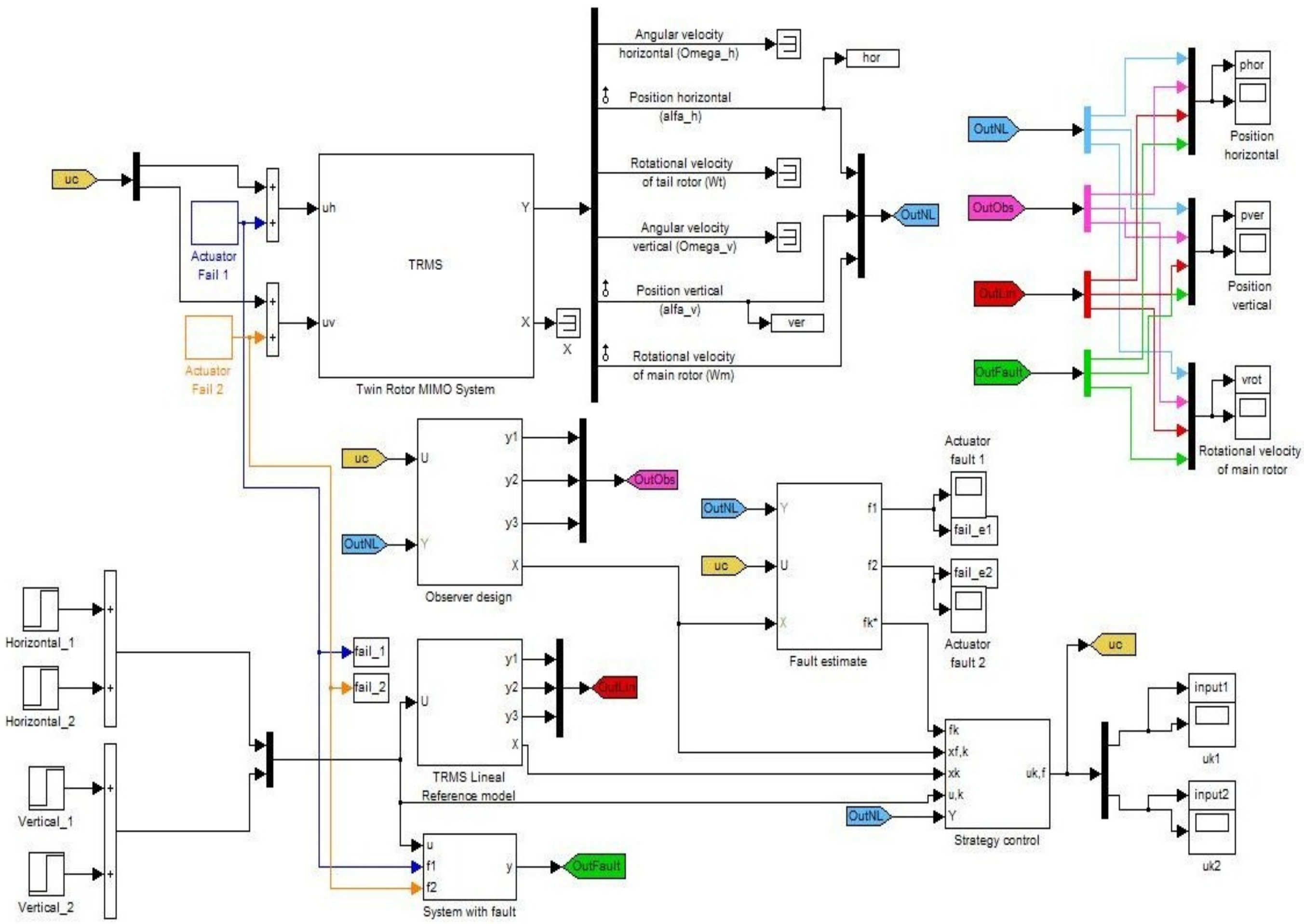

Fig. 7. Completely connected FTC scheme

5. EXPERIMENTAL RESULTS

5.1. Fault of Tail Rotor

The first fault scenario and the results are presented as follows:

$$f_h = f_{k,3} = \begin{cases} 0, & for\ k < 300 \\ 0.1, & for\ 300 \leq k < 340 \\ 0, & for\ k \geq 340 \end{cases}$$

where $k = 0, \ldots, 400$ with $T = 0.05s$.

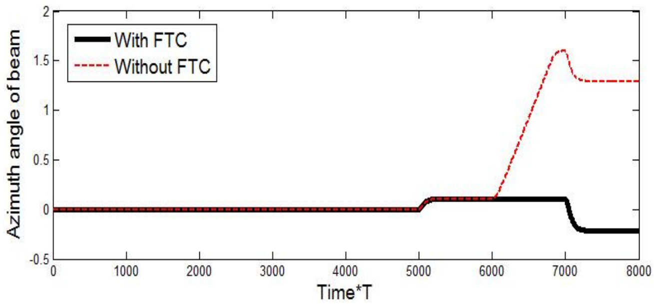

Fig. 8. Azimuth angle of beam (horizontal position) with Fault Tolerant Control and without Fault Tolerant Control

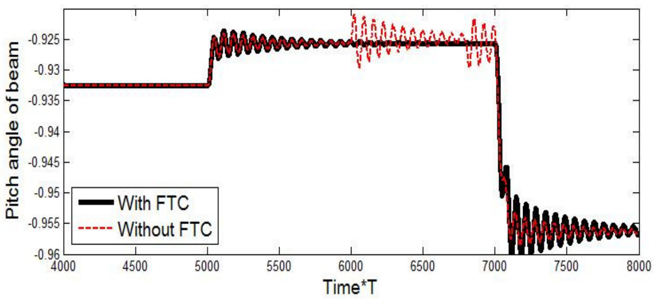

Fig. 9. Pitch angle of beam (vertical position) with FTC and without FTC

Fig. 8. represents the performance of control strategy proposed and the response of the azimuth angle of beam without FTC. The system is stabilized with the FTC in spite of actuator fault. Fig. 9. shows the pitch angle of beam and its trajectory was not changed significantly after the fault.

5.2. Fault of Main Rotor

$$f_h = f_{k,4} = \begin{cases} 0, & for\ k < 220 \\ -0.03, & for\ 220 \leq k < 320 \\ 0, & for\ k \geq 320 \end{cases}$$

where $k = 0, \ldots, 400$ with $T = 0.05s$.

Fig. 10. illustrates an exemplary fault estimation. In particular, it shows that the input voltage fault of the main rotor can be estimated with very high accuracy. Figs. 11-12 represent the azimuth and the pitch angle of the beam, respectively. It can be observed that in the presents of the fault, the performance was not impaired with FTC, unlike without it.

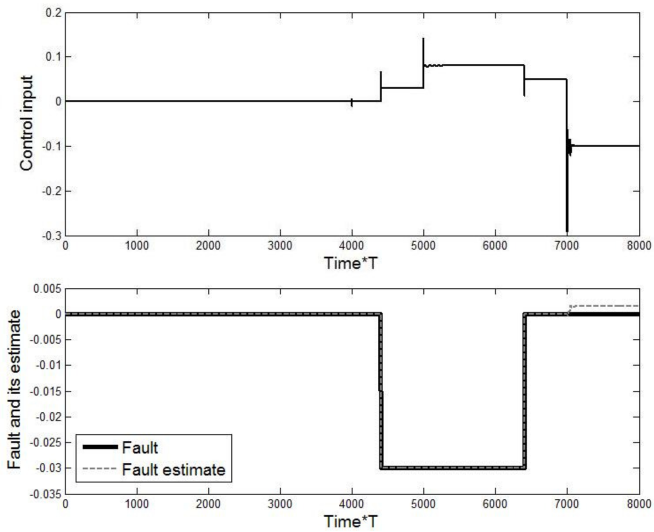

Fig. 10. (Top) Control input voltage of the main rotor. (Bottom) Input voltage fault of the main rotor and its estimate.

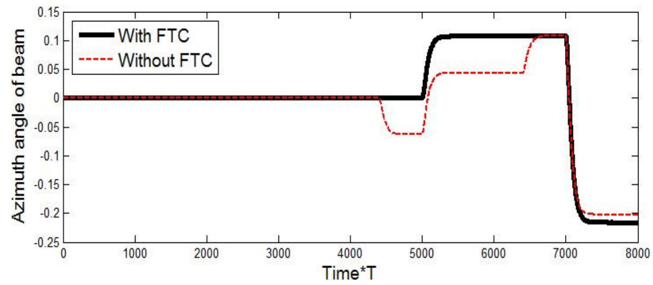

Fig. 11. Azimuth angle of beam (horizontal position) with Fault Tolerant Control and without Fault Tolerant Control

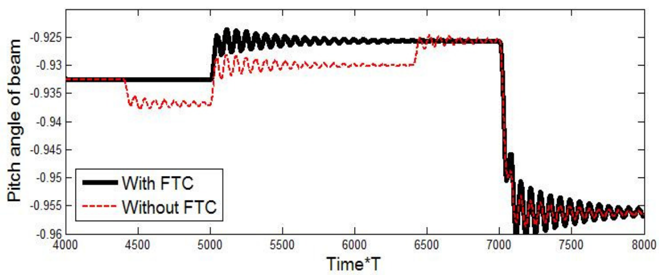

Fig. 12. Pitch angle of beam (vertical position) with Fault Tolerant Control and without Fault Tolerant Control

6. CONCLUSIONS

As a new emerging area in automatic control, fault tolerant control has attracted more and more attention in recent years. The main objective of the thesis was to develop a fault-tolerant control (FTC) toolbox. An active FTC strategy for LPV system was presented in this paper. First, this new approach was presented in the context of linear systems and emphasizes the importance of non-linear system based on LPV representation. Controllers were designed for each separate linear model trough LMIs pole placement. This method was suitable for partial actuator faults. The proposed approach was an integrated FTC design procedure of the fault identification and control scheme. Fault identification was based on the use of the observer. The FTC toolbox for Simulink was designed, included library. The experiments took place in the Laboratory of Advance Control Systems Research in the Automatic Control Department (ESAII) of Technical University of Catalonia (UPC).

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