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Uncertainty Analysis in Two-terminal Impedance Measurements with Residual Correction

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Abstract ***– Residual impedance correction in impedance analyzers when using an asymmetrical test fixture needs three reference measurements, usually open circuit, short circuit, and load (meaning an impedance close to the impedance under test). This paper provides an uncertainty estimate for impedance measurements that apply a simple open/short correction in spite of using an asymmetrical test fixture. Experimental results show that the minimal uncertainty is obtained for impedance values close to the geometric mean of the short-circuit and open-circuit impedances, and that the theoretical prediction is indeed an upper limit for the actual uncertainty.***

Keywords ***– Impedance measurements, Residual correction, Uncertainty analysis.***

I. INTRODUCTION

Accurate impedance measurements require us to consider the effects of residual impedance in test fixtures. Test fixtures can be modeled as two-port networks described by their transmission parameters (*A B C D*) [1]. Symmetrical (balanced) test fixtures they can be described by two ratios between transmission parameters because *A* = *B*. These ratios can be determined from two reference measurements, for example open-circuit and short-circuit conditions (open/short correction), and the results allow us to correct for the unknown residual impedance in electric contacts and cables connecting the impedance under test to the impedance analyzer. When the test fixture is not symmetrical, residual correction needs three reference measurements because we have to determine three ratios between transmission parameters [2]. Usually, a reference impedance close to the impedance under test is measured, hence the name open/short/load correction.

However, it is sometimes difficult to obtain reference impedances close to the impedance under test, for example when measuring electrolytes. Therefore, the question arises about the uncertainty associated to impedance measurements that use an asymmetrical test fixture but implement a simple open/short correction procedure. This paper provides an estimate for the uncertainty in such impedance measurements and substantiates the predictions by experimental results obtained in two-wire impedance measurements.

II. UNCERTAINTY MODEL IN TWO-TERMINAL IMPEDANCE MEASUREMENTS

Fig. 1 shows the impedance under test connected to an impedance analyzer through a test fixture modeled by its transfer parameters *A*, *B*, *C*, and *D*. The voltage and current at the input of the impedance meter can be obtained from

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix} \quad (1)$$

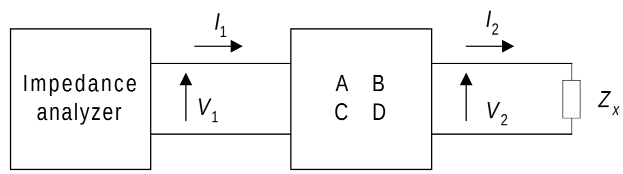

Fig. 1. The test fixture connecting the impedance under test to the impedance analyzer is modeled by a two-port network described by its four transfer parameters *A*, *B*, *C*, *D*.

The impedance measured, disregarding the uncertainty of the impedance meter, will be

$$Z_{xm} = \frac{V_1}{I_1} = \frac{AV_2 + BI_2}{CV_2 + DI_2} \quad (2)$$

whereas the actual impedance under test is

$$Z_x = \frac{V_2}{I_2} \quad (3)$$

In order to estimate Z_x from Z_{xm}, we first rewrite (2) by replacing $V_2/I_2 = Z_x$ to obtain

$$Z_{xm} = \frac{AZ_x + B}{CZ_x + D} \quad (4)$$

Solving for Z_x yields

$$Z_x = \frac{DZ_{xm} - B}{A - CZ_{xm}} \tag{5}$$

Measuring at open-circuit condition ($I_2 = 0$), (2) yields $Z_{\mathrm{om}} = A/C$. Measuring at short-circuit condition ($V_2 = 0$), (2) yields $Z_{\mathrm{sm}} = B/D$. Therefore, (5) can be rewritten as

$$Z_x = \frac{D}{A}\frac{Z_{xm} - Z_{\mathrm{sm}}}{1 - \frac{Z_{xm}}{Z_{\mathrm{om}}}} \tag{6}$$

If we measure a known impedance Z_{l} and obtain a result Z_{lm}, we will have

$$Z_{\mathrm{l}} = \frac{D}{A}\frac{Z_{\mathrm{lm}} - Z_{\mathrm{sm}}}{1 - \frac{Z_{\mathrm{lm}}}{Z_{\mathrm{om}}}} \tag{7}$$

Solving for D/A and replacing it in (6) yields

$$Z_{x\mathrm{u}} = Z_{\mathrm{l}} \frac{1 - \frac{Z_{\mathrm{lm}}}{Z_{\mathrm{om}}}}{Z_{\mathrm{lm}} - Z_{\mathrm{sm}}} \frac{Z_{\mathrm{xm}} - Z_{\mathrm{sm}}}{1 - \frac{Z_{\mathrm{xm}}}{Z_{\mathrm{om}}}} \tag{8}$$

where the subscript "u" has been added to indicate that Z_x has been measured with an unbalanced (asymmetrical) fixture and corrected by three reference readings: Z_{om}, Z_{sm}, and Z_{lm}. If the network connecting Z_x to the impedance analyzer were balanced (symmetrical), $A = D$ and (6) would reduce to

$$Z_{x\mathrm{b}} = \frac{Z_{xm} - Z_{\mathrm{sm}}}{1 - \frac{Z_{xm}}{Z_{\mathrm{om}}}} \tag{9}$$

where the "b" in the subscript stands for balanced. Equation (9) differs from (8) by a gain factor. Therefore, using only two reference measurements when the network connecting the impedance under test to the impedance meter is asymmetrical implies to consider a unity gain factor in (8), which is equivalent to use (9) to calculate the corrected impedance value from the reading Z_{xm} and the two reference measurements Z_{om} and Z_{sm}. Hence, the relative uncertainty is

$$\frac{u(Z)}{Z} = \frac{Z_{x\mathrm{b}} - Z_{x\mathrm{u}}}{Z_{x\mathrm{u}}} = Z_{\mathrm{l}} \frac{1 - \frac{Z_{\mathrm{lm}}}{Z_{\mathrm{om}}}}{Z_{\mathrm{lm}} - Z_{\mathrm{sm}}} - 1 \tag{10}$$

Because it is recommended to select Z_{l} close to Z_x, and we can assume Z_{xm} to be close to Z_{x}, we can approximate $Z_{\mathrm{l}} \approx Z_{\mathrm{x}} \approx Z_{\mathrm{xm}}$ and $Z_{\mathrm{lm}} \approx Z_{\mathrm{xm}}$. Under these assumptions, (10) leads to

$$\frac{u(Z)}{Z} \cong \frac{Z_{\mathrm{om}} Z_{\mathrm{sm}} - Z_{xm}^2}{Z_{\mathrm{om}}\left(Z_{xm} - Z_{\mathrm{sm}}\right)} \tag{11}$$

In summary, disregarding measurement uncertainties in the impedance analyzer, if we use open/short impedance measurements to correct residual impedances for an unbalance test fixture, we can estimate the true impedance value corresponding to an instrument reading Z_{xm} by applying (9) but the calculated result has a maximal relative uncertainty given by (11). Furthermore, because impedance values are complex numbers, the uncertainty because of an asymmetrical test fixture will affect both the amplitude and phase of the result.

From (11), when the result is close to the geometrical mean of the open circuit and short circuit measurements,

$$Z_{x\mathrm{m}}\big|_{\mathrm{opt}} = \sqrt{Z_{\mathrm{om}} Z_{\mathrm{sm}}} \tag{12}$$

the uncertainty because of the unbalanced connecting network is minimal. Nevertheless, this calculated impedance value is not necessarily close to any impedance of interest.

III. EXPERIMENTAL RESULTS AND DISCUSSION

We have applied the analysis above to predict the contribution from incompletely corrected residual impedances to the uncertainty of impedance measurements performed with the HP4294A impedance analyzer. We assessed the influence of an asymmetrical test fixture on the result by measuring several reference impedances, first with a symmetrical test fixture provided by the manufacturer (HP16047A) and then by connecting each impedance under test to the instrument with RG214 cables 1 m long. Both measurement sets used the corresponding open/short correction. The impedance values calculated from (9) when using a symmetrical test fixture and its open/short correction were considered the "true" value. The impedance analyzer averaged eight readings for each impedance value before transferring the results to a computer for calculation and display. The impedance under test was inside a grounded Faraday cage measuring 10 cm × 10 cm × 12.5 cm. Each cable shield was grounded too. Because the impedance analyzer is a self-balanced bridge, the capacitance contributed by the grounded shield did not affect the measurement result [3].

Test impedance values were chosen according to the open and short-circuit measurements when using the asymmetrical test fixture. We obtained $Z_{\mathrm{om}} = 1/(j\omega \times 2\ \mathrm{pF})$ and $Z_{\mathrm{sm}} = 60\ \mathrm{m}\Omega + j\omega \times 100\ \mathrm{nH}$. From these values, (12) yields an "optimal" impedance that is difficult to implement. We have selected two impedances close to that optimum: a 220 Ω resistor (Z_1) and the series combination of that resistor and a 15 nF capacitor (Z_2). The remaining test impedances were off-theshelf resistors: Z_3 = 1 kΩ, Z_4 = 51 Ω, Z_5 = 5.1 kΩ, and Z_6 = 10 Ω.

Fig. 2 shows the maximal absolute value of the relative uncertainty predicted by (11) for Z_1 to Z_6 in the range from 100 Hz to 1 MHz. The theoretical uncertainty is obviously minimal for impedance values Z_1 and Z_2, which are close to the optimum (12), and increases with frequency. The relative uncertainty is similar for Z_3 and Z_4, whose values are (geometrically) symmetrical about that given by (12). The relative uncertainty increases for impedance values that are very different from (12): it is larger for Z_5 and Z_6 than for Z_3 and Z_4. Furthermore, at low frequencies, the relative uncertainty is proportionally larger for Z_6 (small impedance) than for Z_5 (large impedance).

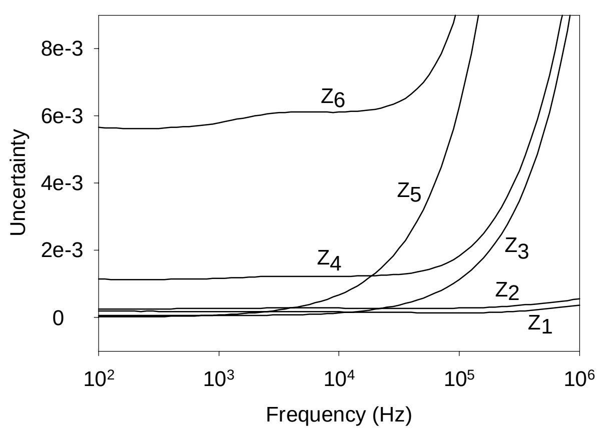

Fig. 2 Maximal absolute value of the predicted relative uncertainty for the six test impedance values according to (11).

Figs. 3 and 4 respectively show the real and imaginary components of the predicted uncertainty calculated from (11). The real component slightly depends on frequency for Z_5 and Z_6 (the largest and smallest impedance values). The imaginary component also increases with frequency for Z_3, Z_4, Z_5, and Z_6, but starts to increase earlier than the real component. Also, the farther the impedance value is from the optimum given by (12), the sooner the frequency dependence starts. By comparing the relative uncertainties in Figs. 3 and 4, we infer that the imaginary component is responsible for most of the dependence of the uncertainty with the frequency.

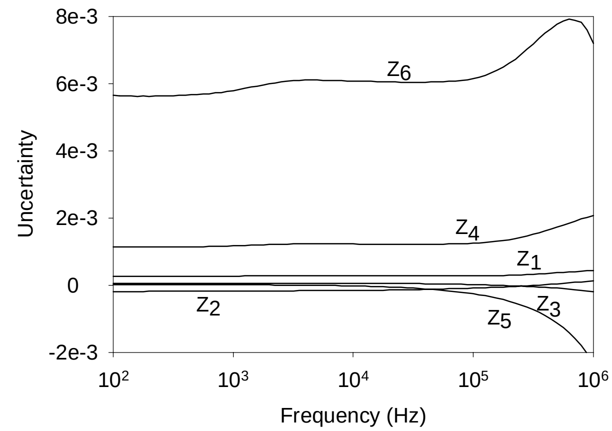

Fig. 3 Real component of the theoretical relative uncertainty predicted from (11) for six test impedance values.

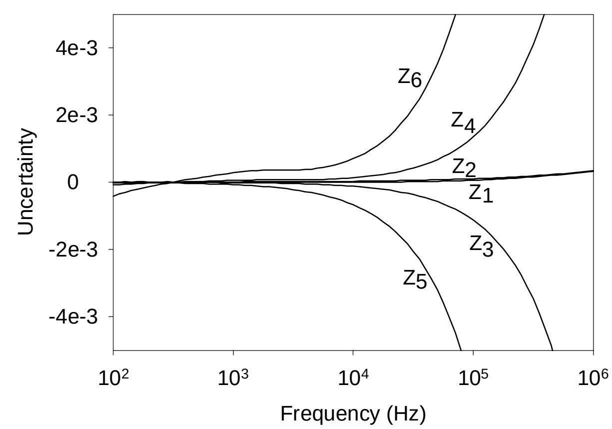

Fig. 4 Imaginary component of the theoretical relative uncertainty predicted from (11) for six test impedance values.

Fig. 5 shows the absolute value of the relative uncertainty of the impedance for these six test impedance values measured in the range from 100 Hz to 1 MHz. The relative uncertainty is minimal for Z_1 and Z_2, as predicted, and is smaller than the predicted uncertainty in Fig. 2. The relative uncertainty increases from 100 kHz up, particularly for the largest and smallest impedance values (Z_5 and Z_6), whereas the predicted relative uncertainty increases above 10 kHz. The uncertainty is similar for Z_3 and Z_4, the same as in Fig. 2. Also, at low frequencies the relative uncertainty is smaller for Z_5 than for Z_6, which is consistent with the theoretical prediction.

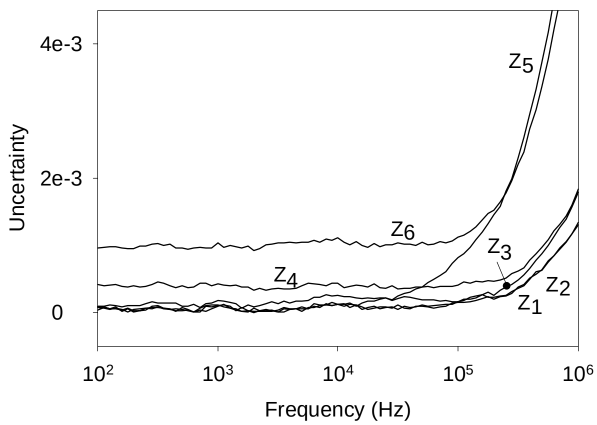

Fig. 5 Absolute value of the relative uncertainty for the six test impedance values measured with the HP4294A.

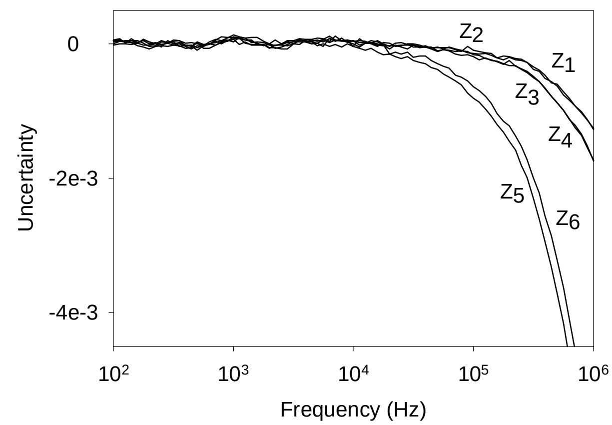

Fig. 7 Imaginary component of the relative uncertainty for the six test impedance values in Fig. 5.

Figs. 6 and 7 respectively show the real and imaginary components of the actual relative uncertainty. They reveal that both the real and the imaginary components of the actual relative uncertainty are smaller than the (maximal) predicted uncertainty (Fig. 3 and Fig. 4) and that their frequency dependence is also smaller than predicted. Furthermore, the increase in relative uncertainty at high frequencies in Fig. 5 is attributable to the frequency dependence of the imaginary component of the relative uncertainty, the same as for the theoretical prediction.

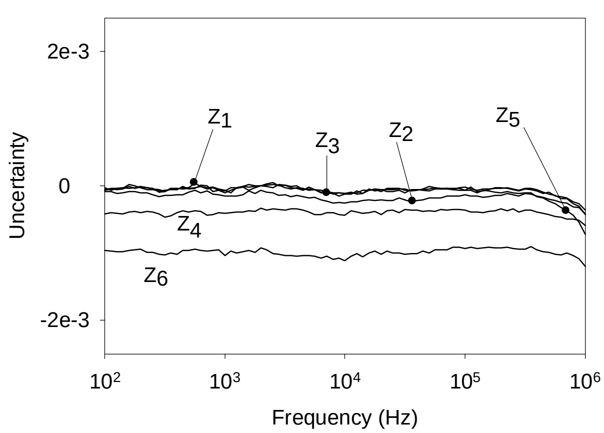

Fig. 6 Real component of the relative uncertainty for the six test impedance values in Fig 5.

IV. CONCLUSION

Accurate measurements using impedance analyzers require us to compensate for impedance residuals in the test fixture. When the test fixture is symmetrical, this correction is usually performed by measuring short-circuit and open-circuit impedance, and using the results to compute the actual impedance according to (9). When the text fixture is asymmetrical, we need a third reference measurement in order to compensate for impedance residuals according to (8). Using simple open/short correction instead adds gain uncertainty to that of the impedance analyzer. Equation (11) estimates the maximal relative uncertainty added and (12) determines the impedance value having the minimal relative uncertainty. Experimental results confirm that: (a) actual relative uncertainties when performing simple open/short correction are smaller than the theoretical limit calculated from (11); (b) relative uncertainties for impedance values close to the optimum calculated from (12) are very small; and (c) at low frequencies, relative uncertainties are larger for small impedance values than for high impedance values.

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