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Taking decisions on assignment of tasks with path dependent learning curves

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Performance in doing a task in the learning stage depends on the previous practice on other tasks –learning curve is path dependent. A mathematical program aimed to assign a set of tasks to a set of workers considering path dependent learning curves is presented. Due dates for task and final knowledge objectives are possible, some tasks can be subcontracted and temporary workers can be hired. The model allows obtaining the cost of due dates and final knowledge.

1. Introduction

The performance is, at least in part, a consequence of experience. A conventional point of view neglects to consider the effect of the experience obtained during the planned horizon on performance in the same horizon period. This effect is sometimes negligible – when we are planning a period too short to generate significant experience in the tasks involved – but often it is not. In effect, when a set of tasks is being performed, the experience acquired in the first stages can obviously influence the capacity of the worker to do this task and other tasks later. Despite this, to the author's knowledge, assigning and scheduling a set of tasks that consider the influence of experience acquired in one task on other tasks has not been dealt with in the literature, probably due to the hard computational problem that this generates.

The relationship between experience and performance has been widely studied and this has led to the emergence of the concept of learning curve, which shows the relationship between experience and performance. It is based on the premise that the performance of a task by an organization or person improves with experience. In addition to experience in the task to be done, other factors influencing the capacity of an individual have been considered. In particular, the influence of the experience acquired in one task on the performance of another was presented by Olivella (2007).

The assignment and scheduling of a set of tasks has been dealt with in the literature (e.g. Alfares and Bailey 1997; Corominas et al. 2006; Pastor and Corominas 2007; Lusa et al. 2008). As far as the authors are aware assignment and scheduling of a set of tasks that consider the influence of experience acquired in one task on other tasks had not been previously discussed in the literature.

The contribution of this paper lies in presenting and modelling the assignment of a set of tasks to a set of workers, when worker's performance on a task depends on the experience of the worker of this task and of the other tasks involved. Due dates for task and final knowledge objectives can be stated. When regular staff cannot cover all the needs some tasks can be subcontracted and temporary workers can be hired. The cost of due dates and final knowledge can be obtained.

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2. Problem

A mathematical programming model will be defined employing the assumptions that follow:

- A set of tasks need to be done by a set of workers. Each task can only be done from a determined moment (ready time), expressed in units of time from the beginning of the planning horizon.
- The efficiency of a worker on a task depends on experience of all tasks involved, and is obtained as a function of the sum of the experience of this task and the equivalent experience, which is obtained by doing other tasks.
- The time is divided into periods of the same length. In each period a worker is assigned to a single task or to none; changes in assignments during a given period are not possible. In a given period a task can be only assigned to one worker.
- The volume of work necessary to complete each task is expressed in units of work. These units are, typically, hours of work when the work is carried out by an experienced worker – i.e. hours of standard work time. The efficiency of a worker in doing a task is the work that he is able to do in relation to standard. In planning the maximum of a worker's efficiency is 1.
- Precedence between tasks is possible.
- Objectives for efficiency of the workers of a task at the end of the planning horizon can be stated –for example, that the worker A achieves efficiency 0.8 in the task X.
- A due date can be established for each task –in concrete, a period at the end of which the task has to be done.
- Some of the tasks can be total or partially outsourced, i.e., entrusted to outside suppliers. This supposes a cost which depends on the volume of work necessary to complete the outsourced workload, measured in units of work.
- Some of the workers are on a temporary contract. As they are supposed to have initial experience zero and no stated objective of final efficiency, they are usually assigned to the tasks whose efficiency is less dependent on experience. Hiring a worker has an initial cost due to selection process, administrative tasks and giving instructions to him, and a cost proportional to the length of the contract. A temporary worker can be hired only for one uninterrupted interval of time, which starts at the beginning of a period and finishes at the end of another.
- The objective function is the costs of outsourcing and the cost of temporary workers, both the cost of hiring and the cost that depends on the time he is in the company.

3. Formulation of the model

Data

J	Number of tasks (j=1..J).
W	Number of workers (i=1..W).
T	Number of periods in the planning horizon (t=1..T).
v_j	Volume of work of task j to be done, measured in units of work, typically hours of work when the work is carried out by an experienced worker –i.e. hours of standard work time.
m_j	Lower bound of the number of periods required to complete task j obtained from the volume of work to be done, the initial experience of the workers and the performance function defined below.
n_j	Upper bound of the volume of work for task j done in a given period, obtained

from the performance function defined below.

r_j — Ready time of task j–the task cannot start before the beginning of period r_j. When a is an immediate predecessor of b, r_b is necessarily equal or greater than r_a+m_a.

e^0_{ji} — Initial experience of task j of worker i, measured in the same units of work as the volume of work.

$q_{jj'}$ — Proportion of the experience of task j' that is equivalent to the experience of task j when doing task j.

b_j — Upper bound of equivalent experience of task j obtained by doing other tasks.

O — Group of tasks that can be outsourced ($O \subseteq \{1..J\}$).

co_j — Cost of outsource the task j ($j \in O$).

H — Group of workers that can be hired on a temporary contract or not contracted ($H \subseteq \{1..W\}$).

ch — Cost of hire a temporary worker.

cp — Cost of a period of work of a temporary worker.

P — Set of pairs (a,b), where task a is an immediate predecessor of task b.

G,g_{ji} — Set of pairs (j,i) and values g_{ji}, for $(i,j)\in G$, where the efficiency in doing the task j of worker i at the end of the planning horizon has to be g_{ji} or more –i.e, g_{ji} is the goal of worker i for task j efficiency.

d_j — Due date of task j –task j has to be done at the end of the period d_j, being $d_j>r_j+m_j$ and $d_j \leq T$.

ε — Small positive number.

Variables

x_{jit} — Binary variable that indicates whether task j is done by worker i in a period of time t ($i=1..W, j=1..J, t=r_j..d_j$).

e_{jit} — Experience of task j of worker i before the beginning of a period of time t, measured in units of work done –typically hours of standard time ($i=1..I, j=1..J, t=r_j+1..T$ for $d_j=T$ and $(i,j)\notin G, t=r_j+1..d_j+1$ otherwise).

e'_{jit} — Equivalent experience of task j of worker i before the beginning of a period of time t, measured in units of work done (obtained by doing other tasks) ($i=1..I, j=1..J, t=r_j+1..d_j+1$ for $(i,j)\in G$, $t=r_j+1..d_j$ otherwise).

s_{jt} — Volume of work in the task j that has been done at the end of the period t ($j=1..J, t=r_j..d_j$).

δ_{jt} — Binary variable that is equal to 1 *if* the task j has been completed in or before the period t ($j=1..J, t=r_j+m_j..d_j-1$).

o_j — Binary variable that indicates whether task j is outsourced j ($j\in O$).

ti_i — Integer positive variable that indicates the initial period of work of the temporary worker i ($i\in H$).

tl_i — Integer positive variable that indicates the length of the contract of the temporary worker i, measured in number of periods ($i\in H$).

th_i — Binary variable that indicates whether temporary worker i is hired ($i\in H$).

Performance function

$\varphi_j(e_{jit}+e'_{jit})$ — Number of units of work of task j that worker i is expected to do in period t, that is function of e_{jit} –experience of the worker i of task j at the beginning of the period t– and e'_{jit} –equivalent experience of the worker i of task j at the beginning of the period t.

Constraints

Constraint (1) means that a worker cannot simultaneously carry out more than one task during period t, (2) applies to the tasks that cannot be outsourced, and means that a task cannot be simultaneously carried out by two or more workers and (3) applies to the task that can be outsourced and is equivalent to (2) when the task is not outsourced (o_j=0) and forces a task not to be assigned when it is (o_j=1).

$$\sum_{j=1..J|t\geq r_j} x_{jit} \leq 1, \quad i=1..W,\ t=1..d_j \tag{1}$$

$$\sum_{i=1}^{W} x_{jit} \leq 1, \quad j\in\{1..J\}-O,\ t=r_j..d_j \tag{2}$$

$$\sum_{i=1}^{W} x_{jit} \leq (1-o_j), \quad j\in O,\ t=r_j..d_j \tag{3}$$

Constraint (4) initializes accumulative variables e_{jit} , while (5) makes e_{jit} equal to the work done by a worker i of a task j until de beginning of a period t, measured in units of work done. When due date (d_j) is less than T, the accumulate work at the beginning of the period d_j+1 has to be obtained because it influences the efficiency on other tasks not finished yet. The values of $e_{ji(T+1)}$ are necessary only for the workers and tasks with a goal of efficiency at the end of the planning horizon; constraint (6) applies then to this cases. Constraints (5) and (6) are not linear for two reasons: it includes a product between a variable and a function and this function is not necessarily linear. The linearization of the model is described later.

$$e_{jir_j} = e^0_{ji}, \quad j=1..J, i=1..W \tag{4}$$

$$e_{jit} = e_{ji(t-1)} + x_{ji(t-1)} \cdot \varphi_j(e_{ji(t-1)} + e'_{ji(t-1)}), \quad j=1..J, i=1..W, t=r_j+1..\min(d_j+1,T) \tag{5}$$

$$e_{ji(T+1)} = e_{jiT} + x_{jiT} \cdot \varphi_j(e_{jiT} + e'_{jiT}), \quad (j,i)\in G, d_j = T \tag{6}$$

Constraints (7) and (8) make variables e'_{jit} the experience in a task acquired by carrying out other tasks. Equivalent experiences are obtained by linear combination of experience in other tasks (7) and are limited by the bounds of equivalent experience b_j (8).

$$e'_{jit} \leq \sum_{j'\in J|j'\neq j \wedge t\leq r_{j'}} q_{jj'} \cdot e^0_{j'i} + \sum_{j'\in J|j'\neq j \wedge t> r_{j'}} q_{jj'} \cdot e_{j'i\min(d_{j'}+1,t)}$$
$$j=1..J, i=1..W, t=r_j..d_j\ (for(i,j)\notin G),\ t=r_j..d_j+1\ (for(i,j)\in G) \tag{7}$$

$$e'_{jit} \leq b_j \quad j=1..J, i=1..W,\ t=r_j..d_j\ (for(i,j)\notin G),\ t=r_j..d_j+1\ (for(i,j)\in G) \tag{8}$$

Constraint (9) makes s_{jt} the accumulated volume of work done at the end of period t. This constraint, as the constraints (5) and (6), is not linear for two reasons: it includes a product between a variable and a function and this function is not necessarily linear. The linearization of the model is described later.

$$s_{jr_j} = \sum_{i=1}^{W} x_{jir_j} \cdot \varphi_j(e_{jir_j} + e'_{jir_j}), \quad j=1..J \tag{9}$$

$$s_{jt} = s_{j(t-1)} + \sum_{i=1}^{W} x_{jit} \cdot \varphi_j(e_{jit} + e'_{jit}), \quad j=1..J, t=r_j+1..d_j \tag{10}$$

Constraint (11) applies to the task that cannot be outsourced and implies that the tasks are completely finished at the end of the planning horizon. Constraint (12) applies to the task that can be outsourced and when the task is not outsourced (o_j=0) implies that the tasks are completely finished, while when the task is outsourced (o_j=1) the inequation is always true. Constraint (13) implies that δ_{jt} is equal to 1 when task *j* is completed at the end of period *t*, and constraint (14) guarantees that work is only assigned to non-completed tasks. Variable δ_{jt} must be equal to 1 when task *j* is completed at the end of the period *t* —as imposed by constraint (13) —and 0 beforehand. This last condition is implicit in the formulation: if δ_{jt} is 1, no more assignments to task *j* in the periods from *t* to *T* are possible; thus, if δ_{jt} was 1 when task *j* had not been completed in period *t*, task *j* would not be completed. The small constant number ε is introduced in the equations (11) and (13) because otherwise a task would be considered completed only when the volume of work to cope with task is exceeded, and not when it is exactly covered.

$$s_{jd_j} \geq v_j - \varepsilon, \quad j \in \{1..J\} - O \tag{11}$$

$$s_{jd_j} \geq v_j \cdot (1 - o_j) - \varepsilon, \quad j \in O \tag{12}$$

$$s_{jt} \leq v_j - \varepsilon + n_j \cdot \delta_{jt}, \quad j = 1..J, t = r_j + m_j..d_j \tag{13}$$

$$\sum_{i=1}^{W} x_{jit} \leq 1 - \delta_{j,t-1}, \quad j = 1..J, t = r_j + m_j + 1..d_j \tag{14}$$

Constraint (15) supposes that the initial period and the length of the contract of a temporary worker are 0 when he is not finally hired and non greater than the number of periods of the planning horizon when he is. Constraint (16) and (17) makes that a temporary worker is not assigned to any task before his hiring or after the end of his contract, respectively.

$$ti_i + tl_i \leq th_i \cdot T, \quad i \in H \tag{15}$$

$$(T - t) \cdot x_{jit} \leq (T - ti_i), \quad j = 1..J, i \in H, \ t = r_j..d_j \tag{16}$$

$$t \cdot x_{jit} \leq ti_i + tl_i - 1, \quad j = 1..J, i \in H, \ t = r_j..d_j \tag{17}$$

Constraints (18) guarantees that the precedence between tasks is respected $-a$ is an immediate predecessor of b and thus $r_a+m_a < r_b$, as defined. Constraint (19) imposes the goals of efficiency at the end of the planning horizon, while constraint (20) imposes the due dates.

$$x_{bit} \leq \delta_{a(t-1)}, \quad (a,b) \in P, i = 1..W, t = r_b..d_j \tag{18}$$

$$\varphi_j(e_{jid_j+1} + e'_{jid_j+1}) \geq g_{ji}, \quad (j,i) \in G \tag{19}$$

$$\delta_{jt} = 1, \quad (j,t) \in D \tag{20}$$

Objective function

The objective function (21) includes the cost of outsourcing, the cost of the fact of hiring temporary workers and the cost proportional to the length of the contract of the temporary workers.

$$\sum_{j \in O} co_j \cdot o_j + \sum_{i \in H} ch \cdot th_i + cp \cdot tl_i \tag{21}$$

4. Linearization and solution of the model

To approximate the solution of the defined mathematical program, constraints (5),(6),(9),(10) and (19) must be approximated by a linear expression. By assuming that function φ_j is concave, we consider a piecewise linear approximation. We consider the data, variables and constraints that follow:

Data

L_j	Number of intervals in the linear approximations of φ_j $(l=1..L_j)$
br_{jl}	Length of intervals in the linear approximations of φ_j $(j=1..J,l=1..L_j)$
α_j	Ordinate intercept of the linear approximation of φ_j corresponding to task j
β_{jl}	Slope of the l interval in the linear approximation of φ_j corresponding to task j,

Variables

y_{jitl}	Variables in the linear approximation of function φ_j corresponding to task j, worker i, period t and interval l
u_{jit}	Number of units of work for task j that worker i does in the period t
z_{jil}	Variables in the linear approximation of function φ_j corresponding to task j, worker i, period $T+1$ and interval l
w_{ji}	Number of units of work for task j that worker i could do in the period $T+1$

Constraints

$$y_{jitl} \le br_{jl}, \quad l=1..L_j, j=1..J, i=1..W, t=r_j..d_j \tag{22}$$

$$y_{jitl} \ge 0, \quad l=1..L_j, j=1..J, i=1..W, t=r_j..d_j \tag{23}$$

$$e_{jit} + e'_{jit} \ge \sum_{l=1}^{L_j} y_{jitl}, \quad j=1..J, i=1..W, t=r_j..d_j \tag{24}$$

$$u_{jit} \le \alpha_j + \sum_{l=1}^{L_j} \beta_{jl} \cdot y_{jitl}, \quad j=1..J, i=1..W, t=r_j..d_j \tag{25}$$

$$z_{jil} \le br_{jl}, \quad l=1..L_j, (j,i) \in G \tag{26}$$

$$z_{jil} \ge 0, \quad l=1..L_j, (j,i) \in G \tag{27}$$

$$e_{jid_j+1} + e'_{jiT+1} \ge \sum_{l=1}^{L_j} z_{jil}, \quad (j,i) \in G \tag{28}$$

$$w_{ji} \le \alpha_j + \sum_{l=1}^{L_j} \beta_{jl} \cdot z_{jil}, \quad (j,i) \in G \tag{29}$$

To linearize the constraints (5),(6),(9) and (10) the expression $x_{jit} \cdot \varphi_j$ has to be replaced by a linear approximation. To do this u_{jit} has to be an approximation of φ_j when x_{jit} is 1 and 0 otherwise. The constraint (30) imposes this condition.

$$u_{jit} \le n_j \cdot x_{jit}, \quad j=1..J, i=1..W, t=r_j..T \tag{30}$$

The expression $x_{jit} \cdot \varphi_j$ is replaced by u_{jit}, giving place to constraints (31),(32),(33) and (34), that replaces constraints (5),(6),(9) and (10) and respectively.

$$e_{jit} = e_{ji(t-1)} + u_{ji(t-1)}, \quad j=1..J, i=1..W, t=r_j+1..\min(d_j+1, T) \tag{31}$$

$$e_{jit} = e_{ji(t-1)} + u_{ji(t-1)}, \quad (j,i) \in G, d_j = T \tag{32}$$

$$s_{jr_j} = \sum_{i=1}^{W} u_{jir_j}, \quad j=1..J \tag{33}$$

$$s_{jt} = s_{j(t-1)} + \sum_{i=1}^{W} u_{jit}, \quad j = 1..J, t = r_j + 1..d_j \tag{34}$$

In addition, constraint (19) is replaced by the constraint (35)

$$w_{ji} \geq g_{ji}, \quad (j,i) \in G \tag{35}$$

Finally, constraints (36), (37) and (38) are included to make the calculations easier.

$$\delta_{jt} \leq \delta_{j(t+1)}, \quad j \in J, t = r_j + m_j..d_j - 2 \tag{36}$$

$$s_{jt} \geq \delta_{jt} \cdot (v_j - \varepsilon), \quad j \in J, t = r_j + m_j..d_j - 1 \tag{37}$$

$$th_i \leq tl_i, \quad i \in H \tag{38}$$

Numerical tests were done and the adequacy of the model was verified.

References

Alfares, H.K. and Bailey, J.E. (1997), "Integrated project task and manpower scheduling" IIE Transactions, 29, 711-717.

Corominas, A., Pastor R. and Rodríguez, E. (2006), "Rotational allocation of tasks to multifunctional workers in a service industry." International Journal of Production Economics 103, 3-9.

Lusa, A., Pastor, R. and Corominas, A. (2008), "Determining the most appropriate set of weekly working hours for planning annualised working time." International Journal of Production Research.

Olivella, J. (2007). An experiment on task performance forecasting based on the experience of different tasks. IN: Proceedings of I-KNOW' 07 . Tochtermann, K.; Maurer, H.:, 2007, p. 305-312.

Pastor, R. and Corominas, A. (2007). “Job assignment” in Nembhard, D. A. (ed.) Workforce Cross Training, Taylor & Francis Ltd., Boca Raton, Fla.