

# Parameter Identification of Large-Scale Magnetorheological Dampers in a Benchmark Building Platform

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**Abstract**—Magnetorheological (MR) dampers are devices that can be used for vibration reduction in structures. However, to use these devices in an effective way, a precise modeling is required. In this sense, in this paper we consider a modified parameter identification method of large scale magnetorheological dampers which are represented using the normalized Bouc-Wen model. The main benefit of the proposed identification model is the accuracy of the parameter estimation. The validation of the parameter identification method has been carried out using a black-box model of an MR damper in a smart base-isolated benchmark building. Magnetorheological (MR) dampers are used in this numerical platform both as isolation bearings as well as semiactive control devices.

## I. INTRODUCTION

Magnetorheological (MR) dampers are devices that change their mechanical properties when they are exposed to a magnetic field. The magnetorheological fluid of these actuators is characterized by a great ability to vary, in a reversible way, from a free-flowing linear viscous liquid to a semi-solid one within milliseconds [1]. Moreover, MR dampers have a low cost and can be controlled with a low voltage at the coils [1]. All these features make MR dampers very attractive and promising as actuators controlled by the voltage that can be used in different engineering fields, such as dampers and shock absorbers (pressure driven flow mode devices), as well as clutches, brakes, chucking, and locking devices (direct-shear mode devices) [5]. From a structural control point of view, MR dampers are usually employed as actuators operated by low voltages. In this respect, semi-active control systems seem to combine the best compromise between *passive* and *active* control: they offer the reliability of passive devices together with the versatility and adaptability of active

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systems [13]. However, the first step in the design of a semi-active control strategy is the development of an accurate model of the MR device. It is worth noting that the system-identification issue plays a key role in this control problem [12]. High-accuracy models can be designed using two different model families: semi-physical models [10], [13], and black-box models [6], [16]. Some of the most known semi-physical models to describe the hysteretic behaviour of MR dampers are the Bingham model and its extended versions, the Bouc-Wen model, the Dahl model, the modified LuGre model and some other non-parametric models [11]. It is important to remark that these models are not linear-in-parameters and, therefore, classical parameter identification methods, such as the gradient or the mean square algorithms, cannot be applied.

In this paper we present a modified parameter identification method of large scale magnetorheological dampers in a benchmark building platform [7]. Using a normalized version of the Bouc-Wen model, Ikhouane et al. [4] present an identification algorithm which is directly used for MR dampers in shear-mode [11]. However, this methodology can produce large parameter identification errors if the viscous friction is much smaller than the dry friction [11]. To cope with this drawback, a modified step was proposed by Rodríguez et al. [11]. To make the process even more accurate and general to apply on large-scale MR dampers, a modified identification method is proposed in this work by considering the extended form of the identified Bouc-Wen model that takes into account three terms instead of just two. The validation of this modified parameter identification method has been carried out using a black-box model of an MR damper in a smart base-isolated benchmark building. The benchmark platform is then considered as a virtual laboratory experiment. The numerical results show that the proposed modified method is able to improve significantly the accuracy of the parameter identification.

The paper is organized as follows. In Section II, the normalized Bouc-Wen model is presented. Section II also presents the key points of the modified identification method. In Section III, the application of the proposed identification method to a large-scale MR damper in a benchmark building platform is considered. Finally, some concluding remarks are stated in Section IV.

## II. THE NORMALIZED BOUC-WEN MODEL

The normalized version of the Bouc-Wen model [11] is an equivalent representation of the original Bouc-Wen model [16]. The normalized Bouc-Wen model of MR dampers in

shear mode is then given by:

$$F(\dot{x}, w)(t) = \kappa_{\dot{x}}(v)\dot{x}(t) + \kappa_w(v)w(t), \quad (1)$$

$$\begin{aligned} \dot{w}(t) &= \rho(\dot{x}(t) - \sigma|\dot{x}(t)||w(t)|^{n-1}w(t) \\ &+ (\sigma - 1)\dot{x}(t)|w(t)|^n), \end{aligned} \quad (2)$$

where  $F(\dot{x}, w)$  is the output force of the MR damper,  $\dot{x}(t)$  and  $v$  are the velocity and voltage inputs, respectively. The voltage input  $v$  is the applied voltage at the coil of the MR. The system parameters are  $\kappa_{\dot{x}}(v) > 0$ ,  $\kappa_w(v) > 0$ ,  $\rho > 0$ ,  $\sigma > 1/2$ , and  $n \geq 1$ . These parameters control the shape of the hysteresis loop and their meaning can be found in [3]. The state  $w(t)$  has not a physical meaning so that it is not accessible to measurements.

Since the normalized Bouc-Wen representation described in equations (1)-(2) is not a linear-in-parameter model, classical parameter identification methods cannot be applied. In this regard, a new parameter identification algorithm has been proposed in [11], at the same time as it is based on the identification method presented in ([4], p. 38). The method is based on applying a periodic input velocity  $\dot{x}(t)$  at a constant voltage coil  $v$  and observing the periodic steady-state force response of the MR damper. The details of this method are omitted here and can be found in the Appendix. Nonetheless, large relative errors in the identification process can be observed when the MR damper has a friction force small enough with respect to the dry friction [11]. To improve the accuracy of this parameter identification, we use the following extended Bouc-Wen model:

$$F_1(x, \dot{x}, w)(t) = \kappa_x(v)x(t) + \kappa_{\dot{x}}(v)\dot{x}(t) + \kappa_w(v)w(t), \quad (3)$$

$$\begin{aligned} \dot{w}(t) &= \rho(\dot{x}(t) - \sigma|\dot{x}(t)||w(t)|^{n-1}w(t) \\ &+ (\sigma - 1)\dot{x}(t)|w(t)|^n), \end{aligned} \quad (4)$$

where the linear force  $\kappa_x(v)x$  has been added keeping its voltage dependence as in the other parameters. At constant voltage, the computation of the parameter  $\kappa_x(v)$  is straightforward. For instance, we can consider the force-displacement diagram of the MR damper under a large enough sinusoidal displacement. With respect to this diagram, the average inclination of the resulting ellipsoid is an estimation of this parameter. As an example, consider the numerical experiment of an MR damper driven with zero coil command voltage. The force-displacement diagram of this numerical experiment is shown in Figure 1 (top). The estimated value of  $\kappa_x(0)$  is 83 kN. To estimate the rest of the parameters, we use the knowledge of  $\kappa_x(0)$  and the following modified dynamic:

$$\begin{aligned} F_2(\dot{x}, w) &= F_1(x, \dot{x}, w) - \kappa_x(v)x(t) \\ &= \kappa_{\dot{x}}(v)\dot{x}(t) + \kappa_w(v)w(t), \end{aligned} \quad (5)$$

$$\begin{aligned} \dot{w}(t) &= \rho(\dot{x}(t) - \sigma|\dot{x}(t)||w(t)|^{n-1}w(t) \\ &+ (\sigma - 1)\dot{x}(t)|w(t)|^n). \end{aligned} \quad (6)$$

Figure 1 (bottom) shows the typical form of the force-velocity diagrams for both  $F_1$  and  $F_2$  dynamics.  $F_2$  is produced by subtraction of the linear force  $\kappa_x(v)x$  from the measured force  $F_1$  (produced by the MR damper). In

summary, an estimation of the Bouc-Wen parameters in equations (3)-(4) can be obtained through the following parameter identification algorithm:

- ▶ Step 1. Keep the coil voltage at a constant value;
- ▶ Step 2. Estimate the value of  $\kappa_x$  (this can be done graphically);
- ▶ Step 3. Using model (5)-(6) and the identification algorithm in [11], estimate the rest of the parameters.

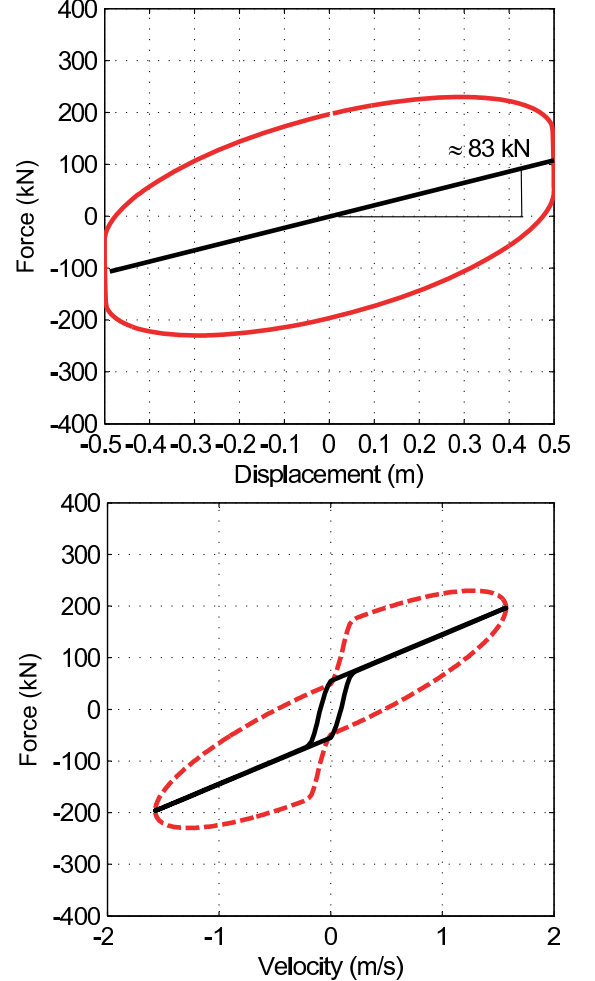


Fig. 1. Force-displacement diagram for an MR damper with zero coil command voltage (red) and estimation of  $\kappa_x(v)$  (black) (top). Force-velocity diagram for both  $F_1$  (dashed) and  $F_2$  (solid) dynamics (bottom).

We remark that with the above algorithm we can obtain the estimation of a complete set of Bouc-Wen model parameters based on a reduced model (1)-(2) (or (5)-(6)). This way we can avoid the extension algorithm given in [4] or in [11] to model (3)-(4), which would be a hard work.

### III. PARAMETER IDENTIFICATION OF MR DAMPERS IN A BENCHMARK PLATFORM

The following section shows the model validation using a fluctuating current and a varying displacement signal. More precisely, the proposed identification algorithm is tested using a black-box model of an MR damper in a smart base-isolated benchmark building.

### A. Smart base-isolated benchmark building

The smart base-isolated benchmark building [8] is employed as an interesting and more realistic example to further investigate the effectiveness of the proposed design approach. This benchmark problem is recognized by the American Society of Civil Engineers (ASCE) Structural Control Committee as a state-of-the-art model developed to provide a computational platform for numerical experiments of seismic control attenuation [9], [14].

The benchmark structure is an eight-storey frame building with steel-braces, 82.4 m long and 54.3 m wide, similar to existing buildings in Los Angeles, California. Stories one to six have an L-shaped plan while the higher floors have a rectangular plan. The superstructure rests on a rigid concrete base, which is isolated from the ground by an isolator layer, and consists of linear beam, column and bracing elements and rigid slabs. Below the base, the isolation layer consists of a variety of 92 isolation bearings. The isolators are connected between the drop panels and the footings below, as shown in Figure 2.

### B. Model validation

Figure 3 shows the response of the MR damper in the benchmark building platform to a periodic wave displacement excitation at zero coil voltage. It is noticeable that the shape of these signals are very close to that observed experimentally in [11].

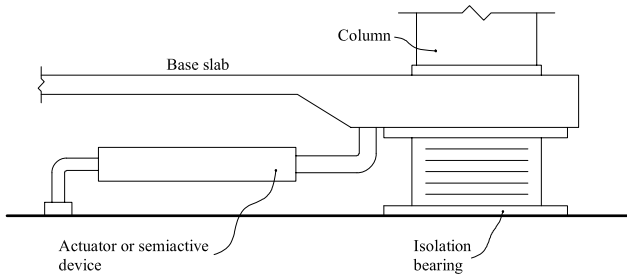


Fig. 2. Elevation view with devices

Following the parameter identification algorithm given in Section II, the resulting values are depicted in Figure 4 for several coil voltages, where parameters are plotted versus voltage along with the best coloration curve computed for

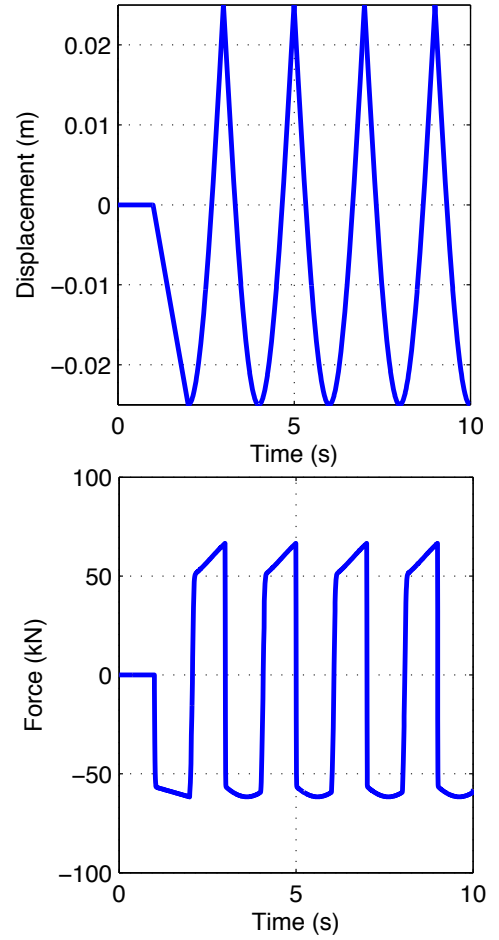


Fig. 3. Response of the MR damper in the benchmark building platform.

each of them. Accordingly the identified parameters are:

$$\kappa_x(v) = \kappa_x \quad (7)$$

$$\kappa_{\dot{x}}(v) = \kappa_{\dot{x},a} + \kappa_{\dot{x},b}v \quad (8)$$

$$\kappa_w(v) = \begin{cases} \kappa_{w_1} + \kappa_{w_2}v^{1.15}, & v \leq 0.3 \\ \kappa_{w_3} + \kappa_{w_4} \sin\left(\frac{\pi(v-0.3)}{0.8}\right) + \kappa_{w_5} \sin\left(\frac{3\pi(v-0.3)}{0.8}\right), & 0.3 \leq v \leq 0.7 \\ \kappa_{w_6} + \kappa_{w_7}v + \kappa_{w_8}v^3 + \kappa_{w_9}v^5, & 0.7 \leq v \end{cases} \quad (9)$$

$$\rho(v) = \rho_a + \rho_b \exp(-14v) \quad (10)$$

$$n(v) = n_a + n_b \exp(-13v) \quad (11)$$

$$\sigma(v) = \sigma_a + \sigma_b \exp(-14v) \quad (12)$$

The parameters in equations (7)-(12) can be easily computed using least-squares fitting algorithms, as can be seen in Table I.

The identification models presented in the literature usually have good accuracy when they consider a constant voltage. Because of the role of MR dampers as a semi-active devices in structural control systems, the final identified model has to be checked under a simulated condition using, for instance, an earthquake record and the corresponding

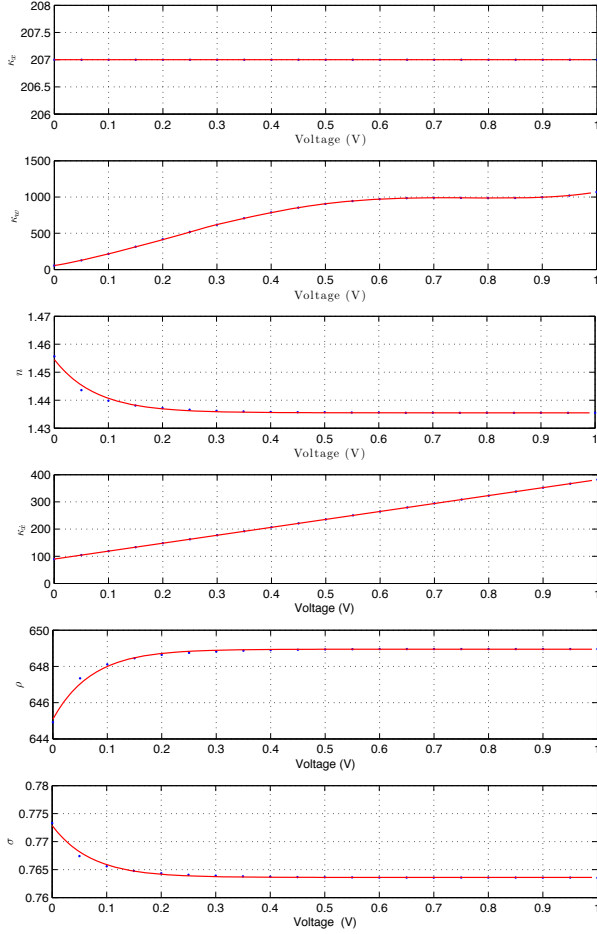


Fig. 4. Results of the parameter identification algorithm.

varying command voltage. To do this, our identified model has to be compared with the corresponding black-box model of MR damper in the benchmark building platform, under exactly the same situation. To measure the discrepancy between the two models, the 1-norm error ( $\varepsilon$ ) is used [11]:

$$\varepsilon = \frac{\|F_{BM} - F_{id}\|_1}{\|F_{BM}\|_1}, \quad (13)$$

$$\|f\|_1 = \int_0^{T_r} |f(t)| dt, \quad (14)$$

where  $F_{BM}$  is the output force of the black-box model (benchmark building platform) and  $F_{id}$  is the resulting force of the identified MR damper based on the Bouc-Wen model. The length in time of each earthquake is denoted by  $T_r$ . The 1-norm is a measure that reflects the average size of a signal and thus it is a good tool for computing the discrepancy between these two models. Based on this 1-norm, if the computed value of the damping force is far from the reference value, the value of  $\varepsilon$  will be large. On the contrary, if it is small, the identified model can calculate forces which are very close to the *real* ones. Table II presents the model errors for several earthquakes ( $FP-X$  and  $FP-Y$  are the estimation errors in the  $x$ -force and  $y$ -force directions).

TABLE I  
RESULTS FOR PARAMETER IDENTIFICATION

parameter	value	
$k_x$	207	
$k_{\dot{x}}$	$k_{\dot{x},a}$	89.64
	$k_{\dot{x},b}$	292
$k_w$	$k_{w1}$	55.38
	$k_{w2}$	2270
	$k_{w3}$	619.85
	$k_{w4}$	387.34
	$k_{w5}$	18.42
	$k_{w6}$	-87.52
	$k_{w7}$	2665
	$k_{w8}$	-3054.7
	$k_{w9}$	1545.4
$\rho$	$\rho_a$	648.95
	$\rho_b$	-3.86
$\eta$	$\eta_a$	1.44
	$\eta_b$	0.02
$\sigma$	$\sigma_a$	0.76
	$\sigma_b$	0.009

A sample earthquake record and the corresponding signal voltage during the control process is presented in Figure 5. It is interesting to compare the resulting model errors in Table II with the resulting model errors when the parameter identification is performed with the model in equations (1)-(2) (see Table III). By analysing and comparing these two tables, the proposed parameter identification algorithm is clearly more accurate than the original method. Figure 6 shows the damper force using our model (3)-(4) and (7)-(12) with the identified parameters for the case when a sinusoidal displacement input is applied at a constant amplitude and varying its frequency, for a constant voltage of 0.1 V.

TABLE II  
ERROR NORM ( $\varepsilon$ ) FOR THE PROPOSED PARAMETER IDENTIFICATION

	Newhall	Sylmar	El Centro	Rinaldi	Kobe	Jiji	Erzinkan
$FP-X$	6.47 %	5.67 %	7.78 %	7.12 %	6.52 %	3.61 %	4.88 %
$FP-Y$	3.84 %	8.44 %	7.90 %	5.67 %	7.85 %	4.02 %	5.35 %

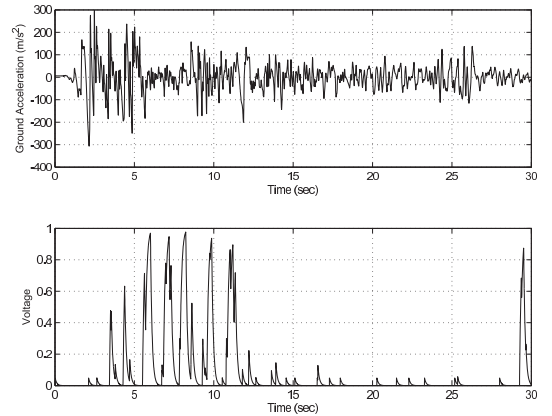


Fig. 5. El Centro, ground acceleration (top) and corresponding command voltage (bottom).

TABLE III  
ERROR NORM ( $\varepsilon$ ) FOR THE METHOD PROPOSED BY RODRÍGUEZ  
ET AL., 2008

	Newhall	Sylmar	El Centro	Rinaldi	Kobe	Jiji	Erzinkan
FP-X	16.15 %	18.06 %	22.89 %	17.55%	18.22 %	14.16 %	14.91 %
FP-Y	15.83 %	24.14 %	19.68 %	18.48 %	24.72 %	20.09 %	18.80 %

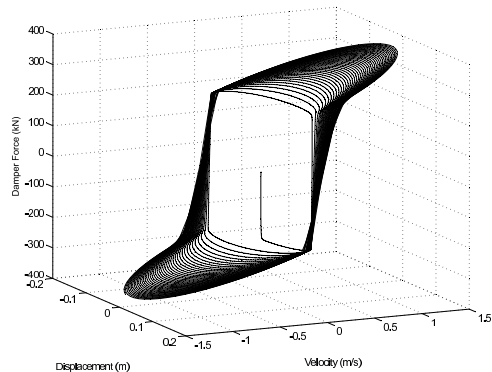


Fig. 6. 3D vision of damper force.

#### IV. CONCLUSION

Magnetorheological (MR) dampers are devices that can be used for vibration reduction in structures. However, to use these devices in an effective way, a precise modeling is required. In this sense, in this paper we have considered a modified parameter identification method of large scale magnetorheological dampers which are represented using the normalized Bouc-Wen model. The main benefit of the proposed identification model is the accuracy of the parameter estimation. The validation of the parameter identification method has been carried out using a black-box model of an MR damper in a smart base-isolated benchmark building. Magnetorheological (MR) dampers are used in this numerical platform both as isolation bearings as well as semiactive control devices.

#### APPENDIX

The normalized version of the Bouc-Wen model [4] is an equivalent representation of the original Bouc-Wen model [16]. This normalized model has less number of parameters thus eliminating the over-parameterizations present in the original model. The parameter identification in [11] departs from the next shear-mode Bouc-Wen model:

$$F(\dot{x})(t) = \kappa_{\dot{x}}(v)\dot{x}(t) + \kappa_w(v)(t) \quad (15)$$

$$\begin{aligned} \dot{w}(t) = & \rho(\dot{x}(t) - \sigma|\dot{x}(t)||w(t)|^{n-1}w(t) \\ & + (\sigma - 1)\dot{x}(t)|w(t)|^n) \end{aligned} \quad (16)$$

where  $\kappa_x > 0$ ,  $\kappa_w > 0$ ,  $\rho > 0$ ,  $\sigma > 1/2$ , and  $n \geq 1$ . For parameter identification, we have to apply a  $T$ -periodic input  $\dot{x}(t)$  (see Figure 2) to the Bouc-Wen system under constant voltage  $v$ . The identification method is based on some instrumental functions which has to deal with the

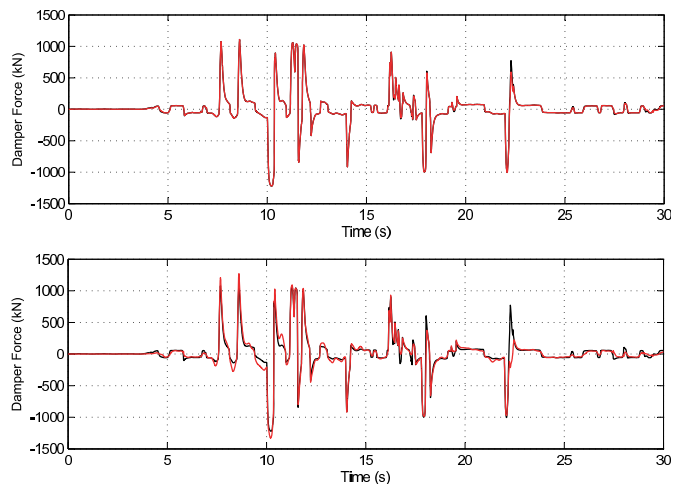


Fig. 7. Comparison of the MR damper force for the model with the proposed parameter identification (top) and for the model with the parameter identification method proposed by Rodríguez et al., 2008 (bottom) (red) and the response of the original black-box model (black) under Kobe ground motion (FP-y).

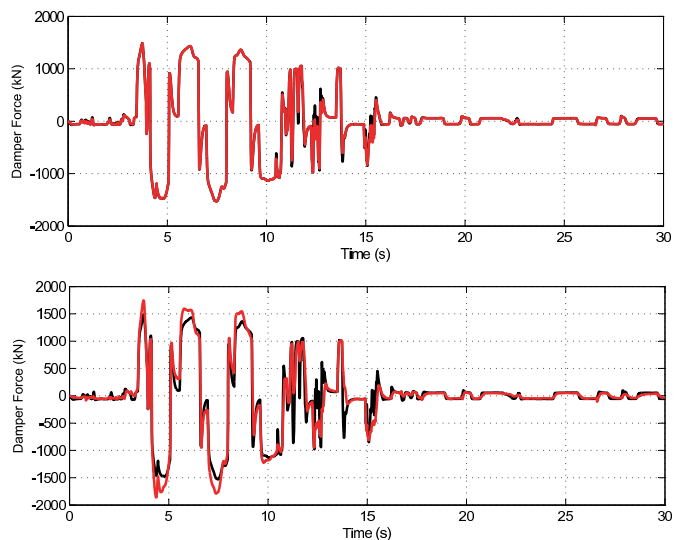


Fig. 8. Comparison of the MR damper force for the model with the proposed parameter identification (top) and for the model with the parameter identification method proposed by Rodríguez et al., 2008 (bottom) (red) and the response of the original black-box model (black) under Sylmar ground motion (FP-y).

hysteretic system. Basically, and because these functions are invertible, they described a limit cycle of the Bouc-Wen model when it is periodically excited [11], i.e. under a  $T$ -periodic input signal  $\dot{x}(t)$ , the output force of the Bouc-Wen model goes asymptotically to a periodic limit function, called  $\bar{F}(t)$ , which corresponds to periodic hysteretic behavior in the internal state, called  $\bar{w}(t)$ . The whole identification process can be summarized as follows.

The parameter  $\kappa_{\dot{x}}$  is first determined using the plastic region of the hysteresis loop by a linear regression for each

constant voltage:

$$\bar{F}(\tau) = \kappa_x(v)\dot{x}(\tau) + \kappa_w(v).$$

To continue with parametric estimation, a function  $\theta$  is computed as:

$$\theta(x(\tau)) = \bar{F}(x(\tau)) - \kappa_x \frac{dx(\tau)}{d\tau}, \quad \tau \in [0, T^+], \quad (17)$$

which has a unique zero, i.e., there exists a time instant  $\tau_* \in [0, T^+]$ , and a corresponding value  $x_* = x(\tau_*) \in [X_{\min}, X_{\max}]$ , such that the function  $\theta$  is zero. Because  $\theta$  is known, then  $\dot{x}_*$  is also known. Define the quantity

$$a = \left( \frac{d\theta(x)}{dx} \right)_{x=x_*}. \quad (18)$$

Then, the parameter  $n$  is determined as:

$$n = \frac{\log \left[ \frac{\left( \frac{d\theta(x)}{dx} \right)_{x=x_{*2}} - a}{\left( \frac{d\theta(x)}{dx} \right)_{x=x_{*1}} - a} \right]}{\log \left( \frac{\theta_{x=x_{*2}}}{\theta_{x=x_{*1}}} \right)} \quad (19)$$

where  $x_{*2} > x_{*1} > x_*$  are design parameters. Define

$$b = \frac{a - \left( \frac{d\theta(x)}{dx} \right)_{x=x_{*2}}}{\theta(x_{*2})^n}. \quad (20)$$

Then, the parameters  $\kappa_w$  and  $\rho$  are computed as follows:

$$\kappa_w = \sqrt[n]{\frac{a}{b}}, \quad (21)$$

$$\rho = \frac{a}{\kappa_w}. \quad (22)$$

The function  $\bar{w}(x)$  can be computed as:

$$\bar{w}(x) = \frac{\theta(x)}{\kappa_w}. \quad (23)$$

Finally, the remaining parameter  $\sigma$  is determined as:

$$\sigma = \frac{1}{2} \left( \frac{\left( \frac{d\bar{w}(x)}{dx} \right)_{x=x_{*3}} - 1}{\frac{\rho}{(-\bar{w}(x_{*3}))^n} + 1} \right) \quad (24)$$

where  $x_{*3}$  is a design parameter such that  $x_{*3} < x_*$ .

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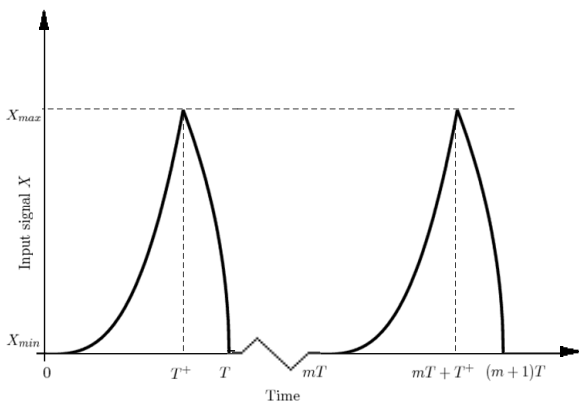


Fig. 9. A sample  $T$ -wave periodic signal