Two-stage Stochastic Programming Model for the Thermal Optimal Day-Ahead Bid Problem with Physical Future Contracts

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Abstract: *The reorganization of electricity industry in Spain has finished a new step with the start-up of the Derivatives Market. Nowadays, all electricity transactions in Spain and Portugal are managed jointly through the MIBEL by the Day-Ahead Market Operator and the Derivatives Market Operator. This new framework requires important changes in the short-term optimization strategies of the Generation Companies. One main characteristic of MIBEL's Derivatives Market is the existence of physical futures contracts; they imply the obligation to settle physically the energy. The market regulation establishes the mechanism for including those physical futures in the day-ahead bidding of the Generation Companies. Thus, the participation in the derivatives market changes the incomes function and it could imply changes in the optimal planning, both in the optimal bidding and in the unit commitment. The goal of this work is the optimization of the coordination between the physical futures contracts and the Day-Ahead bidding following this regulation. We propose a stochastic quadratic mixed-integer programming model which maximizes the expected profits taking into account futures contracts settlement. The model gives the simultaneous optimization for the Day-Ahead Market bidding strategy and power planning production (unit commitment) for the thermal units of a price-taker Generation Company. The uncertainty of the day-ahead market price is included in the stochastic model through a scenario set. There has been applied both simulation and reduction techniques for building this scenario set from a time series ARIMA model. The implementation of the model is done with the modeling language AMPL. Implementation details and some first computational experiences for small real cases are presented.*

1 Introduction

In the last years there is an on-going phase of reorganization of electricity industry. The deregulation of the generation and distribution of electricity carried out in most countries in Europe and the creation of the Electricity Markets has changed the problems that the companies have to face.

The most important change that affects the daily operation is the increase of risk factors. In particular, with the introduction of the Electricity Markets, the electricity price has become a significant risk factor. One of the techniques for hedge against the market-price risk is the participation in futures markets [5] and, for this reason, the creation of Derivatives Electricity Markets has been the next natural step after the deregulation.

In the case of Spain, the Electricity Market was launched in 1998 and it includes a Day-Ahead Market, a reserve market and a set of balancing and adjustment markets. As the introduction of competition and the deregulation process did not behave as expected, the Spanish market has been improved in 2007 with the start-up of the MIBEL[1] and some other new regulations. The MIBEL joins Spanish and Portuguese electricity system and it complements the previous Spanish Electricity Market with a Derivatives Market.

In this work we focus on a generation company (GenCo) and the new challenges introduced by the liberalization in its planning. In a deregulated market, the GenCo objective is the maximization of profits. This profits are defined as the difference between revenues from the Market and generation cost. So, in this new framework, the GenCo has to face all the technical problems

[1] Mercado IBerico de ELectricidad - Iberian Electricity Market

of generation, such as the coordination of the unit's commitment and, also, the uncertainty of electricity prices.

Within the products that the Derivatives Markets offers, this work focus on the futures contracts. A futures contract is an exchange-traded derivative that represents agreements to buy/sell some underlying asset in the future for a specified price [9].

The participation in the Spot Market and in the Derivatives Market has traditionally been studied independently but there are some evidences that indicates it could be interesting a joint approach. Firstly, most of the Derivatives Markets has some short-term futures contracts, as week or day futures contracts, that could be included in short-term strategies of a GenCo. Secondly, futures contracts with physical settlement affect directly GenCo's unit commitment. And finally, the participation of the generation company in those markets changes its incomes function, thus it changes its optimal planning.

This paper is organized as follows. In section 2 we present the concepts needed for understanding the framework of the problem. In sections 3 and 4 we describe the first and second problem approaches, respectively. In section 5 the results of the study case with real data is presented and in section 6 we discuss the conclusions of our work.

2 Previous considerations

Following the idea that the participation in the Spot and the Derivatives Markets has to be studied jointly, the main objective of this work is to build a model which includes the coordination between physical futures contracts and Day-Ahead Market bidding following the MIBEL rules. In other words, we want to see how the inclusion of futures contracts in the model affects the short term strategies of the GenCo in the Day-Ahead Market.

The acronyms used in this work correspond to the MIBEL's operators and units.

2.1 Futures contracts

The derivative product considered in this work is the futures contract. Nowadays futures contracts are traded at organized Derivatives Markets in most Electricity Markets. The GenCos and other participants send the offers for the futures contracts to the market operator who does the clearing process.

The main characteristics of a futures contract are:

- *Procurement*: futures contracts could have physical or financial settlement. Physical futures contracts have cash settlement and physical delivery whereas financial ones have cash settlement only.

- *Delivery period*: the delivery period defines the duration of the contract. In most common products the delivery period is a year, a quarter, a month, a week or a day.

- *Load*: futures contracts could be base or peak load. In base load futures contracts the quantity to procure is constant for all the delivery period intervals. In peak load futures contracts there is procurement only in peak intervals (from 8 am to 24 pm, Monday to Friday).

Suppose the GenCo has a futures contracts portfolio F for day d as a result of the Derivatives Market clearing process. Those contracts are defined by a pair of price and quantity, $(\lambda_j^f, L_j),\ j \in F$. The futures contracts are settled by differences, i.e., each futures contract has daily cash settlement of the price differences between the market reference price and the futures settlement price. The incomes function of the Derivatives Markets at interval i is:

$$I_i^f = \sum_{\forall j \in F} (\lambda_j^f - \lambda_i^d) L_j$$

where λ_i^d is the clearing Day-Ahead Market price.
The futures contracts included in this study are physical and base load, meaning an agreement to sell some constant quantity of electricity at some price with physical delivery and cash settlement along a specific delivery period. Those contracts are available in the MIBEL.

2.2 Day-Ahead Market

Nowadays all GenCos participate in one or more Spot Markets. Typically, Spot Markets are organized as a set of successive market mechanisms: the Day-Ahead Market, the Automatic Generation Control, the Adjustment Market and a market for ancillary services. This work focus on the Day-Ahead Market because it presents large transactions volumes when compared to the other Spot Market mechanisms and because the coordination with the futures contracts is done in it.
Usually, the Day-Ahead Market is form by twenty-four hourly auctions. Both offers from selling agents (i.e. generation companies) and bids from buying agents (i.e. distribution companies) are submitted to each auction. Each agent can submit several offers but it is unaware of the offers submitted by the rest of agents [2].
The offer for each interval and unit is defined by a quantity and a price. To derive the aggregate offer curve, offers are sorted by increasing prices and their quantities are accumulated. The clearing-price is determined by the intersection of the aggregate supply and demand curve.
The incomes that a GenCo takes in from the Day-Ahead Market depends on the results of the clearing process. The offers are called matched if their price is lower or equal to the clearing-price. Only the matched offers produce benefits. The incomes function of the Day-Ahead Market for interval i is:

$$I_i^d = \sum_{\forall t \in T} \lambda_i^d p_{it}$$

where T is the set of thermal units, λ_i^d is the clearing-price and p_{it} is the quantity that has to be produced by unit t.

2.3 Coordination between Day-Ahead and Derivatives Markets

As stated above, in the MIBEL there are two possible settlements for the futures contracts either financial or physical. The difference is that the energy L_j of a physical futures contract j must be allocated through the thermal units of the GenCo. The MIBEL regulation [14] describes the coordination between this physical futures contracts portfolio and the Day-Ahead bidding mechanism, which is shown in figure 1 and described below.
Firstly, for every derivatives product the GenCo is interested in, it has to define the Term Contract Units (UCP[2]) which are the virtual units allowed to offer in the Derivatives Market. Each UCP is formed by the subset of the physical units of the GenCo which will generate the energy to cover the corresponding contract. For each product a physical unit can only participate in one UCP.
Secondly, two days before the delivery date every GenCo receives from the Derivatives Market Operator (OMIP[3]) the quantity of electricity every UCP has to produce for covering the

[2]Unidad Contratación a Plazo - Term Contract Units
[3]Operador Mercado Ibérico de Energía. Polo Portugués - Portuguese Iberian Market Operator

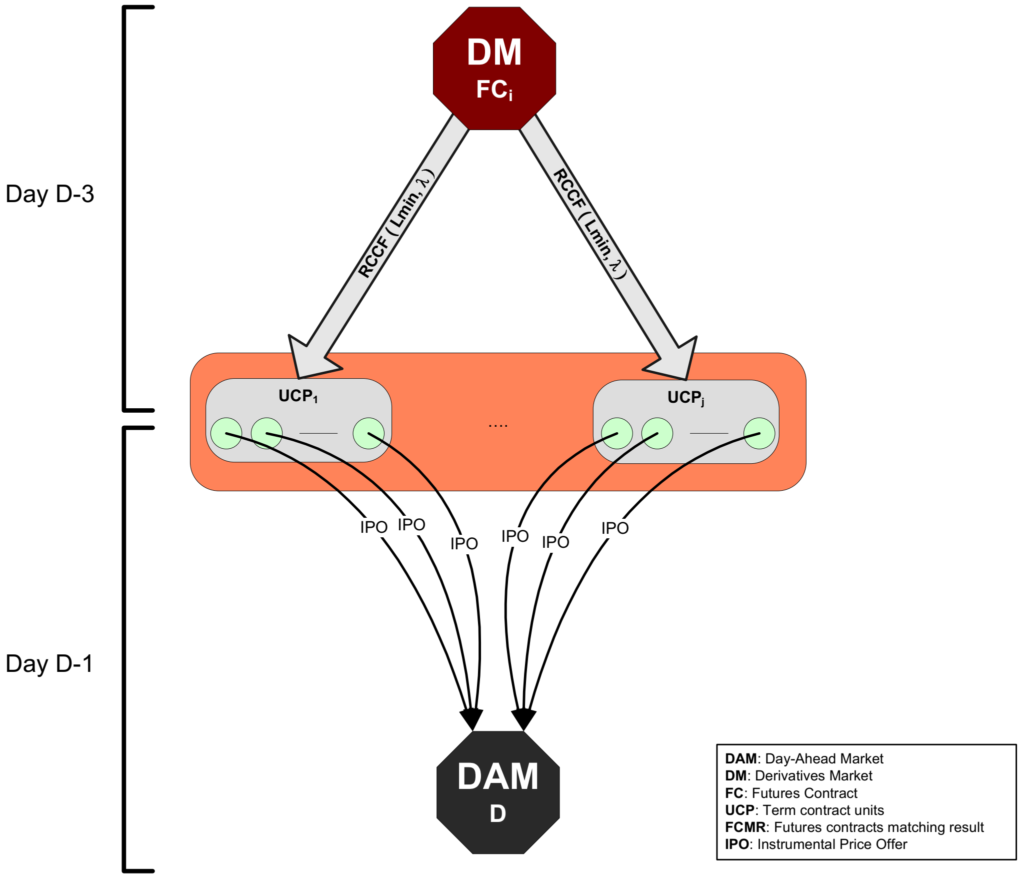

Figure 1: Coordination between Derivatives and Day Ahead Markets

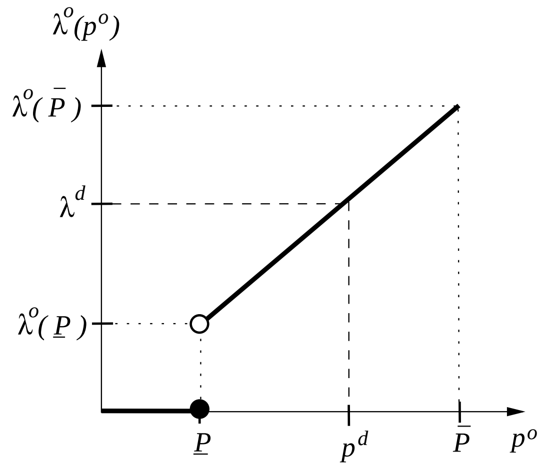

Figure 2: Optimal Offer Curve

matched futures contracts. This information is also send to the Day-Ahead Market Operator (OMEL[4]).
Finally, OMEL demands every GenCo to commit the quantity designed to futures contracts through the Day-Ahead Market bidding of the physical units that form each UCP. This commitment is done by a instrumental price offer.
An instrumental price offer consists in an offer with price equal to 0€/MWh. Due to the algorithm the market operator uses to clear the Day-Ahead Market all offers with those characteristics will be matched in the clearing process and remunerated at the spot price λ_i^d
That regulation implies the GenCo has to determine its unit commitment in order to be able to cover those obligations and it has to determine its optimal offer taking into account those instrumental price offers.

2.4 Day-ahead and futures incomes functions

2.4.1 Day-ahead incomes function for a price-taker

This work focus on the thermal units of a *price-taker* generation company, i.e. a GenCo with no capability to alter market-clearing prices [3]. Therefore, the framework of this kind of GenCos could be equated to a market with perfect competition. Perfect competition is defined as a market structure in which there are large numbers of both buyers and sellers, all of them small, so that all of them act as price-takers. And it is known that in a perfectly competitive market a GenCo would maximize his profits by bidding his true marginal cost function [7].
The optimal offer curve for thermal unit t is the offer to the Day-Ahead Market that ensures a matched generation p_{it}^d with maximal benefit independently of the value of the clearing price λ_i^d. As stated above, in the case of a price-taker GenCo, the function that meet this condition is the *marginal cost curve*. If the cost function of the thermal unit t is represented by the quadratic function $c_t^b + c_t^l p_{it} + c_t^q (p_{it})^2$, then the optimal offer curve for this unit is:

$$\lambda_{it}^o(p_{ti}^o) = \begin{cases} 0 & \text{if} \quad p_{it}^o \le \underline{P}_t \\ 2c_t^q p_{it}^o + c_t^l & \text{if} \quad \underline{P}_t < p_{it}^o \le \overline{P}_t \end{cases} \tag{1}$$

Any offer of the GenCo must consist on pairs $(p_{it}^o, \lambda_{it}^o(p_{it}^o))$ belonging to the optimal offer

[4]Operador Mercado Ibérico de Energía. Polo Español - Spanish Iberian Market Operator

curve (Fig. 2). By sending this offer to the Day-Ahead Market, the matched generation p^d_{it} corresponding to any clearing price λ^d_i will be:

$$p^d_{it}(\lambda^d_i) = \begin{cases} \underline{P}_t & \text{if } p^*_{it}(\lambda^d_i) \leq \underline{P}_t \\ p^*_{it}(\lambda^d_i) & \text{if } \underline{P}_t \leq p^*_{it}(\lambda^d_i) \leq \overline{P}_t \\ \overline{P}_t & \text{if } p^*_{it}(\lambda^d_i) \geq \overline{P}_t \end{cases} \tag{2}$$

where

$$p^*_{it}(\lambda^d_i) = \left(\lambda^d_i - c^l_t\right)/2c^q_t. \tag{3}$$

It is easy to see that, for any clearing price λ^d_i, expression 2 the value that maximizes the benefit function:

$$B_{it}(p_{it}) = \lambda^d_i p_{it} - \left(c^b_t + c^l_t p_{it} + c^q_t (p_{it})^2\right) \tag{4}$$

taking into account the operational limits of the thermal unit.

The optimal offer curve problem for a price-taker GenCo is reduced to as many independent stochastic unit commitment problems as thermal units has the utility. If the optimal unit commitment shows that a given thermal unit must be on at interval i, then (1) represents the optimal offer curve to be sent to the Day-Ahead Market. The total incomes for all the committed unit, T_{on_i}, will be:

$$I^d_i = \sum_{\forall t \in T_{on_i}} \lambda^d_i p^d_{it}$$

2.4.2 Day-ahead and futures incomes function for a price-taker

Nowadays, the generation companies participate in all available market mechanisms in order to maximize their benefits. Suppose a GenCo that trades in both markets, the Derivatives and the Day-Ahead one and that it has a futures contracts portfolio, F, with constant quantities and prices, $(\lambda^f_j, L_j) j \in F$. All contracts are payed by differences, so the incomes from this portfolio at interval i will be:

$$I^f_i = \sum_{j \in F} \left(\lambda^f_j - \lambda^d_i\right) L_j$$

As the MIBEL's regulation describes, the energy L_j of the physical futures contract j must be allocated through the units of the GenCo that participate in this contract, $t \in T_j$, and delivered to the system through the instrumental price offer of each unit. This situation changes the structure of the optimal offer curve. Let f_{itj} be the generation of thermal t at interval i allocated to the futures physical contracts j, that is:

$$\sum_{t | t \in T_j \cap T_{on_i}} f_{itj} = L_j \quad \forall i \in I,\ \forall j \in J \tag{5}$$

And let q_{it} be the instrumental price offer which must not be lesser than the generation of thermal t at interval i allocated to the set of physical contracts in which it participates, $j \in F_t$:

$$q_{it} \geq \sum_{j \in F_t} f_{itj} \tag{6}$$

Following the market rules, each generator sends the amount q_{it} to the Day-Ahead Market through an instrumental price offer. This led to the following redefinition of the optimal offer curve (1) (Fig. 3):

$$\lambda^o_{ti}(p^o_{it}; q_{it}) = \begin{cases} 0 & \text{if } p^o_{it} \leq q_{it} \\ 2c^q_t p^o_{it} + c^l_t & \text{if } q_{it} < p^o_{it} \leq \overline{P}_t \end{cases} \tag{7}$$

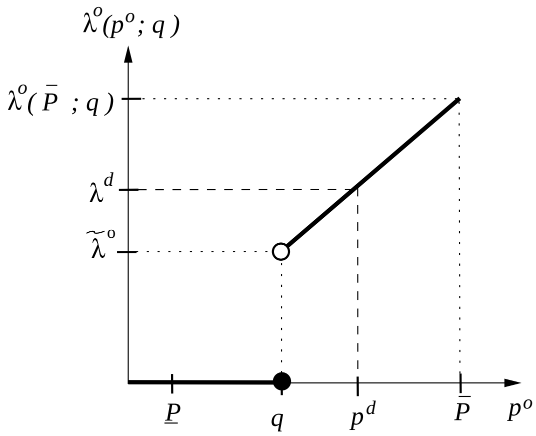

Figure 3: Optimal Offer Curve with Physical Futures Contracts

The value of the matched energy depends now on the value of the market clearing price with respect to the threshold price $\tilde{\lambda}^o_{it}$:

$$\tilde{\lambda}^o_{it} = 2c^q_t q_{it} + c^l_t \tag{8}$$

For any value $\lambda^d_i \leq \tilde{\lambda}^o_{it}$ the matched energy is $p^{df}_{it} = q_{it}$. When $\lambda^d_i > \tilde{\lambda}^o_{it}$ the matched energy coincides with expression (2), that is:

$$p^{df}_{it} = \begin{cases} q_{it} & \text{if } \lambda^d_i \leq \tilde{\lambda}^o_{it} \\ p^d_{it} & \text{if } \lambda^d_i > \tilde{\lambda}^o_{it} \end{cases} \tag{9}$$

Notice that λ^d_i and q_{it} determines completely the amount of matched energy p^{df}_{it} through expressions (8) and (9). Using the definitions of the matched generation (2) and threshold price value (8), this matched generation with futures can be re-expressed as:

$$p^{df}_{it} = \begin{cases} q_{it} & \text{if } q_{it} \geq p^d_{it} \\ p^d_{it} & \text{otherwise} \end{cases} \tag{10}$$

that sets the value of the matched energy as a non-differentiable function of the instrumental price offer q_{it} which is the real decision variable and it will be part of the decision variables of the optimization model.
The incomes function for the Day-Ahead Market with futures contracts for all the committed units at interval i, I^{df}_i, must take into account both the new expression of the matched energy (9) and the revenues coming from the futures portfolio:

$$I^{df}_i = \sum_{\forall j \in F} \left(\lambda^f_j - \lambda^d_i\right) L_j + \sum_{\forall t \in T_{on_i}} \lambda^d_i p^{df}_{it}$$

2.4.3 Day-ahead and futures benefit function for a price-taker

Let's consider now that the Day-Ahead Market has been cleared, with a market price λ^d_i. For all committed thermal unit t at time interval i (set T_{on_i}), the quadratic generation costs associated with the matched energy (10) is:

$$C_i = \sum_{\forall t \in T_{on_i}} \left(c^b_t + c^l_t p^{df}_{it} + c^q_t (p^{df}_{it})^2\right) \tag{11}$$

and the overall benefit function is:

$$
\begin{aligned}
B^{df} &= \sum_{\forall i \in I} \left(I_i^{df} - C_i \right) = \\
&\sum_{\forall i \in I} \left(\sum_{\forall j \in F} \left(\lambda_j^f - \lambda_i^d \right) L_j + \sum_{\forall t \in T_{on_i}} \lambda_i^d p_{it}^{df} - \sum_{\forall t \in T_{on_i}} \left(c_t^b + c_t^l p_{it}^{df} + c_t^q (p_{it}^{df})^2 \right) \right) = \\
&= \sum_{\forall i \in I} \left(\sum_{\forall j \in F} \left(\lambda_j^f - \lambda_i^d \right) L_j - \sum_{\forall t \in T_{on_i}} \left(c_t^b + (c_t^l - \lambda_i^d) p_{it}^{df} + c_t^q (p_{it}^{df})^2 \right) \right) \qquad (12)
\end{aligned}
$$

As it has been shown λ_i^d and q_{it} determine completely the amount of matched energy p_{it}^{df}. Therefore, expression (12) shows the dependency of the benefit function both on the market clearing price λ_i^d and the instrumental price offer q_{it} of the committed units. The market price is a random variable at the moment when the decision process must be undertaken, this fact will be analyzed in next sections.

2.5 Modeling the optimal bid for a price-taker GenCo

There exist different bidding structures depending on the markets rules. Usually, each unit has to send an individual bid for each interval. This bid is compounded by a determined number of blocks, each of them being a pair of (increasing) price and quantity.
As stated above, the main scope of this work is to build the GenCo's optimal bid following the MIBEL's rules and taking into account its futures contracts. At it has been shown, the futures contracts are included through the instrumental price bid. Thus, for each unit and interval, the model will provide us the Day-Ahead Market bidding first block.
The bidding strategies we want to obtain are destined to price-taker GenCos. As we have describe, a price-taker GenCo cannot change the market clearing price through its offers and it would maximize his profits by bidding its true marginal cost function. In summary, the bidding strategy we define for each unit and interval is:

i) To build the first bid block with instrumental price and quantity resulting from the optimization problem.

ii) To divide the rest of the unit capacity into the number of intervals the market rules fixes with price equal to the marginal cost.

2.6 Uncertainties modeling

The optimization model presented in this work is stochastic due to the presence of a random variable, the Day-Ahead Market price. This random variable has the characteristics of a financial time series (Fig. 4) and, in order to formulate a stochastic model, it has to be discretized on a scenario tree. In particular, the optimization model presented in this work is a two-stage stochastic problem and, for this kind of model, it is sufficient a set of individual scenarios. This means the scenario tree will be a fan of scenarios $\lambda^{d,s} = \{\lambda_1^{d,s}, \ldots, \lambda_I^{d,s}\}$ with probabilities $P^s = P(\lambda^{d,s})\ \forall s \in S$, where S is the number of scenarios (Fig. 5). In this work, we have follow the next steps in order to obtain the required scenario set:

i) To fit a time series model for the market-price,

ii) To discretize the random variable by simulating a large number of scenarios,

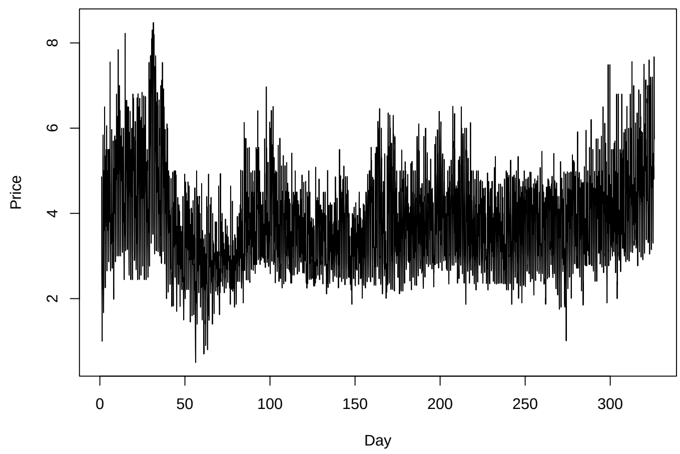

Figure 4: Spanish Day-Ahead Market Prices (2007)

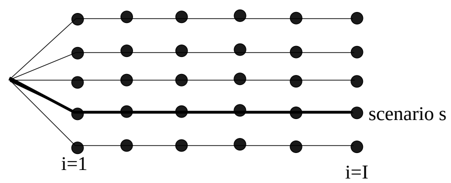

Figure 5: Fan of individual scenarios

iii) To reduce the set of scenarios preserving at maximum the characteristics of the simulated tree

The details are described in section 5.2.

3 Problem formulation: first approach

In order to achieve the objective of this work, the model has to find the instrumental price bid quantity for each interval and its economic dispatch. The solution has to maximize the expected Day-Ahead Market profits taking into account the physical futures contract portfolio.

3.1 Characteristics

Following MIBEL's rules, if we are optimizing today we focus on tomorrow's Day-Ahead Market because we have to submit tomorrow's bidding. So, the optimization horizon is 24 hourly intervals, this set of intervals is denoted as I.
The proposed short-term bidding strategies are addressed to a price-taker GenCo. The generation units to be considered are the thermal units with participation in the auction process. The relevant parameters of a thermal unit are:

- quadratic generation costs with linear and quadratic coefficients, c_t^l and c_t^q respectively, for the t^{th} unit: production cost in €/MWh
- maximum and minimum power capacity, $\overline{P}_t$ and $\underline{P}_t$ respectively, for the t^{th} unit: maximum and minimum power output that the unit can generate in MWh

3.2 Production variables and operational constraints

The discussion in sections 2.4.2 and 2.4.3 established that the offer curve (7) is completely determined by the amount of energy at instrumental price q_{it}. At the scenario s, with clearing price $\lambda_i^{d,s}$, the matched energy is given by expression (10):

$$p_{it}^{df,s} = \begin{cases} q_{it} & \text{if } q_{it} \geq p_{it}^{d,s} \\ p_{it}^{d,s} & \text{if otherwise} \end{cases} \tag{13}$$

where

$$p_{it}^{d,s} = \begin{cases} \underline{P}_t & \text{if } p^{*,s} \leq \underline{P}_t \\ p_{it}^{*,s} & \text{if } \underline{P}_t \leq p_{it}^{*,s} \leq \overline{P}_t \\ \overline{P}_t & \text{if } p^{*,s} \geq \overline{P}_t \end{cases} \tag{14}$$

and $p_{it}^{*,s} = \left(\lambda_i^{d,s} - c_t^l\right)/2c_t^q$, as it is defined in expression (3). Expression (13) sets the value of the matched energy of thermal t at time interval i under scenario s but it does not have to be explicitly introduced in the model because, as we will see in section 3.5, the optimal value of the decision variable p_{it}^s corresponds to $p_{it}^{df,s}$. In our stochastic programming model, this decision variable will be a second stage one because it depends on the scenario since it is related with the market clearing price.
Thus, the variable q_{it} is the real decision variable of the model. In the final formulation, it will be expressed in terms of the allocated energy to each individual physical futures contract, f_{itj}, in order to obtain the economic dispatch of each contract:

$$q_{it} \geq \sum_{\forall j \in F_t} f_{itj} \tag{15}$$

were F_t is the subset of contracts in which unit t participates (parameter). In the stochastic model, those variables f_{itj} are considered first stage variables, as they are needed to configure the generation offer for tomorrow and they are independent of the market clearing price. Therefore, the set of variables that models unit's generation output is:

- f_{itj}: generation of unit $t \in T_{on_i}$ at interval $i \in I$ allocated to the futures contract $j \in F_t$
- q_{it}: instrumental (price acceptant) selling bidding of unit $t \in T_{on_i}$ at interval $i \in I$.
- p_{it}^s: matched energy of unit $t \in T_{on_i}$ at interval $i \in I$ scenario $s \in S$

Remember that variables p_{it}^s and q_{it} can be expressed in terms of f_{itj} through (13) and (15).
The following four set of constraints are the operational ones. The first two set of constraints control the production of the unit, i.e., the unit will not produce above or below operational limits. Constraints (18) and (19) concern also to the operational limits and control the relationship between both set of variables.

$$p_{it}^s \geq \underline{P}^t \qquad \forall i \in I \quad \forall t \in T_{on_i} \quad \forall s \in S \tag{16}$$
$$p_{it}^s \leq \overline{P}^t \qquad \forall i \in I \quad \forall t \in T_{on_i} \quad \forall s \in S \tag{17}$$
$$q_{it} \geq \underline{P}^t \qquad \forall i \in I \quad \forall t \in T_{on_i} \quad \forall s \in S \tag{18}$$
$$q_{it} \leq p_{it}^s \qquad \forall i \in I \quad \forall t \in T_{on_i} \quad \forall s \in S \tag{19}$$

3.3 Futures contracts production variables and associated constraints

As it has been defined, q_{it} represents the energy of futures contract $j \in F$ allocated to unit $t \in T_{on_i}$ at interval $i \in I$. The following set of constraints model the futures contract production:

$$\sum_{t|t\in T_j \cap T_{on_i}} f_{itj} = L_j \qquad \forall i \in I \quad \forall j \in F \tag{20}$$

where T_j is the set of thermal units that participates in contract j and L_j is the quantity that has to be settled for contract j.

3.4 Objective function

The solution has to maximize the expected Day-Ahead Market profits taking into account futures contracts and they depend on the market price which is a random variable at the moment when the decision process must be taken. As has been explained in section 2.6, this random variable is modeled through a set of scenarios $\lambda^{d,s} = \{\lambda_1^{d,s}, \dots, \lambda_I^{d,s}\}$ with probabilities $P^s = P(\lambda^{d,s})\ \forall s \in S$, where S is the number of scenarios.
Following section 2.4.3 the expression of the day-ahead and futures benefit function for scenario s, $B^{df,s}$, is:

$$B^{df,s} = \sum_{\forall i\in I}\left(\sum_{\forall j\in F}\left(\lambda_j^f - \lambda_i^{d,s}\right)L_j - \sum_{\forall t\in T_{on_i}}\left(c_t^b + (c_t^l - \lambda_i^{d,s})p_{it}^{df,s} + c_t^q(p_{it}^{df,s})^2\right)\right) \tag{21}$$

Taking the expected value along the S scenarios we obtain:

$$\begin{aligned} E[B^{df,s}] =& E\left[\sum_{\forall i\in I}\left(\sum_{\forall j\in F}\left(\lambda_j^f - \lambda_i^{d,s}\right)L_j - \sum_{\forall t\in T_{on_i}}\left(c_t^b + (c_t^l - \lambda_i^{d,s})p_{it}^{df,s} + c_t^q(p_{it}^{df,s})^2\right)\right)\right] = \\ &= |I|\sum_{\forall j\in F}\left(\lambda_j^f - E[\lambda_i^{d,s}]\right)L_j - \sum_{\forall i\in I}\sum_{\forall t\in T_{on_i}} c_b^t \qquad (22) \\ &- \sum_{\forall i\in I}\sum_{\forall s\in S} P^s\left(\sum_{\forall t\in T_{on_i}}(c_t^l - \lambda_i^{d,s})p_{it}^{df,s} + c_t^q(p_{it}^{df,s})^2\right) \qquad (23) \end{aligned}$$

Terms in (22) are constants with respect to the decision variables and then, the objective function $f(x)$ reduces to the last terms of $E[B^{df,s}]$:

$$f(p,q,g) = \sum_{\forall i\in I}\left[\sum_{\forall s\in S} P^s\left(\sum_{\forall t\in T_{on_i}}(c_t^l - \lambda_i^{d,s})p_{it}^{df,s} + c_t^q(p_{it}^{df,s})^2\right)\right] \tag{24}$$

This is the objective function to be minimized in our model.

3.5 Complete formulation

The complete formulation can be recast as:

$$\underset{p,q,g}{\text{minimize}} \sum_{\forall i \in I} \sum_{\forall t \in T_{on_i}} \sum_{s \in S} P^s \left[(c_t^l - \lambda_i^{d,s}) p_{it}^s + c_t^q (p_{ti}^s)^2 \right] \tag{25}$$

s.t.

$$\sum_{t | t \in T_j \cap T_{on_i}} f_{itj} = L_j \qquad \forall i \in I,\ \forall j \in J \tag{26}$$

$$q_{it} \geq \sum_{j \in F_t} f_{itj} \qquad \forall i \in I,\ \forall t \in T_{on_i} \tag{27}$$

$$p_{it}^s \in [\underline{P}_t, \overline{P}_t] \qquad \forall i \in I,\ \forall t \in T_{on_i},\ \forall s \in S \tag{28}$$

$$q_{it} \in [\underline{P}_t, p_{it}^s] \qquad \forall i \in I,\ \forall t \in T_{on_i},\ \forall s \in S \tag{29}$$

$$f_{itj} \geq 0 \qquad \forall i \in I,\ \forall t \in T_{on_i},\ \forall j \in J \tag{30}$$

In order to show that problem (25-30) is coherent with the optimal offer curve model developed in the previous sections, it is necessary to demonstrate that the optimal value of p_{it}^s coincides with the expression (13) of the matched energy at each scenario s. To see this equivalence, the Karush-Kuhn-Tucker optimality conditions of problem (25-30) will be used. This problem is separable by intervals. [11] We express the problem associated with the i^{th} time interval in standard form, together with the Lagrange multipliers λ_i and μ_i of each constraint:

$$\underset{p_i,q_i,f_i}{\text{minimize}} \sum_{\forall t \in T_{on_i}} \sum_{s \in S} P^s \left[(c_t^l - \lambda_i^{d,s}) p_{it}^s + c_t^q (p_{it}^s)^2 \right] \tag{31}$$

s.t.

$$\sum_{t | t \in T_j \cap T_{on_i}} f_{itj} - L_j = 0 \qquad \forall j \in J \quad (\lambda_{ij}) \tag{32}$$

$$\sum_{j \in F_t} f_{itj} - q_{it} \leq 0 \qquad \forall t \in T_{on_i} \quad (\mu_{it}^{fq}) \tag{33}$$

$$p_{it}^s - \overline{P}_t \leq 0 \qquad \forall t \in T_{on_i},\ \forall s \in S \quad (\overline{\mu}_{it}^s) \tag{34}$$

$$\underline{P}_t - q_{it} \leq 0 \qquad \forall t \in T_{on_i} \quad (\underline{\mu}_{it}) \tag{35}$$

$$q_{it} - p_{it}^s \leq 0 \qquad \forall t \in T_{on_i},\ \forall s \in S \quad (\mu_{it}^{pq,s}) \tag{36}$$

$$- f_{itj} \leq 0 \qquad \forall t \in T_{on_i},\ \forall j \in J \quad (\mu_{itj}^f) \tag{37}$$

The Karush-Kuhn-Tucker first order optimality conditions of problem (31-37) are:

$$P^s\left[\left(c_l^t-\lambda_i^{d,s}\right)+2c_q^t p_{it}^s\right]+\overline{\mu}_{it}^s-\mu_{it}^{pq,s}=0 \quad \forall t\in T_{on_i},\ \forall s\in S \tag{38}$$

$$-\mu_{it}^{fq}-\underline{\mu}_{it}+\sum_{\forall s\in S}\mu_{it}^{pq,s}=0 \quad \forall t\in T_{on_i} \tag{39}$$

$$\mu_{it}^{fq}+\lambda_{it}-\mu_{itj}^{f}=0 \quad \forall t\in T_{on_i},\ \forall j\in F_t \tag{40}$$

$$\mu_{it}^{fq}\left(\sum_{j\in F_t} f_{itj}-q_{it}\right)=0 \quad \forall t\in T_{on_i} \tag{41}$$

$$\overline{\mu}_{it}^s\left(p_{it}^s-\overline{P}_t\right)=0 \quad \forall t\in T_{on_i},\ \forall s\in S \tag{42}$$

$$\underline{\mu}_{it}\left(\underline{P}_t-q_{it}\right)=0 \quad \forall t\in T_{on_i} \tag{43}$$

$$\mu_{it}^{pq,s}\left(q_{it}-p_{it}^s\right)=0 \quad \forall t\in T_{on_i},\ \forall s\in S \tag{44}$$

$$\mu_{itj}^{f} f_{itj}=0 \quad \forall t\in T_{on_i},\ \forall j\in J \tag{45}$$

$$\mu_{it}^{fq},\ \overline{\mu}_{it}^s,\ \underline{\mu}_{it},\ \mu_{it}^{pq,s},\mu_{itj}^{f}\geq 0 \quad \forall t\in T_{on_i},\ \forall j\in J,\ \forall s\in S \tag{46}$$

It will be proved that at every solution of the above KKT system (38-46) the value of the primal variables p_{it}^s and q_{it} satisfies the same relation as the matched energy (13):

$$p_{it}^s\equiv p_{it}^{df,s}=\begin{cases} q_{it} & \text{if } q_{it}\geq p_{it}^{d,s}\\ p_{it}^{d,s} & \text{if otherwise}\end{cases} \tag{47}$$

where $p_{it}^{d,s}$ given by expression (14).
The following expression of p_{it}^s can be obtained from equation (38):

$$p_{it}^s=\frac{\lambda_i^{d,s}-c_l^t}{2c_q^t}+\frac{\mu_{it}^{pq,s}-\overline{\mu}_{it}^s}{2c_q^t P^s}=p_{it}^{*,s}+\frac{\mu_{it}^{pq,s}-\overline{\mu}_{it}^s}{2c_q^t P^s} \tag{48}$$

To derive the relation (47), the solution of the KKT system will be studied in the following five cases, among which any optimal solution must fall:

Case A , $\underline{P}^t<q_{it}=p_{it}^s=\overline{P}^t$ **:** When $q_{it}=\overline{P}^t$ it is easy to see from expressions (13) and (14) that $p_{it}^{df,s}=q_{it}$, been then $p_{it}^s=p_{it}^{df,s}$

Case B , $\underline{P}^t\leq q_{it}<p_{it}^s=\overline{P}^t$ **:** Condition (44) gives $\mu_{it}^{pq,s}=0$ that, together with the non-negativity of the lagrange multipliers $\overline{\mu}_{it}^s$ and equation (48) gives $\overline{P}^t\leq p_{it}^{*,s}$, been then $p_{it}^{d,s}=\overline{P}^t$. As $q_{it}<p_{it}^{d,s}=\overline{P}^t$ then, equation (13) provides $p_{it}^{df,s}=p_{it}^{d,s}=\overline{P}^t=p_{it}^s$

Case C , $\underline{P}^t<q_{it}=p_{it}^s<\overline{P}^t$ **:** In this case, condition (42) forces $\overline{\mu}_{it}^s=0$ which, in combination with equation (48) and condition $\mu_{it}^{pq,s}\geq 0$ gives $p_{it}^s=q_{it}\geq p_{it}^{*,s}$. Then, it is clear from (13) and (14) that $q_{it}\geq p_{it}^{d,s}$ and $p_{it}^{df,s}=q_{it}=p_{it}^s$

Case D , $\underline{P}^t\leq q_{it}<p_{it}^s<\overline{P}^t$ **:** In this case, conditions (42) and (44) gives $\overline{\mu}_{it}^s=\mu_{it}^{pq,s}=0$ that, together with equation (48) gives $p_{it}^s=p_{it}^{*,s}$. Then, again from (13-14), $p_{it}^{d,s}=p_{it}^{*,s}$, $q_{it}<p_{it}^{d,s}$ and $p_{it}^{df,s}=p_{it}^{d,s}=p_{it}^s$.

Case F , $\underline{P}^t=q_{it}=p_{it}^s<\overline{P}^t$ **:** Here we have $\overline{\mu}_{it}^s=o$ (condition (42)) and then $p_{it}^s=\underline{P}^t\geq p_{it}^{*,s}$. Then equations (13-14) sets $p_{it}^{d,s}=\underline{P}^t$ and $p_{it}^{df,s}=q_{it}=p_{it}^s$.

Problem (25-30) is convex ($c_t^q \geq 0$) and then, any solution of the KKT system (38-46) is a global minimum. Thus, the previous development shows that over any optimal solution of problem (25-30) the value of the variable p_{it}^s corresponds to the value of the matched energy, $p_{it}^{df,s}$, at the i-th Day-Ahead Market, conditioned to scenario s.

4 Problem formulation: second approach

Once we have built and solved the model that maximize the expected Day-Ahead Market profits taking into account the physical futures contracts portfolio, we can generalize it to the case in which the unit commitment has to be also optimized.
This new approach is useful for many real situations, for example:

- if the unit commitment is fixed but it needs to be adjusted with most recent information about the market
- to fix the unit commitment in a medium term approach taking into account the available information of the market
- to do a medium term foresight of the unit commitment and the bidding strategies

In this work we will focus on the first case but it can be extended to the others with the convenient changes in the number of intervals and the uncertainty information.

4.1 Characteristics

This second approach preserves all the characteristics of the first one (section 3.1). It is addressed to a price-maker GenCo that participates in the MIBEL. The model is formulated for the thermal units and the optimization horizon is 24 hourly intervals.
To the relevant parameters of the first model we incorporate the next ones:

- start-up, c_t^{on}, and shut-down, c_t^{off}, costs for the t^{th} unit: costs for the start-up and shut-down process in €
- minimum operation and minimum idle time, min_t^{on} and min_t^{off} respectively, for the t^{th} unit: minimum number of hours that the unit must remain in operation once it is started up and minimum number of hours that the unit must remain idle once it has been shut down before started up again, respectively.

4.2 Unit commitment variables and associated constraints

The formulation of the start-up and shut-down process follows [12]. Let $u_{it} \in \{0, 1\}$ be a binary variable expressing the off-on operating status of the t^{th} unit over the i^{th} interval.
Values of u_{it} and $u_{(i+1)t}$ must obey certain operating rules to take into account the constraints of the minimum in service and idle time. It is necessary to introduce two extra binary variables e_{it} and a_{it} for each u_{it}.
Let $e_{it} \in 0, 1$ be a start-up indicator for the t^{th} unit. It is zero in all intervals where the t^{th} unit has not changed from $u_{(i-1)t} = 0$ to $u_{it} = 1$. Similarly, $a_{it} \in 0, 1$ is a shut-down indicator for the t^{th} unit. It is zero in all intervals where the t^{th} unit has not changed from $u_{(i-1)t} = 1$ to $u_{it} = 0$.
The following three set of constraints model uniquely the binary variables and the star-up and

shut-down process:

$$u_{it} + u_{(i-1)t} - e_{it} + a_{it} = 0 \qquad \forall i \in I \quad \forall t \in T \tag{49}$$

$$e_{it} + \sum_{k=i}^{\min\{i+t_t^{on}, |I|\}} a_{kt} \leq 1 \qquad \forall i \in I \quad \forall t \in T \tag{50}$$

$$a_{it} + \sum_{k=i+1}^{\min\{i+t_t^{off}, |I|\}} e_{kt} \leq 1 \qquad \forall i \in I \quad \forall t \in T \tag{51}$$

4.3 Production variables and operational constraints

The production variables (section 3.2) are identical in the first and in the second formulation of the model. The operational constraints change slightly. The set of constraints (16-18) are reformulated as below:

$$p_{it}^s \geq \underline{P}^t u_{it} \qquad \forall i \in I \quad \forall t \in T \quad \forall s \in S \tag{52}$$

$$p_{it}^s \leq \overline{P}^t u_{it} \qquad \forall i \in I \quad \forall t \in T \quad \forall s \in S \tag{53}$$

$$q_{it} \geq \underline{P}^t u_{it} \qquad \forall i \in I \quad \forall t \in T \tag{54}$$

The futures contracts production variables and its associated constraints (section 3.3) are also identical in both formulations of the model.

4.4 Objective function

The objective function of the second formulation is analogous to the one developed in section 3.4. The differences are that, first, if the thermal unit t is starting-up ($e_{it} = 1$) or shutting-down ($a_{it} = 1$), then the fixed costs c_{on}^t and c_{off}^t must be included respectively, and, second, the constant generation cost c_t^b must be considered. The objective function $f(x)$ for the second formulation is:

$$f(p,q,g,u,a,e) = \sum_{\forall i \in I} \sum_{\forall t \in T} \left(c_t^{on} e_{it} + c_t^{off} a_{it} + c_t^b u_{it} + \sum_{s \in S} P^s \left[(c_t^l - \lambda_i^{d,s}) p_{it}^s + c_t^q (p_{ti}^s)^2 \right] \right) \tag{55}$$

4.5 Complete formulation

The complete problem can be recast as:

$$\underset{p,q,g,a,e,u}{\text{minimize}} \sum_{\forall i \in I} \sum_{\forall t \in T} \left(c_t^{on} e_{it} + c_t^{off} a_{it} + c_t^b u_{it} \qquad + \sum_{s \in S} P^s \left[(c_t^l - \lambda_i^{d,s}) p_{it}^s + c_t^q (p_{ti}^s)^2 \right] \right) \tag{56}$$

s.t.

$$q_{it} \geq \sum_{j \in F_t} f_{itj} \qquad \forall i \in I,\ \forall t \in T \tag{57}$$

$$\sum_{t \in U_j} f_{itj} = L_j \qquad \forall i \in I,\ \forall j \in J \tag{58}$$

$$u_{it} + u_{(i-1)t} - e_{it} + a_{it} = 0 \qquad \forall i \in I,\ \forall t \in T \tag{59}$$

$$e_{it} + \sum_{k=i}^{\min\{i+t_t^{on}, |I|\}} a_{kt} \leq 1 \qquad \forall i \in I,\ \forall t \in T \tag{60}$$

$$a_{it} + \sum_{k=i+1}^{\min\{i+t_t^{off}, |I|\}} e_{kt} \leq 1 \qquad \forall i \in I,\ \forall t \in T \tag{61}$$

$$p_{it}^s \geq \underline{P}^t u_{it} \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{62}$$

$$p_{it}^s \leq \overline{P}^t u_{it} \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{63}$$

$$q_{it} \geq \underline{P}^t u_{it} \qquad \forall i \in I,\ \forall t \in T \tag{64}$$

$$q_{it} \leq p_{it}^s \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{65}$$

$$f_{itj} \geq 0 \qquad \forall i \in I,\ \forall t \in T,\ \forall j \in J \tag{66}$$

$$u_{it}, a_{it}, e_{it} \in \{0, 1\} \qquad \forall i \in I,\ \forall t \in T \tag{67}$$

It is easy to see that any solution of problem (56-66) satisfies the optimal bidding definition (13). First, note that for any given value of the binary variables $\tilde{u}$, $\tilde{a}$ and $\tilde{e}$, problem (56-66) reduces to (25-30), with $T_{on_i} := \{t | \tilde{u}_{it} = 1\}$. Now consider the optimal solution $x^* = [p^*, q^*, f^*; a^*, e^*, u^*]$ to problem (56-66). Then, as the optimal values $[p^*, q^*, f^*]$ corresponds to the solution of the equivalent problem (25-30) associated to $[a^*, e^*, u^*]$, then p^* and q^* should satisfy the optimal bidding definition(13).

5 Case study

In this section is presented first the data sources and the scenario tree construction needed to solve the problem. Then, the computational tests carried out with this data and tree are described.
The model has been implemented in AMPL [6] and solved with CPLEX [4] (with default options) using a SunFire X2200 with two processors dual core AMD Opteron 2222 at 3 GHz and 32 Gb of RAM memory.

5.1 Data sources

The sources for all data used in the case studies are described below. All the data of this work is public and it is either directly available in the web pages indicated or it has been calculated using some other public data.

- **Market data:**
  The main important market data needed for this work is the Day-Ahead Market price, it

t	c_t^b €	c_t^l €/MWh	c_t^q €/MWh^2	$\underline{p}_t$ MW	$\overline{p}_t$ MW	st_t^0 hr	c_t^{on} €	c_t^{off} €	t_t^{on} h	t_t^{off} h
1	151.08	40.37	0.015	160.0	350.0	+3	412.80	412.80	3	3
2	554.21	36.50	0.023	250.0	563.2	+3	803.75	803.75	3	3
3	97.56	43.88	0.000	80.0	284.2	–3	244.80	244.80	3	3
4	327.02	28.85	0.036	160.0	370.7	+3	438.40	438.40	3	3
5	64.97	45.80	0.000	30.0	65.0	+3	100.20	100.20	3	3
6	366.08	-13.72	0.274	60.0	166.4	+3	188.40	188.40	3	3
7	197.93	36.91	0.020	160.0	364.1	+3	419.20	419.20	3	3
8	66.46	55.74	0.000	110.0	313.6	–3	1298.88	1298.88	3	3
9	66.46	55.74	0.000	110.0	313.6	–3	1298.88	1298.88	3	3
10	372.14	105.08	0.000	90.0	350.0	–3	1315.44	1315.44	3	3

Table 1: Operational characteristics of the thermal Units

is available at OMEL's site (*www.omel.es*) since January 1998 until today. In this work we use the data from January 1^{st}, 2004 to October 23^{th}, 2006.
Since information about specific futures contracts between companies is confidential, it is useful to know the quantities and clearing price of the products with the day of study within its delivery period. This data is available at OMIP's site (*www.omip.pt)* and it is used to define some examples of futures contracts.

- **Generation Company data:**
  The information about the units in study belongs to a Generation Company that bids daily in the Day-Ahead Market and also participates in the Derivatives Market. Most of the information about the generation units is available at the CNE's site (*www.cne.es*)[5].
  We have included ten thermal units of this GenCo, two of them are fuel-gas units, one is a fuel-oil unit and the other seven are coal units.

5.2 Scenario set construction

The detail of the steps described in section 2.6 used to build the scenario set are described bellow.

i) **Time series model:**
The first step is fit the model that best describes the random variable. As most competitive electricity market prices, the Spanish Day-Ahead Market price presents the following characteristics: high frequency, nonconstant mean and variance, multiple seasonality, calendar effect, high volatility and high presence of picks [13].
Following previous works [1], the market price has been characterized by an auto-regressive integrated moving average model. We work with the log scale of the price in order to avoid the nonconstant variance, specifically:

$$ln(\lambda^d) \sim ARIMA(5,0,2)(8,0,1)_{24}(3,0,3)_{168}$$

The model is fitted based on the data from 2004 until the day before the day in study. The coefficients are estimated by *maximum likelihood estimation.*

[5] Comisión Nacional de Energía - Spanish Energy Systems Regulator

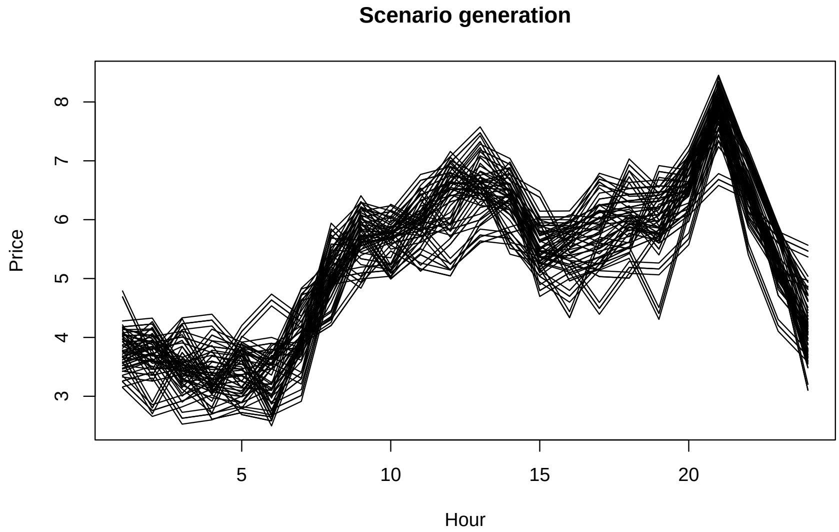

Figure 6: Simulations

ii) Scenario generation:
The next step in the scenario set construction is the discretization of the random variable in order to introduce it in the optimization problem. One of the most usual mechanism for this discretization is the simulation of prices scenarios for the day in study [10]. Following this, once the model has been fitted we have generated 300 simulated scenarios for the 24 hours of the day in study (Fig. 6).

iii) Scenario reduction:
The last step of this process is to reduce the dimensionality of the optimization problem. A set of decision variables is required for each scenario, so the reduction of the number of scenarios will ease the computational resolution. Following the algorithm described in [8], the set of scenarios has been reduced preserving at maximum the characteristics of the simulated set.

5.3 Computational results

A set of computational tests has been performed in order to validate the described models and its results are presented below. The results reported correspond to the second formulation as it is a generalization of the first one. We obtain identical results for the first formulation if we fix the unit commitment to its second formulation's optimal value.
All available real data is used. The computational tests have been done changing the quantity of energy allocated to physical futures contracts and the status of the units at first interval in order to study their influence in the results. As expected, the status of the units at first interval only affects the unit commitment. After testing some of them, the starting status of the units is fixed as all open and allowing them to be closed at the first interval in order to give more freedom to the unit commitment.
In stochastic programming models, the number of scenarios is a critical decision. We deal with this problem increasing the number of scenarios until the stabilization of the objective function optimal value. The original tree has 300 scenarios that have been reduced to sets of 150, 100, 75, 50, 40, 30, 20 and 10 scenarios following the steps described in 5.2. In table 2 the main parameters of each test are summarized: percentage of the total available energy allocated in

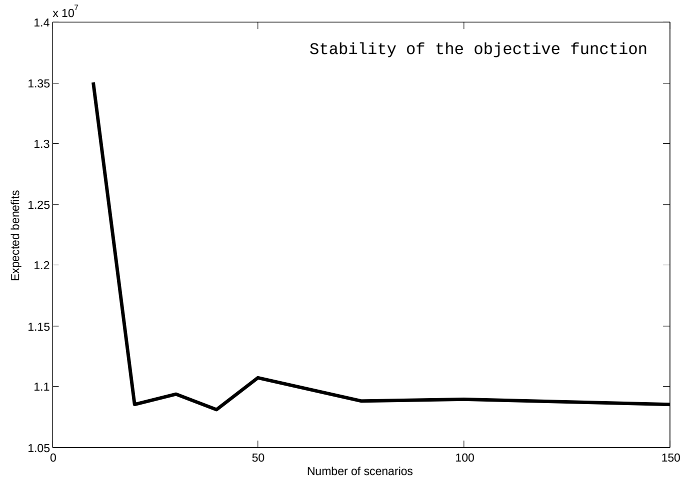

Figure 7: Expected benefits for each reduced set of scenarios

futures contracts ($\%\overline{P}$), number of scenarios (S), number of binary variables (b.v.), number of continuous variables (c.v.), CPU time in seconds (CPU(s)) and the value of the expected benefits (minus the objective function). It is shown the increasing CPU time according to the increase of the number of scenarios because of the proportional relationship between them and the number of continuous variables (the number of binary variables is independent of the number of scenarios). It can be seen also the convergence of the value of the objective function when the number of scenarios grows (Fig. 7) and the convergence of the optimal value of the decision variables (Fig. 8). Approximately, from 75 scenarios both values converge and the computational time is acceptable, therefore this will be the number of scenarios for the next tests.

$\|S\|$	c.v.	CPU(s)	E(benefits)(€)	$\|\|\Delta(€)/\Delta(s)\|\|$
10	3.360	13	1.350.830	
20	5.760	55	1.085.240	6.323,57
30	8.160	112	1.093.900	151,93
40	10.560	216	1.081.010	123,94
50	12.960	444	1.107.110	114,47
75	18.960	2.100	1.087.860	11,62
100	24.960	3.319	1.089.280	1,16
150	36.960	4.244	1.084.880	4,76

$|I| = 24; |T| = 10; \%\overline{P} = 40;$ b.v.= 720

Table 2: Results for different number of scenarios

The quantity allocated to futures contracts is confidential and we have no real data for the units in study. The set of computational tests presented is based on the percentage of the total available energy that the GENCO has allocated in futures contracts. For this case study, we include the 10 available units in one or more of the 3 UCPs created, each of them corresponding to one futures contract. In table 3 are summarized the main parameters of the computational test for each percentage of the available energy studied: 5%, 40% and 75%. Computational time has no relation with the quantity allocated since it does not change the number of variables. The

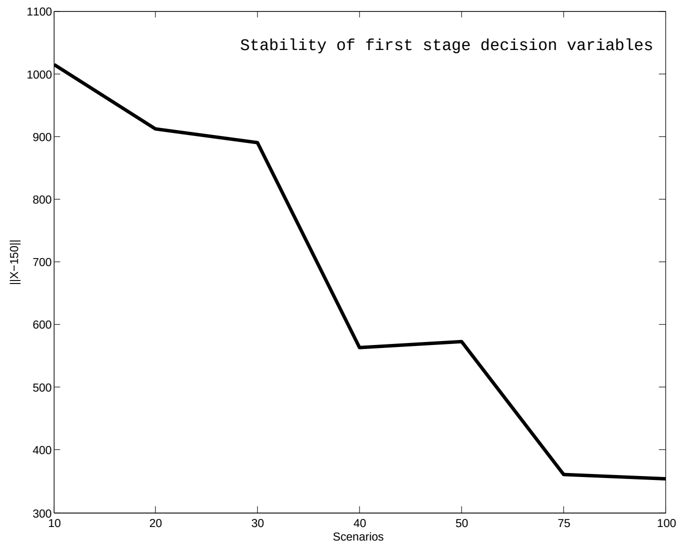

Figure 8: Difference between the value of the first stage variables for the complete set of scenarios and each reduced one

value of the expected benefits is different for each case, notice that when we force to produce the 75% of the available energy for the settlement of the physical futures contracts the GENCO makes a loss.

$\%\overline{P}$	E(benefits)
5	1.823.170
40	1.107.110
75	-2.800.460

$|I| = 24$; $|T| = 10$; $|S| = 75$;
c.v. = 720; b.v. = 12960

Table 3: Results for different quantity allocated to futures contracts

Figure 9 shows the optimal offer of one of the units in study for each percentage of energy allocated to the futures contracts. The plot represents the offer function that the GENCO has to submit to the MIBEL operator: step-wise curve with ten steps, each of them corresponding to a pair *(energy, price)* always with increasing prices. The first step corresponds to the offer at instrumental price (0€) and the following steps follow the marginal curve. Notice that for the first case (blue line) the unit has no energy allocated to futures contracts so the instrumental offer's quantity is the minimum operational limit (160MW) because, as the unit is committed, the matched energy has to be at minimum this quantity. For the other two cases the energy allocated to futures contracts is 186MW (green line) and 256MW (red line). From now on results the percentage of available energy used for physical futures contracts will be fix at 40%.
Next we present the main results of the model: the economic dispatch of the futures contracts, the unit commitment of the units and the bidding curve for each unit. Figure 10 shows the instrumental price bid for each unit and interval, this bid is either the quantity allocated to futures contracts or the minimum operational limit of the unit. It is also represented the unit commitment since if the unit is not producing the minimum operational limit it means the unit

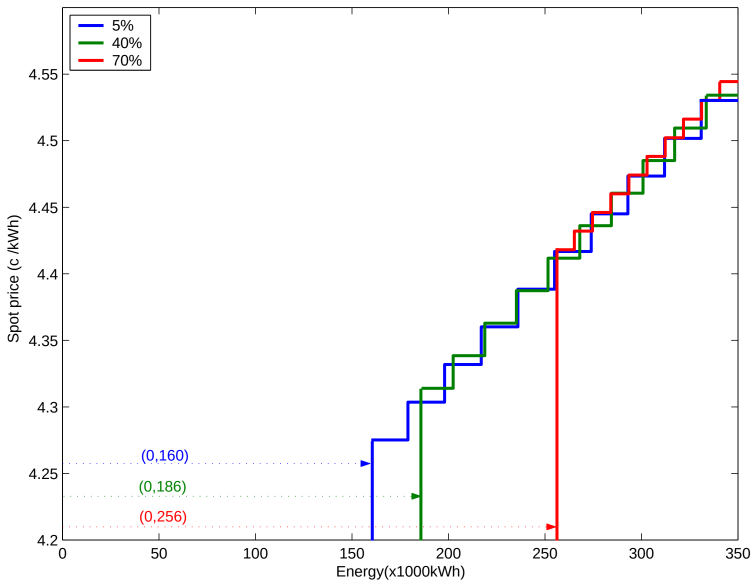

Figure 9: Optimal offer for unit 1 at interval 12

is off.In figure 11 it is represented the economic dispatch of each futures contract, that means, how the contract is settled among the different units of each UCP. Three kind of physical futures contracts have been considered, 200MWh in weekly contracts, 500MWh in monthly contracts and 500MWh in yearly contracts. Finally, the main objective of the study, to obtain the optimal bid curves for all the units of the producer. Figure 12 shows the optimal bid curves for each committed thermal unit at hour 12. The first interval is always the 0 price bid and the next ones correspond to the marginal cost curve. The thermal units 3, 5 and 9 have a straight line as offer curve because they have linear production cost function instead of a quadratic one.

6 Conclusions

We have developed a mixed-integer stochastic programming model for the short-term thermal optimal bidding problem in the Day-Ahead Market of a price-taker Generation Company operating also in the Derivatives Physical Electricity Markets. The optimal solution of our model determines the unit commitment of the thermal units, the optimal instrumental price bidding strategy for the generation company and the economic dispatch of the committed futures contracts for each hour so as to maximize the benefits arising from the Day-Ahead Market while satisfying thermal operational constraints. The model meets the new regulation of the MIBEL. There has been presented two approaches of the optimal bidding problem, in the first one the unit commitment is took as input data and only the optimal bid and the economic dispatch of the futures contracts is decided meanwhile in the second approach also the unit commitment is a decision to optimize. For both models, there has been shown through Karush-Kuhn-Tucker conditions that the optimal value of the decision variables corresponds to the theoretical optimal bidding curve for a price-taker producer.
The computational tests done with real data of the thermal units of a price-taker producer operating in the MIBEL have validated the model and they provide suitable results.

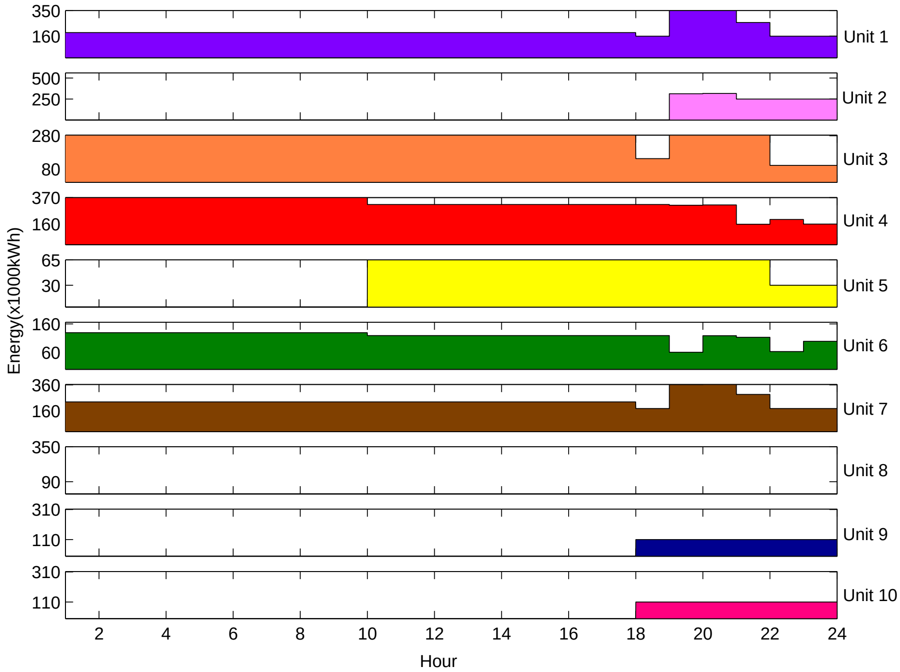

Figure 10: Instrumental price bid quantity for each unit and interval

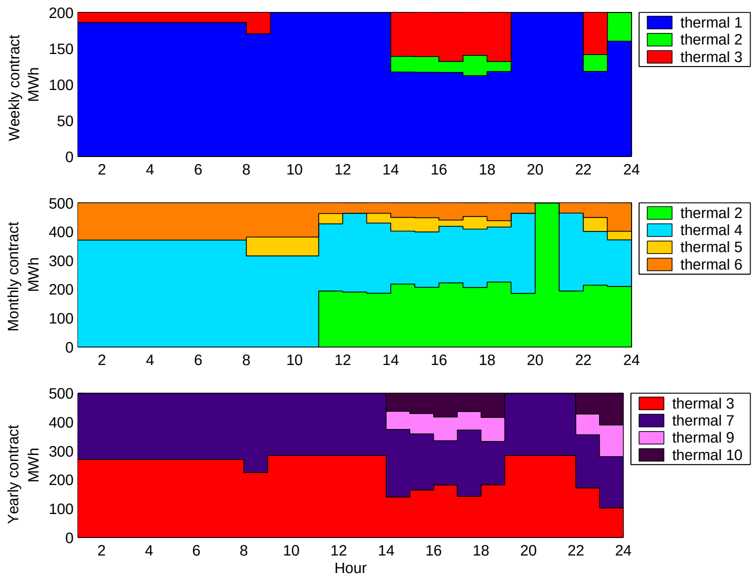

Figure 11: Economic dispatch of each futures contracts

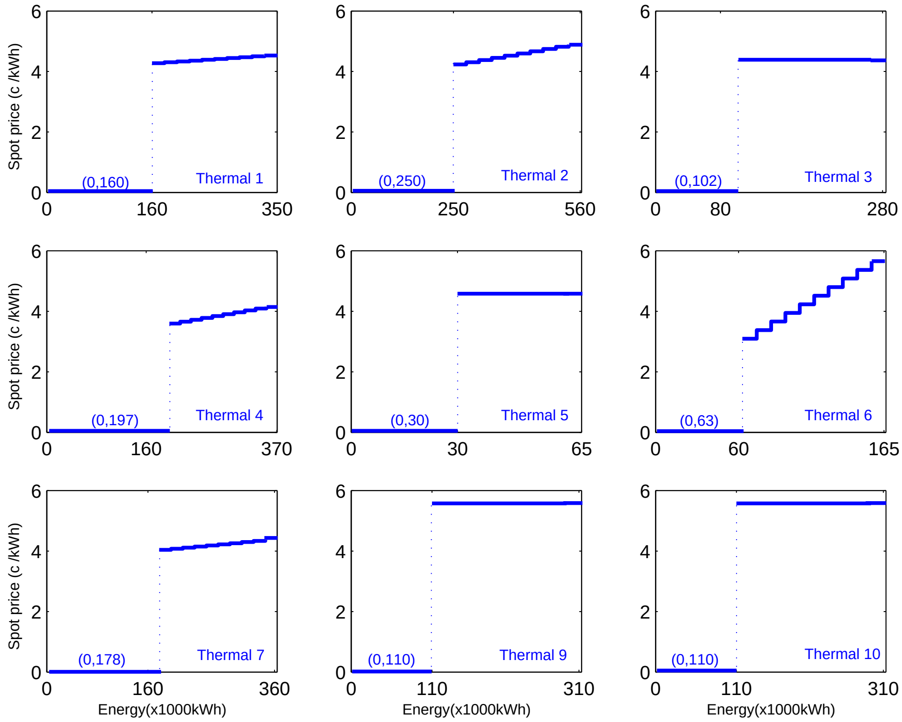

Figure 12: Bidding curve for hour 12

7 Glossary of symbols

a_{it} binary variable, fix the state of unit t during interval i

B^{df} Day-Ahead and Futures Market benefit function

$B_{it}(p_{it})$ Day-Ahead Market benefit function of unit t at interval i

C_i generation costs at interval i

c_t^b cost function intercept of unit t

c_t^l cost function lineal coefficient of unit t

c_t^{off} shut-down cost of unit t

c_t^{on} start-up cost of unit t

c_t^q cost function quadratic coefficient of unit t

e_{it} binary variables, fix the state of the unit t during interval i

F set of futures contracts

F_t set of futures contracts in which unit t participates

g_{itj} energy of contract j allocated to unit t at interval i

I set of intervals

i (subindex) indication of i^{th} interval

I_i^d Day-Ahead Market incomes

I_i^{df} Day-Ahead and Futures Market incomes

I_i^f Futures Market incomes

j (subindex) indication of j^{th} contract

L_j amount of energy of contract j

λ_{ij} Lagrangian multipliers of the set of constraints (32)

λ_i^d Day-Ahead Market price at interval i

$\lambda^{d,s}$ $= \{\lambda_1^{ds}, \dots, \lambda_I^{ds}\}$ Day-Ahead Market price scenario s

λ_j^f price of futures contract j

$\lambda_{ti}^o(p_{ti}^o)$ optimal offer curve

$\lambda_{ti}^o(p_{ti}^o; q_{it})$ optimal offer curve with physical futures contracts

$\tilde{\lambda}_{ti}^o$ threshold price

min_t^{off} operational minimum shut-down time of unit t

min_t^{on} operational minimum start-up time of unit t

$\underline{\mu}_{it}$ Lagrangian multipliers of the set of constraints (35)

μ_{itj}^f Lagrangian multipliers of the set of constraints (37)

μ_{it}^{fq} Lagrangian multipliers of the set of constraints (33)

$\mu_{it}^{pq,s}$ Lagrangian multipliers of the set of constraints (36)

$\overline{\mu}_{it}^s$ Lagrangian multipliers of the set of constraints (34)

P^s $= P(\lambda^{ds})$ probability of scenario s

$\underline{P}_t$ operational minimum limit of unit t

$\overline{P}_t$ operational maximum limit of unit t

p_{it} energy for free-bidding in the Day-Ahead Market for unit t at interval i

p_{it}^d matched energy at Day-Ahead Market of unit t at interval i

$p_{it}^{d,s}$ matched energy at Day-Ahead Market of unit t at interval i scenario s

p_{it}^{df} matched energy at Day-Ahead Market including futures contracts of unit t at interval i

$p_{it}^{df,s}$ matched energy at Day-Ahead Market including futures contracts of unit t at interval i scenario s

p_{it}^s energy for free-bidding in the Day-Ahead Market for unit t at interval i scenario s

p^o_{it} energy bided to the Day-Ahead Market for unit t at interval i

$p^*_{it}(\lambda^d_i)$ marginal offer curve for unit t at interval i

q_{it} energy of futures contracts allocated to unit t at interval i

S set of Day-Ahead Market price scenarios

s (subindex) indication of s^{th} price scenario

T set of thermal units

t (subindex) indication of t^{th} unit

T_j set of thermal units that participates in contract j

T_{on_i} set of committed units at interval i

u^t_i binary variables, fix the state of the unit t, $\forall i \in I^2, \forall t \in T$

Units: costs and prices are in €/MWh and energy terms in MW.

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