NDA Waveform Estimation in the Low-SNR Regime

José A. López-Salcedo, *Associate Member, IEEE*, and Gregori Vázquez, *Senior Member, IEEE*

***Abstract*—This correspondence addresses the problem of nondata-aided waveform estimation for digital communications. Based on the unconditional maximum likelihood criterion, the main contribution of this correspondence is the derivation of a closed-form solution to the waveform estimation problem in the low signal-to-noise ratio regime. The proposed estimation method is based on the second-order statistics of the received signal and a clear link is established between maximum likelihood estimation and correlation matching techniques. Compression with the signal-subspace is also proposed to improve the robustness against the noise and to mitigate the impact of abnormals or outliers.**

***Index Terms*—Channel estimation, low signal-to-noise ratio (SNR), nondata-aided, waveform estimation.**

I. Introduction

Conventionally, most of the digital communication systems assume that the shaping pulses or waveforms are *a priori* known by the receiver. In such cases, the transmitted waveforms are commonly designed in such a way that an intersymbol interference free (ISI-free) detection is possible for an ideal AWGN channel. When the communication channel turns frequency selective, channel estimation techniques must be implemented because the detected information at either the MAP or the ML optimal receiver is affected by the intersymbol interference introduced by the nonideal channel response. Channel estimation techniques can either adopt a *data-aided* (DA) or *nondata-aided* (NDA) approach depending on whether training symbols are available or not. Whereas DA techniques offer the best possible performance, an efficiency penalty is incurred by the transmission of training symbols. Moreover, the receiver is required to be pre-aligned with the piece of incoming data where the training symbols are located. In contrast, flexibility is gained by using NDA techniques since only the statistical characterization of the received signal is required. However, one of the main problems of NDA channel estimation techniques is that a severe degradation is experienced because of abnormals when operating in low-SNR conditions. This is particularly true for traditional channel estimation techniques based on deterministic properties of the noise subspace. In order to circumvent this limitation, the main goal of this correspondence is to formulate the optimal waveform estimation strategy for operation under the low signal-to-noise ratio (SNR) regime. To this end, the low-SNR approximation of the unconditional maximum likelihood criterion (UML) is adopted [1].

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J. A. López-Salcedo was with the Department of Signal Theory and Communications, Technical University of Catalonia (UPC), 08034 Barcelona, Spain. He is now with the Department of Telecommunications and Systems Engineering, Universitat Autònoma de Barcelona (UAB), 08193 Barcelona, Spain (e-mail: jlopezsalcedo@ieee.org; jose.salcedo@uab.es).

G. Vázquez is with the Department of Signal Theory and Communications, Technical University of Catalonia (UPC), 08034 Barcelona, Spain (e-mail: gregori@gps.tsc.upc.edu).

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In the derivation of the proposed estimator, a *waveform* estimation approach is adopted. This is instead of the traditional *channel* estimation approach that assumes the transmit and receive filters to be known at the receiver. The reason is that for some modern wideband communication systems, knowing the actual shape of the transmitted waveform is not possible. This is the case of ultrawideband (UWB) communication systems, also known as *impulsive radio* (IR) [2]. In these circumstances we cannot properly refer to a *channel* estimation problem but rather to a *waveform* estimation problem. At this point, it is interesting to remark that waveform estimation can indeed be avoided by using transmitted-reference (TR) schemes. That is, by transmitting an unmodulated pulse prior to data modulated pulses with the aim of matched filtering at the receiver [3]. Unfortunately, the main drawback here is the efficiency loss and the performance degradation in low-SNR scenarios because of the correlation with a dirty template. The final consideration to be made with respect to the proposed technique is the one regarding the subspace-compressed approach. By compressing the likelihood function with the signal-subspace of the received data, a closed-form implementation is allowed and rank-reduction can be applied. Rank-reduction is interesting in NDA waveform estimation because it can be used to restrict the solution space and thus, to provide a more robust performance in front of the noise and possible ill-conditioning [4].

The structure of the correspondence is the following: Section II defines the signal model for the problem under consideration. Next, Section III presents the UML estimation framework and the resulting low-SNR cost function. Based on the UML formulation, Section IV introduces the subspace-compressed approach and presents the proposed waveform estimation technique. Finally, simulation results are shown in Section V and conclusions are drawn in Section VI.

II. Signal Model

The signal model to be considered herein assumes pulse-amplitude modulation (PAM), pulse-position modulation (PPM) and amplitude-pulse-position modulation (APPM).[1] Let us consider a burst transmission of $L = 2K + 1$ symbols, for some positive integer K. Assuming oversampling of N_{ss} samples per symbol, the observation interval results in a total of N samples that are stacked in a $(N \times 1)$ vector $\mathbf{r}$. The resulting signal model in matrix notation can be expressed as follows:

$$\mathbf{r} = \sum_{p=0}^{P-1} \mathbf{A}_p(\mathbf{g})\mathbf{x}_p + \mathbf{w} \tag{1}$$

where P parallel and independent linear modulations are represented. For the case of PAM modulation, $P = 1$ whereas for M-ary PPM and for $(M_a \times M)$ APPM (i.e., M_a amplitude modulating symbols and M pulse position modulating symbols) we have $P = \log_2(M)$. In (1), wave shaping is carried out by the $(N \times L)$ matrix $\mathbf{A}_p(\mathbf{g})$, transmitted symbols are indicated by the $(L \times 1)$ vector $\mathbf{x}_p$ and the $(N \times 1)$ vector $\mathbf{w}$ stands for the additive white Gaussian noise samples with variance σ_w^2. The symbols are assumed to be zero mean, $E_{\mathbf{x}}[\mathbf{x}_p] = 0$ for any p, and with covariance matrix $E_{\mathbf{x}}[\mathbf{x}_p\mathbf{x}_q^H] = (1/P)\mathbf{I}_L\delta_{pq}$, where $\mathbf{I}_n$ is the $(n \times n)$ identity matrix and δ_{ij} the Kronecker delta. Note that for PPM modulation, the hypothesis of zero mean symbols implies that polarity randomization codes are adopted to avoid the existence of spectral lines. Moreover, for the case of PPM or APPM, inactive pulse-positions are indicated by setting to zero the corresponding entry within $\mathbf{x}_p$ since just one pulse-position can be active for each symbol interval. Finally, the unknown waveform $\mathbf{g}$ is assumed to have a maximum length

[1]Notice that pulse position modulated signals can be expressed as the sum of parallel independent PAM modulations [5].

of N_g samples, and for $N_g > N_{ss}$, N_g is assumed to be a multiple of the number of samples per symbol N_{ss}.

The columns of $\mathbf{A}_p(\mathbf{g})$ are N_{ss}-samples time-shifted replicas of the unknown waveform $\mathbf{g}$. That is $\mathbf{A}_p(\mathbf{g}) \doteq [\mathbf{a}_{-K,p}(\mathbf{g}), \mathbf{a}_{-K+1,p}(\mathbf{g}), \ldots, \mathbf{a}_{K,p}(\mathbf{g})]$, where the n-th column of $\mathbf{A}_p(\mathbf{g})$ is given by $\mathbf{a}_{n,p}(\mathbf{g}) \doteq \mathbf{K}_{n,p}\mathbf{g}$. The set of $(N \times N_g)$ N_{ss}-samples time-shift matrices $\mathbf{K}_{n,p}$ is defined as $\mathbf{K}_{n,p} \doteq \mathbf{J}^n_{N_{ss}}\mathbf{J}^p_{N_\Delta}\mathbf{\Pi}$ with N_Δ the PPM time-shift in samples, $\mathbf{J}_m$ an $(N \times N)$ m-samples shift matrix and $\mathbf{\Pi}$ an $(N \times N_g)$ zero-padding matrix. The general expression for the set of m-samples shift matrices $\mathbf{J}_m$, for $0 \le m \le N$ is given by

$$[\mathbf{J}_m]_{i,j} = \begin{cases} 1 & : \quad (j-i) = m \\ 0 & : \quad (j-i) \neq m \end{cases} \tag{2}$$

whereas the selection matrix is given by $\mathbf{\Pi} \doteq [\mathbf{0}^T_{(N-N_g)/2\times N_g}, \mathbf{I}_{N_g}, \mathbf{0}^T_{(N-N_g)/2\times N_g}]^T$.

From the above considerations, the signal model in (1) can alternatively be expressed as

$$\mathbf{r} = \sum_{p=0}^{P-1}\sum_{n=-K}^{K} x_{n,p}\mathbf{K}_{n,p}\mathbf{g} + \mathbf{w}. \tag{3}$$

The advantage of the formulation in (3) is that it clearly shows the linear dependence of the unknown waveform $\mathbf{g}$ with the received signal $\mathbf{r}$. This linear relationship through the set of matrices $\mathbf{K}_{n,p}$ will be the basis for the derivation of the proposed waveform estimation technique.

III. Low-SNR UML Cost Function for Waveform Estimation

Maximum likelihood (ML) estimation is considered herein for the formulation of the waveform estimation problem. According to the signal model in (1), the likelihood function is based on the Gaussian noise probability density function as follows, $\Lambda(\mathbf{r}|\mathbf{g};\mathbf{x}) = C_0 \exp(-(1/\sigma_w^2)\|\mathbf{r} - \sum_{p=0}^{P-1}\mathbf{A}_p(\mathbf{g})\mathbf{x}_p\|^2)$ with C_0 an irrelevant constant. Since a nondata-aided approach is considered, the transmitted symbols $\mathbf{x}_p$ become nuisance unknown parameters in the ML formulation. By adopting the unconditional maximum likelihood (UML) criterion, we can consider the nuisances as unknown parameters with a known statistical distribution and then obtain the marginal ML function with respect to these unknowns [6]. In this way, the estimate for the unknown waveform $\mathbf{g}$ can be obtained as $\widehat{\mathbf{g}} = \arg\max_{\mathbf{g}} E_{\mathbf{x}}[\Lambda(\mathbf{r}|\mathbf{g};\mathbf{x})]$, with $E_{\mathbf{x}}$ the expectation with respect to the transmitted symbols $\mathbf{x}$. However, this expectation poses insurmountable obstacles and there is usually no choice but to resort to grid search or gradient-based methods to find the solution [7].

In order to circumvent this limitation, the low-SNR assumption allows us to approximate the likelihood function by its Taylor series expansion, $\Lambda(\mathbf{r}|\mathbf{g};\mathbf{x}) \approx C_1[1 + (2/\sigma_w^2)\chi(\mathbf{r};\mathbf{g};\mathbf{x}) + (2/\sigma_w^4)\chi^2(\mathbf{r};\mathbf{g};\mathbf{x})]$ with

$$\chi(\mathbf{r};\mathbf{g};\mathbf{x}) \doteq \sum_{p=0}^{P-1}\mathrm{Re}\left[\mathbf{x}_p^H\mathbf{A}_p^H\mathbf{r}\right] - \frac{1}{2}\sum_{p=0}^{P-1}\sum_{q=0}^{P-1}\mathbf{x}_q^H\mathbf{A}_q^H\mathbf{A}_p\mathbf{x}_p \tag{4}$$

and $C_1 \doteq C_0\exp(-\mathbf{r}^H\mathbf{r}/\sigma_w^2)$. Note that the dependence of $\mathbf{A}$ on $\mathbf{g}$ is omitted for the sake of simplicity. Then, the marginal with respect to the unknown parameters, $\Lambda(\mathbf{r}|\mathbf{g}) = E_{\mathbf{x}}[\Lambda(\mathbf{r}|\mathbf{g};\mathbf{x})]$ can be evaluated. To this end, and after some straightforward manipulations, we have that $E_{\mathbf{x}}[\chi(\mathbf{r};\mathbf{g};\mathbf{x})] = -Tr(\mathbf{M})/2P$ and $E_{\mathbf{x}}[\chi^2(\mathbf{r};\mathbf{g};\mathbf{x})] = (1/2P)\mathbf{r}^H\mathbf{M}\mathbf{r} + (1/4P)\|\mathbf{M}\|_F^2 + (\zeta/4)$ with ζ an asymptotically constant term for a large observation interval and $\mathbf{M} \doteq \sum_{p=0}^{P-1}\mathbf{A}_p\mathbf{A}_p^H$. Alternatively, the log-likelihood function $L(\mathbf{r}|\mathbf{g}) \doteq \ln\Lambda(\mathbf{r}|\mathbf{g})$ can de adopted. This is just a formal consideration but it will allow us to relate the maximum likelihood cost function with some information criteria for determining the signal subspace dimension of the received data. Then, by taking into consideration that $\ln(1+x) \approx x$ when $x \to 0$, we have

$$L(\mathbf{r}|\mathbf{g}) \approx C_2 + \frac{1}{\sigma_w^4 P}\left[Tr\left(\mathbf{M}\left[\mathbf{r}\mathbf{r}^H - \sigma_w^2\mathbf{I}_N\right]\right) + \frac{1}{2}\|\mathbf{M}\|_F^2 + \frac{\zeta}{2}P\right] \tag{5}$$

with $C_2 \doteq \ln C_1$. The expression in (5) can be further simplified by removing all the irrelevant constant terms. Let us denote this simplified expression by $L'(\mathbf{r}|\mathbf{g})$. Next, the initial burst duration of N samples can be split into a set of shorter and truncated observation intervals of N_r samples each,[2] with $N_r = \max\{N_g, N_{ss}\}$. By doing so, the simplified UML cost function results in

$$L'(\mathbf{r}|\mathbf{g}) = Tr\left(\breve{\mathbf{M}}\left[\mathbf{R} - \sigma_w^2\mathbf{I}_{N_r}\right]\right) + \frac{1}{2}\|\breve{\mathbf{M}}\|_F^2 \tag{6}$$

with $\breve{\mathbf{M}}$ the $(N_r \times N_r)$ truncated version of the $(N \times N)$ matrix $\mathbf{M}$. That is

$$\breve{\mathbf{M}} \doteq \sum_{p=0}^{L_p-1}\sum_{n=-K_r}^{K_r}\breve{\mathbf{K}}_{n,p}\mathbf{g}\mathbf{g}^H\breve{\mathbf{K}}_{n,p}^H \tag{7}$$

with $2K_r+1$ the number of N_{ss}-samples shifted replicas of $\mathbf{g}$, and L_p the number of N_Δ-samples shifted replicas of $\mathbf{g}$, both within an observation interval of N_r samples. The matrix $\breve{\mathbf{K}}_{n,p}$ in (7) is the $(N_r \times N_g)$ truncated version of the $(N \times N_g)$ matrix $\mathbf{K}_{n,p}$. That is, in Matlab notation, $\breve{\mathbf{K}}_{n,p} \doteq \mathbf{K}_{n,p}(1:N_r,:)$. Finally, $\mathbf{R}$ is the $(N_r \times N_r)$ synchronous autocorrelation matrix of the received data defined as

$$\mathbf{R} \doteq \lim_{L\to\infty}\frac{1}{L}\sum_{n=0}^{L-1}\mathbf{r}_n\mathbf{r}_n^H \tag{8}$$

with $\mathbf{r}_n \doteq [r(nN_{ss}), r(nN_{ss}+1), \ldots, r(nN_{ss}+N_r-1)]^T$.

From the observation of (6) two important remarks must be made. First, the low-SNR UML cost function projects the synchronous autocorrelation matrix of the received data onto the signal subspace spanned by $\breve{\mathbf{M}}$ in (7). This is in contrast with most channel and waveform estimation methods, where the noise subspace plays the central role. Second, the constant term $\|\breve{\mathbf{M}}\|_F^2$ in (6) is a second-order constraint that allows the UML cost function to be thought as a least-squares problem on the second-order statistics of the received signal. This issue will be illustrated in more detail in Section IV.

IV. Proposed Waveform Estimation Technique

A. Subspace-Compressed Approach

One of the major drawbacks of ML channel/waveform identification methods is that a closed-form solution is often difficult to be obtained. In addition, the solution is further complicated by the possible existence of local minima. This is the case of (6), where a nonlinear optimization with respect to $\mathbf{g}$ is required. However, a valuable help is given when the information regarding the signal subspace is available. This allows the ML estimator to restrict the solution space to a neighborhood around the true waveform. Thus, a valid estimate is still possible even when close to unindentifiable [8].

In this section, a subspace constraint is presented for the cost function derived in (6). To this end, note that the synchronous autocorrelation matrix of the received data is asymptotically given by $\mathbf{R} = \breve{\mathbf{M}} + \sigma_w^2\mathbf{I}_{N_r}$. Then, the linear space spanned by $\breve{\mathbf{M}}$ is the signal subspace where the unknown waveform $\mathbf{g}$ is contained. Moreover, the signal subspace dimension d turns out to be given by the number of linearly independent columns in $\breve{\mathbf{M}}$. Based on this remark, it can be

[2] Notice that this reduced observation interval could have been considered right at the beginning of this correspondence. However, this would complicate the formulation because transmitted symbols for adjacent intervals would be correlated when $N_g > N_{ss}$.

TABLE I
DESCRIPTION OF THE PROPOSED LOW-SNR UML WAVEFORM ESTIMATION TECHNIQUE

1)	Estimate the $(N_r \times N_r)$ synchronous autocorrelation matrix $\mathbf{R}$ in (8).
2)	Determine $\mathbf{U}_{\mathrm{s}}$, the matrix of signal subspace eigenvectors of $\mathbf{R}$.
3)	Construct the vector $\mathring{\mathbf{r}}_{\mathrm{v}}$ in (11) and the matrix $\mathbf{Q}$ in (12).
4.1)	Solve $\widehat{\boldsymbol{\alpha}}_{\mathrm{v}} = \left(\mathbf{Q}^H\mathbf{Q}\right)^{-1}\mathbf{Q}^H\mathring{\mathbf{r}}_{\mathrm{v}}$ in (15).
4.2)	Get $\widehat{\boldsymbol{\alpha}}$, the maximum eigenvector of $vec^{-1}(\widehat{\boldsymbol{\alpha}}_{\mathrm{v}})$.
5)	Obtain the waveform estimate as $\widehat{\mathbf{g}} = \mathbf{U}_{\mathrm{s}}\widehat{\boldsymbol{\alpha}}$.

stated that $\mathrm{range}\{\breve{\mathbf{M}}\} = \mathrm{range}\{\mathbf{U}_{\mathrm{s}}\}$ where $\mathbf{U}_{\mathrm{s}}$ is the $(N \times d)$ matrix with the d signal subspace eigenvectors of $\mathbf{R}$. Consequently, the estimate for the unknown waveform $\mathbf{g}$ can be forced to be contained within the signal subspace as follows:

$$\mathbf{g} = \mathbf{U}_{\mathrm{s}}\boldsymbol{\alpha} \tag{9}$$

for some $(d \times 1)$ vector $\boldsymbol{\alpha}$ such that $\boldsymbol{\alpha}^H\boldsymbol{\alpha} = 1$. Note that the $(d \times 1)$ vector $\boldsymbol{\alpha}$ contains the coordinates of the $(N_g \times 1)$ unknown waveform $\mathbf{g}$ with respect to the basis of signal eigenvectors. The key point here is that by sampling at a rate equal or greater than twice the symbol rate, then $d < N_g$. As a result, projection onto the signal subspace compresses the number of unknowns from N_g to just d. This can be proved by noting that $d = 2\lceil N_r/N_{ss}\rceil - 1$, with $N_r \doteq \max\{N_g, N_{ss}\}$, and thus, $d < N_g \Leftrightarrow N_{ss} \geq 2$. Since the number of subspace coordinates d is smaller than the number of unknown waveform samples N_g, there is an SNR gain and a more robust performance in front of the noise is expected.

B. Application of the vec Operator to the Low-SNR UML Cost Function

By adopting the vec operator, the low-SNR UML cost function in (6) can be expressed as

$$L'(\mathbf{r}|\mathbf{g}) = \boldsymbol{\alpha}_v^H\mathbf{Q}^H\mathring{\mathbf{r}}_v + \frac{1}{2}\boldsymbol{\alpha}_v^H\mathbf{Q}^H\mathbf{Q}\boldsymbol{\alpha}_v \tag{10}$$

where

$$\mathring{\mathbf{r}}_v \doteq vec\left(\mathbf{R} - \sigma_w^2\mathbf{I}_{N_r}\right) \tag{11}$$

$$\mathbf{Q} \doteq \sum_{n=-K_r}^{K_r}\sum_{p=0}^{L_p-1}\left(\breve{\mathbf{K}}_{n,p}\mathbf{U}_{\mathrm{s}}^*\right) \otimes \left(\breve{\mathbf{K}}_{n,p}\mathbf{U}_{\mathrm{s}}\right) \tag{12}$$

$$\boldsymbol{\alpha}_v \doteq vec(\boldsymbol{\alpha}\boldsymbol{\alpha}^H) \tag{13}$$

being $\mathring{\mathbf{r}}_v$ a $(N_r^2 \times 1)$ vector, $\mathbf{Q}$ a $(N_r^2 \times d^2)$ matrix and $\boldsymbol{\alpha}_v$ a $(d^2 \times 1)$ vector resulting from the stacking of the $(d \times d)$ rank-1 matrix $\boldsymbol{\alpha}\boldsymbol{\alpha}^H$. The expression in (10) is indeed the core of the present manuscript and the basis for the proposed waveform estimation technique. In fact, the expression in (10) is a quadratic equation on the quadratic unknown $\boldsymbol{\alpha}_v$ so that the optimization of (10) is equivalent to the optimization of a least-squares problem

$$\begin{aligned}\max_{\boldsymbol{\alpha}_v} L'(\mathbf{r}|\mathbf{g}) &= \max_{\boldsymbol{\alpha}_v}\left\{\boldsymbol{\alpha}_v^H\mathbf{Q}^H\mathring{\mathbf{r}}_v + \frac{1}{2}\boldsymbol{\alpha}_v^H\mathbf{Q}^H\mathbf{Q}\boldsymbol{\alpha}_v\right\} \\ &= \min_{\boldsymbol{\alpha}_v}\|\mathring{\mathbf{r}}_v - \mathbf{Q}\boldsymbol{\alpha}_v\|^2.\end{aligned} \tag{14}$$

Therefore, the low-SNR UML criterion for the waveform estimation problem is equivalent to a least-squares problem on the second-order statistics of the received signal. This is because $\mathring{\mathbf{r}}_v$ in (11) contains the samples of the synchronous autocorrelation matrix of the received signal. Consequently, the low-SNR UML cost function can be understood as a *correlation matching* (CM) method. CM methods have been previously proposed in the literature for the channel/waveform estimation problem. However, the nonlinear solution is usually obtained by numerical evaluation or gradient-based search [7], [8]. In contrast, the approach presented in this correspondence allows us to obtain an analytical formulation for the low-SNR UML criterion which results in a simple CM problem with a closed-form solution.

C. Closed-Form Solution for the vec Low-SNR UML Cost Function

Based on the equivalence with the least-squares problem in (14), and provided that $\mathbf{Q}$ is a full column rank matrix, the low-SNR UML solution for this particular correlation matching problem is given by

$$\widehat{\boldsymbol{\alpha}}_v = \gamma(\mathbf{Q}^H\mathbf{Q})^{-1}\mathbf{Q}^H\mathring{\mathbf{r}}_v \tag{15}$$

with γ an ambiguity constant inherent in the solution of any blind channel/waveform estimator based on second-order statistics. The closed-form solution in (15) has the same structure as a traditional least-squares problem except for the fact that the unknown variables are quadratic. Thus, once $\boldsymbol{\alpha}_v$ is recovered, it is still required to perform another step to undo the vec operator and recover the vector of signal subspace coordinates $\boldsymbol{\alpha}$. However, when the vec operator is undone, noise and possible signal model mismatches may cause the matrix $\boldsymbol{\alpha}\boldsymbol{\alpha}^H$ to be degraded by a perturbation matrix $\boldsymbol{\Delta}$. That is, $vec^{-1}(\widehat{\boldsymbol{\alpha}}_v) = \boldsymbol{\alpha}\boldsymbol{\alpha}^H + \boldsymbol{\Delta}$ with vec^{-1} the inverse vec operation. Therefore, an estimate for $\boldsymbol{\alpha}$ must be obtained from the eigendecomposition of $vec^{-1}(\widehat{\boldsymbol{\alpha}}_v)$, and taking the eigenvector corresponding to the maximum eigenvalue $\lambda_{\max}$

$$vec^{-1}(\widehat{\boldsymbol{\alpha}}_v)\widehat{\boldsymbol{\alpha}} = \lambda_{\max}\widehat{\boldsymbol{\alpha}}. \tag{16}$$

This is similar to what occurs in the channel estimation for CDMA signals proposed in [9]. Finally, the waveform estimate is given by $\widehat{\mathbf{g}} = \mathbf{U}_{\mathrm{s}}\widehat{\boldsymbol{\alpha}}$. For clarity, the required steps for the proposed technique are summarized in Table I.

In summary, the proposed waveform estimation technique can be understood as a CM method because it performs a matching between the synchronous autocorrelation of the received signal and the synchronous autocorrelation of the signal model. Moreover, the proposed technique is especially devoted to cope with low-SNR scenarios and robustness is improved by compressing the likelihood function with the information regarding the signal subspace. This compression is completely natural since for low-SNR scenarios and finite observation intervals, it makes sense to use the signal subspace rather than the noise subspace. By introducing the signal subspace we are forcing the method to concentrate on a neighborhood around the true waveform and thus, a more robust performance is achieved in the presence of noise and ill conditioning. Finally, an interesting property of the proposed technique is that, since it can be understood as a CM method, it benefits from the well-known asymptotic performance of CM methods in [10] and [11]. In particular, the so-called *asymptotic normalized mean square error* (ANMSE) is the lower bound for the performance of any CM method and it shows the superior performance of moment-based estimators in comparison with traditional eigen-based estimators.

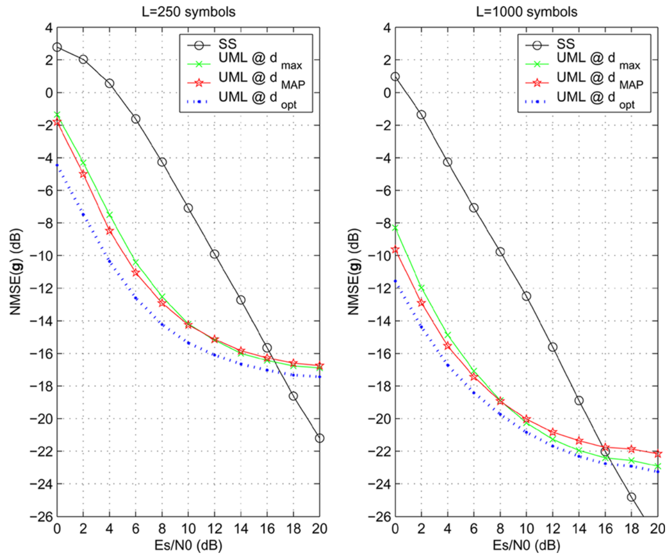

Fig. 1. NMSE as a function of the E_s/N_0 with different criteria for estimating the dimension of the signal subspace.

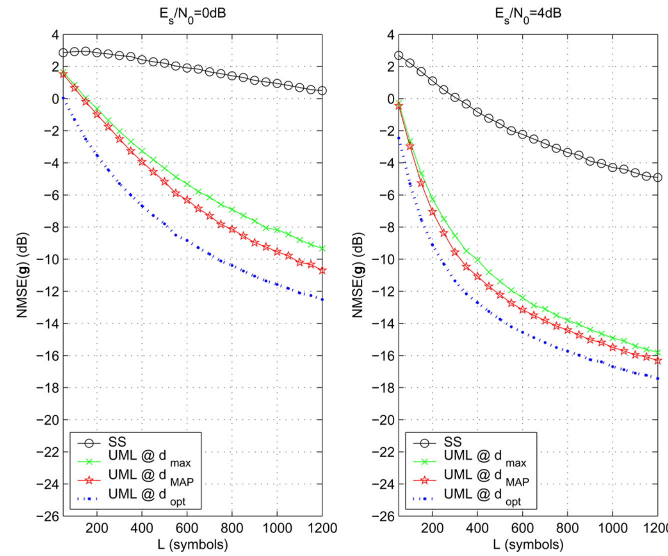

Fig. 2. NMSE as a function of the observation interval with different criteria for estimating the dimension of the signal subspace.

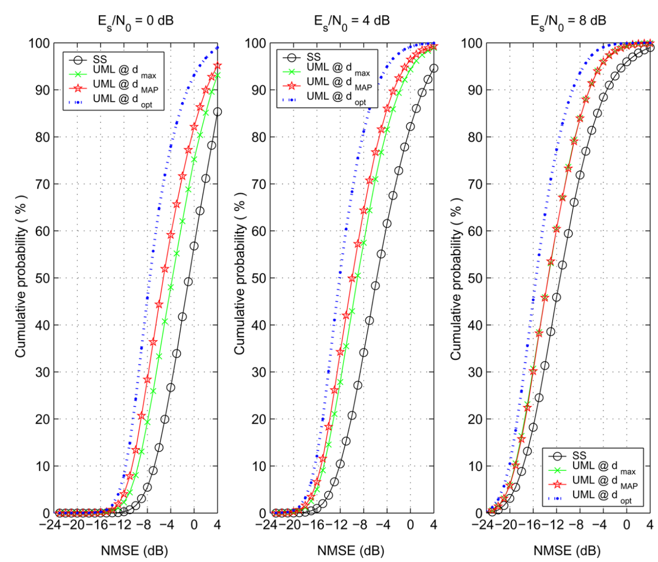

Fig. 3. Cumulative distribution function (CDF) of the NMSE for $E_s/N_0 = \{0, 4, 8\}$ dB.

D. Discussion on the Identifiability of the Proposed Technique

The necessary condition for the waveform $\mathbf{g}$ to be uniquely recovered is that the $(N_r^2 \times d^2)$ matrix $\mathbf{Q}$ in (12) must be a full column rank matrix. In that case there exists a unique solution to the least-squares problem $J(\boldsymbol{\alpha}_v) = \|\mathring{\mathbf{r}}_v - \mathbf{Q}\boldsymbol{\alpha}_v\|^2$. Once the solution to $\boldsymbol{\alpha}_v$ is obtained with (15), then the solution to $\boldsymbol{\alpha}$ and thus, the solution to $\mathbf{g}$, are all unique because the relationship between $\boldsymbol{\alpha}_v$, $\boldsymbol{\alpha}$ and $\mathbf{g}$ is linear. However, a formal proof that guarantees the full column rank condition of matrix $\mathbf{Q}$ is still under investigation.

V. Simulation Results

Computer simulations have been carried out to assess the performance of the proposed technique. For all the simulation scenarios to be considered herein, the unknown waveform is selected at random from a Gaussian distribution with finite time support of $N_g = 8$ samples. The received signal is oversampled with $N_{ss} = 2$ and the observation interval is split in segments of $N_r = N_g$ samples. The modulation format is 16-QAM except for the BER analysis that is set to BPSK for the sake of simplicity. For comparison, the well-known subspace (SS) approach in [12] is considered, which is also a closed-form solution based on second-order statistics but exploiting deterministic properties of the noise subspace.

Experiment 1: The results in Fig. 1 show the normalized mean square error (NMSE) as a function of E_s/N_0 for an observation interval with $L = \{250, 1000\}$ symbols. The estimate of the signal subspace dimension d is obtained with the MAP model order detection rule [13]. For the problem at hand, this results in $d_{\text{MAP}} = \arg\min_m \|\mathring{\mathbf{r}}_v - \mathbf{Q}^{(m)}\boldsymbol{\alpha}_v^{(m)}\|^2 + \ln|\mathbf{Q}^{(m)T}\mathbf{Q}^{(m)}|$. The results with d_{MAP} are compared with the ones when adopting the maximum dimension $d_{\text{MAX}} = 2\lceil N_r/N_{ss}\rceil - 1$ and the optimal dimension d_{opt} that provides the minimum *a posteriori* NMSE in the current simulation.

Experiment 2: The NMSE is evaluated in Fig. 2 as a function of the number of symbols L in the received data record. Clearly, the most remarkable difference can be observed in the right-hand side plot where the NMSE is depicted for the case of $E_s/N_0 = 4$ dB. It can be observed that the gap in terms of NMSE between the proposed method and the SS approach is almost 10 dB when more than 300 symbols are considered.

Experiment 3: Cumulative NMSE is depicted in Fig. 3 to assess the robustness of the proposed technique in front of identifiability issues and possible ill-conditioning. The working points are set to $E_s/N_0 = \{0, 4, 8\}$ dB and a different unknown waveform is randomly selected for each Monte Carlo run. In total, 20 000 different waveforms are generated for each E_s/N_0 working point. A significant gain is also experienced. For the case of $E_s/N_0 = 4$ dB, 20% of the estimated waveforms with the SS approach have a NMSE lower than -10 dB. For the proposed technique, this percentage ranges from 55% up to 75% depending on the way the signal subspace dimension is determined.

Experiment 4: Bit-error rate (BER) is evaluated in Fig. 4 for randomly selected waveforms. The results have been obtained when the estimated waveform is adopted for implementing an MMSE symbol detector. As shown in Fig. 4, the BER significantly improves when increasing the observation interval. This is because the proposed method relies on the signal subspace of the synchronous autocorrelation matrix of the received data, and a long enough observation interval is required to properly estimate this matrix. In terms of E_s/N_0, and under the low-SNR regime with $L = 100$ symbols, there is a 4 or 5 dB loss

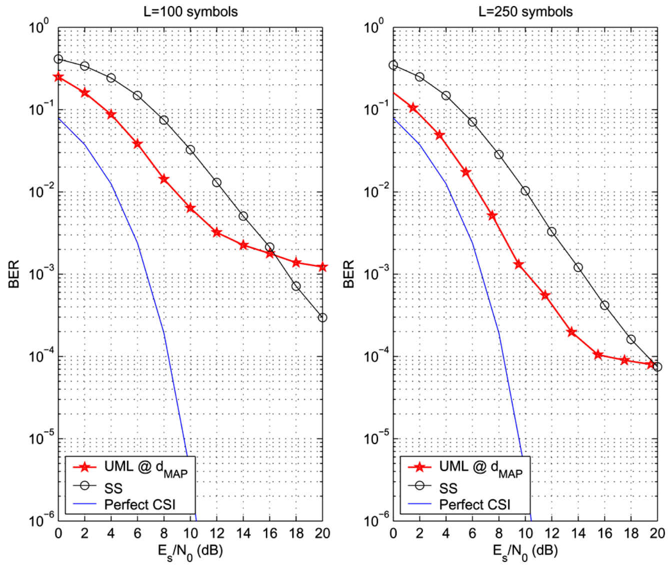

Fig. 4. BER for the MMSE detector as a function of E_s/N_0.

when using the proposed method compared to the case with perfect channel state information (CSI). However this E_s/N_0 loss reduces to just 2 dB with $L = 250$ symbols. In that case, the proposed method also provides a BER that is one order of magnitude lower than that provided by the SS scheme.

VI. Conclusion

A closed-form waveform estimation technique has been proposed based on the low-SNR UML criterion. By introducing a signal subspace constraint and the vec operator, the nonlinear optimization problem is converted into a least-squares problem on the second-order statistics of the received signal. The proposed subspace-compressed approach can be seen as a principal component analysis, and thus, a reduction in the computational burden is obtained through a tradeoff between bias and variance. Moreover, the subspace constraint restricts the solution space and hence, it avoids many of the effects of ill-conditioning and local-maxima of traditional ML channel estimators. Simulation results show the superior performance of the proposed technique with respect to other closed-form methods based on second-order statistics.

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Outage and Diversity of Linear Receivers in Flat-Fading MIMO Channels

Ahmadreza Hedayat, *Member, IEEE*, and
Aria Nosratinia, *Senior Member, IEEE*

***Abstract*—This correspondence studies linear receivers for multiple-input–multiple-output (MIMO) channels under frequency-nonselective (flat) quasi-static Rayleigh fading. The outage probability and diversity gain of minimum mean square error (MMSE) and zero forcing (ZF) receivers are investigated. Assuming M transmit and N receive antennas, the ZF receiver always has diversity $N - M + 1$, unlike the MMSE receiver which may exhibit a rate-dependent behavior. Under separate spatial encoding, where the parallel data streams are not jointly encoded, MMSE is no better than ZF in terms of diversity. Under joint spatial encoding, the MMSE receiver achieves diversity MN at low spectral efficiencies but has diversity only $M - N + 1$ at high spectral efficiencies. These results are established via simulations and an outline for the corresponding analysis is presented.**

***Index Terms*—Diversity, equalization, minimum mean square error (MMSE), multiple-input–multiple-output (MIMO), zero forcing (ZF).**

I. Introduction

In rich scattering conditions, multiple-input–multiple-output (MIMO) wireless channels can support high data rates through spatial multiplexing. Optimal reception, when complexity is not a concern, is through nonlinear nulling-and-cancelling [1]–[3], but when complexity is an issue one may use linear receivers. In this correspondence,

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A. Hedayat is with Navini Networks, Richardson, TX 75083 USA (e-mail: hedayat@ieee.org).

A. Nosratinia is with the Department of Electrical Engineering—MS 33, The University of Texas at Dallas, Richardson, TX 75083 USA (e-mail: aria@utdallas.edu).

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