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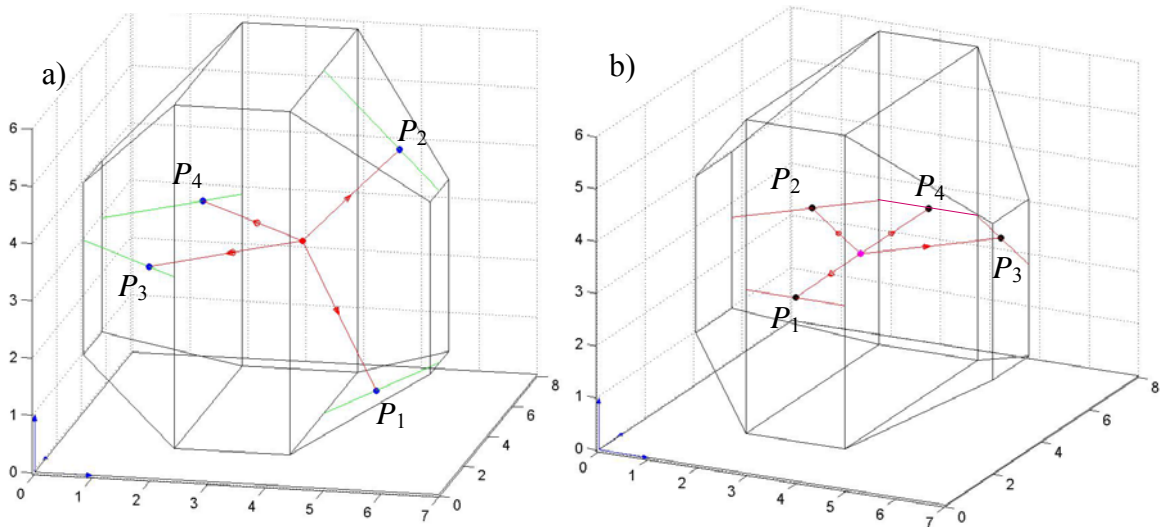


Fig. 1 Two FCG: a) the loss of a finger-object contact implies the loss of the force-closure property, b) the loss of any of the finger-object contacts at P_2 , P_3 y P_4 (one at a time) does not implies the loss of the force-closure property.

I. Introduction

A force-closure grasp (FCG) can balance any external disturbance applied on the object by means of the contact forces applied by the fingers. Different approaches for the computation of force-closure grasps have been presented in the literature and in particular for four object-finger contacts on polyhedral objects [2] [3] [4] [5] [6] [7]. Nevertheless, in the grasps generated with these approaches the lost of a finger-object contact frequently implies the lost of the force-closure property, and the object may fall down (an example of this type of grasps is shown in Figure 1a). As a consequence, these grasps are unlikely to be considered as valid for a manipulation process with finger repositioning on the object (finger gating). The methods to manipulate an object with four fingers and gating techniques start with an initial grasp that allows the individual lost of three contacts without losing the force-closure property [8] [9]; nevertheless, these works do not describe how to obtain such initial grasp.

The approach describe in this paper computes, for a given FCG with three contact points, the regions on the object boundary such that a fourth contact anywhere on them allows the removal of two of the initial contacts without losing the force-closure

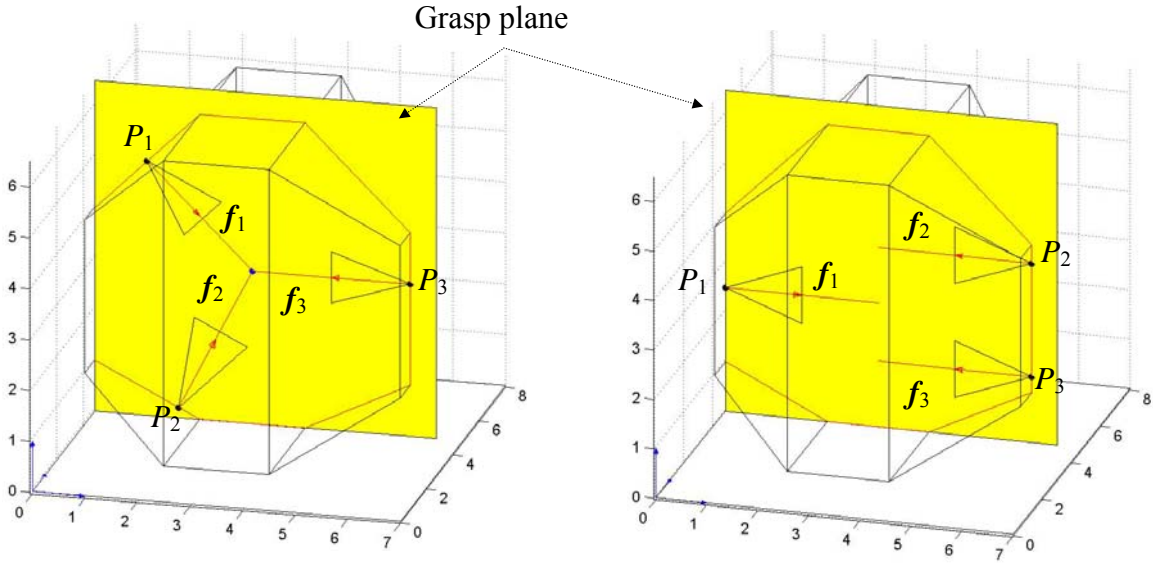


Fig. 2 The two cases of three forces that reach the equilibrium.

property.

First, a necessary and sufficient condition for the determination of a four-finger FCG that includes three three-fingers FCG is presented. Second, given a FCG with three contacts, the procedure to compute the contact regions on the object boundary for the fourth finger is given. Finally, a procedure to select a particular position for the fourth finger is given, trying to generate a good planar grasp or with the fourth finger close to the plane defined by the other three fingers (an example of this type of grasps is shown in Figure 1b); the intention here is to produce a “good enough grasp” [11], which best fits the potential reachability of an anthropomorphic hand.

II. Proposed Approach

A. Necessary and sufficient condition

Three forces f_1, f_2 and f_3 applied on non colinear points on an object reach the equilibrium if and only if one of the following two conditions is satisfied [2] [12] (Figure 2):

1. f_1, f_2 and f_3 are coplanar, span their supporting plane and their supporting lines intersect in a point.

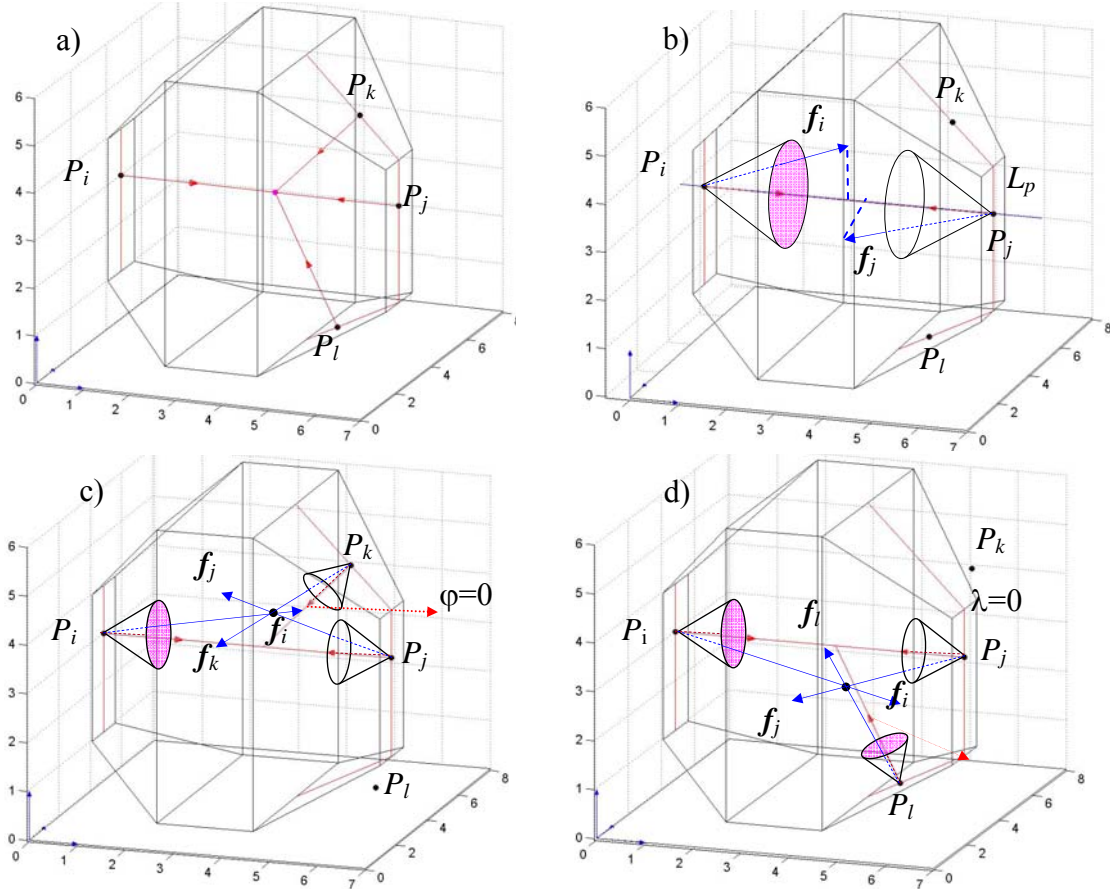


Figure 3 a) FCG with coplanar P_i , $i=1,2,3,4$; b) P_i and P_j satisfy the condition CI ; c) P_i , P_k and P_j allow a FCG ($\varphi=0 \Rightarrow \varphi < \alpha$); d) P_i , P_l and P_j allow a FCG ($\lambda=0 \Rightarrow \lambda < \alpha$).

2. f_1 , f_2 and f_3 are coplanar, parallel and that in the middle of the other two has different sense.

Let C_{f_i} be the friction cone at contact i . It was shown in previous works [2] [12] [14] that if $f_i \in C_{f_i}$ (f_i lies strictly in the interior of C_{f_i}) satisfy one of these conditions then the contact points P_i allow a FCG.

The method proposed in this paper is based on the following proposition. Let n_i be the unitary vector normal to the object boundary at contact point P_i .

Proposition 1. A FCG with contact points P_i , $i=1,2,3,4$, includes at least three FCG with three contact points each one if and only if (Figure 3a):

CI . $\exists P_i, P_j$, with $i, j \in \{1,2,3,4\}$ and $i \neq j$ defining a straight line L_p such that:

1. The projections of \mathbf{n}_i and \mathbf{n}_j on L_p have different sense (Figure 3b).

2. $L_p \subset C_{fi} \cap C_{fj}$ and L_p lies strictly in the interior of C_{fi} and C_{fj} .

C2. The angle φ between \mathbf{n}_k and the plane defined by P_i, P_j and P_k (Figure 3c), and the angle λ between \mathbf{n}_l and the plane defined by P_i, P_j y P_l (Figure 3d), are both smaller than the friction cone half-angle α , and $L_p \cap C_{fk} \cap C_{fl} \neq \emptyset$ being $\{i,j,k,l\} = \{1,2,3,4\}$ and $i \neq j \neq k \neq l$. ■

Proof

Sufficient Condition.

The condition C1 allows the application of forces \mathbf{f}_i and \mathbf{f}_j such that their components on L_p have different senses while their components orthogonal to L_p may have any sense.

The condition C2 assures that $L_p \cap C_{fk} \neq \emptyset$ and $L_p \cap C_{fl} \neq \emptyset$, this means that there exist forces \mathbf{f}_k and \mathbf{f}_l whose supporting lines intersect with L_p and, as $L_p \subset C_{fi} \cap C_{fj}$ then $C_{fi} \cap C_{fj} \cap C_{fk} \neq \emptyset$ and $C_{fi} \cap C_{fj} \cap C_{fl} \neq \emptyset$ (since $\varphi < \alpha$ and $\lambda < \alpha$ then these forces do not belong to the limits of C_{fk} and C_{fl} respectively).

Now, if \mathbf{f}_k is null and \mathbf{f}_l is non null, then a positive lineal combination of the component of \mathbf{f}_i and \mathbf{f}_j on L_p can balance the component of \mathbf{f}_l on L_p (the components of \mathbf{f}_i and \mathbf{f}_j on L_p have different senses), besides, a positive lineal combination of the component of \mathbf{f}_i and \mathbf{f}_j orthogonal to L_p can balance the component of \mathbf{f}_l orthogonal to L_p . This means that $\mathbf{f}_i, \mathbf{f}_j$ and \mathbf{f}_l can reach the equilibrium, and therefore P_i, P_j and P_l allow a FCG. The same reasoning can be applied to the case where \mathbf{f}_k is non null and \mathbf{f}_l is null, obtaining that P_i, P_j and P_k allow a FCG (the forces \mathbf{f}_i and \mathbf{f}_j that balance \mathbf{f}_k are not same that the forces \mathbf{f}_i and \mathbf{f}_j that balance \mathbf{f}_l) and also to the case where \mathbf{f}_j is null and $\mathbf{f}_i, \mathbf{f}_k$ and \mathbf{f}_l non-null, obtaining that P_i, P_k and P_l allow a FCG.

Then $(P_i, P_k, P_l), (P_i, P_j, P_k)$ and (P_i, P_j, P_l) determine, each one, a FCG.

Necessary Condition.

If the projection of \mathbf{n}_i and \mathbf{n}_j on L_p have the same sense then the components of \mathbf{f}_i and \mathbf{f}_j on L_p have also the same sense; this does not allow the application of forces $\mathbf{f}_i, \mathbf{f}_j$

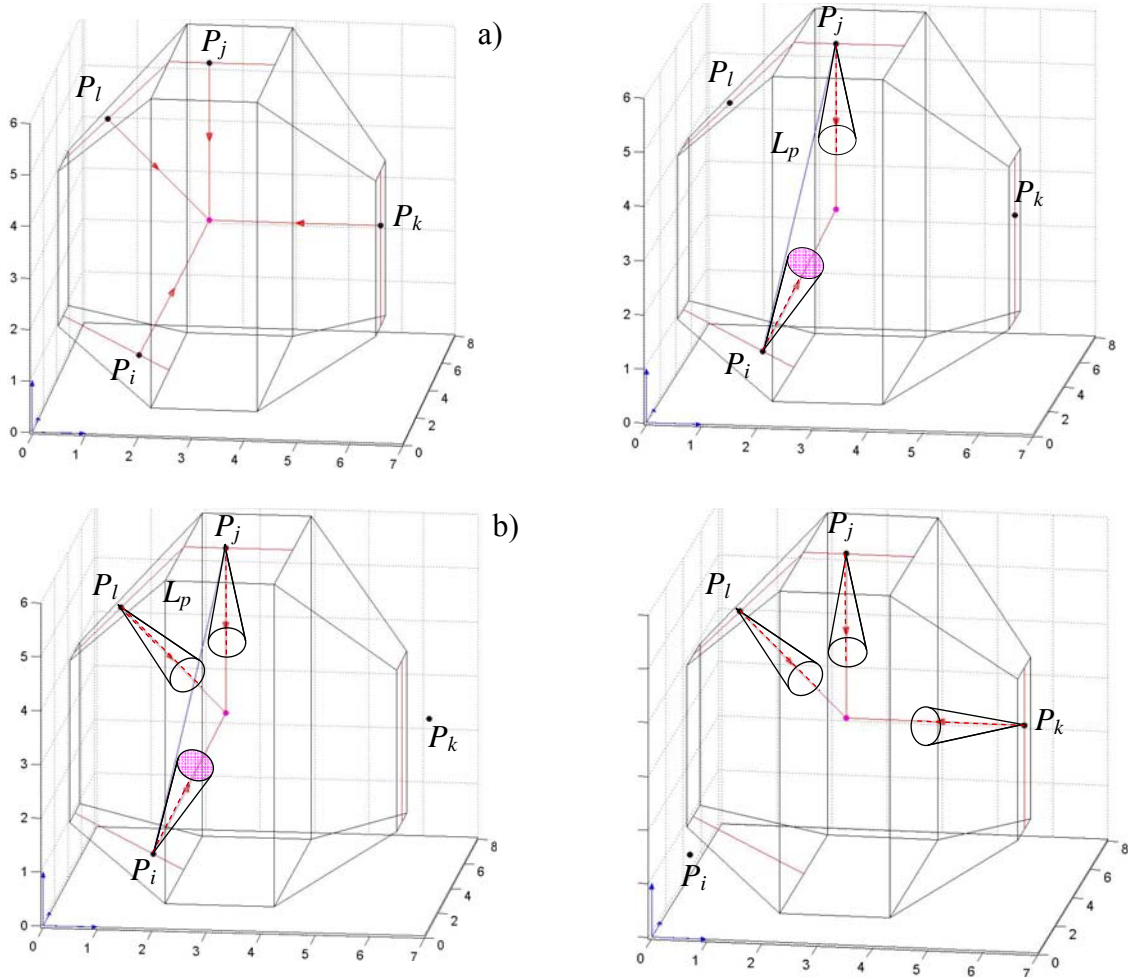


Figure 4 a) There are not two points P_i and P_j satisfying the condition CI ; b) the sets (P_i, P_l, P_j) and (P_l, P_k, P_j) do not allow a FCG.

and f_k (with $f_k = \emptyset$) that are either parallel or coplanar spanning their supporting plane, then, the applied forces do not reach the equilibrium and therefore P_i, P_j and P_k do not allow a FCG. The same reasoning can be applied for f_i, f_j and f_l (with $f_k = \emptyset$), and therefore P_i, P_j and P_l neither allow a FCG. As a consequence, two out of the four possible sets of three contacts do not allow FCG, therefore the first part of CI is necessary for the existence of a FCG that using the points $P_i, i=1,2,3,4$ allow three FCG with three contacts each one.

Now if $L_p \not\subset C_{fi} \cap C_{fj}$ (Figure 4a) and $f_k = \emptyset$ then it is not possible to apply coplanar forces f_i, f_j and f_l that span their supporting plane, then, the applied forces do not reach the equilibrium and therefore P_i, P_j and P_l do not allow a FCG (Figure 4b). As a consequence,

two out of the four possible sets of three contacts do not allow FCG, therefore the second parte of CI is also necessary for the existence of a FCG that, using the points $P_i, i=1,2,3,4$, allows three FCG with three contacts each one.

A necessary condition for the existence of a FCG using $P_i, i=1,2,3$, is that the grasp plane defined by the contact points intersect with $C_{fi}, i=1,2,3$ [13][13], and it is possible only if the angle between n_i and the grasp plane is smaller than α . Therefore, if P_i, P_j and P_k allow a FCG then φ must be smaller than α ; on the same way, if P_i, P_j and P_l allow a FCG then λ must be smaller than α . As a consequence, the condition $C2$ is necessary for the existence of a FCG that using the points $P_i, i=1,2,3,4$ allows three FCG with three contacts each one. ■

Note that from the previous proof it can also be concluded that a sufficient condition for $P_i, i=1,2,3$, to allow a FCG is that two contact points satisfy the condition CI and the friction cone of the third contact point intersect with the line L_p ; this means that $L_p \cap C_{fi} \cap C_{fj} \cap C_{fk} \neq \emptyset$, with $i,j,k \in \{1,2,3\}$ and $i \neq j \neq k$.

Conditions CI and $C2$ in Proposition 1 are used in the next subsection to obtain the desired four contact FCG.

B. Determining the contact regions R_i

Given three contact points $(P_i, P_j, P_k), \{i,j,k\}=\{1,2,3\}$, that allow a FCG, the contact region R_i such that a fourth point $P_4 \in R_i$ makes the sets (P_i, P_j, P_4) and (P_i, P_k, P_4) to also allow a FCG are computed in two steps:

1. Compute the region $R'_i / \forall P_4 \in R'_i, P_4$ and P_i satisfy the condition CI .
2. Compute $R_i \subseteq R'_i / \forall P_4 \in R_i$, the sets (P_i, P_j, P_4) and (P_i, P_k, P_4) satisfy the condition $C2$.

If $\forall i R_i = \emptyset$ then the desired grasp cannot be obtained. A conceptual example of the regions R_i is shown in Figure 5. Note that R_i may be a discontinuous region (e.g. R_2 in Figure 5), having parts of different object faces.

The proposed procedure takes into account this situation checking for all the faces that are candidates to contain a portion of R_i .

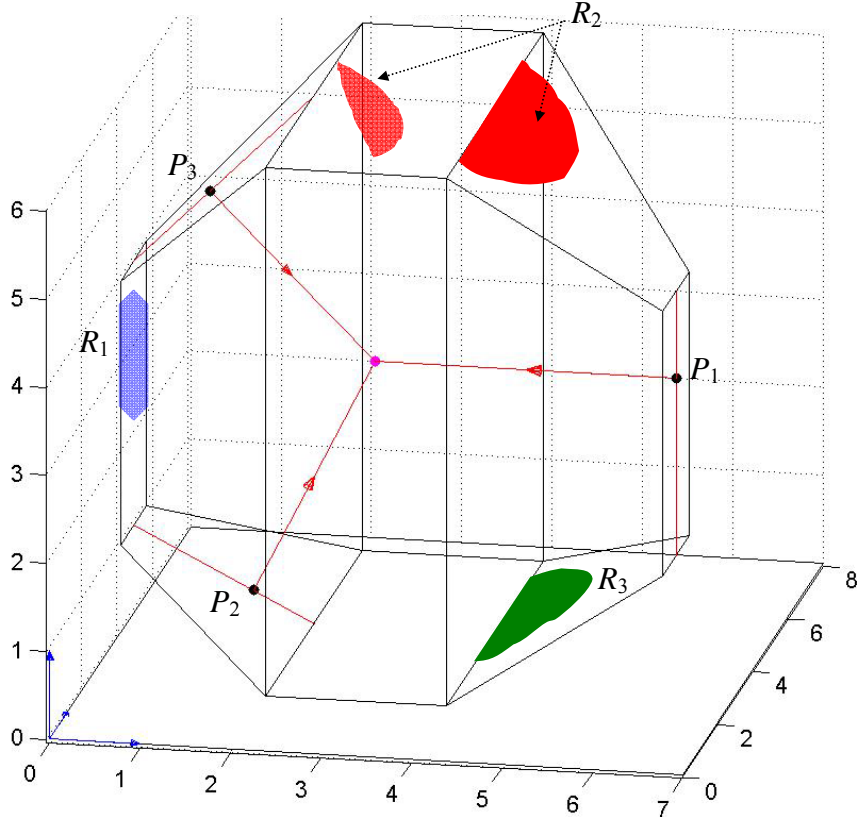


Figure 5 Conceptual representation of the regions $R_i \neq \emptyset$ for the initial FCG given by P_1, P_2 and P_3 .

Let $A_l, l=1, \dots, m$, be the object faces. For each A_l , the following procedure determines, first, whether the position and orientation of A_l allows the existence of the region R'_i (Steps 1 and 2), and second, if $R'_i \neq \emptyset$ then the region $R_i \subseteq R'_i$ (Step 3). Given three contact points P_i, P_j , and P_k (that allow a FCG) the procedure to determine R_i is as follows:

1. Compute the angle γ between \mathbf{n}_l and $-\mathbf{n}_i$. If $\gamma \geq 2\alpha$ then Return(*invalid face*).
If $\gamma \geq 2\alpha$ then at least one of the two parts of condition *CI* is not satisfied.
2. Compute $R'_i = A_l \cap C_{fo} \cap C_{fi}$, where C_{fo} is the cone of half-angle α , axis with the direction of \mathbf{n}_l and origin at P_i (Figure 6). Being $\gamma < 2\alpha$ then $C_{fo} \cap C_{fi} \neq \emptyset$. If $R'_i = \emptyset$ then Return(*invalid face*).

If $R'_i = \emptyset$ then P_i and any P_4 on A_l do not satisfy the second part of the condition *CI*, even when $A_l \cap C_{fi} \neq \emptyset$. If $R'_i \neq \emptyset$ (note that $R'_i \subset C_{fi}$) then P_i and any $P_4 \in R'_i$ satisfy the condition *CI*.

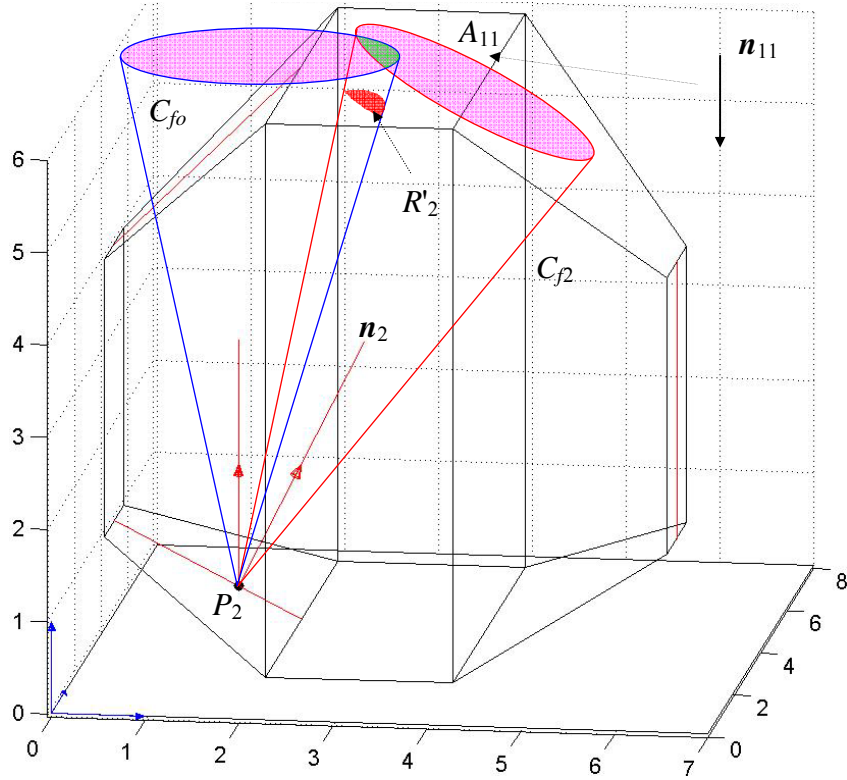


Figure 6 Computation of R'_2 on the face A_{11} (i.e. $i=2$ and $l=11$).

3. Compute R_i according to one of the following four cases:

Let: L_{ij} be the straight line defined by P_i and P_j .

L_{ik} be the straight line defined by P_i and P_k .

- 3.1 If $L_{ij} \subset C_{fj}$ and $L_{ik} \subset C_{fk}$ then $R_i = R'_i$ (Figure 7).

In this case, $P_i \in L_{ij}$ and $L_{ij} \subset C_{fj} \Rightarrow P_i \subset C_{fj}$, therefore $C_{fi} \cap C_{fj} \neq \emptyset$. Now, by the step 1 is satisfied that P_i and any $P_4 \in R'_i$ satisfy the condition CI (remind that $R'_i \subset C_{fi}$), therefore $L_p \subset C_{fi} \cap C_{f4}$ and as $C_{fi} \cap C_{fj} \neq \emptyset$ then $L_p \cap C_{fi} \cap C_{fj} \cap C_{f4} \neq \emptyset$. Then P_i, P_j and P_4 satisfy the sufficient condition to determine a FCG.

With an analogous reasoning, since $P_i \in L_{ik}$ and $L_{ik} \subset C_{fk} \Rightarrow P_i \subset C_{fk}$, and therefore P_i, P_k and P_4 allow a FCG.

As a conclusion, $(P_i, P_j, P_k), (P_i, P_j, P_4)$ and (P_i, P_k, P_4) allow a FCG (remind that P_i, P_j and P_k define the initial FCG).

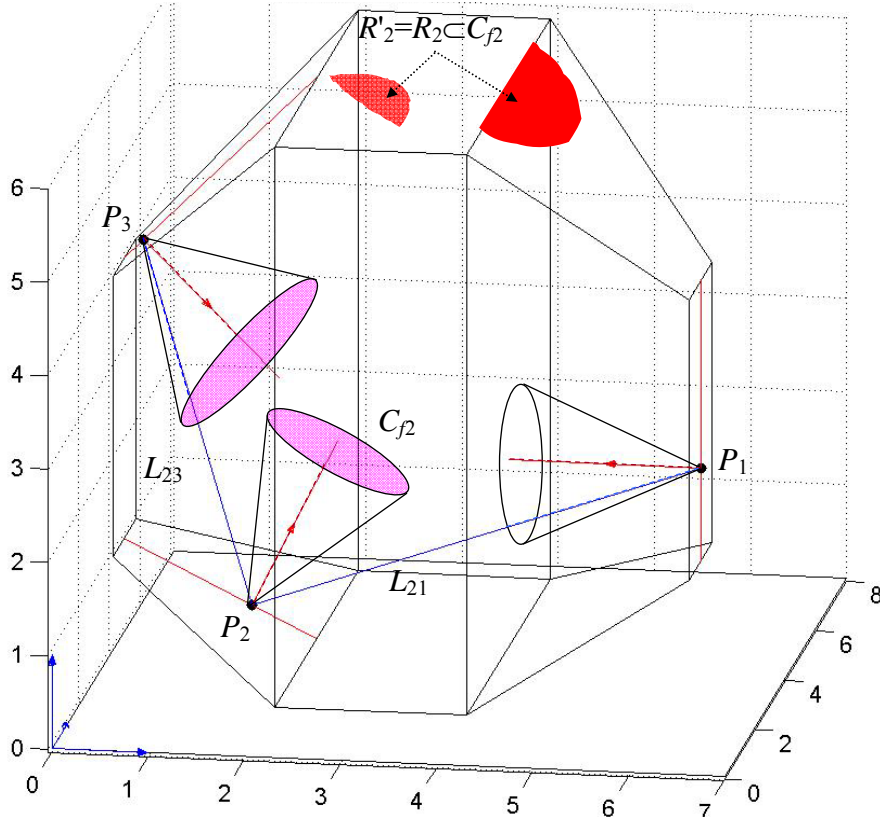


Figure 7 $L_{ij} \subset C_{fj}$ and $L_{ik} \subset C_{fk} \Rightarrow R'_2 = R_2$.

3.2 If $L_{ij} \subset C_{fj}$ and $L_{ik} \not\subset C_{fk}$ then:

- Find the sub-space, S_{ik} , limited by two planes tangent to C_{fk} and containing L_{ik} , such that $C_{fk} \subset S_{ik}$ (Figure 8).

Any plane that intersects with C_{fk} and that contains P_i and P_k always belongs to S_{ik} . Since in a FCG with three contact points the grasp plane (defined by the three contact points) always intersects with the three friction cones, then the grasp plane of P_i , P_k and any other contact point that allow a FCG always is contained in S_{ik} .

- Compute $R_i = R'_i \cap S_{ik}$.

If $P_4 \in R'_i \cap S_{ik}$ then is satisfied that $L_p \subset C_{fi} \cap C_{f4}$ (Step 1) and $L_p \subset S_{ik}$, and as by construction $C_{fk} \subset S_{ik}$ (C_{fk} is tangent to S_{ik}) then $L_p \cap C_{fk} \neq \emptyset$. This implies that $L_p \cap C_{fi} \cap C_{fk} \cap C_{f4} \neq \emptyset$, therefore P_i , P_k and P_4 satisfy the sufficient condition to

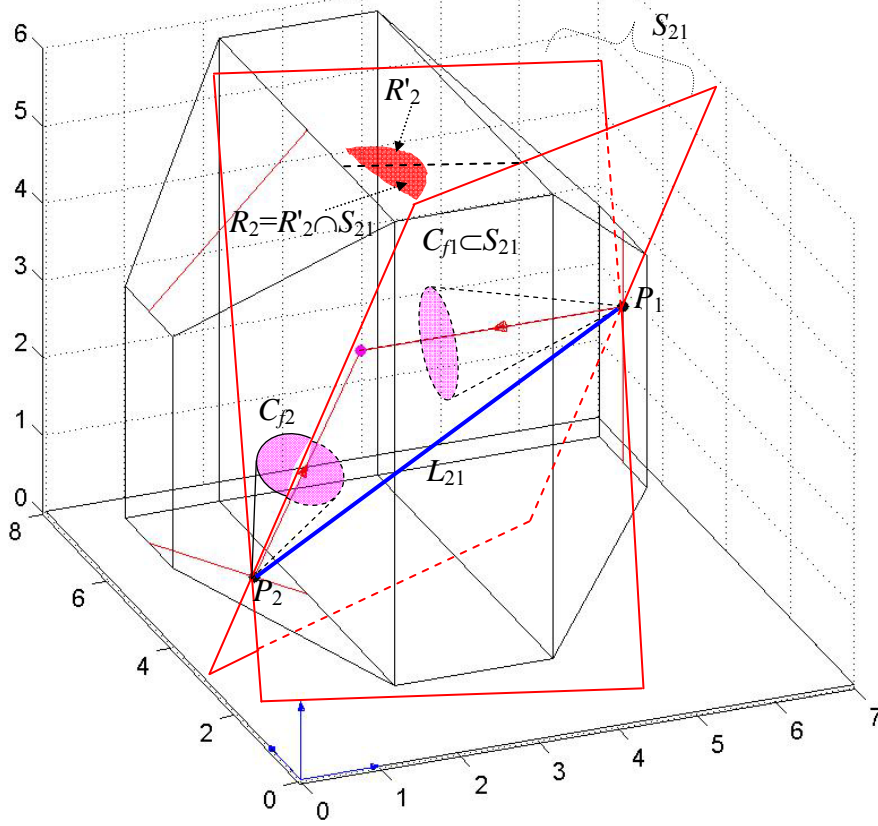


Figure 8 The sub-space S_{21} contains C_{f1} (C_{f1} is tangent to the limits of S_{21}) and $R_2=R'_2 \cap S_{21}$.

determine a FCG (note that the grasp plane defined by P_i , P_k and P_4 lie in S_{ik}).

On the other hand, since $L_{ij} \subset C_{ff} \Rightarrow P_i \subset C_{ff}$ (this condition is similar those of the first case), and as $P_4 \in R'_i \cap S_{ik}$ then $L_{ij} \cap C_{ff} \cap C_{fj} \cap C_{f4} \neq \emptyset$, this implies that P_i , P_j and P_4 determine a FCG.

Therefore (P_i, P_j, P_k) , (P_i, P_j, P_4) and (P_i, P_k, P_4) determine each a FCG.

3.3 If $L_{ij} \not\subset C_{ff}$ and $L_{ik} \subset C_{fk}$ then:

This case is analogous to the previous one, therefore with the same reasoning and swapping C_{fk} by C_{ff} and L_{ij} by L_{ik} results $R_i = R'_i \cap S_{ik}$ (S_{ik} is the equivalent to S_{ij}).

3.4 If $L_{ij} \not\subset C_{ff}$ and $L_{ik} \not\subset C_{fk}$ then:

- Find the sub-space, S_{ik} , limited by two planes tangents to C_{fk} and containing L_{ik} such that $C_{fk} \subset S_{ik}$ (Figure 9), and the sub-space, S_{ij} , limited by two planes tangents to C_{ff} and containing L_{ij} such that $C_{ff} \subset S_{ij}$.

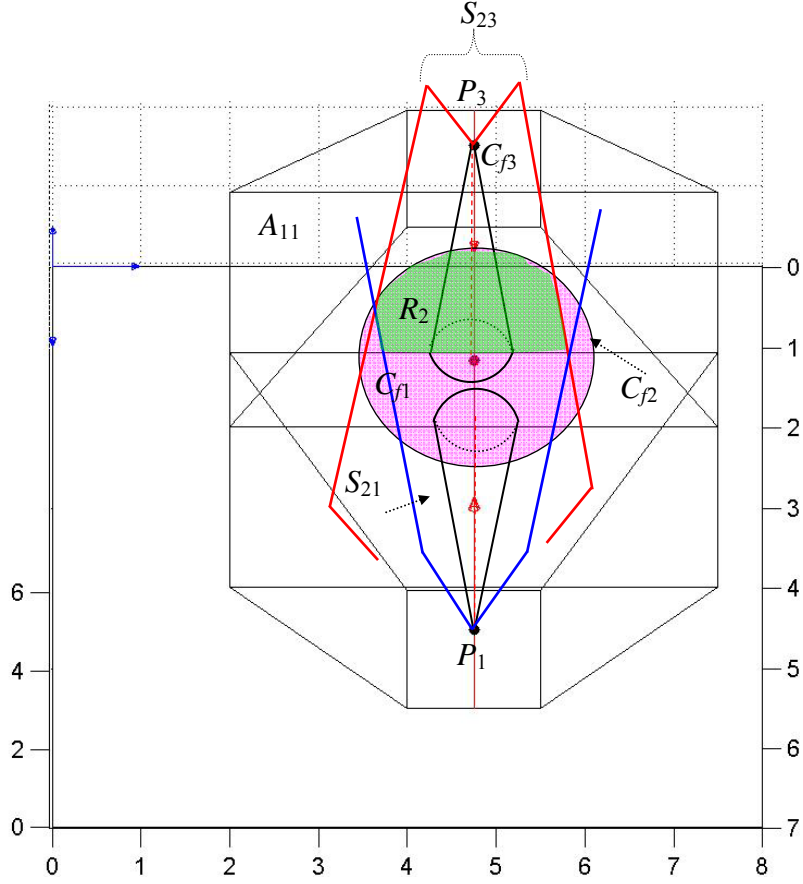


Figure 9 Computation of $R_2 \subset A_{11}$ according the case $L_{ij} \not\subset C_{fj}$ and $L_{ik} \not\subset C_{fk}$.

As it was stated before, any plane that intersects with C_{fk} and that contains P_i and P_k always belongs to S_{ik} , and analogously, any plane that intersects with C_{fj} and that contains P_i and P_j always belongs to S_{ij} .

- Compute $R_i = R'_i \cap S_{ik} \cap S_{ij}$.

If $P_4 \in R'_i \cap S_{ik} \cap S_{ij}$ then is satisfied that $L_p \subset C_{f1} \cap C_{f4}$ (step 1), $L_p \subset S_{ik}$ and $L_p \subset S_{ij}$, and as by construction $C_{fk} \subset S_{ik}$ and $C_{fj} \subset S_{ij}$ (C_{fk} and C_{fj} are tangent to S_{ik} and to S_{ij} , respectively) then $L_p \cap C_{fk} \neq \emptyset$ and $L_p \cap C_{fj} \neq \emptyset$. This implies that $L_p \cap C_{f1} \cap C_{fk} \cap C_{f4} \neq \emptyset$ and $L_p \cap C_{f1} \cap C_{fj} \cap C_{f4} \neq \emptyset$. Therefore (P_i, P_k, P_4) and (P_i, P_j, P_4) satisfy the sufficient condition to determine a FCG.

As a conclusion, (P_i, P_j, P_k) , (P_i, P_j, P_4) and (P_i, P_k, P_4) allow a FCG.

Note that if $P_4 \in R_i \cap R_j \neq \emptyset$, $i \neq j$, then any of the three fingers located contacting on the original contact points P_i, P_j, P_k can be removed without losing the FCG.

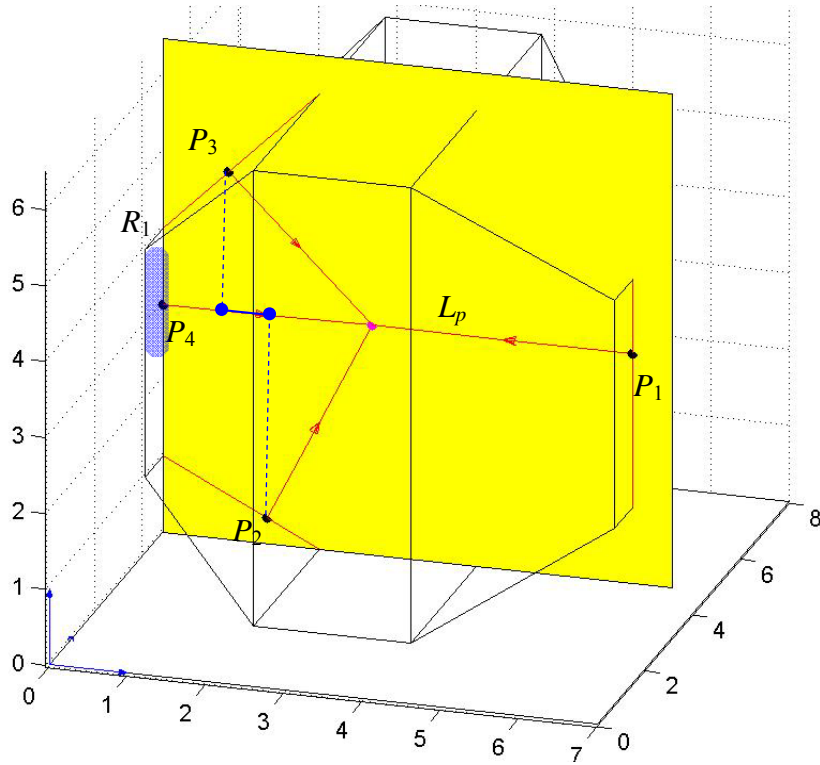


Figure 10 P_4 is selected on the R_i that allow to obtain a coplanar grasp.

Note that there was no assumption about the position of the contact points on the object faces, thus the approach is valid for contact points on two, three or four faces.

C. Determination of the fourth contact point on R_i

In previous works [10] [11] [15] it was shown that if a grasp planner considers (for a precision grasp) the geometry and kinematics of an anthropomorphic hand (the referenced works use four finger anthropomorphic mechanical hands) then the contact points of the best grasp are coplanar or, at least, the contact point of the middle finger is close to the plane defined by the other three contact points. It is also interesting to remark that, frequently, in the examples in these works the line defined by the contact points of the middle finger and the thumb is equidistant of the other two contact points.

Then, even when the work presented here cannot assure the reachability of the resulting grasp by a given hand, considering these results, P_4 is selected on R_i looking for:

- A coplanar grasp (i.e. $P_i, i=1,2,3,4$ are coplanar) (Figure 10), or with P_4 as close as possible to the grasp plane defined by the initial grasp.

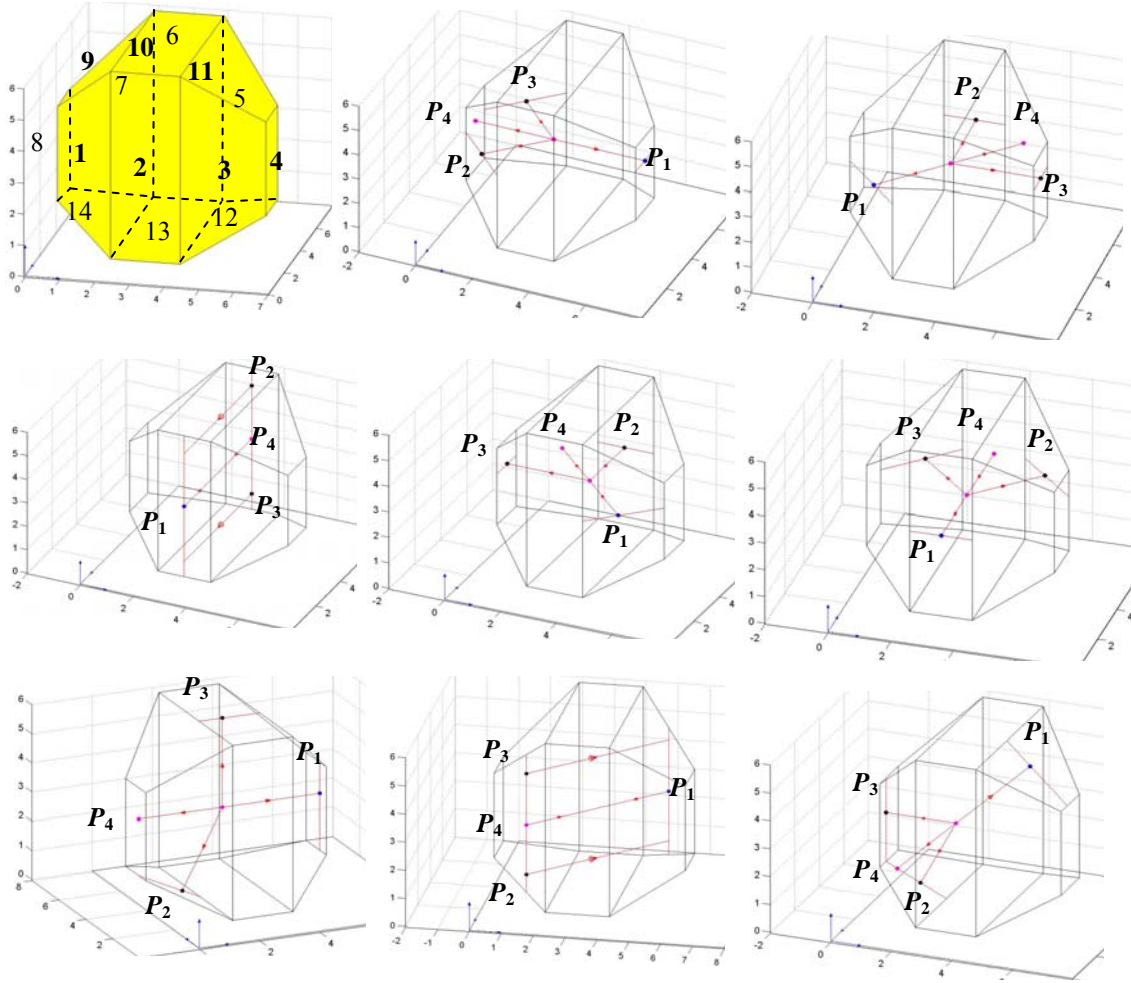


Fig. 11. Different FCG determined by the approach proposed on an object of 14 faces.

- A similar distances between L_p and the two contact points that do not define L_p (i.e. the points that satisfy the condition $C2$).
- A minimum distance between the projections on L_p of the two contact points that do not define L_p).

III. Examples

The application of the proposed methodology is illustrated using two different objects. It is considered a constant friction coefficient $\mu=0,36$. The procedure was implemented in Matlab and run on a server INTEL Biprocessor Pentium III 1,4 GHz. Figures 11 and 12 show several examples of the obtained grasps on the two objects using different initial FCG given by P_1 , P_2 and P_3 .

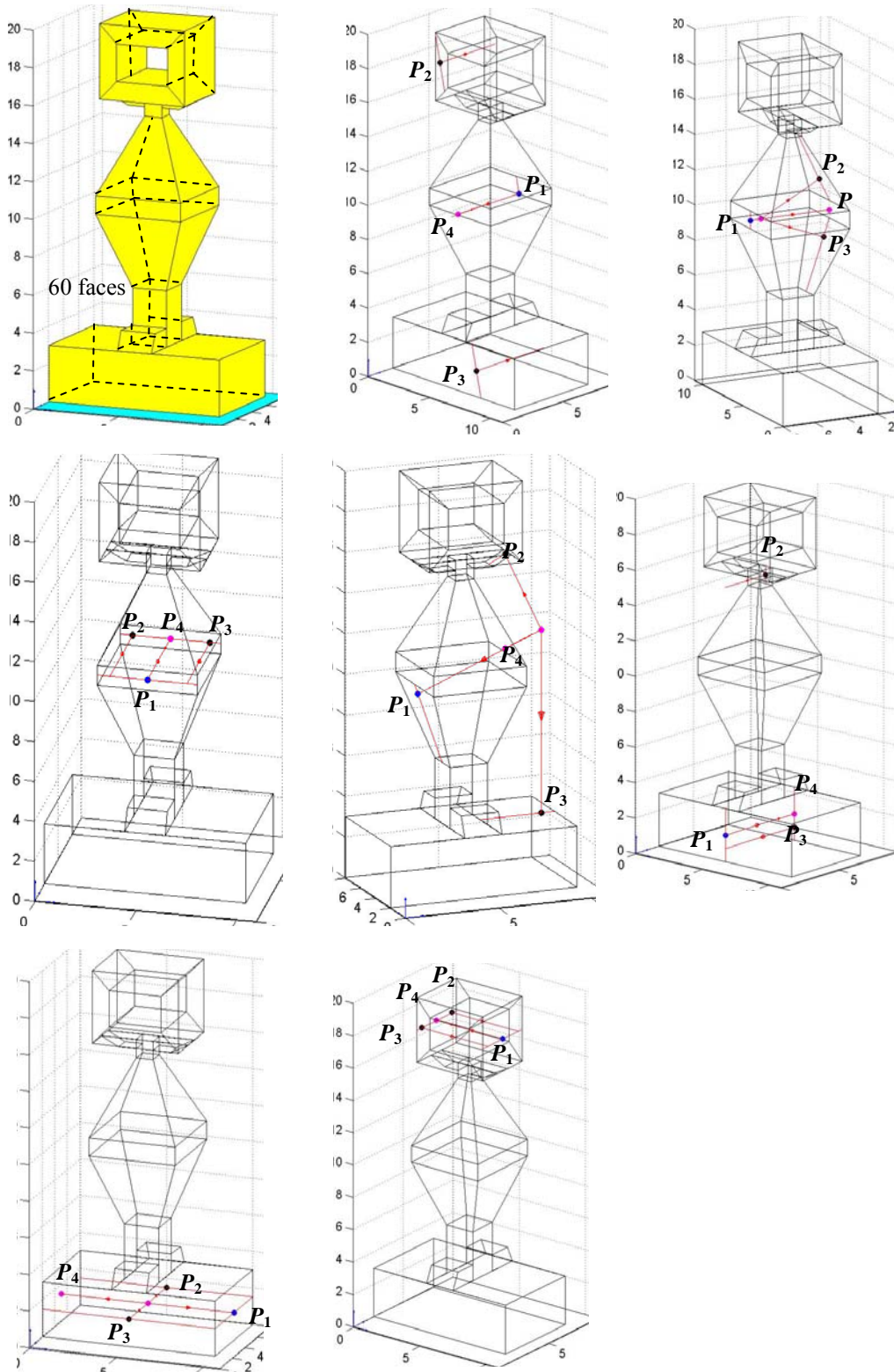


Fig. 12. Different FCG determined by the approach proposed on an object of 60 faces.

IV. Conclusion

The proposed procedure determines, given a FCG with three contact points, all the regions on the object boundary such that if a fourth finger is positioned in any of them results a FCG that allows the loss of contact at two of the three original contact points without losing the force closure property.

This type of grasp can take place on sets of two, three or four object faces, either parallel or non parallel, and is useful to allow the object manipulation by changing the finger positions on the object (*finger gating*). The proposed approach is based on geometric operations in the 3D physical space.

First, a necessary and sufficient condition for the existence of this type of grasp was stated; second, an algorithm for the computation of the contact region for the fourth finger was presented; and, finally, a procedure to determine a convenient contact point within this region was described, trying to obtain a planar grasp reachable with an anthropomorphic hand (but this is not formally assured).

As future work it is considered the development of manipulation strategies based on a sequence of grasps determined with the approach presented in this work.

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