An exact procedure to plan holidays and working time under annualised hours considering cross-trained workers with different efficiencies

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ABSTRACT

Annualising working hours (AH) is a means of achieving flexibility in the use of human resources to face the seasonal nature of demand. Some existing planning procedures are able to minimise costs due to overtime and temporary workers. However, due to the great difficulty of solving the problem, it is normally assumed both that holiday weeks are fixed beforehand and that workers from different categories who are able to perform a specific type of task have the same efficiency. Often reality is different, thus, there is a gap between academic and real problems. In the present paper, those constraints are relaxed and a much more general and close to reality problem is solved in an exact and very efficient way.

Keywords: human resources, manpower planning, annualised hours.

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1. Introduction

Annualising working hours (AH)—i.e. the possibility of irregularly distributing the total number of workers working hours over the course of a year—is a means of achieving flexibility, because AH allows production capacity to be adapted to fluctuations in demand, thus reducing costs (overtime, temporary workers and inventory costs). This flexibility in the use of human resources is especially useful in many service processes (where products cannot be inventoried) and in manufacturing organisations in which holding costs are high.

From workers point of view, it is quite clear that this kind of system implies a worsening of their working conditions, mainly because having to do irregular working hours creates a difficulty for planning their own free time. Hence, annualised working hours systems must be negociated between the company and the workers and, usually, some kind of compensation (e.g. reduction of working time, salary increase, etc.) is offered to the workers in exchange of the flexibility they provide. Furthermore, the distribution of working time must comply with some bounds and rules so that a significant worsening of working conditions is avoided.

AH gives rise to new problems that have hitherto been given little attention in literature. For instance, in Hung (1999a), Hung (1999b), Grabot and Letouzey (2000) and Azmat and Widmer (2004) it is emphasised that the concept of annualised hours is surprisingly absent from literature on planning and scheduling. Due to the great variety of existing production systems, there is a considerable diversity of problems entailed by AH; in Corominas et al. (2004), the characteristics of the planning problem are discussed and a classification scheme is proposed, giving rise to thousands of different cases. Moreover, AH often implies the need to solve a complicated working time planning problem. Some authors deal with different versions of the problem (e.g. Hung, 1999a; Hung, 1999b; Vila and Astorino, 2001; Corominas et al., 2002; Azmat and Widmer, 2004; Azmat et al., 2004), but most papers (e.g. Lynch, 1995; MacMeeking, 1995 and Mazur, 1995) discuss AH only from a qualitative point of view.

Most of the aforementioned papers include some assumptions that may be too restrictive for some real situations. For example, workers' holiday weeks are either not considered or taken as a data. However, in reality, companies try to avoid workers taking their holidays in high demand periods. Somehow, demand, working time and holidays are considered together. Hence, the only reason for not considering the holidays in a working time planning procedure must be the difficulty in modeling and solving the problem. Of course, workers have some rights related to their own holidays and, thus, not only demand has to be considered but also other constraints that may affect the allocation of holiday periods.

In some cases, the agreement between company and workers states that workers can choose when they want to take their holidays, provided that some conditions are satisfied. For example, that summer holidays have to be taken between june and october or that the number of workers that take their holidays in a given period cannot be larger than a certain number (in this case senior workers or high category workers may have priority over others).

In the case of a service centre under an annualised hours system, which normally means that it is not necessary that different workers operate the same working hours, the way in which holidays weeks are determined has a great influence on the way capacity can be adjusted to demand and, hence, in costs. In such a situation, it could be estimated, by means of a planning procedure, the amount of money that the company could save if workers accepted their holidays to be fixed by the company. This money could be partially used to compensate workers in exchange of letting their holidays to be planned in the best moment for the company. Of course, another option would be allowing workers to take more holidays as a compensation. Note that in that situation a planning procedure would be a very useful bargaining tool.

On the other hand, usually there are different categories of workers who are able to perform different types of tasks. Nowadays, cross-trained workers are an additional source of flexibility (Slomp and Molleman 2002, Corominas et al. 2006), and most companies try to contract and train this kind of workers. It is obvious that not all

workers perform the different tasks with the same efficiency. Actually, although workers from different categories may be able to perform a specific type of task, obviously certain categories frequently require more time than others do (Slomp and Molleman, 2002). However, none of AH published papers considers cross-trained workers with different efficiencies.

In this paper, these assumptions are relaxed and a more general and real problem is solved in an exact way. The main aims of this paper are the following: (1) to approach the planning of working hours and holiday weeks over the course of a year in services that employ cross-trained workers who have different efficiencies; and (2) to quantify the improvement in the solution when there is the possibility of determining holiday weeks. The possibility of getting this improvement, in economic terms (cost saving), permits using the proposed planning procedure as a tool in the bargaining process between company and workers. The rest of the paper is organised as follows: section 2 introduces the problem and four models for planning AH over a year; section 3 includes the results of a computational experiment; section 4 shows how results could be used by company and workers to help in the bargaining process; and, finally, section 5 exposes the conclusions.

2. Procedure to plan holidays and working time under AH and cross-trained workers with different efficiencies

Solving the planning problem involves determining the number of weekly working hours and holiday weeks for each worker (in the following, the term "worker" is used to refer only to workers that are members of the staff, but not to temporary workers). Also, it must be determined the number of hours that each category will dedicate to each type of task, taking into account the corresponding efficiencies. The problem, which is inspired on several real cases, is described in the following paragraphs.

A service system carried out on an individual basis is considered. This means that each worker is able to perform a task by his/her own and, hence, that it is not necessary that

different workers operate the same working hours; therefore, the weekly number of working hours can be different from one week to another and also from one worker to another.

Although a thorough discussion about the questions posed by services goes beyond the scope of the present paper, it is necessary, given the heterogeneity inherent to them, to specify the kind of service system that we are dealing with. As it is known, services sector has been often defined as a "residual" (Castells and Aoyoma, 1994; Sampson and Froehle, 2006). This is unsatisfactory from a theoretical point of view and, hence, represents "a barrier to discovering the managerial and operational implications" (Sampson and Froehle, 2006). Academics have proposed many definitions, but they fail, to a greater or lesser extent, to embrace all the activities that are commonly considered as services. Recently, Sampson and Froehle (2006) have proposed a Unified Services Theory; according to it, "with service processes, the customer provides significant inputs into the production process". They also include the most relevant references relative to operations in services and a discussion about the important implications of the quoted definition. For instance, the definition is compatible with the possibility of producing inventory or with the tangibility of the product.

In the present paper, we consider a service process in which neither the capacity that is available nor the demand corresponding to a given period can be transferred to another one. Therefore, if the available capacity is not enough to face the demand, a part of the later will be lost. By means of its price policy and other marketing actions or reservation systems, the company can have an influence on the volume of the demand and its temporal profile. Fitzsimmons and Fitzsimmons (2006), Hurt et al. (2005), Jack and Powers (2004) and Lovelock (1992 and 1996), among others, deal with capacity and demand management in services. It is supposed here that the forecasted demand is that resulting after applying all the appropriate measures. It is also assumed that the required capacity ensues from the forecasted demand and a beforehand established service level (however, the issue of establishing the required capacity given the demand and the service level is partially open; see, e.g., Green et al., 2003).

Is it assumed that there are different types of tasks and that the company forecasts the demand and establishes the capacity requirements for each task.

For the sake of service quality, it must be guaranteed that production capacity in any given week is greater than or equal to what is needed. Hence, if the staff cannot provide entirely this capacity, temporary workers will be hired for the number of hours required. Overtime is admitted but, as usual, its total amount is bounded. Also, it is quite normal to find in some agreements and laws (see, for example, the French law in www.35h.travail.gouv.fr) that overtime hours are classified into two blocks, and that the cost of an hour belonging to the second block is greater than that of an hour belonging to the first block.

The objective is to minimise the total capacity shortage cost. Hence, the objective function is the cost of overtime plus the cost of employing temporary workers.

Usually, although some categories are able to perform different types of tasks, workers are specialised on a specific set of tasks and is preferable that, whenever is possible, other tasks are avoided. The reason can be, for example, that it is not considered appropriate that high qualified categories perform certain tasks. Given that normally there is more than one minimum cost solution, it is possible to break the tie between optimal solutions by considering, for each category of workers, penalties associated with the assignment of tasks for which those workers are not specialised. This is done by adding a penalty function to the first one (the cost), multiplied by a small weight.

As it has been said, workers from different categories may frequently be able to perform a specific type of task, although certain categories may require more time than others. Therefore, cross-trained workers are considered: certain categories can perform different types of tasks and can have different relative efficiencies associated with them (for example, a value of 0.9 means that a worker in that category needs to work 1/0.9 hours to perform a task that a worker with a relative efficiency equal to 1 would perform in 1 hour).

The conditions to be fulfilled by the solution, which arise from real situations and prevent the solution to worsen too much workers working conditions, are the following:

i) the total of annual working hours is fixed (e.g. 1700 hours per worker);

ii) the weekly number of working hours must fall within an interval defined by a lower and upper bound (e.g. [30-48] hours);

iii) for each worker, the average weekly working hours for any set of twelve consecutive weeks is upper bounded (this condition comes from the French law, which considers the possibility of annualising working time and establishes some constraints; see www.35h.travail.gouv.fr);

iv) for each worker, if the average of weekly working hours over a specified number of consecutive weeks ('week-block'; e.g. eight weeks) is greater than a certain value (e.g. 45 hours), then, over a given number of weeks immediately succeeding the week-block (e.g. two weeks), the number of working hours must not be greater than a certain value (e.g. 30 hours). This condition avoids long hard periods and gives some rest weeks after a hard period.

v) if 'strong' and 'weak' weeks are defined as those in which the number of working hours is respectively greater or less than certain specified values, there is, for each worker, an upper bound for the number of strong weeks and a lower bound for the number of weak weeks (for example, no more than 15 weeks with a number of working hours greater than 44 hours and at least eight weeks with a number of working hours not greater than 30 hours).

On the other hand, it is assumed that workers take two holiday periods: two consecutive weeks in winter and four consecutive weeks in summer.

We propose to use mathematical programming to solve the problem. Specifically, four mixed integer linear programming models (MILP) are proposed: (*M1*) minimises the

cost and includes holidays as variables to be determined; (*M2*) minimises the cost and considers holidays as a data fixed beforehand; (*M3*) minimises the cost, regularises the distribution of the working time and includes holidays as variables; and (*M4*) minimises the cost, regularises the distribution of the working time and considers holidays as a data. The details of these four models are given below.

The objective function to be minimised in models *M1* and *M2* has already been specified: cost of overtime plus cost of employing temporary workers (the penalties associated with the assignment of types of tasks to categories are considered in order to break the tie between minimum cost solutions). Cross-trained workers are considered in both models. In *M1,* holiday weeks are determined by the model but, in *M2,* these are fixed a priori. This allows us to compare the results of both situations and to use the proposed planning procedure not only as a human resources management tool but also as a bargaining tool.

Usually, the AH models that minimise the cost may have an infinite number of optimal solutions. In addition, by means of an initial experiment, we realised that, in the optimal solution provided by the optimiser, the number of weekly working hours for a worker over the course of a year and weekly working time provided by temporary workers for each week are usually very irregular. This could create a difficulty when trying to adopt an AH system in a real case. To regularise the profile of workers' working hours over a year and the profile of weekly working time provided by temporary workers, i.e. to obtain the most regular solution from all those that involve the minimum cost, two other models (*M3* and *M4*) are used.

The objective function to be minimised in models *M3* and *M4* is the weighted sum of: i) the sum of the discrepancies between the weekly working hours of employees and the average weekly working hours; and (ii) the sum of discrepancies between the working hours provided by temporary workers and the average of weekly working hours provided by them. The penalties associated with the assignment of types of tasks to categories are again considered to break the tie between optimal solutions. In both models, the minimum cost obtained by *M1* is guaranteed by means of an additional

constraint. The difference between *M3* and *M4* is that in *M3* the holiday weeks are determined by the model but in *M4* these are the ones obtained when solving *M1*.

We use the following notation:

Data

T	Weeks in the planning horizon
C	Set of categories of workers
F	Set of types of tasks
E	Set of workers
ρ_{jk}	Relative efficiency associated with the workers in category j in the accomplishment of tasks of type k ($\forall j \in C$; $\forall k \in F$); $0 \leq \rho_{jk} \leq 1$. If $\rho_{jk}=0$, workers in category j are not able to perform tasks of type k.
$\hat{C}_k$	Sets of categories of workers that can be assigned to tasks of type k ($\forall k \in F$)
$\hat{F}_j$	Sets of types of tasks which can be performed by employees in category j ($\forall j \in C$)
p_{jk}	Penalty associated with an hour of work in a task of type k of a worker in category j ($\forall k \in F$; $\forall j \in \hat{C}_k$). This parameter is used to break the tie between optimal solutions and the units it has depend on the units of the objective function.
λ_1	Parameter to weight the penalties to establish the trade-off between them and the monetary cost of the solution. This parameter has a very small value which is used to break the tie between minimum cost solutions.
$\hat{E}_j$	Set of employees in category j ($\forall j \in C$)
r_{tk}	Required capacity (in working hours) for tasks of type k in week t ($t=1,..,T$; $\forall k \in F$)
H_i	Stipulated ordinary annual working hours of worker i ($\forall i \in E$);
α_1, α_2	Maximum proportions, over the annual amount of ordinary working hours, of overtime corresponding to blocks 1 and 2 respectively.
$\beta1_i, \beta2_i$	Respectively, the cost (in monetary units) of an hour of overtime for block 1

and block 2 for worker i ($\forall i \in E$), with $\beta 1_i < \beta 2_i$

hm_{it}, hM_{it}	Lower and upper bounds of the number of working hours for worker i in week t ($\forall i \in E$; $t=1,...,T$); $hm_{it} < hM_{it}$
L, h_L	L is the maximum number of consecutive weeks in which the average weekly working hours cannot be greater than h_L
B, b, h_B, h_b	b is the minimum number of weeks, after a week-block of B consecutive weeks with a weekly average of working hours greater than h_B, in which the number of weekly hours cannot be greater than h_b
N_S, h_S	N_S is the maximum number of 'strong' weeks, i.e. weeks with a number of working hours greater than h_S
N_W, h_W	N_W is the minimum number of 'weak' weeks, i.e. weeks with a number of working hours not greater than h_W
$hw1_i, hw2_i$	Number of holiday weeks in the first and second holiday periods respectively for worker i ($\forall i \in E$)
$t1_i, t2_i$	First and last week respectively in which worker i can take holidays in the first holiday period ($\forall i \in E$)
$t3_i, t4_i$	First and last week respectively in which worker i can take holidays in the second holiday period ($\forall i \in E$)
γ_k	Cost (in monetary units) of an hour for tasks of type k performed by a temporary worker ($\gamma_k > \beta 2_i$, $\forall i \in \hat{E}_j \mid j \in \hat{C}_k$)

Variables (all the non-binary variables are real and non-negative)

x_{it}	Working hours of worker i in week t $\left(\forall i \in E; t=1,...,T \right)$.
y_{tjk}	Working hours of employees in category j dedicated to tasks of type k in week t ($\forall k \in F$; $\forall j \in \hat{C}_k$; $t=1,...,T$).
d_{tk}	Working hours corresponding to tasks of type k to be supplied in week t by temporary workers ($\forall k \in F$; $t=1,...,T$).
$v1_i, v2_i$	Overtime corresponding respectively to blocks 1 and 2 of worker i ($\forall i \in E$).
$vc1_{it} \in \{0,1\}$	Indicates whether worker i starts the first holiday period in week t ($\forall i \in E$, $t=t1_i,...,t2_i-hw1_i+1$).

$vc2_{it} \in \{0,1\}$ Indicates whether worker i starts the second holiday period in week t
$(\forall i \in E,\ t=t3_i,...,t4_i\text{-}hw2_i+1)$.

$\delta_{i\tau} \in \{0,1\}$ Indicates whether the average working hours of worker i, in a week-block of B weeks that ends with week τ, is (or is not) greater than h_B hours ($\forall i \in E$; $\tau = B,\ldots,T\text{-}b$).

$s_{it} \in \{0,1\}$ Indicates whether worker i has a planned number of working hours greater than h_S hours for week t ($\forall i \in E$; $t=1,\ldots,T$).

$w_{it} \in \{0,1\}$ Indicates whether worker i has a planned number of working hours equal to or less than h_W hours for week t ($\forall i \in E$; $t=1,\ldots,T$).

MODEL 1 (M1)

$$[MIN]z = \sum_{i \in E} \beta 1_i v1_i + \sum_{i \in E} \beta 2_i v2_i + \sum_{k \in F} \sum_{t=1}^{T} \gamma_k d_{tk} + \lambda_1 \sum_{t=1}^{T} \sum_{k \in F} \sum_{j \in \hat{C}_k} p_{jk} y_{tjk} \tag{1}$$

$$\sum_{t=1}^{T} x_{it} = H_i + v1_i + v2_i \qquad\qquad \forall i \in E \tag{2}$$

$$v1_i \leq \alpha_1 H_i \qquad\qquad \forall i \in E \tag{3}$$

$$v2_i \leq \alpha_2 H_i \qquad\qquad \forall i \in E \tag{4}$$

$$\sum_{i \in \hat{E}_j} x_{it} = \sum_{k \in \hat{F}_j} y_{tjk} \qquad\qquad t=1,...,T;\ \ \forall j \in C \tag{5}$$

$$\sum_{j \in \hat{C}_k} \rho_{jk} y_{tjk} + d_{tk} \geq r_{tk} \qquad\qquad t=1,...,T;\ \ \forall k \in F \tag{6}$$

$$\sum_{t=\tau-L+1}^{\tau} x_{it} \leq Lh_L \qquad\qquad \tau = L,...,T;\ \ \forall i \in E \tag{7}$$

$$\sum_{t=\tau-B+1}^{\tau} x_{it} \leq Bh_B + \delta_{i\tau}\left(\sum_{t=\tau-B+1}^{\tau} hM_{it} - Bh_B\right) \qquad \tau = B,...,T-b; \forall i \in E \tag{8}$$

$$\sum_{t=\tau-B+1}^{\tau} x_{it} \leq Bh_B \qquad\qquad \tau = T-b+1,...,T;\ \forall i \in E \tag{9}$$

$$x_{i,\tau+l} \leq hM_{i,\tau+l} - \delta_{i\tau}(hM_{i,\tau+l} - h_b) \qquad \forall i \in E;\ \tau = B,....T-b;\ l=1,...,b \tag{10}$$

$$x_{it} \leq h_S + s_{it}(hM_{it} - h_S) \qquad\qquad \forall i \in E;\ t=1,...T \tag{11}$$

$$x_{it} \leq hM_{it} - w_{it}(hM_{it} - h_W) \qquad\qquad \forall i \in E;\ t=1,...,T \tag{12}$$

$$\sum_{t=1}^{T} s_{it} \leq N_S \qquad \forall i \in E \qquad (13)$$

$$\sum_{t=1}^{T} w_{it} \geq N_W \qquad \forall i \in E \qquad (14)$$

$$\sum_{t=t1_i}^{t2_i-hw1_i+1} vc1_{it} = 1 \qquad \forall i \in E \qquad (15)$$

$$\sum_{t=t3_i}^{t4_i-hw2_i+1} vc2_{it} = 1 \qquad \forall i \in E \qquad (16)$$

$$x_{it} \leq hM_{it} \qquad \forall i \in E; \ t \notin \left([t1_i,...,t2_i] \vee [t3_i,...,t4_i]\right) \qquad (17)$$

$$x_{it} \geq hm_{it} \qquad \forall i \in E; \ t \notin \left([t1_i,...,t2_i] \vee [t3_i,...,t4_i]\right) \qquad (18)$$

$$x_{it} \leq hM_{it}\left(1 - \sum_{\tau=\max(t1_i,\ t-hw1_i+1)}^{\min(t,\ t2_i-hw1_i+1)} vc1_{i\tau}\right) \qquad \forall i \in E; \ t = t1_i,...,t2_i \qquad (19)$$

$$x_{it} \geq hm_{it}\left(1 - \sum_{\tau=\max(t1_i,\ t-hw1_i+1)}^{\min(t,\ t2_i-hw1_i+1)} vc1_{i\tau}\right) \qquad \forall i \in E; \ t = t1_i,...,t2_i \qquad (20)$$

$$x_{it} \leq hM_{it}\left(1 - \sum_{\tau=\max(t3_i,\ t-hw2_i+1)}^{\min(t,\ t4_i-hw2_i+1)} vc2_{i\tau}\right) \qquad \forall i \in E; \ t = t3_i,...,t4_i \qquad (21)$$

$$x_{it} \geq hm_{it}\left(1 - \sum_{\tau=\max(t3_i,\ t-hw2_i+1)}^{\min(t,\ t4_i-hw2_i+1)} vc2_{i\tau}\right) \qquad \forall i \in E; \ t = t3_i,...,t4_i \qquad (22)$$

$$\delta_{i\tau} \in \{0,1\} \qquad \forall i \in E; \ \tau = B,...,T-b \qquad (23)$$

$$s_{it}, w_{it} \in \{0,1\} \qquad \forall i \in E; \ t = 1,...,T \qquad (24)$$

$$vc1_{it} \in \{0,1\} \qquad \forall i \in E; t = t1_i,...,t2_i - hw1_i + 1 \qquad (25)$$

$$vc2_{it} \in \{0,1\} \qquad \forall i \in E; t = t3_i,...,t4_i - hw2_i + 1 \qquad (26)$$

$$v1_i, v2_i \geq 0 \qquad \forall i \in E \qquad (27)$$

$$y_{tjk} \geq 0 \qquad t = 1,...,T; \ \forall k \in F; \ \forall j \in \hat{C}_k \qquad (28)$$

$$d_{tk} \geq 0 \qquad t = 1,...,T; \ \forall k \in F \qquad (29)$$

(1) is the objective function, which includes the cost of overtime plus that of employing external workers and, to break the tie between minimum cost solutions, the (weighted)

penalties associated with the assignment of tasks to non specialist workers (i.e., workers that are not used to those tasks and, therefore, it would not be a desirable assignment); (2) imposes that the total number of worked hours should be equal to the ordinary annual hours stipulated plus overtime, if applicable; (3) and (4) stipulate that overtime for each of the two blocks should not exceed their respective upper bounds; (5) is the balance between the hours provided by specific types of workers and the hours assigned to different types of tasks; (6) expresses that the hours assigned to a type of task that are to be carried out by workers plus, if applicable, the hours provided by temporary workers for that same type of task must not be less than the number of hours required; (7) imposes the upper bound on the average weekly working hours for any subset of L consecutive weeks; (8) implies that variable $\delta_{i\tau}$ is equal to 1 if the average number of working hours in a week-block of B weeks is greater than h_B; (9) prevents the average hours worked from being greater than h_B in the last weeks of the year, when after the week-block of B weeks there are no longer b weeks to 'compensate'; (10) implies that, if variable $\delta_{i\tau}$ is equal to 1, the upper bound of the number of working hours is h_b; (11) imposes that, if the number of working hours is greater than h_S, then variable s_{it} is equal to 1; (12) states that, if the number of working hours is greater than h_W, then variable w_{it} is equal to 0; (13) and (14) stipulate that the number of 'strong' and 'weak' weeks cannot be greater than N_S and less than N_W respectively; (15) and (16) establish that the worker must start holidays in one and only one week; (17) and (18) set the lower and upper bounds of the number of weekly working hours in non-holiday weeks; (19), (20), (21) and (22) set the lower and upper bounds of the number of weekly working hours for possible holiday weeks; (23), (24), (25) and (26) express the binary character of the corresponding variables; and (27), (28) and (29) impose the non-negative character of the rest of the non-binary variables.

MODEL 2 (M2)

M2, which considers holidays as a data, can be obtained by deleting variables $vc1_{it}$ and $vc2_{it}$ and their associated constraints (15, 16 and 19 to 22, 25 and 26) from model *M1* and making several minor and straightforward modifications.

MODEL 3 (M3)

Once model *M1* has been solved, the cost of overtime and temporary workers is stored. The formalisation of *M3* is not included but it may be easily obtained by starting from model *M1* and keeping in mind the following changes:

i) A constraint is added, which requires that the cost of the solution of *M3* cannot exceed that obtained with *M1*.

ii) Variables x_{it} are eliminated using the expression $x_{it} = \bar{x}_i + x_{it}^+ - x_{it}^-$, where $\bar{x}_i$ is the average number of weekly working hours corresponding to worker *i* and x_{it}^+ and x_{it}^- are the positive and negative deviations from the average number of working hours of worker *i* in week *t*.

iii) Variables d_{tk} are eliminated using the expression $d_{tk} = \overline{d_k} + \sigma_{tk}^+ - \sigma_{tk}^-$, being $\overline{d_k}$ the average number of weekly working hours provided by temporary workers for a task of type *k* and σ_{tk}^+ and σ_{tk}^- are the positive and negative deviations from the average number of working hours provided by temporary workers for task *k* in week *t*.

iv) The objective function to be minimised is replaced with a new one that has three weighted components. The first one is the sum of the discrepancies in the number of working hours of workers and the second one is the sum of the discrepancies in the number of working hours provided by temporary workers. Again, to break the tie between optimal solutions, the penalties associated with the assignment of tasks to categories of workers are also considered (multiplied by a very small weight, λ_2):

$$\sum_{i \in E} \sum_{t=1}^{T} \left(x_{it}^+ + x_{it}^- \right) + \sum_{t=1}^{T} \sum_{k \in F} \left(\sigma_{tk}^+ + \sigma_{tk}^- \right) + \lambda_2 \sum_{t=1}^{T} \sum_{k \in F} \sum_{j \in \hat{C}_k} p_{jk} y_{tjk}$$

MODEL 4 (M4)

M4 can be obtained from *M3* by fixing the holiday weeks obtained when solving *M1* (basically, variables $vc1_{it}$, $vc2_{it}$ and their associated constraints have to be deleted).

14

3. Computational experiments

A computational study was performed to evaluate the effectiveness (in terms of computing time and the quality of solutions) of the models. Overall, the results, as it is justified below, were very satisfactory and permit to consider the planning procedure we develop as a management and bargaining tool, as it is shown in section 4.

The basic data used for the experiment are as follows:

– Five MILP models: *M1*, *M2*, *M3*, *M4* and *M4+M3'* (this compound model consists in solving *M4* and, in the remaining computing time, executing *M3'*, which is obtained when a constraint is imposed on *M3* so that the value of the solution of *M3* cannot exceed the value obtained by means of *M4*).

– 10, 40, 70, 100 and 250 workers.

– A time horizon of 52 weeks (46 working weeks and six holiday weeks).

– The holiday weeks for each worker are distributed into two uninterrupted periods, including two weeks in winter and four weeks in summer. In *M2*, the temporary allocation of holidays (for each worker) was fixed at random.

– There are three categories and three types of tasks. There are two patterns of relative efficiency (and penalty). Table 1 and Table 2 show the relative efficiency (and the penalty) values for each pattern.

Table 1. Relative efficiency (and penalty) values for Pattern 1

	Task 1	Task 2	Task 3
Category 1	1 (1)	0.9 (2)	0
Category 2	0	1 (1)	0.9 (2)
Category 3	0	0	1 (1)

Table 2. Relative efficiency (and penalty) values for Pattern 2

	Task 1	Task 2	Task 3
Category 1	1 (1)	0	0

Category 2	0.9 (2)	1 (1)	0
Category 3	0.8 (2)	0	1 (1)

- The capacity (in working hours) required over the year follows three different patterns. Capacity requirements of Type 1 correspond to a non-seasonal capacity requirements pattern with a random noise of $\pm 5\%$. Capacity requirements of Type 2 correspond to a seasonality capacity requirements pattern with one peak, with a random noise of $\pm 5\%$. Capacity requirements of Type 3 correspond to a seasonality capacity requirements pattern with two peaks, with a random noise of $\pm 5\%$. In each case, total required capacity is equal to total available capacity multiplied by 0.99.

For every combination of models, number of workers, type of capacity requirement and pattern of relative efficiency (and penalty), 20 instances were generated (varying at random capacity requirement noise and, in *M2*, holiday weeks), giving 3,000 instances.

In spite of models dimension may be considered large (the average number of variables and constraints are given in Table 3), they were solved to optimality using an ILOG CPLEX 8.1 optimiser and a Pentium IV PC at 1.8 GHz with 512 Mb of RAM. Absolute and relative MIP gap tolerances were set to 0.01. Maximum computing time for all instances was set to 1800 seconds.

Table 3. Average number of variables/constraints

	Number of workers				
	10	**40**	**70**	**100**	**250**
M1	2817/3915	9387/14 715	15 957/25 515	22 527/36 315	55 377/90 315
M2	2310/2567	7357/9319	12 405/16 072	17 452/22 822	42 689/56 572
M3	4169/4592	13 859/16 952	23 549/29 312	33 239/41 672	81 689/103 472
M4	3664/3658	11 835/13 191	20 004/22 718	28 172/32 230	69 035/79 946
M4+M3'	4170/4594	13 860/16 954	23 550/29 314	33 240/41 674	81 690/103 474

For each model and each number of workers, the number of instances for which no feasible solution was obtained, the number of instances with feasible solution and the number of instances with a proven optimal solution are given in Table 4 (for model *M4+M3'*, the number of instances in which there was not enough time to carry out *M3'* is added). Table 5 shows the minimum (t_{min}), the average ($\bar{t}$) and the maximum computing time (t_{max}) (in seconds).

Table 4. Number of instances with no solution, with a feasible solution and with a proven optimal solution

		Number of workers				
		10	**40**	**70**	**100**	**250**
M1	*No solution*	0	0	0	0	0
	Feasible solution	59	57	7	1	0
	Optimal solution	61	63	113	119	120
M2	*No solution*	0	0	0	0	0
	Feasible solution	0	0	0	0	0
	Optimal solution	120	120	120	120	120
M3	*No solution*	1	2	0	0	0
	Feasible solution	109	11	27	112	120
	Optimal solution	10	107	93	8	0
M4	*No solution*	0	0	0	0	0
	Feasible solution	0	0	0	0	22
	Optimal solution	120	120	120	120	98
M4+M3'	*No time for M3'*	0	0	0	0	22
	No solution of M3'	2	11	1	8	98
	Feasible solution of M3'	106	16	3	108	0
	Optimal solution of M3'	12	93	116	4	0

Table 5. Computing times (in seconds)

Number of workers				
10	**40**	**70**	**100**	**250**

M1	t_{min}	24.20	15.55	26.49	42.91	139.94
	$\overline{t}$	935.91	890.23	164.88	82.59	300.02
	t_{max}	1800	1800	1800	1800	1097.71
M2	t_{min}	7.06	7.89	8.75	9.64	16.27
	$\overline{t}$	9.53	9.41	11.21	12.26	30.58
	t_{max}	110.78	14.86	21.63	20.73	198.66
M3	t_{min}	130.03	193.30	671.26	1450.44	1800
	$\overline{t}$	1716.66	716.28	1369.97	1790.47	1800
	t_{max}	1800	1800	1800	1800	1800
M4	t_{min}	6.22	25.52	67.25	105.91	531.20
	$\overline{t}$	9.82	36.49	119.86	265.38	1361.07
	t_{max}	156.08	78.92	258.28	446.29	1800
M4+M3'	t_{min}	79.22	200.92	656.06	1408.45	1800
	$\overline{t}$	1695.76	842.78	1238.95	1793.65	1800
	t_{max}	1800	1800	1800	1800	1800

The maximum computing times are very reasonable considering the problem to be solved (the aim of the models is to establish an annual plan) and its maximum size (two hundred and fifty workers, which is a large enough number). For the models in which costs were to be minimised (*M1* and *M2*), feasible solutions were always obtained and most of these were optimal solutions. Regarding the models which have regularity as objective (*M3*, *M4* and *M4+M3'*), in only one test (of *M3*) no feasible solution was obtained. The variants that were hardest to solve were *M1* and *M3* (or *M3'*), as expected, given that these variants include more constraints and binary variables than others do.

The experiments provided satisfactory results regarding the quality of the solutions of the models. Table 6 shows the minimum (a_{min}), the average ($\overline{a}$) and the maximum (a_{max}) percentage of money saved when *M1* is used versus *M2*. As shown, the possibility of determining holiday weeks with model *M1* provides very good solutions and savings of more than 90%. These values also show how capacity can be adapted to

requirements by determining the holiday weeks of the staff (this is also due to the flexibility provided by the annualisation of working time).

Table 6. Percentage of money saved when using M1 versus M2

		Number of workers				
		10	**40**	**70**	**100**	**250**
M1 vs. M2	a_{min}	65.24	97.17	99.02	99.69	100
	$\overline{a}$	89.53	99.49	99.96	99.99	100
	a_{max}	99.75	100	100	100	100

The way in which capacity is adapted to required capacity can also be seen in Figure 1, in which required capacity, workers capacity and the hours to be provided by temporary workers (shortage) are represented.

[INSERT FIGURE 1]

Figure 1. Capacity vs Required Capacity for task 1, *M1*

From the company point of view (i.e., cost saving) the quality of the solution can be considered very good. The amount and type of conditions imposed to the distribution of the working time guarantee that, from workers point of view, the solution cannot be very bad. However, looking at Figure 2, in which the working hours of a certain worker are represented, it can be observed how much irregular is the distribution of the working time over the year. In a real situation, few workers would easily accept this kind of solution. Fortunately, as it can be observed in Figure 3, this problem is well solved by using those models whose objective is the regularity at minimum cost (models *M3*, *M4* and *M4+M3'*).

[INSERT FIGURE 2]

Figure 2. Distribution of working time (number of working hours for each week) of worker 1 using model M1 (minimum cost)

[INSERT FIGURE 3]

Figure 3. Distribution of working time (number of working hours for each week) of worker 1 using model M3 (minimum cost + regularity)

Table 7 shows the minimum (mr_{min}), the average ($\overline{mr}$) and the maximum (mr_{max}) percentage of improvement of regularity when two models are compared. Models $M3$, $M4$ and $M4+M3'$ were very effective in regularising the workload of staff members and of temporary workers over the course of a year (the two main components in the function of regularity). In all cases, the percentage of improvement of regularity is about 50%. Moreover, if 1800 seconds of computing time can be used, it would seem that the $M4+M3'$ model is slightly better than the $M3$ model.

Table 7. Percentage of improvement of regularity when two models are compared

		Number of workers				
		10	**40**	**70**	**100**	**250**
	mr_{min}	39.74	47.54	47.78	47.05	44.88
M3 vs. M1	$\overline{mr}$	46.02	50.89	51.53	50.73	49.38
	mr_{max}	58.94	59.66	59.00	59.39	58.81
	mr_{min}	35.18	44.01	45.55	45.55	45.98
M4 vs. M1	$\overline{mr}$	44.15	48.99	49.89	49.97	50.30
	mr_{max}	58.02	57.57	58.32	58.62	59.70
	mr_{min}	39.77	46.60	47.68	47.00	45.98
M4+M3' vs. M1	$\overline{mr}$	46.01	50.80	51.65	50.92	50.30
	mr_{max}	59.00	59.62	59.15	59.25	59.70
	mr_{min}	-0.11	-0.80	-2.46	-0.64	-2.10
M3 vs. M4	$\overline{mr}$	1.87	1.97	1.64	0.76	-0.92
	mr_{max}	5.49	4.31	3.73	2.32	0.56
M3 vs. M4+M3'	mr_{min}	-0.88	-1.50	-2.54	-1.35	-2.10

	$\overline{mr}$	0.00	0.13	-0.12	-0.19	-0.92
	mr_{max}	1.58	2.11	1.28	1.00	0.56
M4+M3' vs. M4	mr_{min}	0.00	0.00	0.00	0.00	0.00
	$\overline{mr}$	1.86	1.81	1.76	0.95	0.00
	mr_{max}	5.47	4.06	3.71	2.96	0.00

Finally, another computational experiment was performed with the following new data: total required capacity is equal to total capacity multiplied by 1.05; for each combination, 5 instances were generated (giving 750 new instances).

The results show that if the system is not adequately sized (total capacity is less than total required capacity), solving the problem is a little more difficult (and the number of optimal/feasible solutions obtained decreases); the results, nevertheless, can be considered very good (Table 8 shows the minimum, the average and the maximum percentage of money saved when using *M1* versus *M2*).

Table 8. Percentage of money saved when using M1 versus M2

		Number of workers				
		10	**40**	**70**	**100**	**250**
	a_{min}	8.61	3.78	2.14	1.13	0
M1 vs. M2	$\overline{a}$	10.84	8.81	6.55	5.42	3.54
	a_{max}	40.85	15.87	10.69	10.19	8.95

As in the first experiment, we can conclude that, if 1800 seconds of computing time can be used, the *M4+M3'* model is slightly better than the *M3* model.

4. A tool for a bargaining process

In most countries companies cannot introduce irregular working hours if workers do not agree, so the question is whether workers will really accept an increase in flexibility (and also their holidays being planned by the company). Besides the convincing argument of conserving their jobs even in periods of low demand, companies should offer some kind of compensation that will lead workers to accept more or less flexibility. One of the most difficult things of adopting an annual hours scheme is the great amount of time and effort that are necessaries to reach an agreement between the company and the workers. A planning procedure, like the one proposed in this paper, can be also a useful tool to help in the bargaining process.

Planning working time under different AH scenarios provides the company and the workers with quantitative information that can be very useful for the bargaining process in order to adopt an annual hours scheme. These scenarios may be characterised, for example, by the weekly flexibility accepted by workers, the total amount of annual working hours (the company could eventually reduce the annual working time), the maximum overtime, the conditions related to the strong and weak weeks and, of course, the possibility of, some rules provided, planning the allocation of holiday weeks. For each scenario, the model (for example, *M1* if holidays can be planned and *M2* otherwise) would give the optimal cost of the solution and the company and the workers could agree to satisfactory conditions for both. Obviously, doing this implies solving several instances of each model. Hence, this would be possible only if solving the model requires a reasonable time; this is the case of the models presented in this paper, which give an optimal solution in short times.

Table 9 shows the results of a case in which scenarios are characterised by the total amount of annual hours (first column) and the weekly flexibility (first row). For each scenario, first and second values correspond to the cost obtained by *M1* –holidays fixed by the model– and *M2* –holidays fixed a priori–, respectively. Note that *K* is the cost obtained in a situation without flexibility, without a reduction in working time and with holidays fixed a priori. It can be seen how the cost diminishes when flexibility is high, even when reducing working time.

Two options for reducing the cost by implementing annualised hours might be as follows: (1) by increasing weekly flexibility and reducing working time as a compensation for the workers; or (2), by increasing flexibility and not reducing working time but instead offering financial compensation to the workers. As it is shown in Table 9, in both cases the cost can be further reduced if workers' holidays are planned by the model.

Table 9. Cost of different scenarios (annual hours, weekly flexibility and planning holidays)

MINIMUM COST		Weekly flexibility (h/week)				Model
		[40,40]	**[40, 50]***	**[30, 45]**	**[25, 50]**	
H (annual hours)	**1840 (40 h/week)**	$0.64 \cdot K$	$0.52 \cdot K$	$0.16 \cdot K$	0	*M1*
		K	$0.86 \cdot K$	$0.52 \cdot K$	$0.21 \cdot K$	*M2*
	1748 (38 h/week)	-	-	$0.16 \cdot K$	0	*M1*
		-	-	$0.51 \cdot K$	$0.21 \cdot K$	*M2*
	1610 (35 h/week)	-	-	$0.45 \cdot K$	0	*M1*
		-	-	$0.58 \cdot K$	$0.21 \cdot K$	*M2*

* Note that in this case (1840 h/year) the only way of using the weekly flexibility (40-50 h/week) is by means of overtime.

5. Conclusions and further research

Annualising working hours (AH) is a means of obtaining flexibility in the use of human resources to face the seasonal nature of demand. There are only few papers dealing with the problem of planning working hours under an annualised hours agreement; moreover, most of them include assumptions that can be relaxed in order to solve a less restrictive and more realistic problem.

In this paper, some of those assumptions are relaxed and a much more general problem is solved: planning the working hours and holiday weeks of cross-trained workers who have different relative efficiencies, over the course of a year in a service centre. Our computational study leads us to conclude that mathematical programming is a technique

suitable to deal with the problem in many real situations and, as it was expected, that better results are obtained when the allocation of holiday weeks is determined by the model.

Furthermore, it has been shown how the proposed procedure could be a very useful tool for helping in the bargaining process carried out before the adoption of an annual hours scheme. Considering different AH scenarios, the model provides the company and the workers with the cost of the corresponding optimal plan, so a satisfactory situation for both can be agreed. However, a more precise idea of the real cost could be obtained by considering the financial costs associated with the plan (e.g., if a significant amount of overtime is needed during a given period, the company would perhaps have to ask for a bank loan to finance it). These costs have been traditionally ignored in planning models, and our future research will try to introduce them in AH planning models. The model can be used, also, to evaluate the impact that changes in the profile of the demand or in the level of service may have on the costs.

Another interesting subject to be studied is how to determine the required capacity, which is normally considered as a data of the planning problem. In a service centre, in which queues exist due to irregularities in customers arrivals and in workers' operation times, it is essential to plan a capacity larger than the forecasted demand to ensure a good service level (e.g., that customers do not have to wait too much to be served). Further research may be to develop a procedure to determine the required capacity starting from the forecasted demand and taking into account the desired service level and the stochastic character of features such as arrivals, operation times and workers' absenteeism.

In some service processes, even though production ultimately requires the presence of the customer, preparatory activities can be performed in advance thus shortening the stay of the customer in the system and transferring part of the capacity requirements of a period to preceding periods. This situation fits in the service framework, according to the Unified Services Theory and it is worthy to be studied.

Of course, an inmediate continuation of our research may consist in solving other problems based on the classification scheme proposed in Corominas et al. (2004). As it is mentioned in the introduction, each case needs a specific model and, given the amount of integer variables and constraints that are usually involved, it cannot be assured that all problems can be optimally solved by using MILP.

Finally, the characteristics of the planning problem solved in this paper are the ones corresponding with an annualised hours scheme, which considers a stable group of employees, and each one of them has to work a certain amount of annual hours that can be distributed in an irregular way. Normally, under an AH scheme, a worker's salary is the same each month, regardless of whether the number of working hours has been higher or lower than the average. Of course, there exist other forms of flexibility such as modifying the number of staff workers depending on the period (i.e., a *hiring and firing* scheme) or considering undertime hours (i.e., workers being paid partially to be at home during low demand periods). Even though most companies prefer an AH scheme rather than a hiring and firing scheme (see Oke, 2000), further research could explore the possibility of combining different sources of flexibility in an hybrid scheme and also to compare different schemes regarding different criteria.

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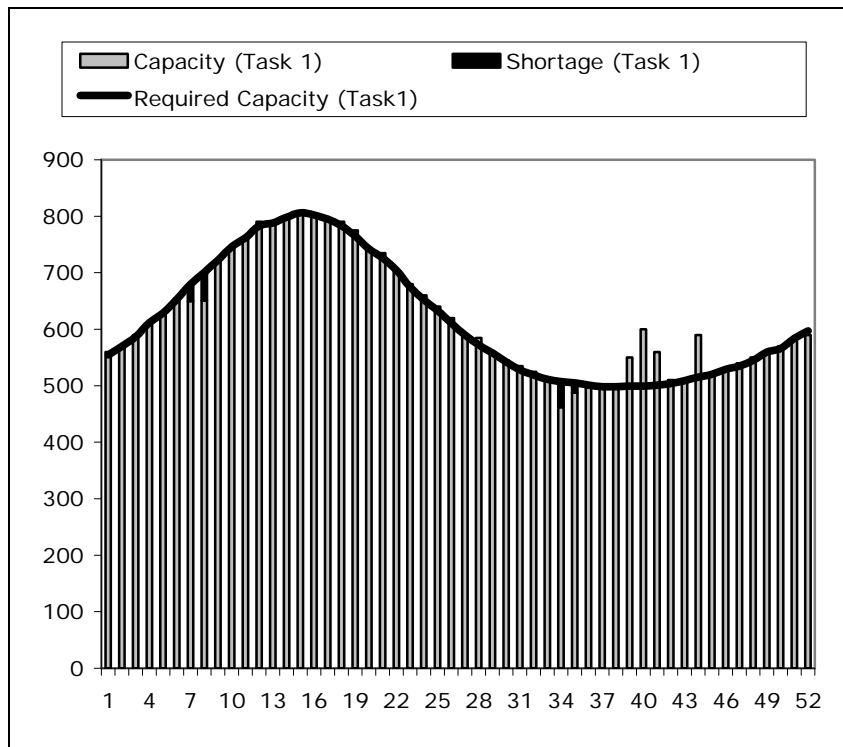
FIGURE 1

Figure 1. Capacity vs Required Capacity for task 1, *M1*
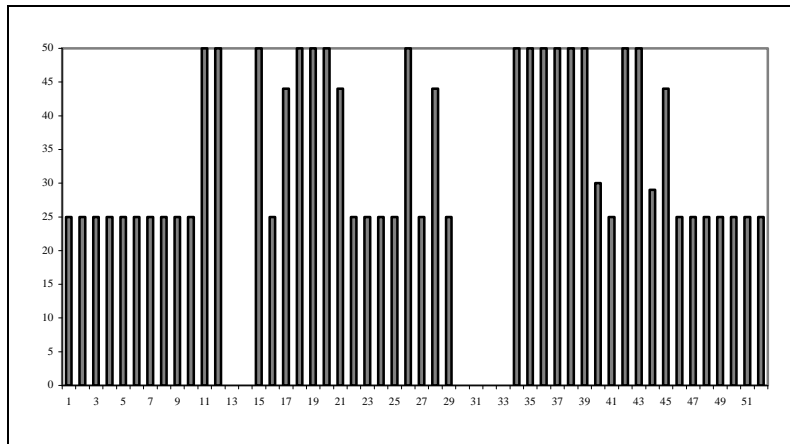
FIGURE 2

Figure 2. Distribution of working time (number of working hours for each week) of worker 1 using model M1 (minimum cost)
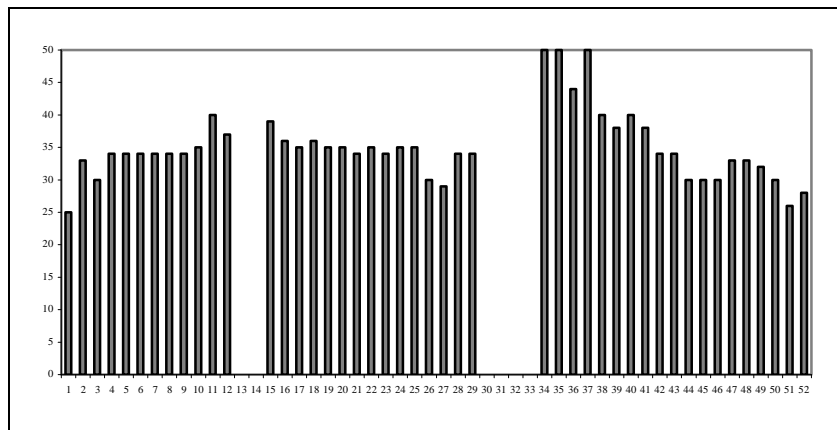
FIGURE 3

Figure 3. Distribution of working time (number of working hours for each week) of worker 1 using model M3 (minimum cost + regularity)