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Contact Force Transitions in Regrasp Tasks of Planar Objects*

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Abstract

This paper presents a simple and fast solution to the problem of finding the time variations of the forces that keep the object equilibrium when a finger is removed from a three contact point grasp or a finger is added to a two contact point grasp, assuming the existence of an external perturbation force (that can be the object weight itself). The procedure returns force set points for the control system of a manipulator device in a regrasping action. The approach was implemented and a numerical example is included in the paper to illustrate how it works.

I. Introduction

The search for flexible end effectors and the development of grasping and manipulation strategies according to different criteria has become a growing research area during the last two decades [2] [3] [6] [9]. One of the problems within this research field lies is the regrasping of an object, i.e. the variation of the contact points on the grasped object while some grasp properties are kept. This particular problem implies finding the original and final grasp contact points, determine the finger movements, and compute the proper forces to be applied by the fingers when a contact is removed or a new contact is established in order to keep the equilibrium conditions and satisfy the dynamic constraints of the system [11] [10]. Regrasping operations are needed is typically needed when the pick-up grasp configuration is not compatible with the actions to be done with the object or the object placement itself, for instance due to physical constraints in the environment or due to the non-holonomic constraints of the finger contacts, or due to the limits in the articulation ranges of the grasping device.

Different approaches have been presented in the regrasping problem, a detailed description including a discussion about the use of two manipulators can be found in [5]. Some relevant works are those of Tournassoud et al. [11], who proposed a system based on polyhedral models for manipulators equipped with parallel jaw grippers, and Kerr et al. [4] who used a multi-finger hand (these end-effectors are expensive and rarely found in industrial manipulators, but are useful in non repetitive tasks in unstructured environments due to their high dexterity). Recent works done in regrasp [1] [7] [8] are focused on algorithms to determine the sequence of grasps configurations to go from an initial state to a desired final state, but they did not deal with the forces needed to perform the regrasp, which is the central point of this paper.

After this brief introduction the paper is organized as follows. In Section II the problem to be solved is described and formalized. In Section s-problem-analysis the problem is analyzed, the behavior of the system dynamics is characterized, and a graphical tool used to find the solution of the problem is introduced. The proposed solution is described in Section IV, and an example is presented in Section V to illustrate how it works. Finally, the last section of the paper gives some conclusions and describes ongoing and future works.

II. Problem Statement

The problem to be solved can be resumed as follows: Given a three contact point grasp of a flat object that balances an external perturbation force (it may be the own object weight), we want to remove one of the contacts while keeping, during the action, the balance of the external force, or, as inverse situation, given a two contact point grasp add a third contact point where a third finger helps in the balance of the external perturbation. Then, the problem to be solved is the determination of the time variation force set point functions for the contact forces that allow the third contact to be removed/added without loosing the force equilibrium during the process.

This type of problems is found in regrasping manipulation of objects, when a finger is removed from one contact point on the object surface to be place in another one. In this particular case the problem stated appears twice, one when retreating the finger and second when replacing it in the desire new contact point.

The following nomenclature will be used throughout the paper.

S_A S_B: two grasp states in equilibrium (forces applied at the contact points balance any external force)
CM : Center of mass of object.
$\boldsymbol{f}_{ext}$: External force acting on the object (may be the own object weight).
P_i : Contact point i on the object.
$\boldsymbol{r_i}$: P_i location referenced to CM.
L_i : Iso-torque lines parallel to $\boldsymbol{r_i}$.
d_i : Distance between L_i and $\boldsymbol{r_i}$.

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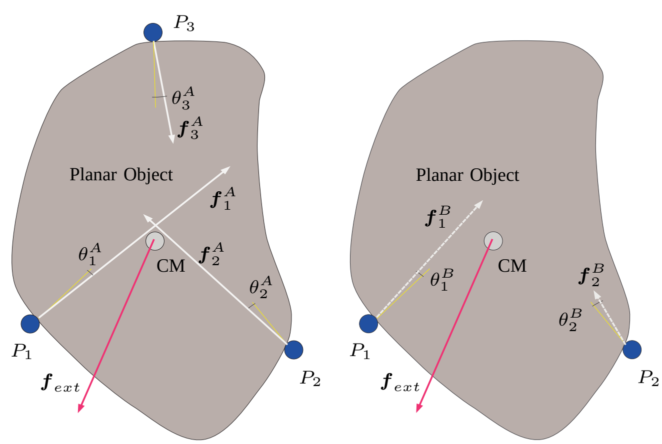
$\boldsymbol{f}_i$	: Force applied on P_i.
C_i	: friction cone at P_i (set of possible forces $\boldsymbol{f}_i$ applicable at P_i).
τ_i	: Torque around CM produced by f_i applied on P_i.
$\boldsymbol{w}_i$	: Generalized force $\boldsymbol{w}_i$ = ($\boldsymbol{f}_i$, τ_i).
Π_0	: Force plane in the wrench space (i.e. null torque plane).
Π_i	: Plane in the wrench space containing any $\boldsymbol{w}_i$ generated at P_i.
$\mathrm{S}\Pi_i$	: Subset of Π_i containing $\boldsymbol{w}_i$ generated at P_i due to forces $\boldsymbol{f}_i$ inside C_i.
$\mathrm{S}\Pi_i{}^{-1}$	: Inverse of $\mathrm{S}\Pi_i$ through the cone origin.

Let S_A be a grasp with three contact points P_i, $i = 1, 2, 3$, on the object boundary (Figure 1a) and S_B be another grasp with only two contact points, which are points P_1 and P_2 from S_A (Figure 1b). It is assumed that in S_A and S_B the finger forces $\boldsymbol{f}_i$ applied at P_i balance an external perturbation force $\boldsymbol{f}_{ext}$, i.e. the summations of the forces and moments applied on the object are null.

The problem to be solved can now be stated as the search of the time variation of the finger forces $\boldsymbol{f}_1(t)$ and $\boldsymbol{f}_2(t)$ that balance $\boldsymbol{f}_{ext}$ while $\boldsymbol{f}_3(t)$ varies from its value in S_A to zero in S_B or vice versa. $\boldsymbol{f}_1(t)$, $\boldsymbol{f}_2(t)$ and $\boldsymbol{f}_3(t)$ are the setpoints values for the finger control system during a manipulation action.



a) State S_A.   b) State S_B.

Fig. 1. Initial (a) and final (b) grasp states.

III. Problem Analysis

A. Torques generated by contact forces

A force $\boldsymbol{f}_i$ applied at P_i produces, with respect to the object center of mass CM, a torque $\tau_i = \boldsymbol{f}_i \times \boldsymbol{r}_i$, where $\boldsymbol{r}_i$ describes the position of P_i with respect to CM.

Consider a line L_i parallel to $\boldsymbol{r}_i$ (see Figure 2). Any $\boldsymbol{f}_i$ applied at P_i such that the vector $\boldsymbol{f}_i$ represented with the tail at P_i has its head on L_i produces the same torque τ_i, thus we refer to the lines L_i as iso-torque lines. The value of τ_i associated to a given L_i is the product of $\|\boldsymbol{r}_i\|$ (which is constant for a given point P_i) times the distance d_i between L_i and P_i, thus τ_i linearly varies with respect to d_i. This linearity means that, in the wrench space, all the wrenches $\boldsymbol{w}_i = (\boldsymbol{f}_i \ \ \tau_i)$ (i.e. the wrenches produced by a force $\boldsymbol{f}_i$ applied at P_i) define a plane Π_i (see Figure 3). Since P_i is a contact point on the object boundary, $\boldsymbol{f}_i$ cannot have any direction, it is constrained to lie inside the friction cone C_i, and therefore only a subset of Π_i, called $S\Pi_i$, can be actually generated. $S\Pi_i$ is the projection along the τ-axis of C_i over Π_i (Figure 3).

B. Wrench loops

The system equilibrium under wrenches $\boldsymbol{w}_i$ in the 3D space due to forces $\boldsymbol{f}_i$ applied on P_i, would be graphically analyzed and characterized. The equilibrium condition is that $\sum \boldsymbol{w}_i = 0$; graphically, this condition can be seen as a closed loop path in the 3D wrench space drawing all the vectors $\boldsymbol{w}_i$ with the tail attached to the head of another one. From now on, this loop will be called “wrench loop”, and the set of all the possible wrench loops produced by the possible wrenches generated at the contact points will be called *”Generic Wrench Loop”* (GWL). The GWL can be graphically constructed as follows (remind that $\boldsymbol{w}_i$ are free vectors so they can be translated in the wrench space with no lose of significance).

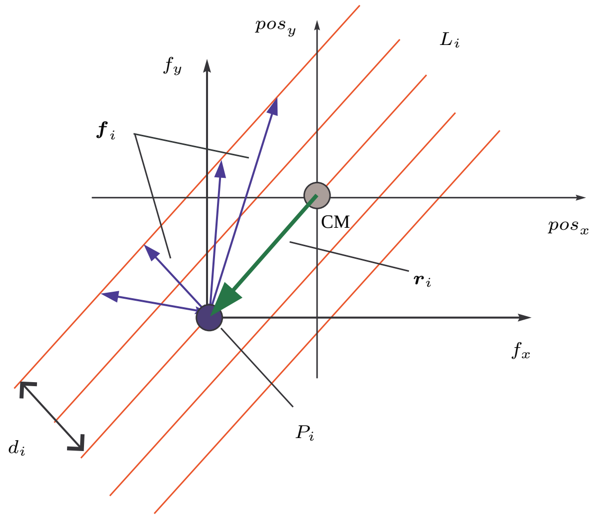


Fig. 2. Lines of constant torque τ_i due to $\boldsymbol{f}_i$ applied at P_i.

1) Consider first the vector representing the external force $\boldsymbol{f}_{ext} = (f_{ext_x}\ f_{ext_y}\ 0)$ (the vector with the tail at the origin in Figure 4).
2) The second vector to be considered is the wrench $\boldsymbol{w}_1$ due to $\boldsymbol{f}_1$ applied on P_1. Since $\boldsymbol{f}_1 \in C_1$ then $\boldsymbol{w}_1 \in S\Pi_1$, thus the entire $S\Pi_1$ is represented displacing its vertex from P_1 to the head of $\boldsymbol{f}_{ext}$ (Figure 4).
3) The third vector to be considered in the path loop is the wrench $\boldsymbol{w}_2$ due to $\boldsymbol{f}_2$ applied on P_2. As in the previous step, $\boldsymbol{f}_2 \in C_2$ then $\boldsymbol{w}_2 \in S\Pi_2$, and the entire $S\Pi_2$ can be represented displacing its vertex from P_2 to the tail of $\boldsymbol{f}_{ext}$ (i.e. the origin of the wrench space)(Figure 4), but this links the tail of the vectors $\boldsymbol{w}_2$ with the tail of $\boldsymbol{f}_{ext}$; in order to make the head of $\boldsymbol{w}_2$ to match the tail of $\boldsymbol{f}_{ext}$, the vectors in $S\Pi_2$ are replaced by their negated ones, which define the set $S\Pi_2^{-1}$ (the inverse of $S\Pi_2$ under the adding operation) represented by the dark cone in Figure 5 (for clarity purpose, from now on the plane Π_0 is not represented in the figures).
Note that $S\Pi_1 \cap S\Pi_2^{-1}$ is the set of points that define all the combinations of $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ that balance $\boldsymbol{f}_{ext}$ (see the enlargement in Figure 5), i.e. they indicate the combinations of forces $\boldsymbol{f}_1$ and $\boldsymbol{f}_2$ applied at P_1 and P_2 that balance $\boldsymbol{f}_{ext}$ and therefore a valid set of forces that generates equilibrium in S_B. We refer to $LS_B = S\Pi_1 \cap S\Pi_2^{-1}$ as the equilibrium loci for S_B.
4) Finally, the vector $\boldsymbol{w}_3$ due to the $\boldsymbol{f}_3$ applied at P_3 is added. Assuming that the value of $\boldsymbol{w}_1$ is known (it is a point inside $S\Pi_1$), $S\Pi_3$ can represented displacing its vertex from P_3 to the head of the given value of $\boldsymbol{w}_1$ inside $S\Pi_1$. Doing this, $LS_A = S\Pi_3 \cap S\Pi_2^{-1}$ is the set of points that define all the combinations of $\boldsymbol{w}_2$ and $\boldsymbol{w}_3$ that balance $\boldsymbol{f}_{ext}$ for the given $\boldsymbol{w}_1$, generating a wrench loop and allowing therefore the equilibrium of S_A (see Figure 6).

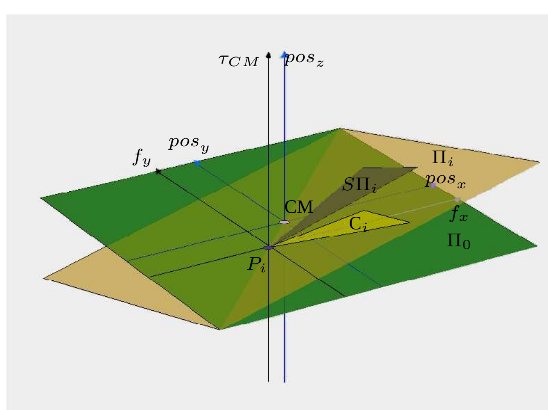


Fig. 3. C_i (light gray cone), and subset $S\Pi_i$ (dark gray cone) of Π_i (C_i and $S\Pi_i$ stretch out from P_i to infinity).

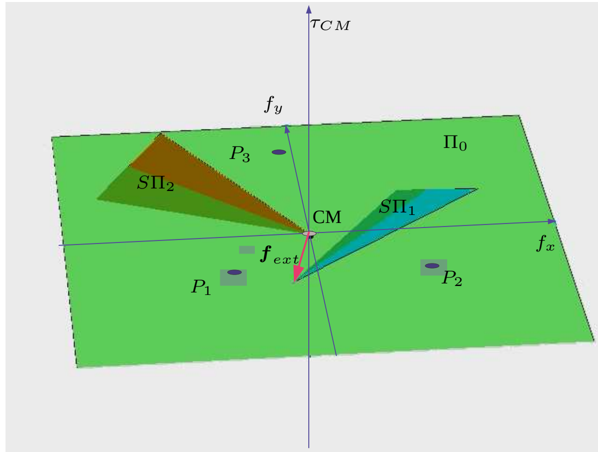


Fig. 4. $\boldsymbol{f}_{ext}$ and two friction cones $S\Pi_1$ and $S\Pi_2$.

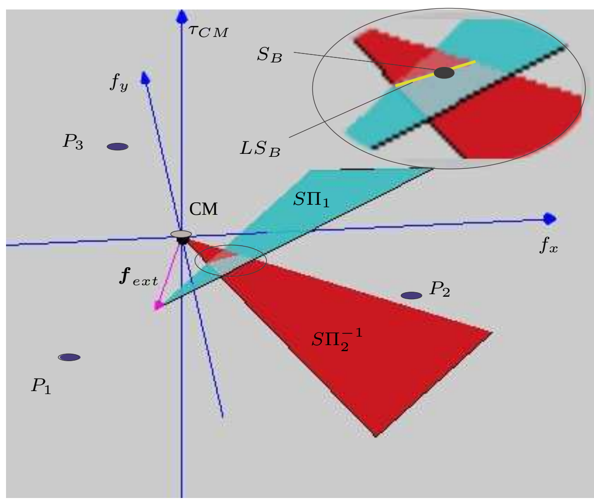


Fig. 5. GWL for S_B, the enlargement shows LS_B.

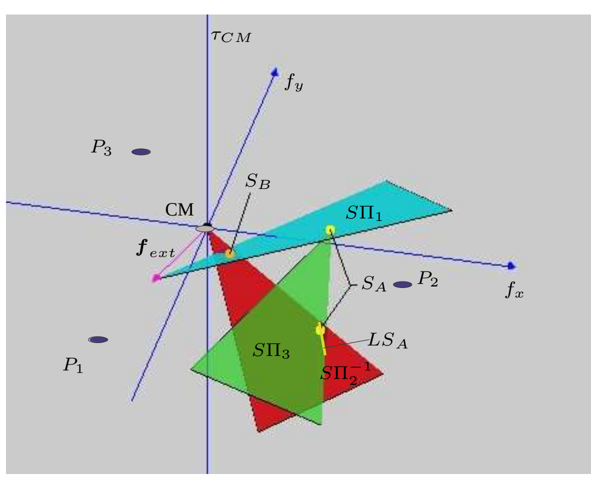


Fig. 6. GWL for S_A showing the three friction cones $S\Pi_1$, $S\Pi_2$ and $S\Pi_3$.

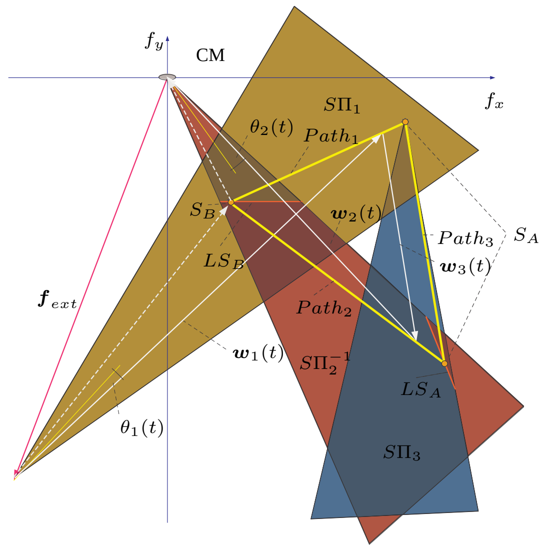


Fig. 7. GWL for S_A, including the initial forces in S_A, the final forces in S_B and the paths for the three forces $\boldsymbol{f}_i$.

IV. Proposed Solution

The graphical representation of the GWL is used now to determine the temporal evolution of $\boldsymbol{w}_1$, $\boldsymbol{w}_2$, and $\boldsymbol{w}_3$, to change from S_A to S_B. The simplest variation of a wrench $\boldsymbol{w}_i$ within the corresponding region $S\Pi_i$ to move from the value in S_A to the value in S_B, is to make it follow a straight line. Consider then that $\boldsymbol{w}_1$ varies on a straight segment Path$_1$ in $S\Pi_1$ and $\boldsymbol{w}_2$ on a straight segment Path$_2$ in $S\Pi_2$. Figure 7) shows an example of the vectors $\boldsymbol{w}_1$, $\boldsymbol{w}_2$ and $\boldsymbol{w}_3$ corresponding to S_J (white vectors), vectors $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ corresponding to S_B (white dashed line vectors), as well as Path$_1$ and Path$_2$. This is always possible, constraining $\boldsymbol{w}_3$ to lie on the plane defined by Path$_1$ and Path$_2$, moreover, if $\boldsymbol{w}_3$ keeps the same direction while its module is reduced then $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ will move along Path$_1$ and Path$_2$ in a proportional way.

Then, using the supraindex A and B to indicated the values of $\boldsymbol{w}_i$ in states S_A and S_B respectively, and letting $T(t)$ be a function that smoothly varies in time between one and zero, we can express the time variations of $\boldsymbol{w}_i$ as

$$\boldsymbol{w}_1(t) = \boldsymbol{w}_1^B + (\boldsymbol{w}_1^A - \boldsymbol{w}_1^B)\ T(t) \quad (1)$$
$$\boldsymbol{w}_2(t) = \boldsymbol{w}_2^B + (\boldsymbol{w}_2^A - \boldsymbol{w}_2^B)\ T(t) \quad (2)$$
$$\boldsymbol{w}_3(t) = \boldsymbol{w}_3^A\ T(t) \quad (3)$$

Note that $\boldsymbol{w}_1$ and $\boldsymbol{w}_2$ move along Path$_1$ and Path$_2$ as linear functions of $T(t)$ while $\boldsymbol{w}_3$ decrease to zero keeping its direction.

V. Example

The proposed approach has been implemented and we describe here an example to illustrate how it works. The problem to be solved is the force transition for the object and the states S_A and S_B shown in Figure 1.

Given the external force $\boldsymbol{f}_{ext} = [-1.5 - 3.5]$, and the contact points $P_1 = [-4\ \ -4]$, $P_2 = [4\ \ -5]$ and $P_3 = [0\ 8]$, the applied forces that produce equilibrium at S_A and S_B are:

$\boldsymbol{f}_1^A = [3.7897\ 3.0034]$
$\boldsymbol{f}_2^A = [-3.0096\ 4.4156]$
$\boldsymbol{f}_3^A = [0.7199\ \ -3.9190]$
$\boldsymbol{f}_1^B = [1.8557\ 2.4555]$
$\boldsymbol{f}_2^B = [-0.3557\ 1.0445]$

With this forces and contact points the following wrenches are produced:

$\boldsymbol{w}_1^A = [3.7897\ 3.0034\ 3.1448]$
$\boldsymbol{w}_2^A = [-3.0096\ 4.4156\ 2.6145]$
$\boldsymbol{w}_3^A = [0.7199\ \ -3.9190\ \ -5.7593]$
$\boldsymbol{w}_1^B = [1.8557\ 2.4555\ \ -2.3930]$
$\boldsymbol{w}_1^B = [-0.3557\ 1.0445\ 2.3930]$

Using these values in equations (1), (2) and (3), and a spline with five control points to define $T(t)$ such that $T'(t_0) = T'(t_f) = 0$ where t_0 and t_f are the initial and final instants. ($T(t)$ is shown in Figure 8), the functions $\boldsymbol{w}_1(t)$, $\boldsymbol{w}_2(t)$ and $\boldsymbol{w}_3(t)$ that allow the object equilibrium were obtained. The results are graphically shown in Figure 9a that shows the variation in the magnitude of $\boldsymbol{f}_i(t)$, $i = 1, 2, 3$, and Figure 9b that shows the variation in the angles between the object normal direction and $\boldsymbol{f}_i(t)$. Note that $\boldsymbol{f}_3(t)$ has no variation in its directions while its module decreases to zero, and that the directions of $\boldsymbol{f}_1(t)$ and $\boldsymbol{f}_2(t)$ remains all the time inside the friction cone limits.

As an additional verification of the system equilibrium, we checked whether $\boldsymbol{f}_{ext}^T - \mathbb{G}\boldsymbol{f}_g^T = 0$ is satisfied, being $\mathbb{G}$ the grasp matrix and $\boldsymbol{f}_g = [\boldsymbol{f}_1^{P_1},\ \boldsymbol{f}_2^{P_2},\ \boldsymbol{f}_3^{P_3}]^T$ and $\boldsymbol{f}_i^{P_i}$ the forces $\boldsymbol{f}_i$ expressed in a coordinate system fixed at P_i; and the condition was satisfied $\forall t$.

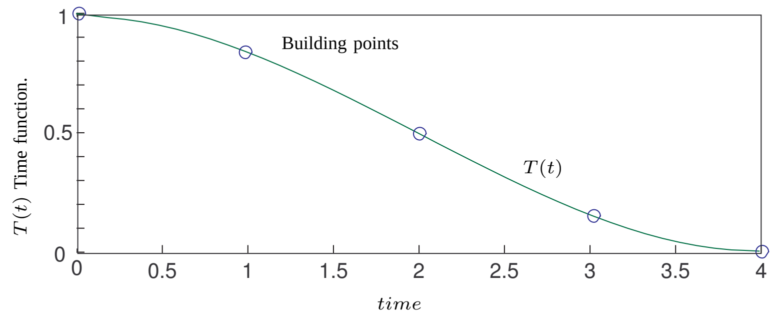
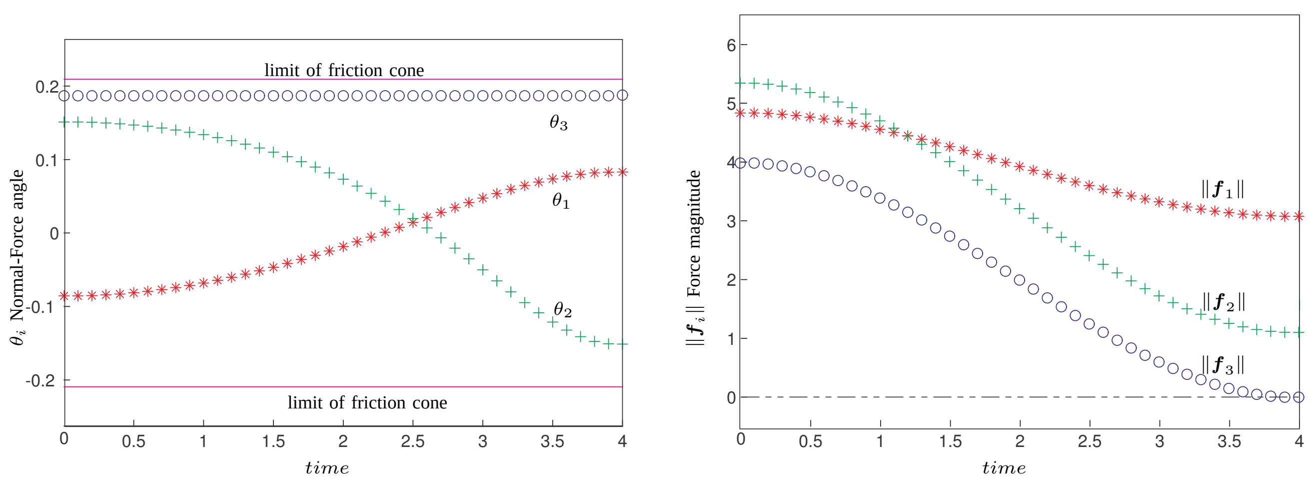


Fig. 8. Time Function $T(t)$.



(a) Angle between Normal and $\boldsymbol{f}_i$.

(b) Magnitude of $\boldsymbol{f}_i$.

Fig. 9. Variation of the directions and modules of $\boldsymbol{f}_i$.

VI. Conclusions and Future Works

A fast non iterative solution to the problem of finding the force variations that keep the object equilibrium when a finger is removed from a three contact point grasp (or added to a two contact point grasp) has been proposed and implemented. The approach is simple and efficient.

The ongoing work includes the determination of a procedure to change from a three contact point grasp, S_A, to another grasp, S_N, with three different contact points (doing in this way a full regrasp of the object), by automatically solving intermediate consecutive grasps S_j that differ in only contact point, and doing object rotations when necessary (in particular when the external force is due to the object weight). Note that the rotation of the object is equivalent to a change in the direction of the external force, and therefore the finger forces that balance it must be recomputed. The whole procedure generates position and force set points for the control system of the grasping device. The problem includes the following subproblems:

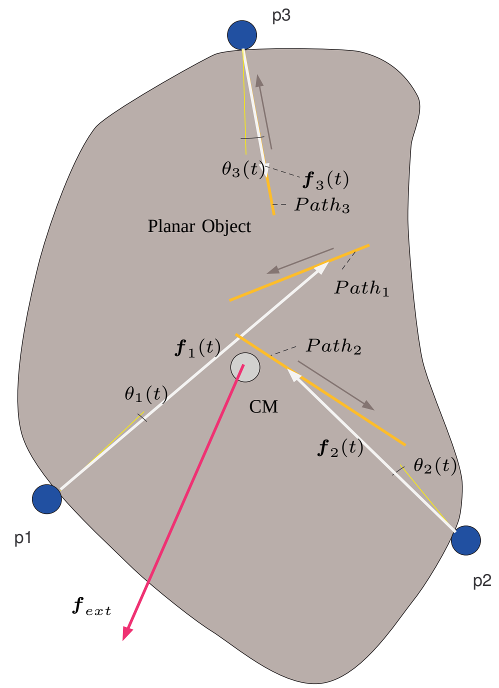


Fig. 10. Object, normals, forces and force paths for the three contact points.

1) Automatic determination of the grasp states that balance the external force with only two fingers between to intermediate consecutive grasps S_j with three contact points (i.e. automatic determination of the grasp state S_B in this paper). The search can be done using a GWL that describes the forces of the two fingers that do not change, and selecting a proper point on the corresponding region LS_j (equivalent to LS_B in Figure 5).
2) Automatic determination of the force variations to keep the equilibrium when the object is rotated. Again, this can be done playing with the GWL representation.
3) Automatic determination of the intermediate consecutive grasps S_j and, if necessary, the rotations of the object to allow the change of a given finger as well as to improve the energy requirements or the system robustness.

A more ambitious future work is to extend the approach to 3D objects considering four frictional contact points.

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