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Containing Aircraft Noise Levels at take-off: A Mathematical Programming Approach

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Abstract The sustained growth of air transportation traffic along the last decades has induced increasing noise exposure for the areas surrounding large and medium airports. This study concentrates on the optimization of the take-off and departure flight phase for a transportation aircraft. A general aircraft trajectory generation problem is first formulated as an optimal control problem where a global cost, including noise penalties, is to be minimized. Since aircraft operators and communities have conflictive objectives, a noise index is introduced. Airlines operations costs and community noise levels are expressed as complex functions of the aircraft trajectory geometry resulting in a complex optimization problem. It is observed that flight dynamics present a differential flatness property and it is discussed how to take advantage of this to solve the trajectory generation problem and assess adequately the resulting surrounding noise exposure. Then, the case of a particular standard take-off/initial climb trajectory is considered. It appears that the noise level constraints turn the resulting mathematical programming problem numerically intricate and that this problem should be approached as a parameter optimization problem through a systematic evaluation process.

1. INTRODUCTION

The sustained growth of air traffic along the last decades has induced increasing noise exposure for the airports surrounding communities. International and national regulations regarding noise exposure have been established by civil aviation authorities incurring in higher operations costs for airlines. So it appears important for airlines to meet the new noise constraints in an efficient way. Since initial climb, including take-off, is critical with respect to aircraft noise exposure, this study, developed under the SONORA cooperation project between French and Spanish teams, concentrates on the optimization of this flight phase.

The general aircraft trajectory generation problem is first formulated as an optimal control problem where a global cost is to be minimized. Since aircraft operators and communities have conflictive objectives, a noise index is introduced to mix the different goals. Airlines operations costs and community noise levels are expressed as complex functions of the aircraft trajectory parameters and geometry resulting in an intricate optimization problem. However, the nature of the dynamics constraints representing flight dynamics present a particular property, differential flatness, which makes interesting an inverse approach where flyable trajectories can be easily assessed.

Then the particular problem of departure trajectory optimization is tackled. In this case the noise constraints are localized at control points while only standard trajectories are considered. Then, this problem can be formulated as a complex mathematical programming problem but with few independent variables. Since these variables must be taken within limited subsets, it appears that a direct evaluation approach can be adopted to avoid numerical difficulties.

2. PROBLEM DEFINITION

For many years now in many urban areas surrounding airports, the noise annoyance related with aircraft activities has reached critical levels. Since air traffic should, in the next decade, be doubled, it becomes imperative to find new solutions to reduce aircraft noise while protecting the economic efficiency of aircraft operations. Noise abatement procedures have been established: partial or complete curfew during the night period, interdiction for noisiest aircraft, prohibition of engine tests and of the use of reverses at landing during the night period, definition of authorized areas for approach and departure operations, definition of daily quotas in terms of number of operations or noise energy.

For an aircraft two main sources of noise can be considered: aeronautical noise and engine noise. The aeronautical noise is the consequence of the friction of the air along the aircraft (wings, fuselage, landing gears, aerodynamic actuators), its power can be considered to be proportional to V_a^3, where V_a is the airspeed. The engine noise is related to the four main components of the engine (fan, turbine, combustion chamber and exhaust nozzle. The power of exhaust noise, which is the more important, is a function of V_e^8, where V_e is the engine exhaust airspeed, and presents a directional distribution.

It appears that adapted take-off and landing procedures can lead to significant decrease in aircraft noise. This can be achieved by modifying approach and climb paths (prohibition of over fly of specific areas, mandatory entry or exit points, concentration over already noisy areas such as highways and industrial areas), by adopting new guidance procedures such as: limitation of speed, new path angles and headings, continuous approaches limiting heading and path angle changes, delayed deployment of the landing gears, adapted or limited thrust at take off and optimized path angle.

Difficulties related with this problem concern on one side the modeling of the aircraft performances (noise emission levels and fuel consumption) and on the other side the evaluation of noise impact over surrounding population. Other difficulties are related with the definition of objectives and the validation conditions of proposed solutions.

3. A GENERAL OPTIMIZATION APPROACH

3.1 Flight Guidance Equations

In this study, only the guidance dynamics of a transportation aircraft are considered since it is assumed that the aircraft of interest are equipped with basic auto-pilots which deal efficiently with their fast dynamics and thus controls their body attitude (θ, ϕ, ψ) (here θ is the pitch angle, ϕ is the bank angle and ψ is the heading angle) and the regime of their engines, N_1. It is also assumed that turn maneuvers are achieved in a coordinated way (the side slip angle remaining approximately null). The aircraft is assumed to fly in a standard atmosphere over a locally flat Earth. Then, neglecting the vertical component of wind speed, the flight guidance equations can be written:

$$\dot{x} = V \sin\psi \cos\gamma \ , \ \dot{y} = V \cos\psi \cos\gamma \ , \ \dot{z} = V \sin\gamma \tag{1}$$

$$m\dot{V} = -D + T - mg \sin\gamma \cos\phi \quad \text{with} \quad L - m\,g \cos\gamma \cos\phi = 0 \tag{2}$$

where x, y and z are the coordinates of the point aircraft, V is the inertial speed, m is the mass, D is the aerodynamic drag force, T is the thrust of the engines and L is the aerodynamic lift force. When perfect coordinated turns are achieved, the heading rate is related to the bank-angle ϕ through the following relation:

$$\dot{\psi} = (g/V)\tan\phi \tag{3}$$

The drag and lift forces are such as:

$$D = 1/2\,\rho(z)\,S\,V_a^{\,2}\,C_D \quad \text{and} \quad L = 1/2\,\rho(z)\,S\,V_a^{\,2}\,C_L \ \text{ with } \ V_a = \sqrt{(\dot{x} - w_x)^2 + (\dot{y} - w_y)^2 + \dot{z}^2} \tag{4}$$

where w_x and w_y are the horizontal components of wind speed. Then, considering the polar expression: $C_D = a + b\,C_L^{\,2}$, the drag force can be expressed as :

$$D = 1/2\,\rho(z)\,S\,a\,V_a^{\,2} + 2\,b\,(m\,g\cos\gamma)^2 / (\rho(z)\,S\,V_a^{\,2}) \tag{5}$$

The thrust can be expressed as :

$$T = \rho(z)\,S_M\,V_a\,(V_e - V_a) \tag{6}$$

where ρ is the air volumic mass, S_M is the engine entry section area and V_e is the exhaust gaz speed. It can be considered as a smooth function of the fan regime N_1, the flight level z and the airspeed V_a :

$$V_e = v(z, V_a, N_1) \tag{7}$$

3.2 Cost Components Associated with Flight Guidance

In this sub section are considered the different components of cost related with the evolution of an aircraft during a time interval $[t_i,\ t_f]$ considering either climb, cruise or descant and approaches flight phases. These components are mainly the fuel cost, the delay cost and the noise effect. Fuel cost can be computed as :

$$C_c = \int_{t_i}^{t_f} \pi_c \ FF(t)\, dt \tag{8}$$

where π_c is the fuel price and where the fuel flow FF , in kg/unit of time, can be estimated as a smooth function of V_a and T :

$$FF = f(V_a, T) \tag{9}$$

The Bada reference data base from Eurocontrol provides for instance the formula :

$$FF = c_{f1}\left(1 + c_{f2} V_a\right) T \tag{10}$$

where c_{f1} and c_{f2} are parameters whose values are characteristic of the aircraft type.

The delay cost represents the different constant rate costs associated with aircraft operations (insurances, traffic control fees, crew salaries, etc). Total delay cost is given by :

$$C_d = \pi_d \left(t_f - t_i\right) \tag{11}$$

where π_d is the cost attached to one unit of time of delay. Here the value of π_d could be a source for controversy, so this cost will be merged in a global noise index, NI, representative of the traffic management policy with respect to noise. Aircraft noise is mainly composed of an omnidirectional source related with the aerodynamic noise and a directional source related with the engine noise.

The level of noise received at point (x_G, y_G, z_G) is given by :

$$P(x_G, y_G, z_G) = (P_a(z, V_a) - P_E(z, V_e)\,\omega(x - x_G, y - y_G, z - z_G, \gamma, \psi))\,\varpi(x - x_G, y - y_G, z - z_G, w_x, w_y) \tag{12}$$

where P_a is the power of aerodynamic noise, P_e is the power of engine noise, ω is related with the directional effect of jet noise and ϖ is related with the noise distortion resulting from the wind. The impact of instant noise over an individual at location (x_G, y_G, z_G) is then given by :

$$\Omega\left(P(x_G, y_G, z_G)/\left((x - x_G)^2 + (y - y_G)^2 + (z - z_G)^2\right)\right) \tag{13}$$

where Ω is a logarithmic function .

Then an aggregated measure of the total effect of instant noise over the ground area surrounding the aircraft can be computed by:

$$\Phi_n(x, y, z) = \int_{X_G}\int_{Y_G}\int_{Z} \sigma(x_G, y_G, z_G)\,\Omega\left(P(x_G, y_G, z_G)/\left((x - x_G)^2 + (y - y_G)^2 + (z - z_G)^2\right)\right) dx_G\, dy_G\, dz_G \tag{14}$$

where σ is a corrected population density function and X_G x Y_G x Z_G is the surrounding area. If it is considered that a penalty, proportional to this measure is applied to the operator of the aircraft, the total cost associated with noise is given by :

$$C_n = \pi_n \int_{t_i}^{t_f} \Phi_n \, dt \qquad (15)$$

The total cost for the operator of the aircraft is given by :

$$C = \int_{t_i}^{t_f} \left(\pi_c \; FF + \pi_d \; + \pi_n \; \Phi_n \right) dt \qquad (16)$$

In order to optimize aircraft trajectories over space and time, a cost index is provided to the flight management system of modern a aircraft. The cost index relates the cost of delay to the price of the fuel : $\pi_d = CI \; \pi_c$. Then economy flights are associated with low values of the cost index while more direct and faster flights are associated with high values of the cost index. In the same way, a noise index NI can be introduced: $NI = \pi_n / \pi_c$. Then trajectories with high noise index will minimize noise impact over surrounding populations, while low noise index will give priority to fuel and/or delay.

3.3 Optimization Problem Formulation and Analysis

A first formulation of the optimization problem of aircraft trajectory with noise impact consideration is proposed here :

$$\text{Min} \int_{t_i}^{t_f} \left(FF + CI + NI \; \Phi_n \right) dt \qquad (17)$$

under the state equations (1) (2) and (3) , with the initial and final conditions where t_f is free :

$$x(t_i) = x_i \,,\; y(t_i) = y_i \,, z(t_i) = z_i \,, V(t_i) = V_i \,, \psi\,(t_i) = \psi_i \qquad (18)$$

$$x(t_f) = x_f \,,\; y(t_f) = y_f \,, z(t_f) = z_f \,, V(t_f) = V_f \,, \psi\,(t_f) = \psi_f \qquad (19)$$

and the following permanent constraints :

$$T_{\min} \le T(z, V_a, V_e) \le T_{\max} \quad , \qquad V_{a\min}(z) \le V_a \le V_{a\max}(z) \qquad (20)$$

and

$$\phi_{\min} \le \phi \le \phi_{\max} \quad , \qquad \gamma_{\min} \le \gamma \le \gamma_{\max} \qquad (21)$$

In this problem, V_e, γ and ϕ are the input variables, they drive the space-time aircraft trajectory.Of course the above problem is far from trivial and cannot today be solved with accuracy by current on board computers. Dynamic programming is one of the possible optimization techniques able to cope with this general optimization problem, but this implies the discretization, either over time or over space, of the whole problem.

3.4 Flatness and Assessment of Candidate Trajectories

One interesting particularity of the considered flight dynamics equations is that, once the time period is fixed, they possess the differential flatness property with respect to the aircraft position *(x, y, z)*.

Definition 1**:** A general nonlinear system given by:

$$\dot{\underline{X}} = f(\underline{X},\underline{U}), \qquad \underline{X} \in R^n,\ \underline{U} \in R^m \tag{22}$$

where f is a smooth mapping, is said *explicitly flat* with respect to the output vector $\underline{Z}$, if $\underline{Z}$ is an m^{th} order vector which can be expressed analytically as a function of the current state, the current input and its derivatives and also such as the state and the input vectors can be expressed analytically as a function of $\underline{Z}$ and its derivatives. Then there exists smooth mappings F_X, F_U, and F_Z such as:

$$\underline{Z} = F_Z\left(\underline{X},\underline{U},\ldots,\underline{U}^{(p)}\right),\ \underline{X} = F_X\left(\underline{Z},\dot{\underline{Z}},\ldots,\underline{Z}^{(q)}\right),\ \underline{U} = F_U\left(\underline{Z},\underline{Z},\ldots,\underline{Z}^{(q+1)}\right) \quad p,q \in N \tag{23}$$

Vector $\underline{Z}$ is called a *flat output* for the considered nonlinear system.

The explicit flatness property is of particular interest for the solution of a control problem when a physically meaningful flat output can be related with its objectives. In many situations, the control problem can be formulated as a flat output trajectory following problem. However, for many systems, no complete analytical models are available to describe their full dynamics. Then some of their components make use of input-output numerical devices derived both from theory and from experimental data. In these cases, available theory provides in general the main mathematical properties of these implicit functions while experimental data is used to build accurate input output numerical devices. This happens for instance when flight dynamics modeling is considered either for control or simulation purposes, since many often, the aerodynamic coefficients are obtained from a set of look-up discrete numerical tables and through complex interpolation computing. In this case, it is possible to define implicit flatness and if it holds, given a trajectory for the flat output $\underline{Z}$, it is still possible to map it numerically into the input space to get an adequate control law so that one of the more helpful property of explicitly flat systems is still maintained. It appears from the above flight equations that the flight variables γ, ϕ and N_1 can be taken as the inputs for the guidance dynamics while they are output variables for the body frame dynamics when controlled by a basic autopilot. By rearranging the kinematical equations, it is convenient to express V, γ and ψ as:

$$V = \left((\dot{x})^2 + (\dot{y})^2 + (\dot{z})^2\right)^{\frac{1}{2}},\ \gamma = -\sin^{-1}(\dot{z}/V) \ \text{ and } \ \psi = \tan^{-1}(\dot{y}/\dot{x}) (24) \tag{24}$$

The state variables obviously can be functions of the position of the aircraft while the control variables satisfy the following relations:

$$\dot{V} - \frac{-D + T - mg\sin\gamma\cos\phi}{m} = 0 \ \text{ with } \ L - m\,g\cos\gamma\cos\phi = 0 \tag{25}$$

and

$$\dot{\psi} - (g/V)\tan\phi = 0 \tag{25}$$

Here:

$$\underline{Z} = (x, y, z)^T \quad \text{and} \quad \underline{U} = (\gamma, \phi, N_1) \tag{26}$$

When, and it is very often the case, no analytical expressions are available for the forces applied to the aircraft, the drag, lift and heading equations can be regarded as implicit functions of the position vector $\underline{Z}$, of its first two derivatives with respect to time and of input $\underline{U}$ .They can be rewritten as:

$$G_{N_1}(\underline{Z},\underline{\dot{Z}},\underline{\ddot{Z}},\underline{U})=0\,,\ G_\theta(\underline{Z},\underline{\dot{Z}},\underline{\ddot{Z}},\underline{U})=0 \quad \text{and} \quad G_\phi(\underline{Z},\underline{\dot{Z}},\underline{\ddot{Z}},\underline{U})=0 \tag{27}$$

These implicit functions are often locally invertible with respect to the input since, for normal flight conditions, the determinant of their Jacobian is not zero. Then, the considered flight guidance dynamics are implicit flat with $\underline{Z}=(x,y,z)^T$ as their flat output vector. The time evolution of these flat outputs represents the trajectory followed by the center of gravity of the aircraft. Then, given a trajectory $\{(x(t),\ y(t),\ z(t)),\ t \in [t_i,\ t_f]\}$, it is possible to reconstruct all the state variables as well as the necessary input values, and more particularly the trajectories of V_a and V_e : $\{(V_a(t),\ V_e(t),\ t \in [t_i,\ t_f]\}$, which are needed to compute the total noise effect history $\{(\Phi_n(x(t), y(t), z(t)),\ t \in [t_i,\ t_f]\}$ and then detect critical instant and aircraft position at which the noise effect is maximum for the surrounding community. Also for each given trajectory, total cost will be computable andvarying the value of the noise index, Pareto efficiency curves relating cost level and noise effect can be drawn.

4. OPTIMIZATION APPROACH FOR DEPARTURE TRAJECTORY

In this section the evaluation of the noise impact from standard take-off procedures and departure trajectories is considered. A no wind condition is considered for an aircraft of known mass *m* climbing in a vertical plane oriented along the runway axis. Here it is assumed that maximum noise levels are imposed at some given control points and that these constraints must be satisfied by the aircraft while performing the climb trajectory.

4.1 Standard Climb Trajectories

Considering a standard departure trajectory for a wide body aircraft, it appears that if the initial segment and engine parameters are prescribed by safety considerations, the other low altitude flight segments present some degree of freedom, giving way to departure trajectory optimization taking into account noise impact. For instance during segment 1, the aircraft may climb with a constant path angle γ_1, the engines are at their max-climb rate so that the aircraft increases its speed from « V_2 », the take-off safety speed, to V_{OM} , the optimal climb speed. During segment 2, the aircraft may perform a climb at constant path angle γ_2 at constant speed V_{OM}, for a max-continue thrust level.

Here the adopted noise indicator is the EPNdB (Effective Perceived Noise dB). There are three reference points used for sound exposure control. A reference point devoted to overfly noise control (point **A**), a reference point (point **B**) in the neighbourhood of the area where take-off noise is maximum and a reference point to check noise level during the approach (point **C**). Other lateral points can be taken into account to cover properly the take-off area. Regulations specify that the noise level at the lateral reference point (take-off) cannot exceed 103 EPNdB for an aircraft with a max take off weight over 400 tons. This threshold decreases linearly with the logarithm of the mass of the aircraft to reach a minimum value 94 EPNdB for 35 tons. At point **B** (overfly), 101 EPNdB for single engine, 104 EPNdB for two engines and 106 EPNdB for three engines and max take-off mass over 385 tons. Here also, these values decrease of 4 EPNdB when mass is devided by two, reaching a lower limit of 89

EPNdB. At point **C** (approach), the limit is 105 EPNdB for single engine, 104 EPNdB for two engines and 106 EPNdB for three engines and max take-off mass over 280 tons. Here also, these values decrease of 4 EPNdB when mass is devided by two, reaching a lower limit of 98 EPNdB at a height of 35 ft. During departure, if the maximum noise level exceeds the threshold, it must done with an overshoot smaller than 3 EPNdB, while excess at another lateral point should not be larger than 2 EPNdB.

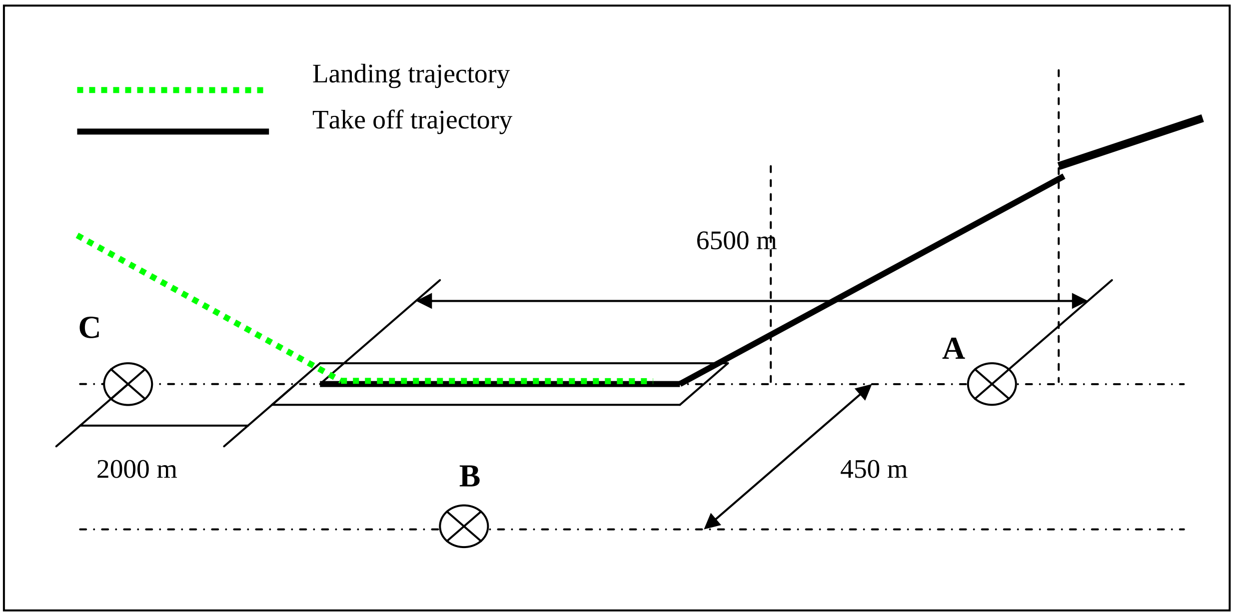

Figure 1 : Position of the control points

4.2 Statement of a Specific Trajectory Optimization Problem

Here we consider a more general case than the one described in the previous sub-section : During segment *1*, altitude goes from z_0 to z_1 at a constant path angle γ_1, while speed is increasing from V_0 to a speed V_1, less or equal to V_{OM}, then during segment *2*, the aircraft climbs from z_1 to z_2 at a constant path γ_2 and reaches V_{OM}. It is also supposed that during each segment the fan regime of the engines remain constant (N_{1i} , $i = 1,2$). Then the propagation equations through segment i are such as:

$$\dot{x} = V\cos\gamma_i \quad , \quad \dot{z} = V\sin\gamma_i \tag{28}$$

and

$$m\dot{V} = -D + T - mg\sin\gamma_i \quad \text{with} \quad L - mg\cos\gamma_i = 0 \tag{29}$$

where equations (5) and (6) hold, with the conditions:

$$x(t_0) = x_0\,,\quad z(t_0) = z_0\,,\quad V(t_0) = V_0\,,\quad z(t_1) = z_1\,,\quad z(t_2) = z_2\,,\quad V(t_2) = V_{OM} \tag{30}$$

where x_0, z_0, V_0, z_1, z_2, and V_{OM} are given. It is Assumed that m and $\rho(z)$ remain practically constant during the two segments, that relation (7) turns out to $V_e = v(V, N_1)$. Then, V obeys to a non linear differential equation such as :

$$\dot{V} = A(\gamma_i) + B(V_{ei}(V, N_{1i}))\, V + C\, V^2 + E(\gamma_i)/V^2 \qquad \text{or} \qquad \dot{V} = \Gamma(V, V_{ei}, \gamma_i) \tag{31}$$

where

$$A(\gamma_i) = -\, mg\sin\gamma_i,\; B(V_e) = (\rho/m) S_M V_{ei},\, C = -\,(\rho/m)(S_M + aS/2),\; E(\gamma_1) = -\left(2bg^2 \cos\gamma_1{}^2 / S\right)/(\rho/m) \tag{32}$$

Here speed constraint (20) is rewritten as :

$$V_{\min} \le V \le V_{\max} \tag{33}$$

4.3 Problem Analysis and Solution Approach

Here V_1 which is an intermediate unknown of importance for operationsis solution of the integral equation :

$$\int_{V_0}^{V_1} \frac{V \sin\gamma_1}{\Gamma(V, V_{e1}, \gamma_1)}\, dV = z_1 - z_0 \tag{34}$$

Then t_1 and x_1 are given by the relations :

$$t_1 = t_0 + \int_{V_0}^{V_1} \frac{dV}{\Gamma(V, V_{e1}, \gamma_1)} \quad , \quad x_1 = x_0 + \int_{V_0}^{V_1} \frac{V \cos\gamma_1}{\Gamma(V, V_{e1}, \gamma_1)}\, dV \tag{35}$$

while t_2 and x_2 are given by:

$$t_2 = t_1 + \int_{V_1}^{V_{OM}} \frac{dV}{\Gamma(V, V_{e2}, \gamma_2)} \quad , \quad x_2 = x_1 + \int_{V_1}^{V_{OM}} \frac{V \cos\gamma_2}{\Gamma(V, V_{e2}, \gamma_2)}\, dV \tag{36}$$

Then the total operator cost for the two segments is given by :

$$C = \pi_c \left(\int_{t_0}^{t_1} FF(V, V_{e1}, z)dt + \int_{t_1}^{t2} FF(V, V_{e2}, z)dt + CI\,(t_2 - t_0) \right) \tag{37}$$

The noise constraints at control points points **A** and **B** are (see (13)):

$$\Omega\left(P(x_A)/\left((x(t) - x_A)^2 + (y(t))^2 + (z(t))^2\right)\right) \le L_{A\max}(m) \qquad t_0 \le t \le t_2 \tag{38}$$

$$\Omega\left(P(x_B, y_B)/\left((x(t) - x_B)^2 + (y(t) - y_B)^2 + (z(t))^2\right)\right) \le L_{B\max}(m) \qquad t_0 \le t \le t_2 \tag{39}$$

Then, the optimization problem will minimize (37) under initial and final conditions (30), state equations (28) and (31), speed constraint (33) and noise level constraints (38) and (39). Here the true independant parameters, for a given aircraft configuration, are the path angles γ_1 and γ_2 , and the fan regimes N_{11} and N_{12}. Observe that in (30), condition $z(t_2) = z_2$**,** ties together these parameters:

$$z_2 = z_0 + \int_{V0}^{V_1} \frac{V \sin\gamma_1}{\Gamma(V, V_e(V, N_{11}), \gamma_2)}\, dV + \int_{V_1}^{V_{OM}} \frac{V \sin\gamma_2}{\Gamma(V, V_e(V, N_{12}), \gamma_2)}\, dV \tag{40}$$

So, choosing three of these four flight control parameters, the fourth will be deduced from the others. For instance, choosing the value of γ_1, γ_2 and N_{11}, it will be possible to solve equation (40), with respect to N_{12}. If the value of N_{12}. is such that the first condition of (20) is satisfied everywhere, (γ_1, γ_2, N_{11}, N_{12}) is a candidate quadruplet. Taking into account the nature of the different constraints and criterion (implicit, integral, non convex, infinite dimension, etc) of this optimization problem, it appears that a progressive optimization approach based on local considerations will be at least problematic. To cope with the infinite dimension constraints (38) and (39), they can be considered only at discrete instants or they can be replaced by single majored constraints which are stated at a given position of the aircraft along the trajectory.
Since few flight control parameters have to be chosen and since they evolve in a well known limited domain, a systematic search over this domain should be performed. Preliminary numerical results have shown that the time response of this search can be improved significantly by using some basic genetic algorithm technique. Once best solutions have been found for many different situations, it can be of interest to integrate within a single neural structure, such as a feed forward all the optimal input-output pairs so that neural interpolation can be used on board the aircraft to generate in real time, adequate flight control parameters for a given situation.

5. CONCLUSIONS AND PERSPECTIVES

The formulation of an aircraft trajectory optimization problem which takes into account noise effect appears to be a complex task requiring specific aerodynamics, propulsive and noise emission models. The resulting optimization problem is in general intractable by standard optimization techniques. Since it is common for commercial air transportation activities to face in normal operation very different conditions (mass, winds, runway length, etc), it appears convenient to integrate its solver within the on-board Flight Management System. However, since on board computing must be limited and guaranteed in terms of response time, numerical convergence, accuracy and stability, extensive off-line computation should be performed and local results integrated within a simpler computing device, such as a neural structure, for on board on-line operation.

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