Supermode analysis of the three-waveguide nonlinear directional coupler: the critical power

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Abstract

The three-waveguide nonlinear directional coupler switching characteristic is studied using the eigenmodes of the structure (i.e. supermodes). This model, which is more exact than the one used in previous works, has allowed to confirm the three waveguide configuration as an alternative to the two waveguide nonlinear coupler. Moreover, an analytical expression for the critical power at which the switching between the output branches occurs has been found.

1. Introduction

The nonlinear directional coupler (NLDC) proposed by Jensen [1] has been one of the all-optical devices that has attracted more interest [2–5]. The extension from two to three [6] or multiple [7,8] waveguide nonlinear directional couplers has been the obvious step to be considered. These works showed that the switching characteristic curves between the two outermost output ports of the three-waveguide NLDC are remarkably sharper than the one obtained for the two-waveguide NLDC. As it was advanced for the two core coupler [10], the dynamics of the three core coupler can be also investigated by finding the nonlinear modes of the device [11,12] and by representations in the phase space [13]. The results of these works give a good qualitative understanding of the switching characteristics.

However, some discrepancies about the usefulness of the three-waveguide coupler have arisen recently [14]. In this work, contrary to previous publications [6–8], all the nonlinear terms were considered, pointing out that for some device configurations the switching characteristic disagreed with the results obtained in those works that only took into account the self- and cross-phase modulation terms. One of the objectives of this work is to clarify the situation. This has impelled us to extend the supermode theory proposed by Silberberg and Stegeman [15] to the three waveguide nonlinear directional coupler. This theory holds for any coupler configuration and all the nonlinear terms are considered. Our model confirms the use of the three core coupler as a switching unit as it was proposed in previous works [6–8].

Also, when one of the outermost waveguides is initially excited, which is the case studied in this work, the use of supermodes has allowed us to find an analytical expression for the critical power, expression that, to our knowledge, was not available so far. The so found critical power expression corresponds to the case in which
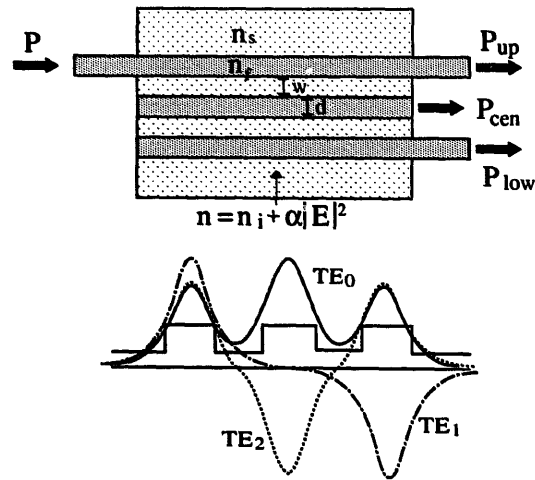
Fig. 1. The three-waveguide nonlinear directional coupler and the three lower order supermodes of the structure.

switching from the central to the output port of the same excited (upper) waveguide occurs. Moreover, we comment on the impossibility of finding an expression for the case in which switching between the central and the lower branch (the outermost nonexcited one) occurs.

2. Theory

The three waveguide nonlinear directional coupler studied in this work is depicted in Fig. 1. The device consists of three waveguides of linear refractive index n_f, thickness d and separation w surrounded by a cladding of linear index n_c. All the media have a Kerr-like nonlinear intensity-dependence given by

$$n^2 = n_i^2 + \alpha |E|^2, \tag{1}$$

with $i = f, c$ depending on the medium and α being the nonlinear coefficient. Loss free media are also assumed. Following Ref. [15] we take as a solution the three lowest-order supermodes of the linear structure, where nonlinear effects appear in the variation of the complex amplitude A_i. The electric field then is written as

$$E = \sum_{i=0}^{2} A_i(z) E_i(x) \exp(-j\beta_i z), \tag{2}$$

$E_i(x) \exp(-j\beta_i z)$ being the transversal electric mode (supermode) of order i, z the propagation direction and x the transverse axis. In order to generalize the results, we use normalized quantities for the distances and the amplitudes in a way similar to that done in a previous work [16]. In this sense, the transverse direction is normalized with respect to the thickness of the waveguide so that $\mathcal{X} = x/d$, the propagation distance with respect to the wavelength in free space, $\mathcal{Z} = 2\pi z/\lambda = k_0 z$ and the amplitude as $\mathcal{E}_i = E_i/E_c$. E_c is an arbitrary constant defined later on.

In the nonlinear regime, by introducing (1) and (2) in the nonlinear wave equation using the previous normalization, the differential coupled-mode equations are written in the form

$$\frac{dA_0}{d\mathcal{Z}} = -j\mathcal{I} \left[A_0(A_0 A_0^* C_{00} + 2A_1 A_1^* C_{01} + 2A_2 A_2^* C_{02}) + A_2(2A_0 A_0^* Q_{02} + 2A_1 A_1^* R \right.$$

$$+ A_2 A_2^* Q_{20}) e^{-j(N_2-N_0)\mathcal{Z}} + A_2^* A_0 A_0 Q_{02} e^{-j(N_0-N_2)\mathcal{Z}} + A_0^* A_1 A_1 C_{01} e^{-2j(N_1-N_0)\mathcal{Z}}$$

$$\left. + A_2^* A_1 A_1 R e^{-j(2N_1-N_0-N_2)\mathcal{Z}} + A_0^* A_2 A_2 C_{02} e^{-2j(N_2-N_0)\mathcal{Z}} \right], \tag{3}$$

$$\frac{\mathrm{d}A_1}{\mathrm{d}\mathcal{Z}} = -j\mathcal{I}\Big[A_1(2A_0A_0^*C_{01} + A_1A_1^*C_{11} + 2A_2A_2^*C_{12}) + A_0A_0A_1^*C_{01}\,\mathrm{e}^{-2j(N_0-N_1)\mathcal{Z}}$$

$$+ 2A_2^*A_1A_0R\,\mathrm{e}^{-j(N_0-N_2)\mathcal{Z}} + 2A_0^*A_1A_2R\,\mathrm{e}^{-j(N_2-N_0)\mathcal{Z}}$$

$$+ 2A_0A_1^*A_2R\,\mathrm{e}^{j(2N_1-N_0-N_2)\mathcal{Z}} + A_1^*A_2A_2C_{12}\,\mathrm{e}^{-2j(N_2-N_1)\mathcal{Z}}\Big], \tag{4}$$

$$\frac{\mathrm{d}A_2}{\mathrm{d}\mathcal{Z}} = -j\mathcal{I}\Big[A_2(2A_0A_0^*C_{02} + 2A_1A_1^*C_{12} + A_2A_2^*C_{22}) + A_0(A_0A_0^*Q_{02} + 2A_1A_1^*R$$

$$+ 2A_2A_2^*Q_{20})\,\mathrm{e}^{j(N_2-N_0)\mathcal{Z}} + A_0^*A_2A_2Q_{20}\,\mathrm{e}^{j(N_0-N_2)\mathcal{Z}} + A_2^*A_0A_0C_{02}\,\mathrm{e}^{-2j(N_0-N_2)\mathcal{Z}}$$

$$+ A_0^*A_1A_1R\,\mathrm{e}^{-j(2N_1-N_0-N_2)\mathcal{Z}} + A_2^*A_1A_1C_{12}\,\mathrm{e}^{-2j(N_1-N_2)\mathcal{Z}}\Big], \tag{5}$$

$N_i = \beta/k_0$ being the effective index, C_{mn}, Q_{jk} and R are normalized overlapping integrals defined as

$$C_{nm} = \int\limits_{-\infty}^{\infty} \mathcal{E}_n^2(\mathcal{X})\,\mathcal{E}_m^2(\mathcal{X})\,\mathrm{d}\mathcal{X}, \quad n,m = 0,1,2,$$

$$Q_{jk} = \int\limits_{-\infty}^{\infty} \mathcal{E}_j^3(\mathcal{X})\,\mathcal{E}_k(\mathcal{X})\,\mathrm{d}\mathcal{X}, \quad j,k = 0,2,$$

$$R = \int\limits_{-\infty}^{\infty} \mathcal{E}_0(\mathcal{X})\,\mathcal{E}_1^2(\mathcal{X})\,\mathcal{E}_2(\mathcal{X})\,\mathrm{d}\mathcal{X},$$

and $\mathcal{I} = E_c^2\alpha/2$. The meaning of $\mathcal{I}$ and the amplitudes A_i depends on the definition of E_c. We choose $E_c^2 = 2P\eta_0/d$ where P is the total input power and η_0 the free space impedance. With this election, $|A_i(\mathcal{Z})|^2 = P_i/P$, i.e. the fraction of power carried by each mode, P_i being the power of the mode i, and $\mathcal{I} = \alpha P\eta_0/d$. If we pay attention to the definition of $\mathcal{I}$, it is a dimensionless parameter which depends on the total power and the nonlinear coefficient of the material α, $\mathcal{I}$ being our power measure.

The system (3)–(5) has two conservative magnitudes. The first one, excluding losses, is the total power, that in normalized parameters is written as

$$|A_0|^2 + |A_1|^2 + |A_2|^2 = 1. \tag{6}$$

The second conservative magnitude is the Hamiltonian of the system. The Hamiltonian is related to the complex amplitudes as follows,

$$j\frac{\partial H}{\partial a_i^*} = \frac{\mathrm{d}a_i}{\mathrm{d}\mathcal{Z}},$$

where $a_i = A_i\,\mathrm{e}^{-jN_i\mathcal{Z}}$. Then the Hamiltonian can be written from (3)–(5) as

$$H = N_0|A_0|^2 + N_1|A_1|^2 + N_2|A_2|^2 + \mathcal{I}\left[\frac{1}{2}(|A_0|^4 C_{00} + |A_1|^4 C_{11} + |A_2|^4 C_{22})\right.$$

$$+ 2|A_0|^2|A_1|^2 C_{01} + 2|A_0|^2|A_2|^2 C_{02} + 2|A_1|^2|A_2|^2 C_{12} + \frac{1}{2}(A_0^{*2}A_1^2 e^{j2(N_0-N_1)\mathcal{Z}} + A_0^2 A_1^{*2} e^{-j2(N_0-N_1)\mathcal{Z}})C_{01}$$

$$+ \frac{1}{2}(A_0^{*2}A_2^2 e^{j2(N_0-N_2)\mathcal{Z}} + A_0^2 A_2^{*2} e^{-j2(N_0-N_2)\mathcal{Z}})C_{02} + \frac{1}{2}(A_1^{*2}A_2^2 e^{j2(N_1-N_2)\mathcal{Z}} + A_1^2 A_2^{*2} e^{-j2(N_1-N_2)\mathcal{Z}})C_{12}$$

$$+ (A_1^2 A_0^* A_2^* e^{j(2N_1-N_0-N_1)\mathcal{Z}} + A_1^{*2}A_0 A_2 e^{-j(2N_1-N_0-N_1)\mathcal{Z}})R$$

$$\left. + (A_0^* A_2 e^{j(N_0-N_2)\mathcal{Z}} + A_0 A_2^* e^{-j(N_0-N_2)\mathcal{Z}})(|A_0|^2 Q_{02} + |A_1|^2 Q_{20} + 2|A_2|^2 R)\right]. \tag{7}$$

System (3)–(5) has been solved using a Runge–Kutta procedure, where the conservative magnitudes (6) and (7) were used to control the accuracy of the solution. However, it is more useful to express the results in terms of the fraction of power carried by each supermode P_i/P and their phase difference θ_i. Then the complex amplitude of mode i can be expressed as $A_i(\mathcal{Z}) = \sqrt{P_i(\mathcal{Z})/P} \exp \phi_i(\mathcal{Z})$, ϕ_i the nonlinear phase. Then the two phase shifts are $\theta_1(\mathcal{Z}) = (N_0 - N_1)\mathcal{Z} - (\phi_0(\mathcal{Z}) - \phi_1(\mathcal{Z}))$ and $\theta_2(\mathcal{Z}) = (N_0 - N_2)\mathcal{Z} - (\phi_0(\mathcal{Z}) - \phi_2(\mathcal{Z}))$. In order to compare with previous works, it is useful to calculate the power carried by each waveguide. The power in waveguide s is obtained using

$$\sum_{i=0}^{2} \int_{-\infty}^{\infty} A_i(\mathcal{Z})\mathcal{E}_i(\mathcal{X}) \exp(-jN_i\mathcal{Z}) a_s(\mathcal{X}, \mathcal{Z}) \, d\mathcal{X} = \sum_{j=1}^{3} \int_{-\infty}^{\infty} a_j(\mathcal{X}, \mathcal{Z}) a_s(\mathcal{X}, \mathcal{Z}) \, d\mathcal{X}. \tag{8}$$

Both terms of this equation are the integration of the total field profile in the transverse direction multiplied by the zero-order mode a_s of the single waveguide s. The total field in the left hand term is introduced using the supermodes of the structure, whereas in the right hand term we use the sum of the modes a_j of each individual waveguide. One of the integrals on the right hand term is the total power carried by the branch s (when $j = s$). The other two integrals (when $j \neq s$) do not vanish due to the nonorthogonality of the basis. Using (8) with $s = 1, 2, 3$ the power in each waveguide can be obtained.

3. Results

3.1. The coupling length

The linear coupling length, in this model, should be that at which the modes TE_0 and TE_2 are phase-matched, and the phase difference between the mode TE_0 and TE_1 is π. This implies a relation between effective indices, which must be $2N_1 - N_0 - N_2 = 0$. Although this relation is fulfilled for other waveguide configurations [17], this is not the case for three-waveguide couplers. Such a fact has two consequences: the total power exchange between the upper and lower branches does not take place, and the coupling length can not be defined as previously [18]. These two effects are not taken into account in the simplified analysis when only the modes of the individual waveguides are used [5–14]. Both effects are just a consequence of the use of supermodes. Thereupon, the coupling length L_c used in this work is that one at which the power exchange between outermost branches is maximum. The exact value of L_c is found numerically.

3.2. The critical power

As a matter of fact, the so-called saddle points, which are unstable critical points of our system, are responsible for the switching dynamics. Both, the stable and unstable fixed points are found when the right side term in the differential-equation system (3)–(5) is set to zero (there is no variation in the amplitudes along the propagation). Solving numerically the resulting system the dependence of the fixed points with respect to the power parameter $\mathcal{I}$ can be found.
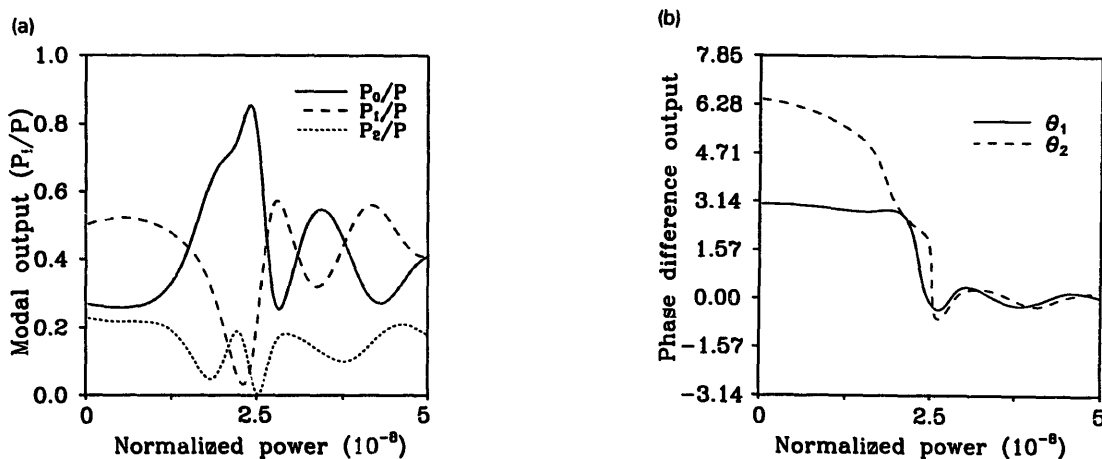
Fig. 2. Output of our variables at the end of a coupler of length $L = L_c$; (a) corresponds to the power carried by each mode, and (b) to the phase difference.

Fig. 2 shows the output for P_i/P and θ_i in terms of the power parameter $\mathcal{I}$ when a coupler of length $L = L_c$ is excited in its upper branch. For this particular coupler, a transition of behavior at $\mathcal{I} = 2.5 \times 10^{-8}$ can be seen. In Fig. 2a, where the fraction of power carried by each mode at the end of the coupler is shown, this transition is characterized by a lost of all the power carried by the mode TE_2, i.e. $P_2(L_c)/P = 0$. Such behavior is similar to the one observed in a two-waveguide coupler excited with the critical power, in this case for the mode TE_1. So, it seems that the behavior of couplers excited with the critical power in one of the outermost branches consists of a complete loss of the power carried by the higher-order mode. On the other hand, in Fig. 2b, the phase difference at the coupler output, is characterized, for the same value of $\mathcal{I}$, by passing through $\theta_1(L_c) = 0$.

If $P_2/P = 0$ and $\theta_1 = 0$ are introduced in the system which gives the fixed points, an analytical expression for the remaining variables can be obtained in terms of the overlapping integrals and the propagation constants. Then, the fixed point is given by

$$P_0/P = (N_1 - N_0)3R/\mathcal{I}[3R(C_{00} - 3C_{01}) + Q_{02}(C_{11} - 3C_{01})],$$

$$P_1/P = (-P_0/P)Q_0/3R, \qquad P_2/P = 0, \qquad \theta_1 = 0, \qquad \forall \theta_2. \tag{9}$$

As Fig. 3 shows, if the coupler is excited with the values given in (9), the initial values for our variables P_i/P (Fig. 3a) and θ_i (Fig. 3b) remain unaltered throughout the propagation until, due to some perturbation, this point loses its equilibrium, which leads to a power exchange between modes and a variation in the phase differences. In Fig. 3b, while the perturbation does not act, θ_2, which in principle was arbitrary (see (9)), evolves from the initial value, which for simplicity was set to be $\theta(0) = 0$, towards a final value $\theta(\mathcal{Z}) = \pi/4$, this one being its real value. These figures clearly show that excitation (9) corresponds to an unstable fixed point. Taking into account that the sum of the fraction of power carried for each mode adds to one, an expression for the power parameter $\mathcal{I}_c$ for this fixed point can be found:

$$\mathcal{I}_c = \frac{(N_1 - N_0)(3R - Q_{02})}{R(C_{00} - 3C_{01}) + Q_{02}(C_{11} - 3C_{22})}. \tag{10}$$

This fixed point has been found using the information given in Fig. 2, which corresponds to the excitation in the upper branch. Therefore, this same expression could correspond to the critical power at which we have the switching between branches. However, in our model, we use the modes of the whole structure. Then, the accuracy of expression (10) depends on the choice of the criteria to define which distribution of power among modes means exactly excitation in one branch. It is possible to find an initial condition for which (10) is
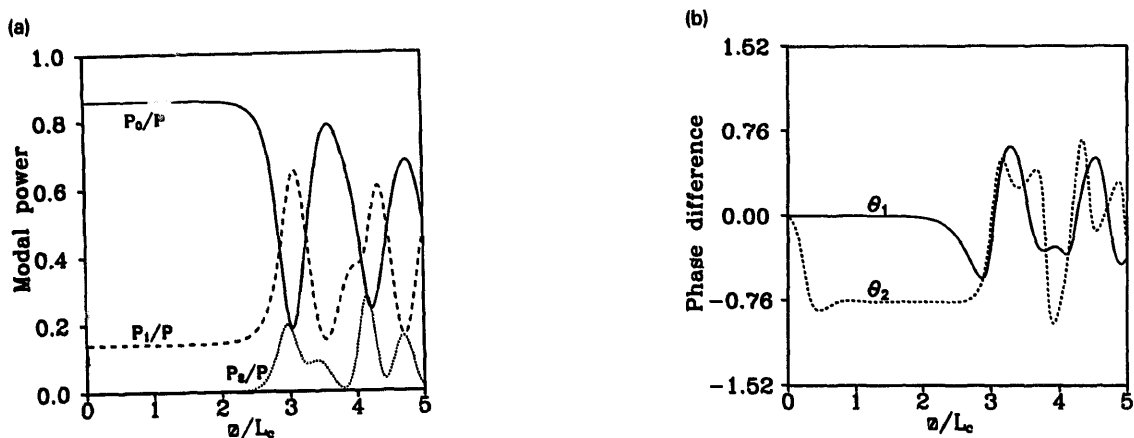
Fig. 3. Evolution of our variables along the propagation when the excitation corresponds to a fixed point; (a) corresponds to the power carried by each mode, and (b) to the phase difference.
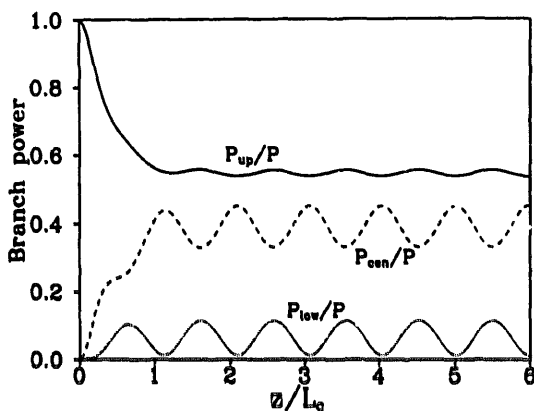

Fig. 4. Power ratio in each branch of the coupler along the propagation in the case of excitation with the critical power.

exact. If this initial distribution is compared with the values obtained using different criteria, the variation in the power distribution among modes is less than 5%. As a consequence, this distribution, that, in the coupler was used to obtain the results, corresponds to $P_0(0)/P \simeq 0.19$, $P_1(0)/P \simeq 0.49$ and $P_2(0)/P \simeq 0.31$, can be considered as excitation in one branch.

In Fig. 4, the power in each branch of the coupler along the propagation has been plotted taking as total power the one given in (10). The figure shows the evolution from the initial point where all the power is in one branch (upper one) to a final oscillatory steady state. The fact that the fixed point is not reached is caused by a previous Hopf bifurcation into a unstable periodic orbit which "encircles" the fixed point. This orbit is the final state towards which the system tends.

3.3. Behavior

Fig. 5a shows the well known coupler transmittance, where the input power has been normalized with respect to the critical power found. This critical power corresponds to the transition from the central output port to the output of the upper waveguide. This figure, where calculations have been done for sufficiently separated waveguides ($w/d = 2$), basically coincides with the transmittance curve obtained in Refs. [6–8] so that, for this geometry, the coupled-mode theory (CMT) using modes of the individual waveguides is sufficiently
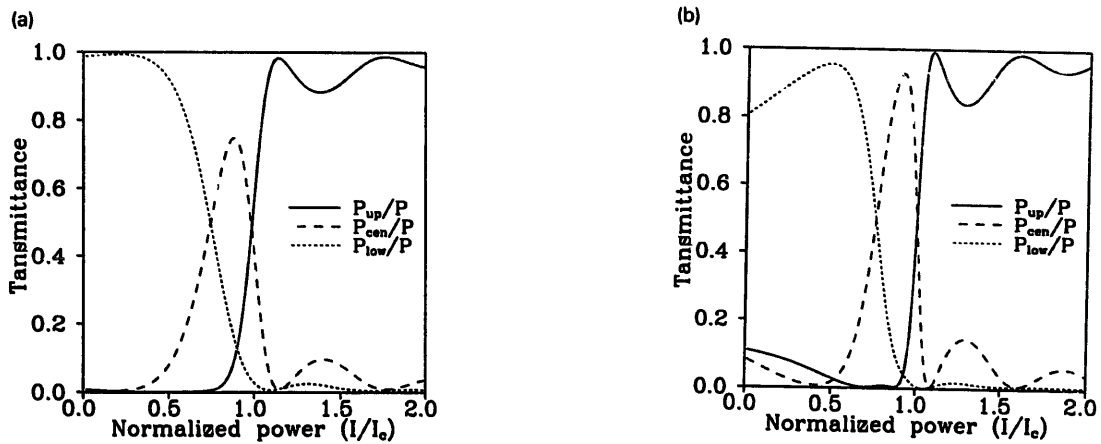
Fig. 5. Power transmittance for each branch of a NLDC of length $L = L_c$ with (a) $w/d = 2$ and (b) $w/d = 1$.

accurate. This theory does not hold if distances between waveguides are reduced, i.e. in a strong coupled configuration, giving the same results of Fig. 5a. We have reduced the distances to $w/d = 1$, where the standard CMT starts to fail. Our model (Fig. 5b) predicts the influence of the incomplete power exchange between outermost waveguides at low input powers, and a sharper transition at a value of the critical power parameter $\mathcal{I}_c = 2.6 \times 10^{-7}$ that is one order of magnitude higher than in the previous case. Contrary to what was stated in Ref. [14], the transmittance characteristic is suitable for using this device as a switching unit. Probably, the fact of ignoring the nonorthogonality of the basis used in Ref. [14] causes the disagreement with our more exact model.

Normally, in previous works, the power at which there is switching between the lower and the central branches is also called critical power. Here, in all the process followed to find the critical power, we have assumed implicitly that the definition of critical power is that for which the field in the coupler evolves to meet a final state, i.e. a fixed point or a periodic orbit, which is unstable. Then, the critical power divides different behavior regimes. For powers below the critical one found previously, the only unstable state corresponds to the mode TE_1 [12,13]. Then this should be the final state to be reached. In order to reach this state, a necessary but not sufficient condition is that the value of Hamiltonian (7) using the initial conditions and its value at the final state must be equal. This, after some mathematics, implies that $N_1(1 - P_1/P) - N_0 P_0/P - N_2 P_2/P$ must be positive. In the case of exciting one of the outermost branches this condition is never fulfilled. In this sense, the power at which switching from the lower to the central branch occurs, cannot be considered to be a critical one because a final unstable state is not reached. Fig. 6a corresponds to the power exchange between branches of the coupler for $\mathcal{I} = 0.4\mathcal{I}_c$ and Fig. 6b for $\mathcal{I} = 0.8\mathcal{I}_c$. This values correspond to an initial power at which the output in Fig. 5a is respectively in the lower and the central waveguide. In Fig. 6a, the power flows successively from the upper to the central branch, and from this to the lower one. However, in Fig. 6b, the power flows from the upper branch to a distribution between the central and the lower one. As these figures show, there is a change in behavior, but the transition between both behaviors is slow and progressive. The power at which there is switching between the lower and the central branches is a switching power, but not a critical one as it is usually defined in nonlinear dynamics.

We have already mentioned that in the linear regime, there is no total power exchange between the outermost branches. However, Fig. 6a shows that in the nonlinear regime total exchange is possible. This only happens for a certain input power, for which at some points of the device, the nonlinear phase differences are simultaneously $\theta_1 = 0$ and $\theta_2 = \pi$.
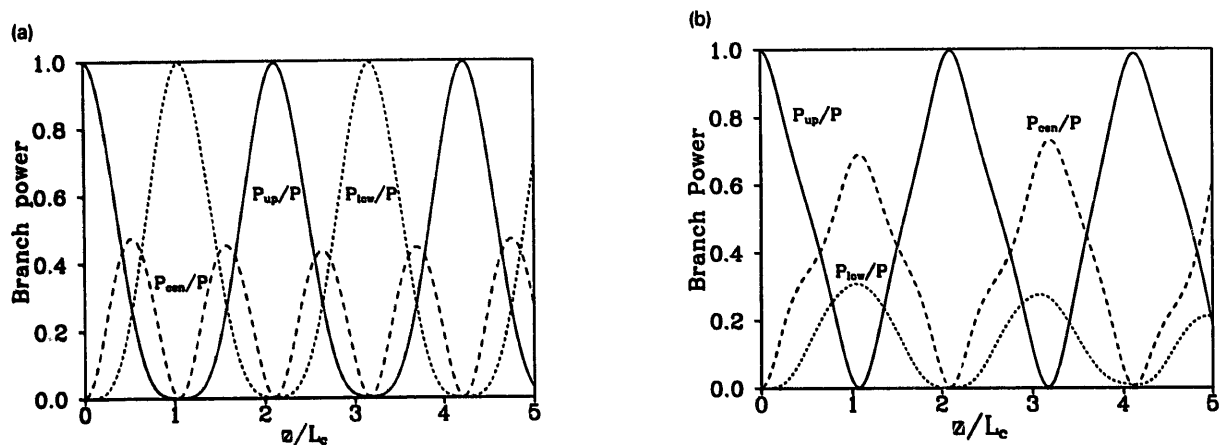
(a)

(b)

Fig. 6. Power ratio in each branch of the coupler; (a) corresponds to a power $P = 0.4P_c$ and (b) to $P = 0.8P_c$.

4. Conclusions

In this work, we have studied the three-waveguide nonlinear directional when excited in one of the outermost branches. To our knowledge, this is the first time that this device has been analyzed using the supermodes of the seven-layer structure. This model can be used in those cases where the separation between branches is too small to use the modes of the individual waveguides. Our results allow to ratify the use of this device as a switching unit. Moreover, we have derived an approximate analytical expression for the critical power at which there is switching from the central to the upper branch. This expression was not previously available. The impossibility of finding an expression for the power at which switching happens from the lower to the central branch has been discussed.

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