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A CONCEPTUAL FRAMEWORK FOR MULTI-SCALE CELLULAR MODELING OF SPATIAL CHANGE

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Abstract: The use of cellular models for simulating spatial change phenomena has been a subject of intensive theoretical and applied research for the last two decades. This type of models was developed during the 1940s to devise mathematical rules for the evolution of biological systems. They were introduced in urban geography by Waldo Tobler in the 1970s because of their spatial structure and their ability for describing complex behaviors from sets of simple rules. The research effort made since then led to many different spatial change cellular models, some of which are being applied in practice. However, these models are based on a single-scale approach, since they focus on metropolitan or urban areas. In this paper, we present a conceptual framework for multi-scale cellular modeling of spatial change, involving the regional and the metropolitan/urban level, as well as the interactions between them. Several important issues regarding the formulation of a multi-scale cellular model are discussed in detail, such as cell dimension, neighborhood format, transition potential measures, calibration procedures, and policy testing.

Keywords: Urban Change, Cellular Automata, Irregular Cells, Multi-Scale Approach.

Introduction

The use of simulation models to study complex urban phenomena has been one of the most important areas of research in urban studies since the 1950s. A large set of simulation approaches have been developed and applied to a wide set of urban contexts and different urban areas around the world. Cellular Automata (CA) was used in many of those approaches and undergoes intensive theoretical and operational research since the 1990s. The concept of CA was introduced in the late 1940s by John von Neumann and Stanislaw Ulam, who where facing (independently) the problem of devising sets of mathematical rules for simulating the self-reproduction and evolution of biological systems. In a simple definition "*an automaton is a processing mechanism with characteristics that change over time based on its internal characteristics, rules and external input*" (Benenson and Torrens, 2004). It is a concept based on the universal Turing machine that considers an agent capable of processing a given code under a set of rules. This agent can have a spatial form which results on the final term of cellular automaton. The simplicity of the concept of CA and its intrinsic spatial nature were at the basis of its introduction to geography by Waldo Tobler (Tobler, 1979). After the 1980s, the evolution of processing capacities, computer science, and database management made possible a series of developments on the geographical application of

CA (Couclelis, 1985, Batty et al., 1997, Couclelis, 1997, Batty, 2005, Couclelis, 2005). In a brief description, CA are based on a discrete set of spatial units called cells that together form a cell structure. The cell is a portion of the territory, being usually a regular square in the large majority of applications. Each cell has a given (cell) state from a finite set of states during each time step. Cell states represent common land uses in urban applications of CA. Time is considered in a discrete manner and the time step can vary from one to several years. Each cell will operate state changes over time according to a finite set of transition rules that can be of various types (deterministic, stochastic, unconstrained, or constrained). State transition results from the application of these rules to each cell considering the neighboring cells. The concept of neighborhood is on the base of CA: the interaction between cells within a given neighborhood is determinant for the state transition.

CA is often pointed out as a good modeling approach for dealing with complex systems. Starting from a simple set of rules that operate on a simple spatial structure, CA is able (INE, 2001)to model evolving systems that become increasingly complex during their evolution. The current trend for developing bottom-up planning approaches is an attempt to deal with complexity in more and more complex urban systems. However, these systems are based on a wide set of individual agents (that can be considered automata). One of the possibilities is to consider small size spatial agents – cells – as a unit for the territory. This suggests that CA can be a powerful tool for simulating urban change, thus promoting a bottom-up modeling approach for understanding its driving forces.

Common CA are applied to single-scale problems (White and Engelen, 1993, Batty and Xie, 1997, Clarke et al., 1997, Barredo et al., 2003). These models consider the entire territory they are modeling as one single problem where cells are small units that usually derive from remote sense images. Other applications consider a multi-scale modeling approach where CA is used only to model local scale urban change (Ward et al., 2003). The use of CA models that work separately in regional/metropolitan scale and local scale is the basic premise of the work we present. Different scales have different driving forces and the use of two different (but connected) CA models can contribute to better simulate those phenomena.

The paper is divided into two main parts. The first part is dedicated to describing the main characteristics we foresee for a multi-scale CA model. The main components of the classic formulation are discussed from a multi-scale point of view. The second part is devoted to introducing issues regarding the integration of simulation models and common planning processes. Finally, some conclusions are drawn and future developments of this research are identified.

Conceptual Framework

A multi-scale approach

Different urban phenomena can be observed and modeled considering, on the one hand, cities as parts of larger regional systems and, on the other hand, cities themselves, where the driving forces of urban change depend on local variables. The issue of scale is currently under debate among the modelers community and some models are already facing the problem of inter-scale interaction (Kocabas and Dragicevic, 2006, Benenson, 2007, White, 2007). The use of both regional and local scales is considered useful to correctly simulate a wide set of complex urban phenomena that occur on different spatial scales. The consideration of a regional scale of approach is expected to improve the simulation of large scale urban phenomena. At this scale, cities within broader urban systems compete with each other in order to attract population, employments, and public investments. The evolution of population and employment observed during large periods and the flows between cities can be used to estimate degrees of relationship between them. Considering that these relationships depend largely on neighborhood conditions and that large scale cells can be observed with a useful amount of reliable data (municipalities, cities) it is clear that a CA-based model may be used to simulate urban evolution at a regional, therefore macro-scale. At this scale, the assessment of aggregate land use demand – for housing, industrial, or tertiary land uses – through population and employment growth can be more representative of urban growth than the disaggregate amount determined for each land use, which is traditionally used by common CA models. In opposition, at a city/neighborhood scale – the local scale – land use growth results from the distribution of each land use (considered in a disaggregate level). In fact, land use demand can be estimated as the amount of land necessary for each land use considering population and employment growth. Then, a set of disaggregate land uses can be assigned to different cells in order to meet land use demand. This multi-scale approach is the consideration of two levels of simulation: the regional level, in which the aim is to assess land use demand from population and employment evolution through time; the local level, in which the aim is to assign different land uses to different locations (cells or parts of cells) in order to meet land uses demand.

Cells

The first CA component to address regards the cell. Common CA models operate with regular square cells that are usually obtained from remote sense maps (White and Engelen, 1993, Clarke et al., 1997, Barredo et al., 2003, Li and Liu, 2006). This approach was the natural evolution from the 2D mathematical CA models because of its similarity. It is also useful for modelers because allows the collection of reliable historical data on land use, irrespective of the ability of this framework for fitting real-world processes (Benenson, 2007). However, these cells are not directly related with urban form because of their regularity. The use of irregular cells in CA models is reduced (Semboloni, 1997, Vandergue et al., 2000, O'Sullivan, 2001, Pinto and Antunes, 2007, Stevens et al., 2007). The potential of using irregular cells results from the possibility of combining urban form and reliable data (Pinto and Antunes, 2007). This approach has produced promising results and is one of the main pillars of the proposed multi-scale approach. The concept of using irregular cells will be extended to the regional scale problem. At this scale, municipalities or other intermediate administrative partitions can be considered as a natural option; at a local scale cells should be as close to the urban structure as possible. Cells could be obtained from established spatial units such as administrative units (municipalities, boroughs) at a regional level or census tracts and sub-tracts at a local level. Census tracts are normally drawn considering the urban structure and contain extensive and reliable information. Nonetheless, the consideration of irregular cells must follow well defined criteria. Land use demand must be assessed through population and employment density, hence depending on the area. A great difference in area should produce distortions on the simulation. To tackle this problem, there are two possible ways: (1) cells that could be assigned for each land use should have some homogeneity in terms of area; (2) the model must be able to assign some land use to a fraction of a cell generating a separation of the remaining cell with the creation of a new one. At a regional level, this problem does not seem to exist, as the cell is determined by a very strict structure of administrative frontiers. These cells are not as close to urban structure as irregular cells obtained from census tracts. However, this is not an important issue because they are strongly representative of the functional structure of the territory at the regional scale, thus being suited for consideration in this type of approach. The use of irregular cells is proposed as the basis for the simulation at regional and local scales. There is a great amount of processed data of various types for both scales for these cells. The possibility of crossing reliable data and spatial structure is the main gain obtained from using irregular cells. This feature is expected to result as a significant advance of this approach.

Neighborhood

Neighborhood is a critical issue for every type of spatial models, especially for CA. Neighborhood is commonly (if not exclusively) considered by the strict concept inherited from the mathematical formulation of CA. This concept is based only on the consideration of a set of physical neighbors to one cell: these neighboring cells could be those which are directly connected to the cell considered or they could be the group of cells that are within a given range from that cell (which can be set by a radius or a number of cells in each direction) (White and Engelen, 1993, Benenson and Torrens, 2004, Ménard and Marceau, 2005). But this mathematical perspective is far from being representative of how cities work. The concept of neighborhood must be able to reproduce how agents interact, considering both spatial and functional levels of interaction. On a macro-scale approach, neighborhood is influenced by both these levels of interaction. A municipality is strongly influenced not only by its directly connected neighbors – in terms of employment and economical relationship – but also by the main functional centers of the region, where administrative and economical decision centers are located. At a local level, this dual influence is also observed. The choice of location for a given land use is influenced both by the surroundings (a good example is the relationship between residential and industrial land uses) and by the distance to the main functional and employment centers in the city. Neighborhood must shift from the concept of a limited area to a larger and possibly disconnected part of the territory. The interaction between two cells will be assessed if they are spatially and functionally related. The main goal is to enhance neighborhood representativeness at both regional and local scales. There are physical and ecological barriers that limit or break the shape of a neighborhood. At a regional scale, a natural park, a mountain system, a river, or delta can act like boundaries for many types of interactions. Locally, topographic features, urban infrastructures (road and railways), and city form may function as barriers for defining real neighborhoods. The concept must be flexible, allowing the consideration of different shapes and extents for similar situations at both levels of approach.

Transition rules and cell states

Transition rules are also an important component of CA. They define the way a CA model evolves throughout time and they are intimately related to the way the model is able to correctly acquire behaviors that are the subject of simulation. Formal CA uses probabilistic sets of transition rules (Wolfram, 1984). Although this probabilistic approach has been used in some theoretical urban CA models, transition rules are commonly designed to reproduce complex urban behavior (White and Engelen, 1993, Clarke et al., 1997, Barredo et al., 2003). These rules can also be used to reproduce planning regulations constraints under given demand behaviors (Pinto and Antunes, 2007). Again, a multi-scale approach requires some attention on the definition of transition rules for both scales. At a regional level of analysis, it is important to notice that the goal is to simulate the macro interactions that are observed within a region. These interactions can be assessed using macro-scale indicators of population, employment, and commuting flows, for example. Transition rules and cell states will relate to this indicators and to measures of interaction. Cell states can be defined as a degree of urbanization taken in aggregate or disaggregate form. A binary state of urbanization, occupied or non-occupied, can be used; an alternative approach can use a disaggregate range of urbanization. It is also possible to consider the main land uses (residential, industrial) in an aggregate or disaggregate way. At a local scale, the goal is to simulate the distribution and growth of land use in a disaggregate manner. These phenomena depend on a series of factors related with land suitabilities, land use demand, accessibility, among many others. At this scale, transition rules will relate to measures of urban potential for each land use considering all the other land uses. Cell states are then the traditional set of disaggregate cell states that are commonly used in regular CA models. Another important issue regarding cell states is homogeneity. The CA concept of cell state implies the consideration of a finite set of acceptable states for each cell. This concept is far from reality in urban studies. Homogeneity is very difficult to observe in any parcel of land of any dimension. There are some possibilities for tackling this problem using composite measures of activities that reproduce the relative importance of land uses other than the main cell state in the formation of that cell's potential (White, 2007). This issue will be carefully addressed during the research.

Land suitability

Land suitability is also an important issue to attend. Different lad uses demand different land suitabilities for choosing a given location. Although this is not a classical CA component, it assumes great importance when we are dealing with constrained models. At a regional level, the analysis is made from an aggregate point of view and land suitability must be taken into account as a measure of comparison for particular issues as general environmental quality or wildlife protection policies. The goal is to capture the influence of general environmental and physical characteristics in location choice, both for residential and for non-residential land uses. There are municipalities that have more demand for residential land uses due not only to their environmental characteristics but also to the distance to large industrial centers or polluted areas. At a local level, the comparison between land suitabilities for every land use is decisive for the assessment of demand for different and/or competitive land uses. Land suitability is an important constraint to the occupation of a given land parcel by a given land use. Therefore, at this level of analysis it is imperative to develop a robust set of land suitability indicators. The goal is to allow the model to better differentiate two neighboring cells in terms of their ability for allocate a given land use.

Accessibility

Accessibility is strongly linked to land use. Any attempt to simulate complex behaviors that occur within an urban system must take into account accessibility, as it may be considered as one of the most important driving force of urban growth. Accessibility is also strongly dependent on the scale of analysis. In order to correctly simulate accessibility conditions and evolution throughout time, different transportation modes must be considered and their scopes of influence must be attended. Air and maritime transportation modes have a regional scale of influence, as the number of network nodes located in a given region is always of only a few. On the contrary, road and rail transportation modes have both a regional scope (regional highways and roads and railroads systems) and a local one (city road network, subway rail systems). The main goal is to develop a good measure of multi-modal accessibility that can be used as an input for a CA model at different scales of analysis.

Assessing model performance

Another important issue regards the use of performance measures. One of the most widespread methods for assessing CA models performances is based on the use of contingency matrices and associated *kappa* (*k*) measures (see Couto (2003) for a detailed reading on the subject). Contingency matrixes allow the comparison of two maps through cell-by-cell overlapping, one map representing a reference situation, and other map resulting from the output of a model. The *k* index, which estimates the degree of agreement, is expected to be as close to 1 (which means total agreement) as possible. This technique is highly representative of accuracy for raster based CA models, in which cells are regular and have the same shape and area. However, the use of these measures for assessing the performance of CA models using irregular cells needs more research, and the use of the *k* statistics must be optimized. Irregular CA must use population or employment densities for dealing with land use demand and the allocation of, say, population varies according to the area. So, when comparing the same cell with two distinct values for population density, a distortion on the value of agreement can be observed. There are other issues regarding the use of contingency matrices that need to be addressed. Pinto and Antunes (2007) proposed a simple and effective modification for the kappa index measure by not considering cells that do not change their states. This modification gives more reliability to the measure because it avoids an important distortion on the global agreement. The use of contingency matrixes can also be made from the perspective of minimizing the differences instead of maximizing the accuracy, as suggested by Pontius (2000). But accuracy must be considered carefully. Urban form is as important as accuracy. Simulation aims to capture complex urban phenomena that are strongly dependent of initial conditions. A slight variation on these conditions may produce significant differences on the result. Therefore, it is important to assess how feasible are the futures predicted by simulation rather than expect that simulation can be able to exactly define those futures. Hagen-Zanker (2006) suggests the combined use of categorical comparison through contingency matrices and the use of pattern measurement. It is necessary to simultaneously assess the extent and severity of differences and the reasons that led to them. The use of fractal analysis is a strong possibility for assessing pattern evolution (White and Engelen, 1993, White, 2006). Another possibility that will be deepened in the research is based on the development of heuristic-based procedures for minimizing differences in urban pattern, considering the relationship between land uses of neighboring cells. Physical analogies (for example the mass centre of the population/employment distribution) will be tested as performance measures.

Performing model calibration

The use of modeling tools to simulate any kind of real-world phenomena strongly depends on the quality of its calibration. Urban growth simulation is difficult to calibrate and validate due to its complexity. There are too many variables at stake and their behaviors are extremely complex and highly interdependent. For each phenomenon there could be a series of calibration parameters and the first effort is to simplify and to reduce the number of parameters. But no matter how simple a simulation can get, it will always depend on a significant number of parameters. Therefore, the use of simple trial and error procedures for calibrating these complex behaviors will not ensure the necessary search for good or quasi-optimum solutions. While varying a parameter with all the other parameters fixed one can not guarantee the convergence to a good solution nor the correct simulation of the interdependence between the behaviors modeled. The choice for calibration based on optimization is then fully justified. Calibration has been one of the most important subjects of research on CA models. There are several calibration approaches for CA models. SLEUTH (Clarke et al., 1997, Silva and Clarke, 2002) has a first stage based on visual calibration after which automatic procedures based on brute force computing search for a quasi-optimum set of calibration parameters. Li and Yeh (2001) applied artificial neural networks for the calibration of transition rules. Barredo et al. (2003) uses sensitivity analysis based on a visual comparison of modeled and reference maps to calibrate weighting parameters for neighborhood interaction. Pinto and Antunes (2007) use an optimization procedure based on the particle swarm algorithm (Eberhart and Kennedy, 1995, Parsopoulos and Vrahatis, 2002) for calibrating a large set of interdependent parameters at the same time. In order to obtain the most reliable simulation results possible, it is desirable to consider other calibration procedures based on different techniques. The multi-scale model proposed will comprise a modular set of calibration procedures based on different concepts that will allow the comparison of calibration results. The importance of calibration for the acceptance of urban simulation models among planning practitioners imposes a deep research on the development of this particular topic.

Bringing models closer to planning practice

The most important goal of developing urban simulation models is to create reliable tools for assisting planning processes. Models are not meant to produce planning by themselves. They can be considered very useful for providing tools for explaining past evolutions, future plausible spatial scenarios, and desirable evolutions to the agents involved in planning activities. But the relationship between modeling and planning has always suffered from doubts among planners about the usefulness of models as reliable decision support tools. This natural tension between modelers and planners was boosted by Douglass Lee’s criticisms in the middle 1970s (Lee, 1973), in which large scale, comprehensive modeling was strongly criticized because of its dissociation from the real-world complex phenomena they were trying to simulate. This issue of reliability is often discussed and still is one the most important issues with which model developers must deal with in our days. The fact that models are grounded on the simplification of complex phenomena suggests, at first, that they tend to have serious difficulties in simulating social and economical behaviors that are highly stochastic. "*Models are based on science; planning is about policy. Models are much better (…) at dealing with natural science problems; planning is mired in difficulties most often due to issues in the purview of social sciences. Models are usually developed from within particular disciplinary perspectives; planning must integrate across all domains. Models are about information and facts; planning is about interpretation and values. (…) Models codify uncertain knowledge; planning must lead to certain action. (…)*" (Couclelis, 2005). This dichotomy illustrates how difficult and challenging it is to bring models closer to the needs of planning. However, the simplicity that is in the very definition of the concept of model must be not be taken as an obstacle to the development of more and more sophisticated simulation systems. The rapid evolution of computational science and processing capacity and the increasing amount of available data provided by GIS allows us to expect that models will improve their simulation resolution to new and significant levels of representation in a near future. Automata based simulation has a strong potential in capturing agent behaviors, not only for individual agents but also for cellular ones. For this to happen, it is very important to consider the issue of policy testing as a major pillar of planning-oriented modeling.

Policy testing

It is widely accepted that policy testing is the ultimate goal of urban simulation. The use of simulation by itself is often considered redundant because it tends to produce futile results to decision makers. Simulation must be able to incorporate the practical needs of planners and of the planning process. There are different degrees of implementation of simulation-based methodologies across the world, which relates with different contexts of planning, regulation, and available information. Simulation is oriented for understanding and reproducing real phenomena in a controlled environment, aiming to explain how these phenomena work and how can they be manipulated. However, these features do not provide planning with the necessary tools for dealing with issues that are strongly influenced by uncertainty. Therefore, simulation must be more oriented for creating flexible modeling tools which allow the configuration of a wide set of scenarios. The goal is to enhance the ability of models for controlling all the parameters of the problems at hand, enabling a helpful use of models by planners. One of the major goals of the research is to effectively incorporate policy testing in the simulation package to be developed. This process is based on the identification of candidate issues to be tested for every component of the model. Its implementation must be an ongoing process that is expected to last even after the end of the development stage: every time a new issue or policy is set to be tested the model must be able to change in order to meet specific simulation needs.

Enhancing simulation capacities

This type of simulation is strongly dependent on the historical data gathered for a given problem. Every simulation technique can be considered captive of this data, reducing its ability of forecasting an uncertain future. This is probably the most important shortcoming that is pointed out to modeling by common planners. Therefore, it is imperative to develop modeling tools that are able to correctly identify historical trends and to break with them under given conditions. Major urban transformations are usually the result of single decisions localized in time as the large urban renovation operations or the organization of important sporting events (for example the Olympic Games). These types of transformations are almost impossible to capture by any model because they are not explicit on historical data. This will be a matter of intensive research. CA work with a set of transition rules that are calibrated considering the historic evolution of a problem. The model must be able to generate new transition rules after observing a trend that can not be explained by past evolution. Another possibility is the calibration of transformation thresholds. An illustrative example is the change of a cell to land uses that are forbidden by planning regulations. Once those thresholds are crossed a given cell is allowed to change to new cell states that were forbidden before.

Concluding remarks

The work presented is the starting point for the development of a modular simulation package that aims to model urban change phenomena with a multi-scale approach. The main goal is to create a flexible modeling tool that incorporates policy testing capacities so that it can assist planning processes in scenario comparison and in the evaluation of desirable evolutions. The use of multiple CA models operating on different scales is considered an important feature that can produce better model calibrations because of the ability of capturing different driving forces that take place at different scales. The research is expected to generate important developments for some components of CA such as neighborhood, transition rules, performance assessment, and calibration procedures.

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