# Application of Optimal Delays Selection on Parallel Cascade Hammerstein Models for the Prediction of RF-Power Amplifier Behavior

D. D. Silveira<sup>1</sup>, H. Arthaber<sup>1</sup>, P. L. Gilabert<sup>2</sup>, G. Magerl<sup>1</sup>, and E. Bertran<sup>2</sup>

<sup>1</sup>Institute of Electrical Meas. and Circuit Design, Vienna University of Tech., Gusshausstrasse 25/354, A-1040, Vienna, Austria, E-mail: daniel.silveira@tuwien.ac.at

<sup>2</sup>Department of Signal Theory and Comm., Universitat Politécnica de Catalunya (UPC), Av Canal Olímpic s/n, 08860, Barcelona, Spain, E-mail: plgilabert@tsc.upc.edu

Abstract — In this work two different black-box modeling strategies are combined in order to achieve improved RF Power Amplifier (PA) models based on measured input/output signals. The model validation is here done within the low input back-off (IBO) region of the power amplifier, thus showing significant nonlinear operation. The Figure of Merit (FoM) applied is the Normalized Mean Square Error (NMSE). A rectangle constellation modulation, 16-QAM, is used as input signal, and the nonlinearities caused by the low IBO operating conditions of the PA distort the output constellation. The PA output signal used to estimate the model coefficients presented high distortion levels, requiring a model capable to also care for memory effects, as it will be seen in this article. It has been observed that the combination of a selection of the delavs values with parallel-cascade Hammerstein pseudo-inverse based models can improve the identification accuracy, leading to precise models. This condition is important regarding the design of efficient pre-distorters, once its performance is very sensitive to the model quality.

Index Terms — Black-box models, nonlinear estimation, nonlinear models, power amplifiers.

#### I. INTRODUCTION

Several modeling approaches have been used to characterize RF power amplifiers. The existence of multiple minima in the error surface is a challenging problem, so the model estimation using more than one approach increases the possibilities to find a suitable model. Parallel-cascade of block structured models structured has been successfully used in [1]-[3], showing acceptable results. Improvements in the estimations of the single linear blocks of these structures are presented in [4]. In [5] a reliable technique to estimate the optimal delays is presented. The unification of two modeling approaches is a promising strategy, when dealing with nonlinear iterative estimation problems.

The objective of this paper is to present an amplifier model fitted by using two distinct techniques together: the optimal selection of delays for a FIR filter at the PA model input, combined with the parallel-cascade Hammerstein model.

#### II. BACKGROUND

This section covers the definition of the parallel cascade Hammerstein structure model, the explanation about the optimal delays structure inclusion in this model, and the figures of merit applied to validate the extracted results.

#### A. Parallel Cascade Hammerstein Model

This model is composed of parallel-cascaded nonlinear and linear blocks as shown in Fig. 1.

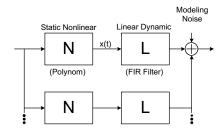


Fig. 1 Parallel Cascade Hammerstein Model

The output can be represented as:

$$y(t) = \sum_{p=1}^{P} \sum_{\tau=0}^{T-1} h_{P}(\tau) \left( \sum_{q=1}^{Q} e_{p}^{q} u^{q}(t-\tau) \right)$$
(1)

where P is the maximum number of paths, Q denotes the maximum order of the polynomial used,  $c_p^q$  are the path polynomial coefficients, T is the memory length, t and  $\tau$  are discrete indexes of the sampling interval and  $h_P(\tau)$  is the path impulse response.

# B. Optimal Delays Included in the Parallel Cascade model

A pre-filtering using the most significant delays is applied at the input of the parallel cascade Hammerstein model, as shown in Fig. 2. This characterizes then a Wiener-Hammerstein model, which is expected to have a good potential for the identification of nonlinear memory effects.

## C. Model Evaluation

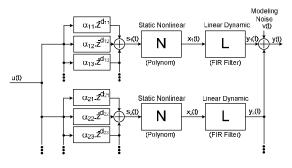


Fig. 2 Parallel Cascade Hammerstein structure with prefiltering considering optimal delays

The statistically based FoM used in this article to specify how well a model describes an unknown system is the *normalized mean squared error* (NMSE) [6]:

NMSE =

$$10 \log \left\{ \frac{\sum_{k=1}^{M} \left[ \left( y_{I,k}^{\text{meas}} - y_{I,k}^{\text{mod}} \right)^{2} + \left( y_{Q,k}^{\text{meas}} - y_{Q,k}^{\text{mod}} \right)^{2} \right]}{\sum_{k=1}^{M} \left[ \left( y_{I,k}^{\text{meas}} \right)^{2} + \left( y_{Q,k}^{\text{mod}} \right)^{2} \right]} \right\}$$
(2)

# III. MODEL FITTING TECHNIQUES

This section describes the techniques used in the identification of the nonlinear and linear blocks of the Hammerstein cascade and in the selection of the optimal delays.

# A. Nonlinear Block

The signals used for the estimation of the nonlinear block are the measured input/output signals. A linear regression in the minimum mean square error sense is applied between this signal and the measured output signal. The polynomial order is chosen to minimize the overall error. Only the inband distortion components of the signal are considered. This was achieved multiplying the complex considered signal for its absolute value

raised to the desired power minus 1, when constructing the regression least squares matrix U. This is better visualized as:

$$U = \begin{bmatrix} u & u |u|^2 & u |u|^4 & u |u|^6 & \dots \end{bmatrix}$$
 (3)

#### B. Linear Block

Applying the singular value decomposition (SVD), it is possible to find a suitable form for the least mean square estimator of the impulse response function used to estimate the linear block of the Hammerstein model. This is called pseudo-inverse and is derived for complex data in [7].

# C. Pre-filtering and Selection of Optimal Delays

Taking into account the pre-filtered parallel cascade model depicted in Fig. 2, the relation between every intermediate variable is presented in the following equations:

$$y_{\text{Model}}(k) = \sum_{b=1}^{\text{Branches}} y_b(k) + v(k) \tag{4}$$

Each branch of this parallel-cascade model (denoted by the subindex "b") contributes to the final  $y_{\text{Model}}$  output. The output of each individual branch can be expressed as:

$$y_{b}\left(k\right) = \sum_{m=0}^{M} \beta_{bm} \cdot x_{b}\left(k - m\right) \tag{5}$$

being  $\beta_{bm}$  the FIR filter coefficients and M the number of considered delayed samples. The nonlinear block is implemented by means of a polynomial of order P and  $\gamma_{bp}$  coefficients.

$$x_{b}(k) = f(s_{b}(k)) = \sum_{p=0}^{p} \gamma_{bp} \cdot s_{b}(k) |s_{b}(k)|^{p}$$
 (6)

Then the pre-filtering is carried out by a FIR filter with  $\alpha_{bn}$  being the filter coefficients and N the number of selected taps. These taps ( $d_{bn}$ ) do not have to be necessary consecutive, but they are selected (as detailed in [5]) for being the most significant delays contributing at minimizing the estimation error.

$$s_{b}(k) = \sum_{n=0}^{N} \alpha_{bn} \cdot u(k - d_{bn})$$
 (7)

In order to estimate the first filter coefficients (  $\alpha_{bn}$  ), we split the identification process into two steps. First, considering that the Hammerstein cascade can be described like:

$$\begin{aligned} y_{b}\left(k\right) &= f\left(s_{b}\left(k-m\right)\right) = \\ &= \sum_{m=0}^{M} \beta_{bm} \cdot \sum_{p=0}^{P} \gamma_{bp} \cdot s_{b}\left(k-m\right) \left|s_{b}\left(k-m\right)\right|^{p} \end{aligned} \tag{8}$$

where  $f(s_b(k-m))$  is a dynamic nonlinear function, as it is expressed in (8). Now, by taking (9), it is possible to consider without lost of generality  $\alpha_{b0} = 1$ ,

$$\begin{split} s_{b}\left(k\right) &= \sum_{n=0}^{N} \alpha_{bn} \cdot u\left(k - d_{bn}\right); \left\{\alpha_{b0} = 1\right\} \\ s_{b}\left(k\right) &= u\left(k\right) + \sum_{n=1}^{N} \alpha_{bn} \cdot u\left(k - d_{bn}\right) \end{split} \tag{9}$$

So then, now it is possible to rewrite the following expression for the variable  $s_{b}$ :

$$s_b(k) = f^{-1}(y_b(k-m))$$
 (10)

And combining it with (7) remains,

$$\widehat{u}(k) = f^{-1}(y_b(k-m)) - \sum_{n=1}^{N} \alpha_{bn} \cdot u(k-d_{bn}) \quad (11)$$

What permits identifying the filter coefficients defining a cost function where the estimation error to be minimized is defined as:

$$\begin{split} &e(k) = u(k) - \widehat{u}(k) \\ &= u(k) - f^{-1}(y_b(k-m)) - \sum_{k=1}^{N} \alpha_{bn} \cdot u(k-d_{bn}) \end{split} \tag{12}$$

Once the filter coefficients ( $\alpha_{bn}$ ) are estimated, the intermediate variable  $s_b(k)$  can be calculated. Finally, by using the pseudo-inverse technique defined in [7] the cascade Hammerstein coefficients are also estimated.

## IV. MEASUREMENT SETUP

The measurement system is presented in Fig. 3. The 16-QAM modulated input signal was loaded in

the I/Q Modulation Generator R&S AMIQ and up converted by a Vector Signal Generator R&S SMIQ.

The class AB main amplifier has the following nominal characteristics: frequency range of

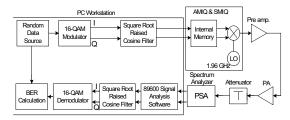


Fig. 3 Measurement system used

1.93...1.96 GHz, maximum output power of +48 dBm and 36 dB Gain.

The PA's output signal was measured at a center frequency of 1.96 GHz using an Agilent Performance Spectrum Analyzer (PSA) and processed by the Agilent 89600 Signal Analysis Software.

The amplifier measured gain characteristics curves are shown in Fig. 4. The gain amplitude and phase present significant changes in the area near the 1 dB compression point.

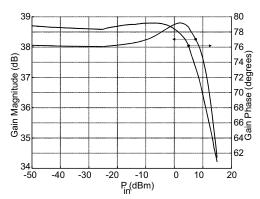


Fig. 4 Amplifier gain characteristics curves

# V. RESULTS

From a previous study it is possible to observe how many delays are necessary in each branch to improve the final contribution to this model.

Table I presents the NMSE results for each branch separately, and Fig. 5 shows the spectral results of the model. A reasonable agreement between AM-AM curves is shown in Fig. 6 and a good agreement between AM-PM curves is seen in Fig. 7.

TABLE I SUMMARY OF NMSE RESULTS

Branches	No. Delays	Taps	NMSE
1	4	[5 1 2 23]	-30.37
2	2	[14 30]	-30.50
3	1	[14]	-30.58

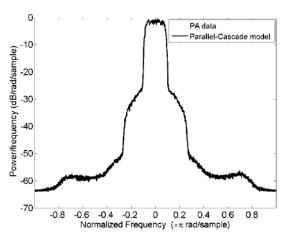


Fig. 5 Power spectral results for the estimated model

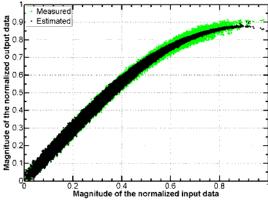


Fig. 6 AM-AM modeling results

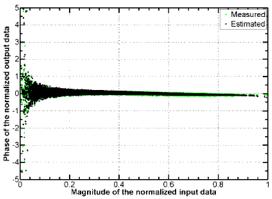


Fig. 7 AM-PM modeling results

#### VI. CONCLUSION

The addition of sparse delays to the Wiener model input has added a degree of freedom, leading to more possibilities in characterizing a PA. Although this is an initial work implying this modeling methodology, some reasonable and promising results have been obtained. Nonlinear dynamics near compression is the weakest point of the model, and future research will be aimed at better characterizing this region using these techniques.

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#### REFERENCES

- M. J. Korenberg, "Parallel Cascade Identification and Kernel Estimation for Nonlinear Systems", Annals of Biomedical Engineering, vol. 19, pp.429-455, 1991.
- [2] H. Ku, M. McKinley and J. S. Kenney, "Quantifying Memory Effects in RF Power Amplifiers", IEEE Transactions on Microwave Theory and Techniques, vol. 50, no. 12, pp. 2843-2849, 2002.
- [3] D. Silveira, M. Gadringer, H. Arthaber, M. Mayer, and G. Magerl, "Modeling, Analysis and Classification of a PA based on Identified Volterra Kernels", 13th GAAS Symposium, pp. 405-408, 2005.
- [4] D. Silveira, M. Gadringuer, H. Arthaber, and G. Magerl, "RF-Power Amplifier Characteristics Determination using Parallel Cascade Wiener Models and Pseudo-Inverse Techniques," APMC-05, pp.204-208, 4-7 December, Suzhou, China, 2005.
- [5] P. L. Gilabert, G. Montoro and E. Bertran, "On the Wiener and Hammerstein Models for Power Amplifier Predistortion," APMC-05, vol. 2, pp.1191-1194, 4-7 December, Suzhou, China, 2005.
- [6] C. P. Silva, C. J. Clark, A. A. Moulthrop, and M. S. Muha, "Survey of Characterization Techniques for Nonlinear Communication Components and Systems", IEEEAC, IEEE, Pages 1-25, 2005
- [7] D. Silveira, M. Gadringer, M. Mayer, and G. Magerl, "Analysis of RF-Power Amplifier Modeling Performance using a 16-QAM Modulation over AWGN Channels", Proceedings of INMMiC 2006. Aveiro, Portugal. January 2006.