Optimal Bidding Strategies for Thermal and Generic Programming Units in the Day-ahead Electricity Market

F.-Javier Heredia, Marcos Rider and Cristina Corchero

Dept. of Statistics and Operations Research
Universitat Politècnica de Catalunya, Barcelona (Spain)

DR 2008/13
17 December 2008

Copies of this report may be downloaded at `http://hdl.handle.net/2117/2468`

Corresponding author: F. Javier Heredia
Dept. Statistics and Operations Research,
Technical University of Catalonia, Campus Nord
c. Jordi Girona 1-3, 08034 Barcelona,
tel. (0034) 934017335 fax. (0034) 934015855
email: `f.javier.heredia@upc.edu`

Optimal Bidding Strategies for Thermal and Generic Programming Units in the Day-ahead Electricity Market

F. Javier Heredia, Marcos J. Rider, *Member, IEEE,* and Cristina Corchero.

***Abstract*—This paper develops a stochastic programming model that integrates the day-ahead optimal bidding problem with the most recent regulation rules of the Iberian Electricity Market (MIBEL) for bilateral contracts, with a special consideration for the new mechanism to balance the competition of the production market, namely virtual power plants auctions (VPP). The model allows a price-taker generation company to decide the unit commitment of the thermal units, the economic dispatch of the bilateral contracts between the thermal units and the generic programming unit (GPU) and the optimal sale/purchase bids for all units (thermal and generic) observing the MIBEL regulation. The uncertainty of the spot prices is represented through scenario sets built from the most recent real data using scenario reduction techniques. The model was solved with real data from a Spanish generation company and spot prices, and the results are reported and analyzed.**

***Index Terms*—Short-term electricity generation planning, virtual power plants auctions, bilateral contracts, electricity spot market, optimal bidding strategies, stochastic programming.**

NOTATION

The notation used throughout the paper is reproduced below for quick reference.

Sets:

$\mathcal{I}$	Set of intervals.
$\mathcal{S}$	Set of scenarios.
$\mathcal{B}$	Set of bilateral contracts.
$\mathcal{T}$	Set of thermal generation units.
$\mathcal{M}_i^S$	Set of scenarios with conditioned accepted GPU's sale bid.
$\mathcal{M}_i^P$	Set of scenarios with conditioned accepted GPU's purchase bid.
$\mathcal{U}^T$	Set of initial condition of unit commitment binary variables.

Constants:

P^s	Probability of scenario s.
c_t^b	Base procurement cost of unit t (€).
c_t^l	Linear procurement cost of unit t (€/MWh).
c_t^q	Quadratic procurement cost of unit t (€/MWh^2).
$p_{it}^{D,s}$	bilateral-free day-ahead matched energy, unit t, interval i, scenario s (MW).
L_{ij}^B	Energy of the bilateral contract j at interval i (MW).
λ_{ij}^B	Unit profit of the bilateral contract j at interval i (€/MWh).
λ^S	Unit profit of the sale bilateral contract after the day-ahead market (€/MWh).
$\overline{b}^S$	Maximum energy that can be sold through the bilateral contract after the day-ahead market (MWh).
λ^P	Unit cost of the purchase bilateral contract after the day-ahead market (€/MWh).
$\overline{b}^P$	Maximum energy that can be purchased through the bilateral contract after the day-ahead market (MWh).
λ^V	Virtual power plant exercise price (€/MWh).
$\overline{p}^V$	Capacity of the virtual power plant (MW).
$\overline{p}_t$	Maximum generation of unit t (MW).
$\underline{p}_t$	Minimum generation of unit t (MW).
c_t^{on}	Start-up cost of unit t (€).
c_t^{off}	Shut-down cost of unit t (€).
st_t^0	Initial state of unit t (Hours).
t_t^{on}	Operational minimum in service time of unit t (Hours).
t_t^{off}	Operational minimum idle time of unit t (Hours).
$\lambda_i^{D,s}$	Day-ahead (spot) market price at interval i, scenario s (€/MWh).

Functions:

λ_{it}^O	Optimal sale bid function of unit t at interval i (€/MWh).
$B^{T,s}$	bilateral-free benefit function

First stage continuous variables:

b_{itj}^T	Generation of unit t at interval i allocated to the bilateral contract j (MW).
p_i^V	Virtual power plant capacity used at interval i (MW).
b_{ij}^G	Generic programming unit's generation at interval i allocated to the bilateral contract j (MW).
w_i^S	Auxiliary variable used in the definition of the sale matched energy of the generic programming unit.
w_i^P	Auxiliary variable used in the definition of the purchase matched energy of the generic programming unit.
w_i^R	Auxiliary variable used in the definition of the residual purchase matched energy of the generic programming unit.

First stage binary variables:

This work was supported by the Ministry of Science and Technology of Spain through CICYT Project DPI2005-09117-C02-01.

F. J. Heredia, M. J. Rider and C. Corchero are with the Statistics and Operations Research Department, Universitat Politècnica de Catalunya, Edifici C5 Campus Nord, Jordi Girona 1-3, 08034, Barcelona, Spain. (e-mail: {f.javier.heredia, marcos.rider, cristina.corchero}@upc.edu).

Digital Object Identifier

x_i^V Equal to 1 if the VPP rights are exercised, 0 otherwise.

u_{it} Equal to 1 if the thermal unit t must be committed at interval i, 0 otherwise.

a_{it} Equal to 1 if the thermal unit t must be turned-on at interval i, 0 otherwise.

e_{it} Equal to 1 if the thermal unit t must be shut-down at interval i, 0 otherwise.

y_i^S Auxiliary variable used in the definition of the sale matched energy of the generic programming unit.

y_i^P Auxiliary variable used in the definition of the purchase matched energy of the generic programming unit.

y_i^R Auxiliary variable used in the definition of the residual purchase matched energy of the generic programming unit.

Second stage continuous variables:

v_{it}^s Auxiliary variables used in the definition of the matched energy of the thermal units at period i, scenario s.

$b_i^{S,s}$ Sale bilateral contract after the day-ahead market at interval i and scenario s (MW).

$b_i^{P,s}$ Purchase bilateral contract after the day-ahead market at interval i and scenario s (MW).

p_{it}^s Total thermal generation of unit t at interval i, scenario s (MW).

$p_{it}^{T,s}$ Matched energy of thermal unit t at interval i, scenario s (MW).

$p_i^{S,s}$ Sale matched energy of the generic programming unit at interval i, scenario s (MW).

$p_i^{P,s}$ Purchase matched energy of the generic programming unit at interval i and scenario s (MW).

$p_i^{R,s}$ Mandatory accepting-price purchase bid of the generic programming unit at interval i, scenario s (MW).

Second stage binary variables:

z_{it}^s Auxiliary variables used in the definition of the matched energy of the thermal units at interval i, scenario s.

I. Introduction

THE new rules of the electrical energy production market operation of the Iberian Electricity Market MIBEL (mainland Spanish and Portuguese systems), for the daily and intradaily market from June 2007 [1], introduces new mechanisms to encourage the competition of the production market (physical futures contracts, bilateral contracts and virtual power plants capacity), and brings new challenges in the modelling and optimization of the market operation.

Aiming to increase the proportion of electricity that is purchased through bilateral contracts with a duration of several months and intending to stimulate liquidity in forward electricity markets, the Royal Decree 1634/2006, dated December 29th, 2006 [3] imposes on Endesa and Iberdrola (the two dominant utility companies in the Spanish electricity market) to hold a series of five auctions offering virtual power plant (VPP) capacity to any party who is a member of the MIBEL.

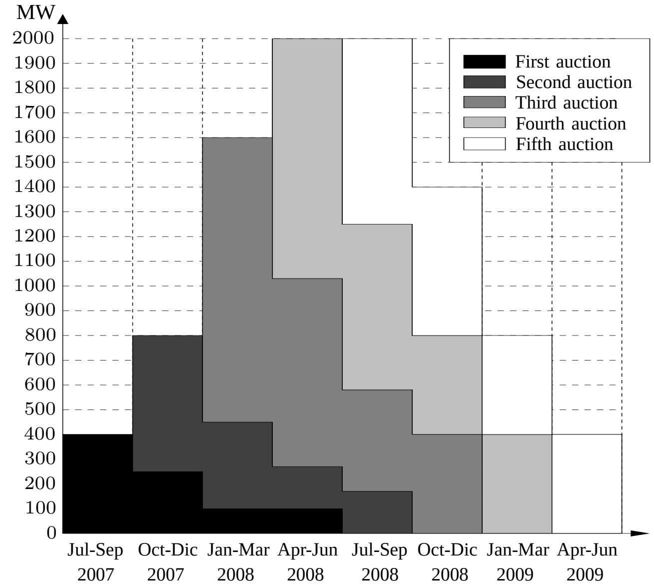

Figure 1. Five auctions of the VPP capacity of the Spanish peninsular electricity market

Both Endesa and Iberdrola have a diversified portfolio of contracts and power plants, increasing the security of their supply and reducing their vulnerability to price uncertainties. In 2006, the total installed capacity of both companies was around 47 GW, being that the total installed capacity of the Spanish electricity system was 78.3 GW. Fig. 1 shows the volumes to be auctioned by Endesa and Iberdrola accordingly with the Real Decree 1634/2006. Observe that the greatest volumes of auctioned VPP capacity will be reached from April to September 2008, with a total amount of 2000 MW [4].

Other experience of the application of VPP auctions can be found in France, where the Electricité de France (EDF) has made available, since September 2001, 5.4 GW of generation capacity in France to facilitate the liberalization of the French electricity market [5]. On July 4, 2003, the Belgian Competition Council approved various transactions leading to the appointment of Electrabel Customer Solutions, a subsidiary of Electrabel, as the default supplier for the customers of several intermunicipal distribution companies, subject to certain undertakings. As part of these undertakings, Electrabel has agreed to offer, to actual or potential competitors, up to a maximum of 1.2 GW of VPP capacity in Belgium [6]. On 19 September 2007, E.ON Sales & Trading GmbH (EST) offered 250 MW to the electricity market in Germany of the VPP product in a first auction. EST will consider conducting further auctions for one or more similar products on an annual basis [7].

In Spain, the VPP capacity means that the buyer of this product will have the capacity to generate MWh at his disposal. The buyer can exercise the right to produce against an exercise price, set in advance, by paying an option premium. So, although Endesa and Iberdrola still own the power plants, part of their production capacity will be at the disposal of the buyers of VPP. There will be baseload and peakload contracts with different strike prices that are defined a month before the

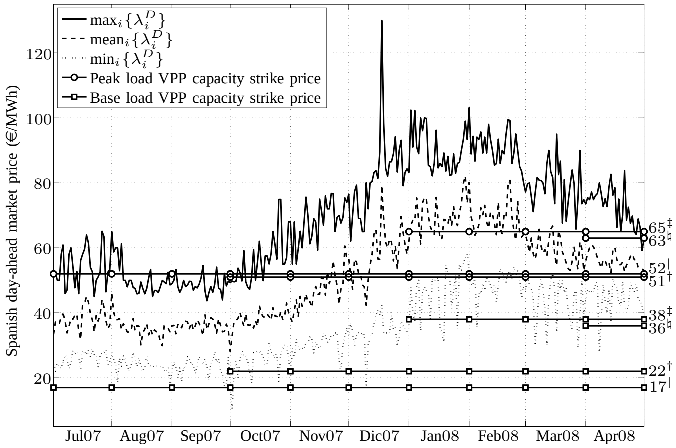

Figure 2. The Spanish day-ahead price market and strike price for the base and peak load VPP capacity (|First auction, †Second auction, ‡Third auction, ♮Fourth auction)

auction. In each case contracts with a duration of 3, 6 and 12 months will be offered. It is planned that all products will be offered simultaneously using an electronic auction. Fig. 2 shows the evolution of the day-ahead market price λ_i^D of day i (daily maximum, mean and minimum) from July 2007 to April 2008, and the three first pre-defined strike price for base and peak load VPP capacity, respectively. The energy resulting from the exercise of the VPP options can be used by buyers both to contribute to the covering of the national and international bilateral contracts prior to the day-ahead market or to sell it to the day-ahead market. In this last case, the unmatched VPP energy, if any, can be sold through national bilateral contracts after the day-ahead market. These new bilateral contracts after the day-ahead market are negotiated previously between the agents and must not be confused with the intradaily markets (see [1] and [8] for more information about the AGC and balancing intradaily markets).

A. Literature Review

The VPP capacity auctions attempt to reduce the influence of the dominant agents through financial tools in order to increase the competition in the market. This kind of regulation wants to converge in the perfect market which is integrated by all price-taker operators. Because of those reasons and the fact that it is very difficult to model the influence of a price-maker operator in the clearing price, the majority of the publications are focused on price-takers generation companies. General considerations about the bidding process in these electricity markets can be found in [9]–[11].

Several authors have proposed optimal bidding models in the day-ahead market for thermal units under the price-taker assumption, with or without bilateral contracts. The authors in [12] present a mixed integer programming model to optimize the production scheduling of a single unit with a simple bidding strategy. The approximation of the step-wise bidding curves by linear functions based on the marginal costs was already considered in [13], although in a context without bilateral contracts. In [14] the concept of *price-power function*, which is similar to the *matched energy function* defined in this paper, is used to derive the optimal offer curves of a hydro-thermal system under the assumption that the spot prices for the day-ahead and reserve markets behave as a Markov Chain. The mixed-integer stochastic programming model presented in [15] distinguishes between variables corresponding to *bid energy* and those representing the *matched energy*, although in a price-maker framework and without bilateral contracts. A model very related in some aspects to the one presented here is [16] where a stochastic unit commitment problem with bilateral contract is solved maximizing the day-ahead market benefit. Stochasticity in the spot prices is introduced through a set of scenarios, giving rise to a two-stage stochastic programming problem. In [17] the authors present a mixed integer stochastic optimization model for scheduling thermal units, the production plans are optimized in the presence of stochastic market clearing prices. Nevertheless the models in [16] and [17] did not propose any explicit modellization of the optimal bidding. To our knowledge, there are no publications which consider either the bilateral contracts after the day-ahead market or the modelization of the VPP.

B. Contributions

This paper develops a stochastic mixed-integer quadratic programming model for a price-taker generation company (GenCo) to find the optimal bidding strategy of a pool of thermal units and a VPP in the Spanish day-ahead electricity market under the most recent MIBEL regulation regarding the bilateral contracts rules. The energy of the VPP options is integrated in the production system through the so called *generic programming unit* which will be described in the next section. The model allows a price-taker generation company to decide the unit commitment of its thermal units, the economic dispatch of the bilateral contracts between the thermal and generic units, and the optimal bid for both thermal and generic units, observing the MIBEL regulation. The model was tested with real data from a Spanish generation company and spot market prices. It has been implemented with AMPL and solved with CPLEX.

The main contributions of this paper are:

- A new model for the optimal thermal bid function and matched energy which takes into account the presence of bilateral contratcs.
- The mathematical modelization of the generic programming unit and the VPP.
- The modelization of the optimal bid functions and matched energy of the generic programming unit.
- The inclusion in the optimization model of the bilateral contracts after the day-ahead market.
- The consideration of the most recent regulations of the MIBEL energy market.

This paper is organized as follows. Section II describes the MIBEL's energy production system around the day-ahead market and the relevance of the generic programming unit. In Section III the stochastic programming model for the optimal bidding strategy is developed. In Section IV the market price scenario generation procedure is described. In Section V a detailed case study is presented and solved with the proposed

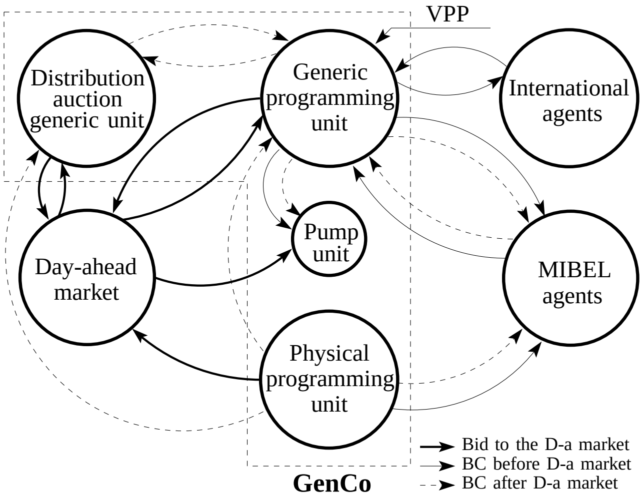

Figure 3. Generic programming unit bilateral contracts

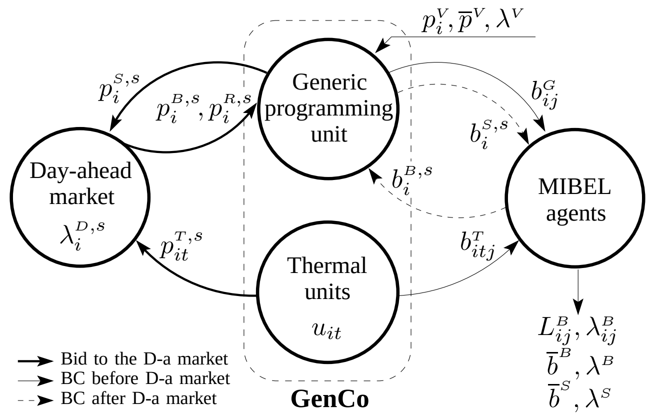

Figure 4. Case study

stochastic programming model, reporting and analyzing the computational results. Finally some relevant conclusions are drawn in Section VI.

II. MIBEL'S ENERGY PRODUCTION SYSTEM

Fig. 3 depicts the MIBEL's energy production system, focused on the GenCo's energy operation problem. Each arc represents an energy transaction between the GenCo's physical and generic programming units and the pool as well as the remaining MIBEL agents. There are four different kinds of transactions: exercised VPP energy (VPP arc), buying/selling bids to the pool (thick arcs), national and international bilateral contracts before the day-ahead market (thin arcs) and national bilateral contracts after the day-ahead market (dashed arcs). A GenCo operating in the MIBEL has to decide all the optimal energy allocations and bids pictured in Fig. 3, where the continuous arcs correspond to decisions to be taken prior to day D market clearing (9:35h and 10:00h of day D-1, respectively, for the BC before the D-a market and bids) and the dashed arcs correspond to decisions to be taken in a short period (typically half an hour) just after the day D market clearing.

A GenCo is represented in Fig. 3 by its physical production units (hydro, thermal, combined cyle, pumping) and two non-physical units: the *Distribution Auction Generic Unit* and the *Generic Programming Unit* (GPU). The distribution auction generic unit administers the bilateral contracts, under regulated tariffs, to the main distribution companies in Spain and Portugal. The regulated tariff and the amount of the bilateral contract are obtained by an auction. By law, a GenCo holding such a bilateral contract must send an accepting price purchase bid to the pool for the entire amount of the contract, and, therefore, there is no room for optimization. For more information on this kind of bilateral contract see [18]. The generic programming unit (GPU) administers the exercised energy of the VPP, and brings more flexibility to the GenCo operations in the MIBEL. With the GPU the utility can:

- Integrate the VPP exercised energy into the energy production system, both offering this energy to the pool through purchase bids or allocating it among the GenCo's portfolio of national and international bilateral contracts.
- Act as a purchase agent, both sending purchase bids to the pool and acquiring energy through national and international bilateral contracts.

After the market clearing the generation program of the GPU must be allocated among the GenCo's physical production units and bilateral contracts, in such a way that the net energy balance of the GPU must be zero. The existence of national bilateral contracts after the day-ahead market prevents violation of the aforementioned netting energy balance condition as a consequence of possible unmatched GPU's sale or purchase bids.

III. THE STOCHASTIC PROGRAMMING MODEL

Fig. 4 represents the part of the whole MIBEL energy production system (Fig. 3) considered in this study. This system will be modeled based on the following assumptions:

- The GenCo is a price-taker operating in the MIBEL a set $\mathcal{T}$ of thermal units (coal, nuclear, fuel) and a GPU.
- The thermal units in $\mathcal{T}$ have linear or convex quadratic generation cost function, constant start-up/shut-down costs and minimum generation/down time.
- The GPU is associated to a VPP with known capacity ($\overline{p}^V$ MWh) and exercise price (λ^V €/MWh).
- Both thermal units and GPU bid to the $i \in \mathcal{I} = \{1, 2, \ldots, 24\}$ hourly auctions of the day-ahead market. The stochasticity of the spot price λ_i^D, $i \in \mathcal{I}$ is represented by a set of $\mathcal{S}$ scenarios.
- There is a portfolio $\mathcal{B}$ of bilateral contracts duties before the day-ahead market with the rest of the MIBEL agents, with known energy (L_{ij}^B MWh) and price (λ_{ij}^B€/MWh).
- There is an agreement for selling (purchase) bilateral contracts after the day-ahead market up to a quantity $\overline{b}^S$ MWh ($\overline{b}^P$ MWh) at a price λ^S€/MWh (λ^P€/MWh). We assume that it is not possible to obtain net gain from those contracts ($\lambda^P > \lambda^S$).

The objective of this study is to find how to optimally manage the thermal units $\mathcal{T}$ and the GPU in order to take the maximum benefit from the day-ahead market (the pool) while covering the bilateral contracts agreements. This problem has been modelled in this work as a mixed integer quadratic two stage stochastic optimization problem. Among the complete list of variables of this model (see the Notation section), the main information provided by the model (*here and now* decisions or first stage variables) for each period $i \in \mathcal{I}$, are:

- For each thermal unit $t \in \mathcal{T}$ the unit commitment (u_{it}), the energy allocated to each bilateral contract b^T_{itj}, $\forall j \in \mathcal{B}$ and the optimal sale bids, expressed as a function of b^T_{itj} (see section III-C).
- For the generic programming unit, the exercised VPP energy (p^V_i), the energy allocated to the bilateral contracts before the day-ahead market (b^G_{ij}, $\forall j \in \mathcal{B}$) and the optimal sale/purchase bids, expressed in terms of b^G_{ij} and p^V_i (see section III-D).

A. Bilateral Contracts Constraints

The GenCo has agreed to physically provide the energy amounts L^B_{ij} at hour $i \in \mathcal{I}$ of day D for each one of the $j \in \mathcal{B}$ bilateral contracts with the rest of the MIBEL participants. This energy L^B_{ij} can be provided both by the real thermal units $\mathcal{T}$ and the virtual GPU:

$$\left.\begin{array}{l} \sum_{t\in\mathcal{T}} b^T_{itj} + b^G_{ij} = L^B_{ij} \\ b^T_{itj} \geq 0 \quad \forall t \in \mathcal{T} \\ b^G_{ij} \geq 0 \end{array}\right\} \begin{array}{l}\forall i \in \mathcal{I} \\ \forall j \in \mathcal{B}\end{array} \quad (1)$$

B. Thermal unit commitment

Following [19], Eq. (2) is used to formulate the minimum up and down times for thermal unit t

$$\left.\begin{array}{ll} u_{it} - u_{(i-1)t} - e_{it} + a_{it} = 0 & \text{(a)} \\ a_{it} + \sum_{j=i}^{\min\{i+t^{off}_t, |\mathcal{I}|\}} e_{it} \leq 1 & \text{(b)} \\ e_{it} + \sum_{j=i+1}^{\min\{i+t^{on}_t, |\mathcal{I}|\}} a_{it} \leq 1 & \text{(c)} \\ u_{it}, a_{it}, e_{it} \in \{0,1\} \cap \mathcal{U}^T & \end{array}\right\} \begin{array}{l}\forall i \in \mathcal{I} \\ \forall t \in \mathcal{T}\end{array} \quad (2)$$

where Eq. (2a) and (2b) define the auxiliary binary variables a_{it} and e_{it} to be $a_{it} = 1$ iff $u_{(i-1)t} = 1$ and $u_{it} = 0$, and $e_{it} = 1$ iff $u_{(i-1)t} = 0$ and $u_{it} = 1$. Then, the minimum in service (t^{on}_t) and iddle (t^{off}_t) times are guaranteed by Eq. (2b) and Eq. (2c) respectivley. $\mathcal{U}^T$ represents the value of the variables u_{it}, a_{it} and e_{it} set by the initial state of the thermal units.

C. Optimal thermal bidding model

In the MIBEL, a simple day-ahead sale bid consists of a step-wise non-decreasing curve defined with up to 10 price/power blocks. Similarly to [13], this step-wise sale bid will be approximated in our model through the *optimal thermal bid function* $\lambda^O_{it}(p^O_{it})$, a piece-wise discontinuous linear non-decreasing function that gives the value of the optimal bid price λ^O_{it} at which the thermal generation p^O_{it} would be bid at

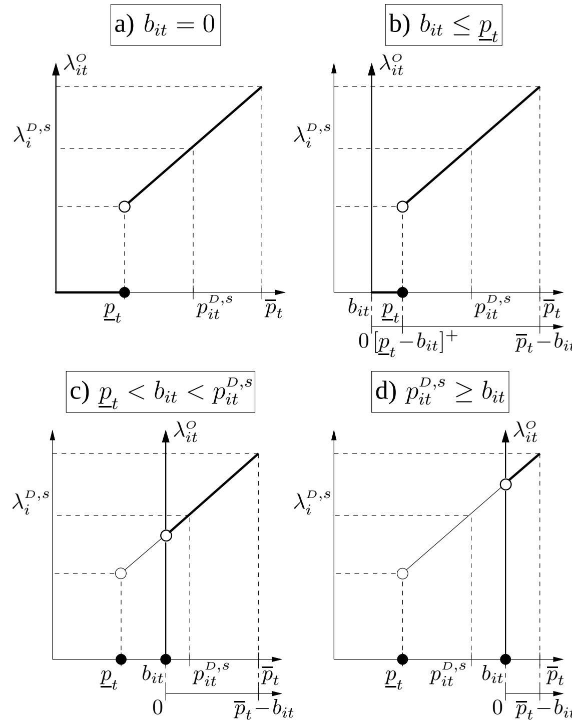

Figure 5. Representation of the optimal thermal bid function $\lambda^O_{it}(p^O_{it}, b^T_{it})$

the i-th day-ahead auction. It can be shown [20] that under the price-taker assumption and the MIBEL bid rules, the optimal thermal bid function, i.e., the bid function that maximizes the day-ahead benefit function for any given value b^T_{it}, regardless of the value of the clearing price, can be expressed as:

$$\begin{array}{l} \lambda^O_{it}(p^O_{it}, b^T_{it}) = \\ \begin{cases} 0 & \text{if } 0 \leq p^O_{it} \leq [\underline{p}_t - b^T_{it}]^+ \\ 2c^q_t(p^O_{it} + b^T_{it}) + c^l_t & \text{if } [\underline{p}_t - b^T_{it}]^+ < p^O_{it} \leq \overline{p}_t - b^T_{it} \end{cases} \\ \forall i \in \mathcal{I}, \ \forall t \in \mathcal{T} \end{array} \quad (3)$$

where $[a-b]^+ = \max\{0, a-b\}$ and variable b^T_{it}, the total energy production of unit t assigned to the whole portfolio of bilateral contracts, is defined as:

$$b^T_{it} = \sum_{j\in\mathcal{B}} b^T_{itj} \quad \forall t \in \mathcal{T}, \forall i \in \mathcal{I} \quad (4)$$

Expression (3) can be interpreted with the help of Fig. 5 which represents the optimal thermal bid function (thick line) corresponding to four representative values of the bilateral contracts energy b^T_{it}.

- **Case a)** this is the case when $b^T_{it} = 0$ (the committed thermal unit t doesn't contribute to the bilateral contract covering) and coincides with the classical self-commitment problem treated by several authors ([13], [12]). In this case, it is well known that the optimal bid strategy for a price-taker GenCo is to bid at the true marginal cost of the unit. Assuming a quadratic thermal generation cost $C^T(p^O_{it}) = c^b_t + c^l_t p^O_{it} + c^q_t (p^O_{it})^2$, then the optimal bidding policy consists of an instrumental ($\lambda^O_{it} = 0$) sale bid up to the operational minimum limit $\underline{p}_t$, to guarantee their acceptance, and the rest of the plant capacity at the marginal price $2c_l^q p_{it}^O + c_t^l$, the slope of the cost function $C^T(p_{it}^O)$. If this sale bid is submitted to the pool, the *bilateral-free day-ahead matched energy under scenario* s, $p_{it}^{D,s}$ will be (see Fig. 5a):

$$p_{it}^{D,s} = \begin{cases} \underline{p}_t & \text{if } p_{it}^{*,s} \le \underline{p}_t \\ \overline{p}_t & \text{if } p_{it}^{*,s} \ge \overline{p}_t \\ p_{it}^{*,s} & \text{otherwise} \end{cases} \quad \begin{matrix} \forall t \in \mathcal{T} \\ \forall i \in \mathcal{I} \\ \forall s \in \mathcal{S} \end{matrix} \tag{5}$$

where $p_{it}^{*,s} = \left(\lambda_i^{D,s} - c_t^l\right)/2c_t^q$ is the unconstrained maximum of the benefit function

$$B^{T,s}(p_{it}^O) = \lambda_i^{D,s} p_{it}^O - C^T(p_{it}^O) \tag{6}$$

for a given thermal t, period i and scenario s. Please note that $p_{it}^{D,s}$ are constant parameters of the model.

- **Cases b) and c)** in both cases the energy b_{it}^T to be allocated to the bilateral contracts is below the bilateral-free day-ahead matched energy $p_{it}^{D,s}$, but strictly positive. The MIBEL rules exclude this allocated energy b_{it}^T from the sale bid of the thermal unit, giving rise to the optimal bid curve associated with the second coordinate system of Fig. 5 b) and c) (thick line), starting at a value b_{it}^T of the original x-axis. In both cases the matched energy will be the difference $p_{it}^{D,s} - b_{it}^T$
- **Case d)** In this last case the allocated energy b_{it}^T exceeds the quantity $p_{it}^{D,s}$. Looking at the optimal bid curve it can be observed that the minimum price asked from the market $\lambda_{it}^O(0, b_{it}^T) = 2c_t^q b_{it}^T + c_t^l$ is greater than the represented spot price $\lambda_i^{D,s}$ and, consequently, the sale bid will remain unmatched.

The *matched energy function under scenario* s $p_{it}^{T,s}$ associated with the optimal thermal bidding function (3) (also called *price-power function* in [14]) will be:

$$p_{it}^{T,s}(b_{it}^T, u_{it}) = \begin{cases} [p_{it}^{D,s} - b_{it}^T]^+ & \text{if } u_{it} = 1 \\ 0 & \text{if } u_{it} = 0 \end{cases} \quad \begin{matrix} \forall i \in \mathcal{I} \\ \forall t \in \mathcal{T} \\ \forall s \in \mathcal{S} \end{matrix} \tag{7}$$

Fig. 6 represents the function $p_{it}^{T,s}(b_{it}^T, u_{it})$ (thick line), for a fixed value of the spot price $\lambda_i^{D,s}$. With the help of the auxiliary variables z_{it}^s (binary) and v_{it}^s (continuous) (see Fig. 6) the non-differentiable expression (7) can be shown to be equivalent to the following mixed-integer linear system [20]:

$$\left.\begin{array}{l} p_{it}^{T,s} = p_{it}^{D,s} u_{it} + v_{it}^s - b_{it}^T \\ p_{it}^{D,s}(z_{it}^s + u_{it} - 1) \le b_{it}^T \\ b_{it}^T \le p_{it}^{D,s}(1 - z_{it}^s) + \overline{p}_t(z_{it}^s + u_{it} - 1) \\ 0 \le p_{it}^{T,s} \le p_{it}^{D,s}(1 - z_{it}^s) \le p_{it}^{D,s} u_{it} \\ 0 \le v_{it}^s \le (\overline{p}_t - p_{it}^{D,s})(z_{it}^s + u_{it} - 1) \\ b_{it}^T \in [0, \overline{p}_t] \\ z_{it}^s \in \{0, 1\} \end{array}\right\} \begin{matrix} \forall i \in \mathcal{I} \\ \forall t \in \mathcal{T} \\ \forall s \in \mathcal{S} \end{matrix} \tag{8}$$

Finally, we define the second stage variables p_{it}^s that represent the total generation of thermal unit t at period i conditioned to scenario s, expressed as:

$$p_{it}^s = p_{it}^{T,s} + b_{it}^T \tag{9}$$

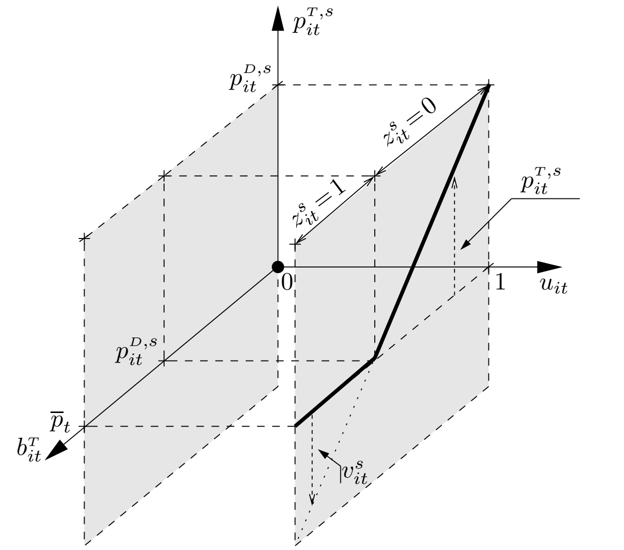

Figure 6. The thermal unit matched energy function $p_{it}^{T,s}$ for a fixed spot price $\lambda_i^{D,s}$

D. Optimal Generic Programming Unit bidding model

In this section the optimal bidding and the matched energy functions for a GPU will be derived. First, variable b_i^G will represent the total contribution of the GPU to the coverage of the bilateral contracts before the day-ahead market:

$$b_i^G = \sum_{j \in \mathcal{B}} b_{ij}^G \quad \forall i \in \mathcal{I} \tag{10}$$

Second, we assume that p_i^V, the exercised energy of the VPP, depends on the value of the binary variable x_i^V as follows:

$$p_i^V = \overline{p}^V x_i^V \quad \forall i \in \mathcal{I} \tag{11}$$

Under this assumption, the expression of the optimal GPU bid function can be developed analyzing the two cases $x_i^V = 0$ and $x_i^V = 1$:

- $x_i^V = 0$: VPP rights are not exercised, and then, the energy b_i^G must be either acquired to the pool or provided by the bilateral contracts after the day-ahead market at an agreed price λ^P, which is the maximum price we were willing to pay to the pool for that amount of energy. Therefore the optimal purchase bid (*energy*, *price*) pair is:

$$(b_i^G, \lambda^P) \quad \text{if} \quad x_i^V = 0 \tag{12}$$

- $x_i^V = 1$: the VPP rights have been exercised and the exercise price has been paid. Then, two different situations must be considered:
  - $b_i^G \le \overline{p}_i^V$: after covering the energy b_i^G with the VPP, there is an energy surplus of $[\overline{p}_i^V - b_i^G]$ that can be sold either to the pool, at unknown spot price λ_i^D, or to the bilateral contracts after the day-ahead market, at known sale price λ^S. Then, the energy surplus should be offered to the pool at a price not less than λ^S, being the optimal sale bid:

$$([\overline{p}_i^V - b_i^G], \lambda^S) \quad \text{if} \quad x_i^V = 1 \text{ and } b_i^G \le \overline{p}_i^V \tag{13}$$

– $b_i^G > \overline{p}_i^V$: analogously to the case $x_i^V = 0$, in order to fulfill the uncovered part of the bilateral contracts duty the following optimal purchase bid must be submitted:

$$([b_i^G - \overline{p}_i^V], \lambda^P) \quad \text{if} \quad x_i^V = 1 \text{ and } b_i^G > \overline{p}_i^V \qquad (14)$$

As a result of the precedent analysis, the *optimal sale and purchase bid for the GPU* (Eq. (12-14)) can be expressed in the following compact form:

$$\begin{aligned} \text{OSB}_i &= \left([p_i^V - b_i^G]^+, \lambda^S\right) && (15)\\ \text{OPB}_i &= \left([b_i^G - \overline{p}^V]^+ + \min\{b_i^G, \overline{p}^V - p_i^V\}, \lambda^P\right) && (16)\end{aligned}$$

It can be easily verified that for any given value of the first stage variables b_i^G and p_i^V, Eq. (15-16) correspond to the optimal bidding rules developed in Eq. (12-14). Eq. (15-16) can be used to derive the expressions of the matched energy at each scenario $s \in \mathcal{S}$, as functions of the first stage variables p_i^V and b_i^G. First, consider the two following sets of scenarios:

$$\begin{aligned} \mathcal{M}_i^S &:= \left\{ s \in \mathcal{S} \mid \lambda_i^{D,s} \geq \lambda^S \right\} \\ \mathcal{M}_i^P &:= \left\{ s \in \mathcal{S} \mid \lambda_i^{D,s} < \lambda^P \right\} \end{aligned} \qquad \forall i \in \mathcal{I} \qquad (17)$$

The set $\mathcal{M}_i^S$ includes those scenarios where, at the i-th day-ahead auction, the optimal sale bid (15), if any, will be accepted. Then, after Eq. (15), the *matched sale energy function* will be:

$$p_i^{S,s}(b_i^G, p_i^V) = \begin{cases} [p_i^V - b_i^G]^+ & \text{if } s \in \mathcal{M}_i^S \quad (a) \\ 0 & \text{if } s \notin \mathcal{M}_i^S \quad (b) \end{cases}$$
$$\forall i \in \mathcal{I}\,,\ \forall s \in \mathcal{S} \qquad (18)$$

Analogously the set $\mathcal{M}_i^P$ includes those scenarios where, at the i-th day-ahead auction, the optimal purchase bid (16), if any, will be accepted. For convenience, the two terms of the total matched purchase energy of Eq. (16) will be represented by two separate matched functions, the *matched purchase energy function*

$$p_i^{P,s}(b_i^G, p_i^V) = \begin{cases} \min\{b_i^G, \overline{p}^V - p_i^V\} & \text{if } s \in \mathcal{M}_i^P \quad (a) \\ 0 & \text{if } s \notin \mathcal{M}_i^P \quad (b) \end{cases}$$
$$\forall i \in \mathcal{I}\,,\ \forall s \in \mathcal{S} \qquad (19)$$

and the *residual matched purchase energy function*

$$p_i^{R,s}(b_i^G) = \begin{cases} [b_i^G - \overline{p}^V]^+ & \text{if } s \in \mathcal{M}_i^P \quad (a) \\ 0 & \text{if } s \notin \mathcal{M}_i^P \quad (b) \end{cases}$$
$$\forall i \in \mathcal{I}\,,\ \forall s \in \mathcal{S} \qquad (20)$$

Observing Eq. (18,19,20) it becomes evident that actually, the value of the matched sale energy will be the same for any scenario in $\mathcal{M}_i^S$, and the same happens with the matched purchase energies and the scenarios in $\mathcal{M}_i^P$. Nevertheless, the supraindex "s" will be conserved for the sake of clarity and to strengthen the fact that these are actually second-stage variables, as there will be scenarios with non-zero matched energies while in others those energies will be zero. Another issue to mention is that, as we are assuming that $\lambda^S < \lambda^P$, the intersection set:

$$\mathcal{M}^{SP} := \mathcal{M}_i^S \cap \mathcal{M}_i^P = \{ s \in \mathcal{S} \mid \lambda_i^{D,s} \in [\lambda^S, \lambda^P] \} \qquad (21)$$

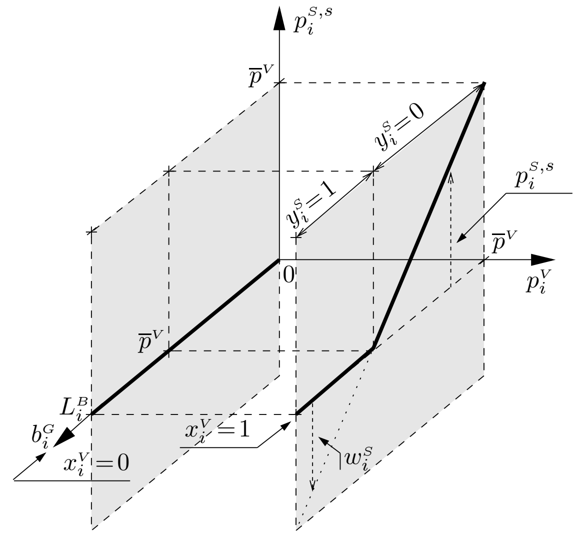

Figure 7. The GPU matched sale energy function (18a) for $s \in \mathcal{M}_i^S$.

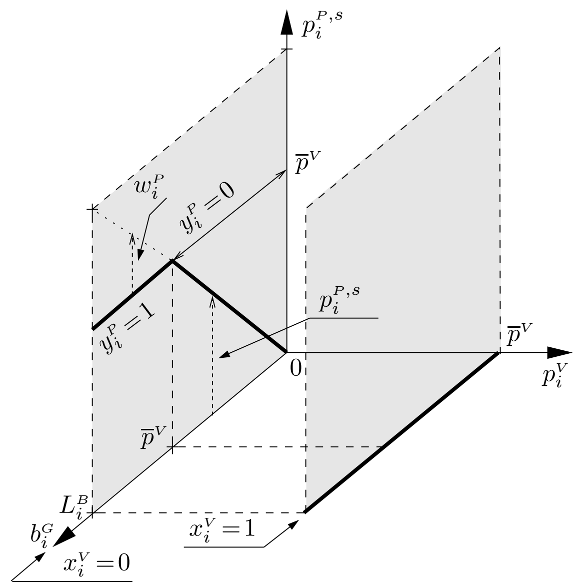

Figure 8. The GPU matched purchase energy function (19a) for $s \in \mathcal{M}_i^P$.

could be nonempty. This fact doesn't reveal any inconsistency of the model, because Eq. (18,19,20) are formulated in a way that, for any $s \in \mathcal{M}^{SP}$, only the matched sale energy $p_i^{S,s}$ or the total matched purchase energy $p_i^{P,s} + p_i^{R,s}$ can be greater than zero, but never both simultaneously. Then, for those scenarios in $\mathcal{M}^{SP}$, only a sale bid or a purchase bid will be submitted, depending on the value of the variables b_i^G and p_i^V.

The non-differential functions (18,19,20) can be conveniently incorporated into the optimization model through an equivalent mixed-linear modelization. Eq. (18), which expresses the matched sale energy $p_i^{S,s}$ as a function of variables p_i^V and b_i^G (see Fig. 7 for a graphical representation of this function) can be incorporated into the optimization model through the equivalent set of linear constraints (22), using the auxiliary variables w_i^S (continuous) and y_i^S (binary):

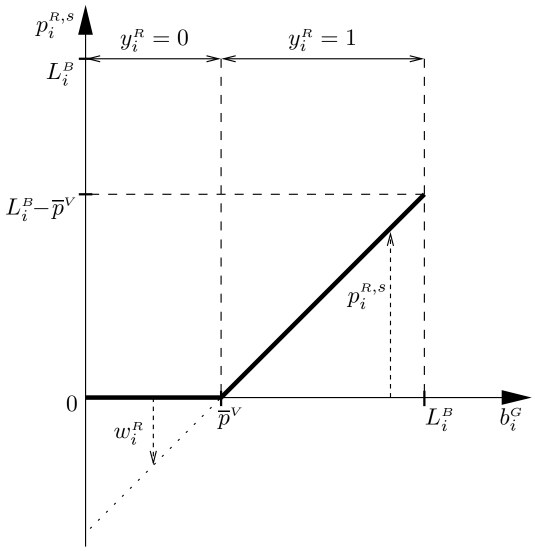

Figure 9. The GPU residual matched purchase energy function (20a) for $s \in \mathcal{M}_i^P$.

$$\left.\begin{array}{ll} p_i^{S,s}=0 & \forall s \notin \mathcal{M}_i^S \\ p_i^{S,s}=p_i^V+w_i^S-b_i^G & \forall s \in \mathcal{M}_i^S \\ 0 \le p_i^{S,s} \le \overline{p}^V(1-y_i^S) \le p_i^V & \forall s \in \mathcal{M}_i^S \\ \overline{p}^V(y_i^S-1)+p_i^V \le b_i^G & \\ b_i^G \le \overline{p}^V(1-y_i^S)+L_i^B y_i^S & \\ 0 \le w_i^S \le (L_i^B-\overline{p}^V)y_i^S+\overline{p}^V-p_i^V & \\ y_i^S \in \{0,1\} & \end{array}\right\} \forall i \in \mathcal{I} \quad (22)$$

where

$$L_i^B=\sum_{\forall j \in \mathcal{B}} L_{ij}^B \quad \forall i \in \mathcal{I} \quad (23)$$

Analogously, the matched purchase energy function (19), represented graphically in Fig. 8, can be formulated as the system of linear constraints (24), with the help of the auxiliary variables w_i^P (continuous) and y_i^P (binary).

$$\left.\begin{array}{ll} p_i^{P,s}=0 & \forall s \notin \mathcal{M}_i^P \\ p_i^{P,s}=b_i^G-w_i^P & \forall s \in \mathcal{M}_i^P \\ \overline{p}^V y_i^P \le p_i^{P,s} \le \overline{p}^V-p_i^V & \forall s \in \mathcal{M}_i^P \\ \overline{p}^V y_i^P \le b_i^G & \\ b_i^G \le \overline{p}^V(1-y_i^P)+L_i^B(y_i^P+x_i^V)-p_i^V & \\ 0 \le w_i^P \le (L_i^B-\overline{p}^V)y_i^P+L_i^B x_i^V & \\ y_i^P \in \{0,1\} & \end{array}\right\} \forall i \in \mathcal{I} \quad (24)$$

Finally, the residual matched purchase energy function (20), represented in Fig. 9, is introduced in the model through the following set of linear constraints:

$$\left.\begin{array}{ll} p_i^{R,s}=0 & \forall s \notin \mathcal{M}_i^P \\ p_i^{R,s}=b_i^G+w_i^R-\overline{p}^V & \forall s \in \mathcal{M}_i^P \\ 0 \le p_i^{R,s} \le L_i^B y_i^R & \forall s \in \mathcal{M}_i^P \\ \overline{p}^V y_i^R \le b_i^G & \\ b_i^G \le \overline{p}^V(1-y_i^R)+L_i^B y_i^R & \\ 0 \le w_i^R \le \overline{p}^V(1-y_i^R) & \\ y_i^R \in \{0,1\} & \end{array}\right\} \forall i \in \mathcal{I} \quad (25)$$

where, once again, w_i^P (continuous) and y_i^P (binary) are introduced as auxiliary variables.

E. GPU's net energy balance

Any GPU operating in the MIBEL must satisfy at each hour $i \in \mathcal{I}$ that the net energy balance of the GPU must be zero, with the help, if necessary, of the bilateral contracts after the day-ahead market (see section III). Following this rule we assume that, for each scenario $s \in \mathcal{S}$, energies $b_i^{P,s}$ and $b_i^{S,s}$ are purchased and sold through these new bilateral contracts up to a given maximum quantity at known prices λ^P and λ^S (remember that $\lambda^S < \lambda^P$) respectively. Then the GPU's net energy balance constraints for each hour i and scenario s are:

$$\left.\begin{array}{l} p_i^V+p_i^{P,s}+p_i^{R,s}+b_i^{P,s}=p_i^{S,s}+b_i^{S,s}+b_i^G \\ 0 \le b_i^{P,s} \le \overline{b}^P \\ 0 \le b_i^{S,s} \le \overline{b}^S \end{array}\right\} \begin{array}{l}\forall s \in \mathcal{S} \\ \forall i \in \mathcal{I}\end{array} \quad (26)$$

F. Objetive function

The expected value of the benefit function B can be expressed as:

$$\mathrm{E}_{\lambda^D}\left[B(u,a,e,p,p^T,p^V,p^S,p^P,p^R,b^S,b^P;\lambda^D)\right]=$$
$$\sum_{\forall i \in \mathcal{I}}\sum_{\forall j \in \mathcal{B}} \lambda_{ij}^B L_{ij}^B \quad (27)$$
$$-\sum_{\forall i \in \mathcal{I}}\sum_{\forall t \in \mathcal{T}}\left[c_t^{on}e_{it}+c_t^{off}a_{it}+c_t^b u_{it}\right]-\sum_{\forall i \in \mathcal{I}}\lambda^V p_i^V \quad (28)$$
$$+\sum_{\forall i \in \mathcal{I}}\sum_{\forall t \in \mathcal{T}}\sum_{\forall s \in \mathcal{S}} P^s\left[\lambda_i^{D,s}p_{it}^{T,s}-c_t^l p_{it}^s-c_t^q(p_{it}^s)^2\right] \quad (29)$$
$$+\sum_{\forall i \in \mathcal{I}}\sum_{\forall s \in \mathcal{S}} P^s\left[\lambda_i^{D,s}(p_i^{S,s}-p_i^{P,s}-p_i^{R,s})\right] \quad (30)$$
$$+\sum_{\forall i \in \mathcal{I}}\sum_{\forall s \in \mathcal{S}} P^s\left[\lambda^S b_i^{S,s}-\lambda^P b_i^{P,s}\right] \quad (31)$$

The term (27) represents the total income of the bilateral contracts before the day-ahead market (constant) and can be ignored in the optimization. The term (28) does not depend on the realization of the random variable $\lambda_i^{D,s}$, and corresponds to the on/off fixed cost of the unit commitment and the exercise cost of the VPP energy. The expressions (29-31) are respectively the expected value of the benefit coming from the day-ahead market's bids of the thermal units (29) , from the day-ahead market's bids of the generic programming unit (30) and from the bilateral contracts after the day-ahead market (31). All the functions appearing in (28-31) are linear excepting the generation costs of the thermal units (29), which are concave quadratic ($c_t^q \ge 0$, see Table II).

G. Final model

The final model developped in the previous sections is:

$$\begin{cases} \max \mathrm{E}_{\lambda^D}\left[B(u,a,e,p,p^T,p^V,p^S,p^P,p^R,b^S,b^P;\lambda^D)\right] \\ \text{s.t. :} \\ \quad \text{Eq. } (1) \quad \text{Bilateral contracts } \mathcal{B} \text{ covering} \\ \quad \text{Eq. } (2) \quad \text{Unit commitment const.} \\ \quad \text{Eq. } (4) \quad \text{Thermal's } b^T_{it} \text{ def.} \\ \quad \text{Eq. } (8) \quad \text{Thermal's matched energy } p^{T,s}_{it} \text{ def.} \\ \quad \text{Eq. } (9) \quad \text{Thermal's total generation } p^s_{it} \text{ def.} \\ \quad \text{Eq. } (10) \quad \text{GPU's } b^G_i \text{ def.} \\ \quad \text{Eq. } (11) \quad \text{VPP's energy nomination } p^V_i \text{ def.} \\ \quad \text{Eq. } (22) \quad \text{GPU's matched sale energy } p^{S,s}_i \text{ def.} \\ \quad \text{Eq. } (24) \quad \text{GPU's matched purchase energy } p^{P,s}_i \text{ def.} \\ \quad \text{Eq. } (25) \quad \text{GPU's residual matched energy } p^{R,s}_i \text{ def.} \\ \quad \text{Eq. } (26) \quad \text{GPU's net energy balance const.} \end{cases} \tag{32}$$

Taking into account the parameters and sets defined in Eqs. (5), (17) and (23). The resulting deterministic equivalent of the proposed two-stage stochastic problem is a mixed continuous-binary linearly constrained concave quadratic maximization problem that can be solved efficiently with the help of standard optimization software, as will be illustrated in Section V.

IV. The Market Price Scenario Generation

The two-stage stochastic model (32) requires a characterization of the market price through a set of scenarios, also known as *scenario fan*. Many scenario generation methods are available, see [23] or [24] for a review of them.

The creation of new bilateral contracts and the application of VPP auctions started up at June 2007. As the behavior of the prices depends on the market rules, a complete set of 261 equiprobable scenarios has been obtained using all available market prices from June 2007 [24].

Given that the size and computational cost of the stochastic programming models depends on the number of scenarios, some scenario reduction techniques have to be applied in order to reduce the original set of scenarios into a smaller but representative one. We apply the scenario reduction algorithm explained in [25], which determines a subset of the initial scenario set and assigns new probabilities to the preserved scenarios.

In our model, a scenario is a set of 24 hourly market prices. The original number of scenarios was 261. The reduction technique is applied resulting in subsets of 10, 25, 50, 75, 100, 150, 200 and 250 scenarios. Fig. 10 shows how the optimal objective function value changes as the number of scenarios increases. It also contains (right axis) the difference in percentage between expected benefits of the complete group of 261 scenarios and each reduced set (ΔE[Benefits](%)). Observe how from 75 scenarios any additional increase of the number of scenarios improves the expected benefits by less than 0.09% while the CPU time increases more than 15 times (from 442s with 75 scenarios to 6554s with 100 scenarios). As a consequence, model (32) will be tested by a fan with 75 scenarios for which the objective function value becomes stable and the computational time cost remains acceptable.

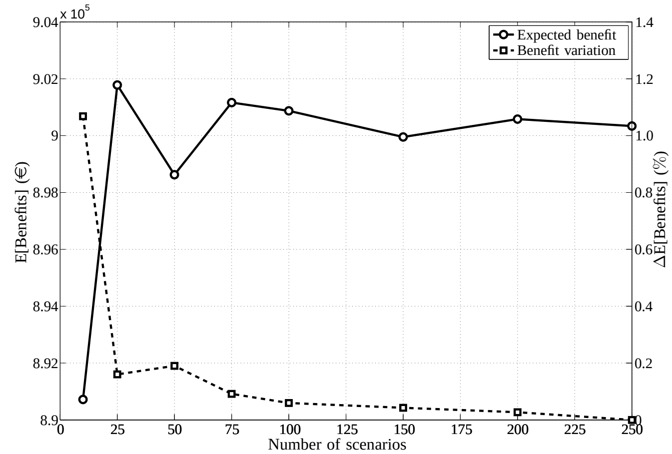

Figure 10. Expected benefit value and difference between the expected benefit of the complete set and each reduced one, as function of the number of scenarios

Table I
STOCHASTIC PROGRAMMING INDICATORS

Monday, April 28, 2008	
RP	901.164 €
EEV	848.528 €
VSS	52.636 €

In Table I the stochastic programming indicators needed to evaluate the goodness of the stochastic approximation [26] are reported. VSS, the measure of the advantage of using the stochastic programming model instead of the deterministic one, shows that it is possible to increase the expected benefits by 52.636 € (6.02%) by using the stochastic optimal solution.

V. Test and Results

The model (32) has been tested with real data from a Spanish generation company and market prices [2] and the results are reported in this section. The day under study is Monday, May 05, 2008. As explained in the previous sections, a fan with 75 scenarios has been used to represent the spot price stochasticity. The characteristics of the thermal units, bilateral contracts and VPP capacity are shown in Tables II, III and IV, respectively. The model (32) has been implemented in AMPL [21] and solved with CPLEX [22] (called with default options) using a SunFire X2200 with two dual core AMD Opteron 2222 processors at 3 GHz and 32 Gb of RAM memory.

A set of computational tests has been performed to evaluate the influence of the GPU and VPP in the GenCo's optimal bidding strategy in the MIBEL. For this reason, the proposed stochastic programming model was tested for three different cases: (a) a GenCo with GPU and VPP capacity; (b) a GenCo with GPU but without VPP capacity; and (c) a GenCo without GPU (see Table V for a summary of the optimization problem's dimensions and solutions). The worst expected profit is obtained in case (c), where the thermal units are the only responsibility for fulfilling the BCs before the day-ahead market. Case (b) obtains a greater expected profit than case (c),

Table II
OPERATIONAL CHARACTERISTICS OF THE THERMAL UNITS

t	c_t^b	c_t^l	c_t^q	$\underline{p}_t$	$\overline{p}_t$	st_t^0	c_t^{on}	c_t^{off}	t_t^{on}	t_t^{off}
	€	€/MWh	€/MWh^2	MW	MW	hr	€	€	hr	hr
1	151.08	40.37	0.015	160.0	350.0	+3	412.80	412.80	3	3
2	554.21	36.50	0.023	250.0	563.2	+3	803.75	803.75	3	3
3	97.56	43.88	0.000	80.0	284.2	–3	244.80	244.80	3	3
4	327.02	28.85	0.036	160.0	370.7	+3	438.40	438.40	3	3
5	64.97	45.80	0.000	30.0	65.0	+3	100.20	100.20	3	3
6	366.08	-13.72	0.274	60.0	166.4	+3	188.40	188.40	3	3
7	197.93	36.91	0.020	160.0	364.1	+3	419.20	419.20	3	3
8	66.46	55.74	0.000	110.0	313.6	–3	1298.88	1298.88	3	3
9	66.46	55.74	0.000	110.0	313.6	–3	1298.88	1298.88	3	3
10	372.14	105.08	0.000	90.0	350.0	–3	1315.44	1315.44	3	3

Table III
CHARACTERISTICS OF THE BILATERAL CONTRACTS

j	$L^B_{1\ldots24j}$	$\lambda^B_{1\ldots24j}$
	MW	€/MWh
1	1100	52
2	400	63

due to the possibility of being able to buy cheaper energy from the pool to cover the BCs and to avoid the use of expensive thermal units. The greatest expected profit is obtained in case (a) where the VPP capacity is used to sell in the day-ahead market and to cover part of the BCs, using the same advantages of case (b).

The optimal management of the GPU in case (a) can be analyzed with the help of Fig. 11 and 12. Fig. 11 shows the aggregated economic dispatch of the two BCs (1.500MWh) by the thermal units (b_i^T, white bars) and the GPU (b_i^G, black bars), together with the exercised VPP energy p_i^V (small circles). Fig. 12 shows the optimal GPU's sale bid (OSB$_i$, positive values) and purchase bid (OPB$_i$, negative values) for both cases (a) and (b) (black and white bars respectively). Observing both graphs along the whole 24h optimization horizon, it is clear that the GPU exhibits a differentiated behaviour depending on the time period considered:

- In time periods $i \in \{5, 6, 7\}$, the GenCo doesn't exercise its VPP rights ($p_i^V = 0$). For those time periods, all the energy b_i^G allocated to the BCs must be purchased to the day-ahead market (purchase bids, black negative bars in Fig. 12) or from the BCs after the day ahead market. For the rest of the time periods the GenCo does exercise completely its VPP rights ($p_i^V = \overline{p}_i^V$).
- There is only one time period ($i = 16$) where the exercised energy coincides with the energy allocated to the BCs ($b_{16}^G = p_{16}^V$).
- For periods $i \in \{3, 4, 8, 21\}$ the allocated energy exceeds the exercised one ($b_i^G > p_i^V$). The surplus energy $b_i^G - p_i^V$ must be obtained either from the day-ahead market (see the purchase bids for those time periods, black negative bars in Fig. 12) or from the BCs after the day ahead market.
- For periods $i \in \{1, 2, 9-15, 17-20, 22-24\}$, only part of the exercised VPP energy is used to satisfy the BCs, and the rest is submitted to the day-ahead market (sale bids for those time periods, black positive bars in Fig. 12)

Table IV
CHARACTERISTIC OF THE VPP CAPACITY AND THE BC'S AFTER DAY-AHEAD MARKET

$\overline{p}^V$	λ^V	λ^S	$\overline{b}^S$	λ^B	$\overline{b}^B$
MW	€/MWh	€/MWh	MW	€/MWh	MW
800	38	20	200	100	200

Table V
OPTIMIZATION CHARACTERISTICS OF THE STUDY CASES

Case	Constraints	Real variables	Binary variables	E(Benefits) €	CPU s
(a)	134034	56002	18816	901.164	442
(b)	128503	52364	18792	665.530	214
(c)	119399	46895	18720	610.264	142

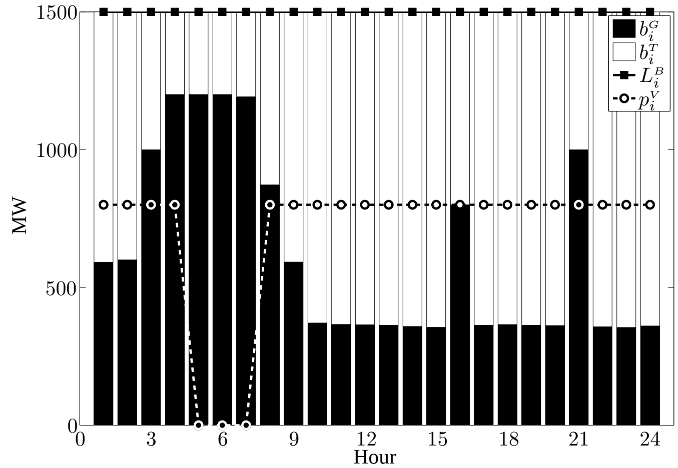

Figure 11. Aggregated economic dispatch of the two BCs between the thermal units and the GPU for study case (a). Exercised VPP energy is also shown.

Case (b) corresponds to those GenCos operating in the MIBEL which are not allowed to acquire any VPP capacity rights in order to prevent these GenCos from becoming price-makers. Under the assumptions of model (32), such a GenCo can use the GPU to purchase energy from the day-ahead market at its best convenience, resulting in an optimality purchase bid pattern that is depicted by the white bars in Fig. 12. The energy of the optimal purchase bid coincides in this case with the contribution of the GPU to the BCs at each time period, b_i^G.

Finally, the optimal thermal unit's bidding is analyzed. The thick line in Fig. 13 shows the optimal thermal bid function $\lambda_{it}^O(p_{it}^O, b_{it}^{T*})$ of three thermal units (3, 4 and 6) for all case studies in each interval. Remember that b_i^T is the energy allocated to the BCs, in such a way that the submitted bidding comprises energies between b_i^T and $\overline{p}_t$. The symbol b_*^T is used to point out the BCs contribution for the remaining hours not shown explicitly in each sub-figure. Observing Fig. 13 it is

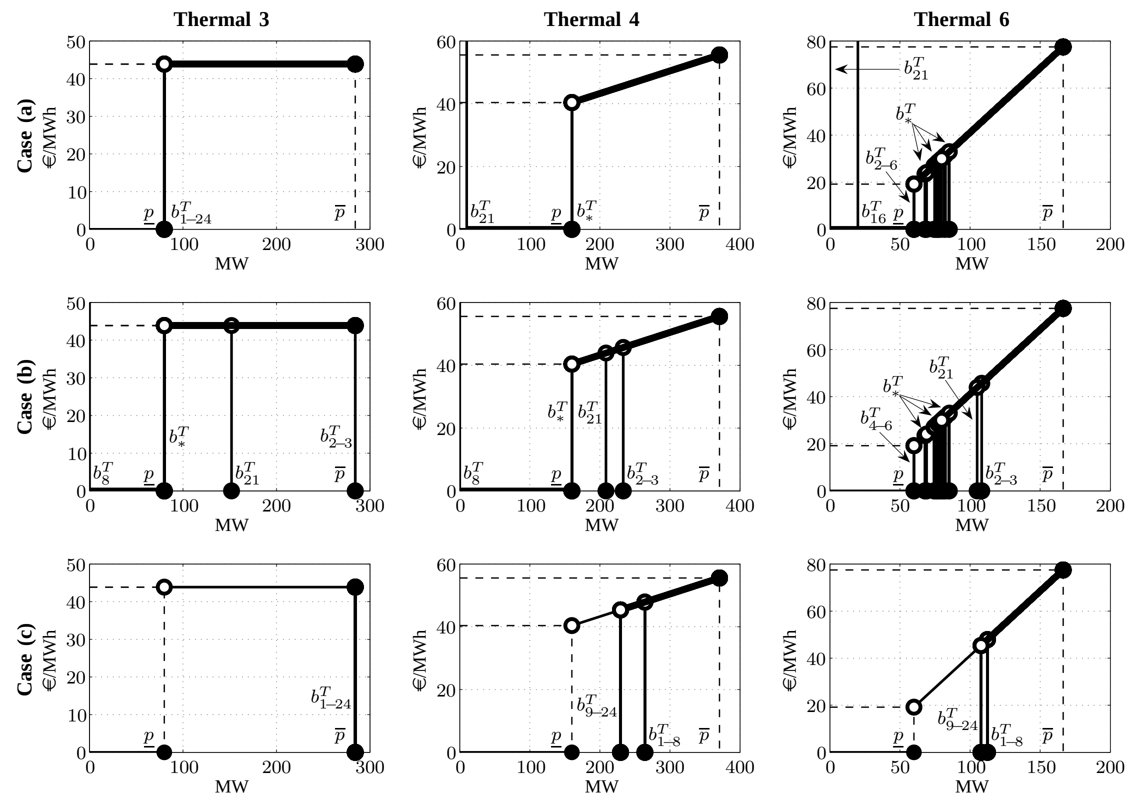

Figure 13. Bidding curve of thermal programming units 3, 4 and 6 for all study cases.

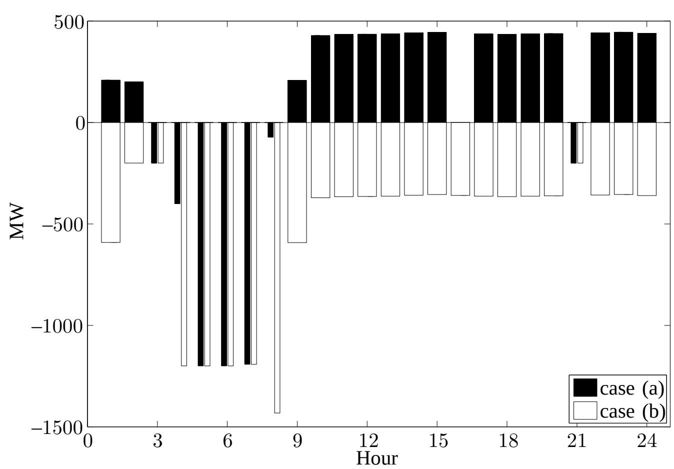

Figure 12. Sold and bought optimal bidding of the generic programming unit for the study cases (a) and (b).

clear that the presence of the GPU and VPP capacity allows the thermal units to submit more energy to the pool. See, for instance, the extreme case of thermal unit 3: without GPU (case (c)) the generation of this unit is exclusively dedicated to the BCs ($b^T_{i,3} = \overline{p}_3 \ \forall i$), while with GPU and VPP capacity (case (a)) all the production output between the operation limits are submitted to the pool ($b^T_{i,3} = \underline{p}_3 \ \forall i$). The rest of the thermal units exhibit a similar behaviour. Observe also how the availability of the GPU allows the bidding of the thermal unit 6 to adapt itself to the different periods in contrast to case (c), where the bidding is almost identical in all time periods. In general, Fig. 13 shows that the optimal thermal unit's bidding is affected significatively when a GPU is considered, changing drastically the optimal bidding in a non-trivial way that increases the opportunity of the GenCo to take benefits from the pool.

VI. Conclusions

This paper provides a procedure for a price-taker generation company operating under the most recent regulations of the MIBEL Iberic Electricity Market to optimally manage a pool of thermal units and a generic programming unit. The proposed technique is built within the versatile decision framework provided by the stochastic programming methodology. A two-stage stochastic mixed quadratic programming problem is proposed to decide the optimal unit commitment of the thermal units, the optimal economic dispatch of the bilateral contracts between the thermal and generic programming units and the optimal bid for thermal and generic programming units observing the MIBEL regulation. The objective of the producer is to maximize the expected profit from its involvement in the spot market, bilateral contracts and virtual power plant capacity. The set of scenarios representing the uncertainty of the spot prices is built applying reduction techniques to the tree obtained from real data of the MIBEL system. The model was implemented and solved with commercial optimization packages and tested with real data of a Spanish generation company and market prices. The results of the computational experiments are reported and analyzed.

ACKNOWLEDGMENT

We thank Dr. Narcis Nabona, Dr. Adela Pagès, and Mr. Matteo Tesser from the *Group on Numerical Optimization and Modelling* at the Technical University of Catalonia; Mr. Antonio Canoyra from Union Fenosa, and Ms. Teresa Sanz and Mr. Emilio Cortes, from Gas Natural, for their valuable comments and suggestions.

REFERENCES

[1] (2007) *Procedimiento de Operación 3.1* Resolución de 26 de junio de la Secretaria General de Energía (B.O.E. núm. 155, de 29 de junio de 2007).

[2] (2008) Market Operator of the Electricity Market of Mainland Spain. OMEL. [Online]. Available: http://www.omel.es/.

[3] (2007) Real Decreto de 29 de diciembre, disposición adicional vigésima *Emisiones Primarias de Energia* (B.O.E. núm. 312, de 30 de diciembre de 2006).

[4] (2008) Endesa & Iberdrola Virtual Power Plant Auctions. [Online]. Available: https://www.endesa-iberdrola-vpp.com/.

[5] (2008) Electricité de France Capacity Auctions. [Online]. Available: http://capacityauctions.edf.fr/.

[6] (2008) Belpex's Virtual Power Plant Auctions. [Online]. Available: http://www.belpexvpp.be/.

[7] (2008) E.ON Energy Sales Virtual Power Plant Auctions. [Online]. Available: http://www.eon-energy-sales.com/.

[8] M. A. Plazas, A. J. Conejo and F. J. Prieto, "Multimarket optimal bidding for a power producer," *IEEE Trans. on Power Syst.*, vol. 20, no. 4, pp. 2041-2050, Nov. 2005.

[9] E. J. Anderson and A. B. Philpott, "Optimal offer construction in electricity markets," *Mathematics of Operations Research*, vol. 27, no. 1, pp. 82-100, 2002.

[10] E. J. Anderson and A. B. Philpott, "Using supply functions for offering market generation into an electricity market," *Operations Research*, vol. 50, no. 3, pp. 477-489, 2003.

[11] P. Neame, A. B. Philpott and G. Pritchard, "Offer stack optimisation in electricity pool markets," *Operations Research*, vol. 51, no. 3, pp. 397-408, 2003.

[12] A. J. Conejo, F. J. Nogales and J. M. Arroyo, "Price-taker bidding strategy under price uncertainty," *IEEE Trans. on Power Syst.*, vol. 17, no. 4, pp. 1081-1088, Nov. 2002.

[13] V. P. Gountis, A. G. Bakirtzis, "Bidding Strategies for Electricity Producers in a Competitive Electricity Marketplace," *IEEE Trans. on Power Syst.*, vol. 19, no. 1, pp. 356-365, Feb. 2004.

[14] E. Ni, P. B. Luh and S. Rourke, "Optimal Integrated Generation Bidding and Schedulling With Risk Management Under a Deregulated Power Market," *IEEE Trans. on Power Syst.*, vol. 19, no. 1, pp. 600-1088, Nov. 2004.

[15] M. Nowak, R. Schultz and M. Westphalen, "A Stochastic Integer Programming Model for Incorporating Day-Ahead TRading of Electricity into Hydro-Thermal Unit Commitment," *Optimization and Engineering*, vol. 6, pp. 163-176, 2005.

[16] G. B. Shresta, S. Kai and L. Goel, "An efficient stochastic self-scheduling technique for power producers in th ederegulated power market," *Electric Power Systems Research*, vol. 71, pp. 91-98, 2004.

[17] C. Triki, P. Beraldi and G. Gross, "Optimal capacity allocation in multi-auction electricity markets under uncertainty," *Computers & Operations Research*, vol. 32, no. 2, pp. 201-217, 2005.

[18] (2008) Auction of Bilateral Contracts for Regulated Supply. [Online]. Available: http://www.subasta-cesur.eu/.

[19] N. Nabona and A. Pages, "A three-stage short-term electric power planning procedure for a generation company in a liberalized market," *International Journal of Electric Power Systems*, vol. 29, no. 5, pp. 408-421, 2007.

[20] F. J. Heredia, M. J. Rider and C. Corchero, "A stochastic programming model for the optimal electricity market bid problem with bilateral contracts for thermal and combined cycle units," submitted to *Annals of Operations Research*, E-prints UPC - http://hdl.handle.net/2117/2282 2008.

[21] R. Fourer, D. M. Gay and B. W. Kernighan, *AMPL: A modeling language for mathematical programming*. CA: Brooks/Cole-Thomson Learning, Pacific Grove, 2nd Ed., 2003.

[22] "CPLEX Optimization subroutine library guide and reference, version 11.0," CPLEX Division, ILOG Inc., Incline Village, NV, USA, 2008.

[23] M. Kaut and S. W. Wallace, "Evaluation of scenario-generation methods for stochastic programming," *SPEPS*, Working Paper vol. 14, 2003.

[24] J. Dupacová, G. Consigli and S. W. Wallace, "Scenarios for multistage stochastic programs," *Annals of Operations Research*, vol. 100, pp. 25-53, 2000.

[25] N. Gröwe-Kuska, H. Heitsch and W. Römisch, "Scenario reduction and scenario tree construction for power management problems," in *Proc. IEEE PowerTech*, vol. 3, Bologna, Italy, 2003.

[26] J. R. Birge and F. Louveaux, *Introduction to Stochastic Programming*. New York: Springer-Verlag, 1997.

F. Javier Heredia received the B.Sc. degree in Physics from the Universitat de Barcelona (Spain) in 1988, and the PhD degree in Operations Research from the Universitat Politècnica de Catalunya (UPC, Barcelona, Spain) in 1995. He is currently Professor of Optimization and Operations Research at the Department of Statistics and Operations Research of the UPC.

His current research interests include modeling and optimization of power systems and electricity markets as well as optimization methods as stochastic and nonlinear programming, dual methods and networks flows.

Marcos J. Rider (S'97, M'06) received the B.Sc. (Hons.) and P.E. degrees in 1999 and 2000, respectively, from the National University of Engineering, Lima, Perú, the M.Sc. degree in 2002 from the Federal University of Maranhão, Maranhão, Brazil, and the Ph.D. degree in 2006 from University of Campinas, Campinas, Brazil, all in electrical engineering. Currently, he is carrying out post doctorate work at the Universitat Politècnica de Catalunya, Barcelona, Spain.

His areas of research are the development of methodologies for the optimization, planning and control of electrical power systems, electrical power system analysis in competitive environment and applications of artificial intelligence in power systems.

Cristina Corchero received the B.Sc. degree in Statistics and Operations Research from the Universitat Politecnica de Catalunya (Spain) in 2004. She is currently working toward the Ph.D. degree at the Universitat Politecnica de Catalunya.

His research interests are in the fields of power system economics, stochastic programming, and electricity markets.