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Feedback passivity of nonlinear discrete-time systems with direct input–output link^{*1, *2}

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Abstract

This paper is devoted to the study of the feedback passivity property in nonlinear discrete-time systems. The relative degree and zero dynamics of the non-passive system are related to the feedback passivity of the system. Two main results are presented. First, some relative degree-related properties of passive systems in general form are stated. Second, sufficient conditions in order to render a multiple-input multiple-output (MIMO) system passive by means of a static state feedback control law are obtained.

Author Keywords: Discrete-time systems; Nonlinear systems; Energy control; Passive elements; Feedback passive systems

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1. Introduction, motivations

The study of dissipativity-related concepts in the nonlinear discrete-time setting is an interesting field in which many problems remain unsolved. This is the case of the problem of rendering nonlinear discrete-time systems dissipative (passive) by means of a static state feedback control law or the study of the relative degree properties of nonlinear discrete-time dissipative (passive) systems.

The action of rendering a system dissipative (passive) by means of a static state feedback is known as feedback dissipativity (feedback passivity or passification). Systems which can be rendered dissipative (passive) are regarded as feedback dissipative (feedback passive) systems. In this paper, the feedback passivity problem is considered for MIMO nonlinear discrete-time systems in general form. Sufficient conditions for this class of systems to be feedback passive are obtained by means of the relative degree and zero dynamics of the original system. These results are also rewritten for those systems which are affine in the input.

This paper follows the same approach given in Byrnes and Lin 1993 and Byrnes and Lin 1994, Lin (1993) and Navarro-López and Fossas-Colet (2002) in the sense that the feedback passivity problem is based on the properties of the relative degree and zero dynamics of the non-passive system, with the difference that in Byrnes and Lin 1993 and Byrnes and Lin 1994, Lin (1993) and in Navarro-López and Fossas-Colet (2002) the problem of feedback losslessness and the problem of feedback passivity is treated, respectively, for affine-in-input nonlinear systems with the restriction of considering storage functions V such that $V(f(x)+g(x)u)$ are quadratic in u . Therefore, the results here presented can be considered as an extension to the passivity general case of those given in Byrnes and Lin (1994) and Navarro-López and Fossas-Colet (2002). In addition, the feedback passivity methodology presented in this paper is an alternative to those proposed in Navarro-López (2002), Navarro-López, Sira-Ramírez, and Fossas-Colet (2002b) and Navarro-López, Fossas-Colet, and Cortés (2002a). In Navarro-López (2002), Navarro-López et al. (2002b) and Navarro-López et al. (2002a), the feedback dissipativity problem is treated for single-input single-output (SISO) nonlinear discrete-time systems which are non-affine in the states and the control input. Furthermore, the conditions proposed in these works are different in essence to those given here; the methodologies presented in Navarro-López (2002), Navarro-López et al. (2002b) and Navarro-López et al. (2002a) are based on the basic dissipativity inequality. In Navarro-López (2002), the discrete-time version of the speed-gradient algorithm is used. Two methodologies of approximate type are given in Navarro-López et al. (2002a), the underlying idea is that the control which makes the system dissipative is based on the control that makes the storage energy function V invariant or on the control that renders the system lossless.

Conditions for a system to be feedback passive can be obtained by means of the properties of the relative degree and the zero dynamics of the system. This idea is inherited from the continuous-time setting, where the study of the properties of the relative degree and zero dynamics of a passive system has played an important role in understanding problems such as feedback passivity or the stabilization of passive systems (see Byrnes, Isidori, & Willems, 1991). As the passivity property is an input–output property, the relative degree and zero dynamics of a passive system will present distinctive features. For general discrete-time systems, the implications of dissipativity and passivity in the relative degree and the zero dynamics have not been established yet; they have, however, been studied for the losslessness and passivity nonlinear affine-in-input case (see Byrnes & Lin, 1994; Navarro-López & Fossas-Colet, 2002, respectively) and the passivity linear case (see

Byrnes & Lin, 1994; Monaco & Normand-Cyrot, 1999). In this paper, the properties of the relative degree and the zero dynamics of passive nonlinear discrete-time systems in general form will be related to the feedback passivity property.

The paper is organized as follows. Section 2 revisits the basic definitions about passive systems for the discrete-time case. Section 3 is devoted to the properties of the relative degree and zero dynamics of passive nonlinear discrete-time systems. Section 4 deals with the feedback passivity problem through the relative degree and zero dynamics properties. In addition, the results given are rewritten for the affine-in-input case. Conclusions and comments on future works are given in the last section.

2. Basic definitions

This section introduces some basic definitions concerning the notions of passivity in the discrete-time setting. These concepts are an adaptation of those given in the continuous-time case (Willems, 1972).

Let a system of the form

$$x(k+1)=f(x(k),u(k)), \quad (1a)$$

$$y(k)=h(x(k),u(k)), \quad (1b)$$

where $f : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$ and $h : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^m$ are smooth maps, and $k \in \mathbb{Z}_+ := \{0, 1, 2, \dots\}$. Let (\bar{x}, \bar{u}) be a fixed point of the system. There is no loss of generality in considering $(\bar{x}, \bar{u}) = (0, 0)$, $f(0,0)=0$ and $h(0,0)=0$.

Definition 1. A positive definite \mathcal{C}^2 function $V : \mathbb{R}^n \rightarrow \mathbb{R}$ such that $V(x)=0 \Leftrightarrow x=0$ is addressed as storage function.

From now on, $\det(\text{Hess}(V(x))|_{x=0}) \neq 0$ is assumed.

Definition 2. System ((1a) and (1b)) is said to be locally passive if there exists a storage function V such that

$$V(f(x,u)) - V(x) \leq h^T(x,u)u, \quad \forall (x,u) \in \mathcal{X} \times \mathcal{U} \quad (2)$$

with $\mathcal{X} \times \mathcal{U}$ a neighbourhood of $x=0, u=0$.

Lemma 3. Let a system of form ((1a) and (1b)) be locally passive. Then, there exists a storage function $V : \mathcal{X} \rightarrow \mathbb{R}$ such that the functions $\phi_1 : \mathcal{X} \times \mathcal{U} \rightarrow \mathbb{R}$, $\phi_2 : \mathcal{X} \times \mathcal{U} \rightarrow \mathbb{R}$:

$$\phi_1(x,u)=V(x)+h^T(x,u)u-V(f(x,u)) \quad (3)$$

$$\phi_2(x,u)=V(x)+h^T(x,u)u$$

have a local minimum at $x=0, u=0$.

Proof. $\forall(x, u) \in \mathcal{X} \times \mathcal{U}$:

$$V(x) + h^T(x, u)u \geq V(x) - V(f(x, u)) + h^T(x, u)u \geq 0.$$

The first inequality is due to the positiveness of V . The second one to the local passivity of the system. Then, $\varphi_2(x, u) \geq \varphi_1(x, u)$, $\forall(x, u) \in \mathcal{X} \times \mathcal{U}$. Since $\varphi_1(0, 0) = \varphi_2(0, 0) = 0$, φ_1 and φ_2 attain a local minimum at $x=0$, $u=0$.

Definition 4. Let $\eta : \mathcal{X} \times \mathcal{U} \rightarrow \mathcal{U}$ be a smooth function. A nonlinear static state feedback control law $u = \eta(x, v)$ is regular if for all $(x, v) \in \mathcal{X} \times \mathcal{U}$ it follows that $\partial\eta/\partial v$ is invertible. The system $x(k+1) = f(x(k), \eta(x(k), v(k)))$ is referred to as the *feedback transformed system*.

Definition 5. Consider system ((1a) and (1b)) and assume that there exists a storage function $V(x)$. The system is said to be locally feedback passive if there exists a regular static state feedback control law of the form $u = \eta(x, v)$, with v as the new input, defined in a neighbourhood $\mathcal{X} \times \mathcal{U}$ of $x=0, v=0$, such that the feedback transformed system is locally passive.

3. Relative degree and zero dynamics of passive nonlinear discrete-time systems

In this section, the relative degree and zero dynamics of nonlinear passive discrete-time systems are analysed.

The relative degree and zero dynamics for nonlinear discrete-time systems have been studied for systems with outputs independent of the inputs (see Monaco and Normand-Cyrot 1987 and Monaco and Normand-Cyrot 1988). This paper focuses on systems with outputs dependent of the inputs. The definition for relative degree is the same as that given in Byrnes and Lin (1994).

Definition 6. A system of form ((1a) and (1b)) is said to have local relative degree zero for all the outputs at $x=0$, $u=0$ if $\left. \frac{\partial h(x, u)}{\partial u} \right|_{\substack{x=0 \\ u=0}}$ is non-singular.

Definition 7. Consider system ((1a) and (1b)). Let \mathcal{X}_1 and \mathcal{U}_1 be neighbourhoods of $x=0$ and $u=0$, respectively, such that $f(x, u) \in \mathcal{X}_1$, $\forall(x, u) \in \mathcal{X}_1 \times \mathcal{U}_1$. Consider $u^* : \mathcal{X}_1 \rightarrow \mathcal{U}_1$ the control which makes $h(x, u^*(x)) = 0$. The zero dynamics of the system is defined in \mathcal{X}_1 by the map $f^*(x) = f(x, u^*(x))$.

Definition 8. A system of form ((1a) and (1b)) has locally passive zero dynamics if there exists a storage function V locally defined in a neighbourhood \mathcal{X}_1 of $x=0$ in \mathbb{R}^n such that

$$V(f(x, u^*)) \leq V(x), \quad \forall x \in \mathcal{X}_1 \quad (4)$$

with u^* as given in Definition 7.

The properties of the relative degree and zero dynamics of locally passive systems of form ((1a) and (1b)) are established as follows:

Proposition 9. *Let system ((1a) and (1b)) be locally passive. Then,*

(i) *if $\forall j=1, \dots, m$, there exists $i \in \{1, \dots, n\}$, such that $\left. \frac{\partial h_j(x, u)}{\partial x_i} \right|_{\substack{x=0 \\ u=0}} \neq 0$, then the system has local relative degree zero at $x=0, u=0$.*

(ii) *If the system has local relative degree zero at $x=0, u=0$ then the zero dynamics of system ((1a) and (1b)) locally exists at $x=0$ and is locally passive.*

Proof.

(i) Since the system is locally passive, there exists a storage function $V(x) > 0$ such that $V(x) = 0 \Leftrightarrow x = 0$ and

$$\begin{aligned} V(f(x, u)) - V(x) - h^T(x, u)u &\leq 0, \\ \forall (x, u) &\in \mathcal{X} \times \mathcal{U}. \end{aligned} \quad (5)$$

From Lemma 3 function ϕ_2 has a local minimum at $x=0, u=0$. Consequently, the eigenvalues of the Hessian matrix of ϕ_2 at $x=0, u=0$ must be positive or zero. This condition will now be checked. The Hessian matrix of ϕ_2 at $x=0, u=0$ takes the following form:

$$\begin{pmatrix} \left. \frac{\partial^2 V}{\partial x^2} \right|_{x=0} & \left. \left(\frac{\partial h(x, u)}{\partial x} \right)^T \right|_{\substack{x=0 \\ u=0}} \\ \left. \frac{\partial h(x, u)}{\partial x} \right|_{\substack{x=0 \\ u=0}} & \left. \frac{\partial h(x, u)}{\partial u} \right|_{\substack{x=0 \\ u=0}} + \left. \left(\frac{\partial h(x, u)}{\partial u} \right)^T \right|_{\substack{x=0 \\ u=0}} \end{pmatrix} \quad (6)$$

By linear changes of coordinates $\tilde{x} = \varphi(x) = Mx$ and $\tilde{u} = \psi(u) = Nu$, Eq. (6) is rewritten as

$$\begin{aligned} &\text{Hess}(\phi_2(\tilde{x}, \tilde{u})) \Big|_{\substack{\tilde{x}=0 \\ \tilde{u}=0}} \\ &= \begin{pmatrix} D_1 & \left. \left(\frac{\partial \tilde{h}(\tilde{x}, \tilde{u})}{\partial \tilde{x}} \right)^T \right|_{\substack{\tilde{x}=0 \\ \tilde{u}=0}} \\ \left. \frac{\partial \tilde{h}(\tilde{x}, \tilde{u})}{\partial \tilde{x}} \right|_{\substack{\tilde{x}=0 \\ \tilde{u}=0}} & D_2 \end{pmatrix} \end{aligned} \quad (7)$$

where $\tilde{h}(\tilde{x}, \tilde{u}) = h(\varphi^{-1}(\tilde{x}), \psi^{-1}(\tilde{u}))$ and D_1, D_2 stand for the diagonal matrices $(\partial V(\varphi^{-1}(\tilde{x}))/\partial \tilde{x}), (\partial \tilde{h}(\tilde{x}, \tilde{u})/\partial \tilde{u})$, respectively, evaluated at $\tilde{x} = 0, \tilde{u} = 0$. By hypothesis, V has a local minimum at $x=0$ and $\det(\text{Hess}(V(x))|_{x=0}) \neq 0$. Furthermore, Sylvester's condition applied to the minor of the Hessian matrix of φ_2 corresponding to $\{\tilde{x}_1, \dots, \tilde{x}_n, \tilde{u}_j\}$ yields

$$\lambda_1 \cdots \lambda_n \left. \frac{\partial \tilde{h}_j(\tilde{x}, \tilde{u})}{\partial u_j} \right|_{\substack{\tilde{x}=0 \\ \tilde{u}=0}} - \sum_{i=1}^n \left. \frac{\partial \tilde{h}_j(\tilde{x}, \tilde{u})}{\partial \tilde{x}_i} \right|_{\substack{\tilde{x}=0 \\ \tilde{u}=0}}^2 \lambda_1 \cdots \hat{\lambda}_i \cdots \lambda_n > 0, \quad (8)$$

where $\lambda_1, \dots, \lambda_n$ are the eigenvalues of D_1 and the hat symbol denotes that the term is not present. Since $\exists i \in \{1, \dots, m\}$ such that

$$\left. \frac{\partial \tilde{h}_j(\tilde{x}, \tilde{u})}{\partial \tilde{x}_i} \right|_{\substack{\tilde{x}=0 \\ \tilde{u}=0}} \neq 0,$$

the function $\left. \frac{\partial \tilde{h}_j(\tilde{x}, \tilde{u})}{\partial \tilde{u}_j} \right|_{\substack{\tilde{x}=0 \\ \tilde{u}=0}}$ must be different from zero; otherwise, (8) would be strictly negative. Note that conditions

$$\exists i \in \{1, \dots, m\} \text{ such that } \left. \frac{\partial \tilde{h}_j(\tilde{x}, \tilde{u})}{\partial \tilde{x}_i} \right|_{\substack{\tilde{x}=0 \\ \tilde{u}=0}} \neq 0,$$

and

$$\exists i \in \{1, \dots, m\} \text{ such that } \left. \frac{\partial h_j(x, u)}{\partial x_i} \right|_{\substack{x=0 \\ u=0}} \neq 0$$

are equivalent.

(ii) If the system has locally relative degree zero at $x=0, u=0$, it follows that, $\left. \frac{\partial h(x, u)}{\partial u} \right|_{\substack{x=0 \\ u=0}}$ is non-singular and $h(0,0)=0$, then by the implicit function theorem, there exists $u^* : \mathcal{X} \rightarrow \mathcal{U}$ with \mathcal{X}, \mathcal{U} neighbourhoods of $x=0$ and $u=0$, respectively, such that $h(x, u^*(x))=0, \forall x \in \mathcal{X}$ and the set $\mathcal{Z}^* = \{(x, u) | h(x, u) = 0\}$ is not empty. Consequently, the zero dynamics of system ((1a) and

(1b)) locally exists in a neighbourhood of $x=0$ in \mathbb{R}^n . As system ((1a) and (1b)) is locally passive, relation (5) is met. Setting $u=u^*$ relation (5) yields relation (4).

Remark 10. If there exists a storage function V such that the zero dynamics of a system is locally passive, the system does not have to be locally passive with the same storage function. On the contrary, if there exists a storage function V such that the system is locally passive, then the zero dynamics of the system is locally passive for the same storage function V , see Proposition (9).

Remark 11. The hypothesis in Proposition (9.1) is necessary. There are SISO systems of form ((1a) and (1b)) with relative degree zero at $x=0, u=0$ which are locally passive and

$$\left. \frac{\partial h(x, u)}{\partial u} \right|_{\substack{x=0 \\ u=0}}$$

is singular. For instance,

$$x(k+1)=ax(k)+bu^2(k),$$

$$y(k)=h(x(k),u(k))=u^3(k),$$

where $a, b \in \mathbb{R}$ and $a^2+b^2 < 1$. Consider $V(x)=x^2$; then, the system is locally passive. Namely, the function

$$\begin{aligned} \phi(x, u) &= V(f(x, u)) - V(x) - h(x, u)u \\ &= (a^2 - 1)x^2 + 2abxu^2 + (b^2 - 1)u^4 \\ &= - \left(\sqrt{1 - a^2}x - \frac{ab}{\sqrt{1 - a^2}}u^2 \right)^2 \\ &\quad + \left(\frac{a^2b^2}{1 - a^2} - (1 - b^2) \right) u^4 \\ &= - \left(\sqrt{1 - a^2}x - \frac{ab}{\sqrt{1 - a^2}}u^2 \right)^2 - \frac{1 - a^2 - b^2}{1 - a^2} u^4 \end{aligned}$$

has a local maximum at $x=0, u=0$ for some neighbourhood of $x=0, u=0$. Nevertheless,

$$\left. \frac{\partial h(x, u)}{\partial u} \right|_{\substack{x=0 \\ u=0}}$$

4. The feedback passivity problem

This section is devoted to rendering a system of form ((1a) and (1b)) passive by means of a static state feedback control law.

The following Lemma is introduced for the sake of clarity.

Lemma 12. *If $\lambda_i > 0, \forall i=1, \dots, n$, then there exists $k_s > 0$ such that $\forall k \in (0, k_s)$*

$$D = \begin{vmatrix} k\lambda_1 & \cdots & 0 & ka_{11} & \cdots & ka_{1s} \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & k\lambda_n & ka_{n1} & \cdots & ka_{ns} \\ ka_{11} & \cdots & ka_{n1} & 2 + kb_{11} & \cdots & kb_{1s} \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ ka_{1s} & \cdots & ka_{ns} & kb_{s1} & \cdots & 2 + kb_{ss} \end{vmatrix} > 0. \quad (9)$$

Proof. The determinant can be written as

$$D = k^n (2^s \lambda_1 \dots \lambda_n + \alpha_1 k + \dots + \alpha_s k^s),$$

where α_j are constants related to $\lambda_i, a_{il}, b_{rl}$, with $j=1, \dots, s, i=1, \dots, n, r=1, \dots, s, l=1, \dots, s$. Then, taking

$$k_s = \min \left\{ 1, \frac{2^s \lambda_1 \dots \lambda_n - \varepsilon}{|\alpha_1| + \dots + |\alpha_s|} \right\}$$

with $0 < \varepsilon \ll 1$, it is concluded that

$$\begin{aligned} D &\geq k^n (2^s \lambda_1 \dots \lambda_n - |\alpha_1|k - \dots - |\alpha_s|k^s) \\ &\geq k^n (2^s \lambda_1 \dots \lambda_n - |\alpha_1|k - \dots - |\alpha_s|k) \\ &= k^n [2^s \lambda_1 \dots \lambda_n - k(|\alpha_1| + \dots + |\alpha_s|)] = k^n \varepsilon > 0. \end{aligned}$$

Theorem 13. Let a system of form ((1a) and (1b)) with locally passive zero dynamics and local relative degree zero at $x=0, u=0$. Consider V as the storage function on the zero dynamics. Assume that $\det(\text{Hess}(V(x) - V(f(x,0)))|_{x=0}) \neq 0$, then the system is locally feedback passive.

Proof. Since $\text{rank}((\partial h(x,u))/\partial u) = m$ in a neighbourhood of $x=0, u=0$ and $h(0,0)=0$, the implicit function theorem applied to $h(x,u)=v$ guarantees the existence of a function $\eta(x,v)$ defined in a neighbourhood of $x=0, v=0$ such that $h(x,\eta(x,v))=v$. Thus, system ((1a) and (1b)) can be rewritten as

$$x(k+1) = f(x(k), \eta(x(k), v(k))) = \tilde{f}(x(k), v(k)),$$

$$y(k) = h(x(k), \eta(x(k), v(k))) = v(k). \quad (10)$$

The goal is to prove that system (10) is locally passive at $x=0, v=0$ with respect to the new input variable v . Note that the zero dynamics is defined by

$$x(k+1) = \tilde{f}(x(k), 0).$$

By hypothesis, there exists a storage function $V(x)$ in a neighbourhood of $x=0$ for which

$$V(\tilde{f}(x, 0)) - V(x) \leq 0. \quad (11)$$

Let $\tilde{V}(x) = kV(x)$, with $k>0$ a constant. It will be shown that, for an appropriate k ,

$$\tilde{V}(\tilde{f}(x, v)) - \tilde{V}(x) - v^T v \leq 0$$

in a neighbourhood of $x=0, v=0$. This is equivalent to proving that the function

$$\phi(x, v) = \sum_{i=1}^m v_i^2 + \tilde{V}(x) - \tilde{V}(\tilde{f}(x, v))$$

has an isolated local minimum at $x=0, v=0$. Since $V(x)$ has an isolated local minimum at $x=0$, $\phi(x, v)$ has a critical point at $x=0, v=0$. Namely, the partial derivatives are

$$\left. \frac{\partial \phi(x, v)}{\partial x_i} \right|_{\substack{x=0 \\ v=0}} = \left[\frac{\partial \tilde{V}}{\partial x_i} - \sum_{h=1}^n \frac{\partial \tilde{V}}{\partial z_h} \bigg|_{z=0} \frac{\partial \tilde{f}_h(x, v)}{\partial x_i} \right]_{\substack{x=0 \\ v=0}} = 0,$$

$$\left. \frac{\partial \phi(x, v)}{\partial v_r} \right|_{\substack{x=0 \\ v=0}} = \left[2v_r - \sum_{h=1}^n \frac{\partial \tilde{V}}{\partial z_h} \bigg|_{z=0} \frac{\partial \tilde{f}_h(x, v)}{\partial v_r} \right]_{\substack{x=0 \\ v=0}} = 0,$$

with $i=1, \dots, n, r=1, \dots, m$. In order to obtain the Hessian matrix of $\phi(x, v)$ at $x=0, v=0$, the following terms are computed:

$$\left. \frac{\partial^2 \phi(x, v)}{\partial x_j \partial x_i} \right|_{\substack{x=0 \\ v=0}} = k \left[\frac{\partial^2 V}{\partial x_j \partial x_i} - \sum_{h,l} \frac{\partial^2 V}{\partial z_h \partial z_l} \frac{\partial \tilde{f}_h}{\partial x_i} \frac{\partial \tilde{f}_l}{\partial x_j} \right]_{\substack{x=0 \\ v=0}},$$

$$\left. \frac{\partial^2 \phi(x, v)}{\partial v_r \partial x_i} \right|_{\substack{x=0 \\ v=0}} = -k \left[\sum_{h,l} \frac{\partial^2 V}{\partial z_h \partial z_l} \frac{\partial \tilde{f}_h}{\partial x_i} \frac{\partial \tilde{f}_l}{\partial v_r} \right]_{\substack{x=0 \\ v=0}},$$

$$\left. \frac{\partial^2 \phi(x, v)}{\partial v_s \partial v_r} \right|_{\substack{x=0 \\ v=0}} = 2\delta_{rs} - k \left[\sum_{h,l} \frac{\partial^2 V}{\partial z_h \partial z_l} \frac{\partial \tilde{f}_h}{\partial v_r} \frac{\partial \tilde{f}_l}{\partial v_s} \right]_{\substack{x=0 \\ v=0}},$$

with $z_h = \tilde{f}_h(x, v), z_l = \tilde{f}_l(x, v), h=1, \dots, n, l=1, \dots, n, i=1, \dots, n, j=1, \dots, n, r=1, \dots, m, s=1, \dots, m$. δ_{rs} is equal to one if $r=s$, and otherwise zero. Note that

$$\left. \frac{\partial^2 \phi(x, v)}{\partial x_i \partial x_j} \right|_{\substack{x=0 \\ v=0}} = \left. \frac{\partial^2 \tilde{\phi}(x)}{\partial x_i \partial x_j} \right|_{x=0}, \quad (12)$$

where $\tilde{\phi}(x) = \phi(x, 0)$. Furthermore, matrix (12) is symmetric, thus it can be diagonalized by an appropriate linear change of coordinates. The eigenvalues of (12) are positive because $\tilde{\phi}(x)$ has a

local minimum at $x=0$ and the determinant of the Hessian matrix at $x=0$ is different from zero by hypothesis. In the new coordinates, the Hessian matrix of $\varphi(x,v)$ evaluated at $x=0, v=0$ is

$$\begin{pmatrix} k\lambda_1 & \cdots & 0 & ka_{11} & \cdots & ka_{1m} \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & k\lambda_n & ka_{n1} & \cdots & ka_{nm} \\ ka_{11} & \cdots & ka_{n1} & 2 + kb_{11} & \cdots & kb_{1m} \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ ka_{1m} & \cdots & ka_{nm} & kb_{m1} & \cdots & 2 + kb_{mm} \end{pmatrix} \quad (13)$$

with $\lambda_i, i=1, \dots, n$ the eigenvalues of matrix (12). The first n diagonal leading minors are positive since $\lambda_i > 0, \forall i=1, \dots, n$. Furthermore, using Lemma 12, it is obtained

$$\begin{vmatrix} k\lambda_1 & \cdots & 0 & ka_{11} & \cdots & ka_{1s} \\ \vdots & \ddots & \vdots & \vdots & \cdots & \vdots \\ 0 & \cdots & k\lambda_n & ka_{n1} & \cdots & ka_{ns} \\ ka_{11} & \cdots & ka_{n1} & 2 + kb_{11} & \cdots & kb_{1s} \\ \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\ ka_{1s} & \cdots & ka_{ns} & kb_{s1} & \cdots & 2 + kb_{ss} \end{vmatrix} > 0,$$

$\forall k \in (0, k_s)$. Finally, consider $\hat{k} = \min\{k_1, \dots, k_m\}$ and $\tilde{V} = \hat{k}V$. Then, by applying Sylvester's test to (13), $\varphi(x,v)$ has a strict local minimum at $x=0, v=0$. To conclude with, the existence of a new storage function for the feedback transformed system to be locally passive at $x=0, v=0$ has been shown.

Remark 14. In Theorem 13, the condition of local relative degree zero of the system is essential in order to prove the existence of a control which passifies the system. If this condition was removed, the feedback passivity of the system would not be assured.

Corollary 15. Consider a system of the form

$$x(k+1) = f(x(k)) + g(x(k))u(k), \quad (14a)$$

$$y(k) = h(x(k)) + J(x(k))u(k), \quad (14b)$$

where $x \in \mathbb{R}^n, u, y \in \mathbb{R}^m, f, g, h, J$ are smooth maps and $f(x) \in \mathbb{R}^n, g(x) \in \mathbb{R}^{n \times m}, h(x) \in \mathbb{R}^m, J(x) \in \mathbb{R}^{m \times m}, f(0)=0, h(0)=0$. Let

$$u = \alpha(x) + \beta(x)v \quad (15)$$

be a static state feedback control law with $\alpha(x)$ and $\beta(x)$ smooth functions such that $\alpha(0)=0$ and $\beta(x)$ is invertible for all $x \in \mathbb{R}^n$. Consider V a storage function on the zero dynamics and assume that $\det(\text{Hess}(V(x)-V(f(x)))|_{x=0}) \neq 0$. If system ((14a) and (14b)) has locally passive zero dynamics and local relative degree zero at $x=0$, then the system is locally feedback passive by means of a regular feedback control law of form (15).

5. Conclusions

The characteristics of the relative degree and zero dynamics of discrete-time nonlinear systems in general form have been related to the feedback passivity property. These characteristics have been used to give sufficient conditions to render systems of this class passive by means of a static state feedback control law. The conditions given for the general case have been rewritten for nonlinear systems which are affine in the control input. The relative degree and zero dynamics properties of passive systems have also been given.

All these results can be considered as the extension to the passivity general case of those given in Byrnes and Lin 1993 and Byrnes and Lin 1994, Lin (1993), Navarro-López and Fossas-Colet (2002) where the feedback losslessness (Byrnes and Lin 1993 and Byrnes and Lin 1994; Lin, 1993) and feedback passivity (Navarro-López & Fossas-Colet, 2002) problems are treated for affine-in-input nonlinear systems requiring $V(f(x)+g(x)u)$ to be quadratic in u .

There is a great variety of dissipativity-related problems remaining unsolved in the discrete-time setting. Feedback dissipativity is an appealing problem to be solved in order to give desirable energy-like properties to systems whose output does not depend on the input. The analysis of the relative degree and zero dynamics of dissipative systems would be required for this purpose.

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Vitae

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