

High Performance Control of a Single-Phase Shunt Active Filter

R. Costa-Castelló, R. Griñó, R. Cardoner, E. Fossas

Abstract—Shunt active power filters are devices connected in parallel with nonlinear and reactive loads which are in charge of compensating these characteristics in order to assure the quality of the distribution network. This work analyzes the dynamics of boost-converter used as an active filter and proposes a control system which guarantees closed-loop performance (power factor close to 1 and current harmonics compensation). Proposed controller is hierarchically decomposed in two control loops, one in charge of shaping the current and the other in charge of assuring the power balance. Differently from other works both control loops are analytically tuned.

The work describes both the analytical development and the experimental results showing the good performance of the closed-loop system.

Index Terms—Active power filters, digital repetitive control

I. INTRODUCTION

Active filters are devices which allow to coexist nonlinear loads and good energy quality in distribution networks. A principal effort in the design and control of these devices has been developed in the past years. One research line deals with topologies and architectures [1], [2], several types of topologies have been proposed including parallel (shunt active filters), serial connections and hybrid serial-parallel connections. Besides the architecture, the behaviour principle has also been a research topic: passive, active (using switching converters) and mixed passive-active devices have been proposed. Additionally, converter based active filters may be based on a voltage or a current bus [3]. Most used active filters are connected in parallel and correspond to active components based on a voltage bus, although a passive serial connected filter is usually added to compensate switching noise.

Another important research line related with active filters is their control, many approaches have been proposed [4], [5], [6], [7], [8], [9], [10]. Most of the proposed control schemes are based on two hierarchical control loops, an inner one in charge of assuring the desired current and an outer one in charge of determining the required sinusoidal shape as well as the appropriate power balance and converter operation point. The current control loop needs to be fast and precise in order to assure the desired energy flow quality. Several approaches have been proposed: hysteresis based control [11], [12], [13], deadbeat controllers [14], Park transformation combined with linear controllers [15] and repetitive control [16], [17] are the most relevant options.

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The authors are with the *Instituto de Organización y Control de Sistemas Industriales, Universitat Politècnica de Catalunya*, Barcelona, Spain. E-mail: {ramon.costa,roberto.grino,rafel.cardoner,enric.fossas}@upc.edu

Several approaches have been proposed for the outer control loop: selecting the desired switching pattern by optimization, the optimization can be based on genetic algorithms [18], [19], neural networks [14] or Fourier series analysis [13]; using Lyapunov Functions [20]; or using a PI controller to determine the amplitude of the network sinusoidal current. Using a PI controller is the most common approach by far but, since the plant is nonlinear, this PI controller is usually experimentally tuned.

This work presents a new controller for a single-phase shunt active filter that uses the traditional two control loops decomposition. In our work the current controller is composed by a feedforward action in charge of assuring very fast transient response and a feedback control law in charge of assuring closed-loop stability and very good harmonic performance. The feedback control law is based on the use of a repetitive odd-harmonic controller [21]. The outer control law is based on the exact computation of the sinusoidal current network amplitude; in order to improve robustness this computation is combined with a feedback control law with a PI controller. One of the contributions of this paper is the analytical tuning of the PI, which is unusual. The complete system results in a simple control law which offers very good results, both in transient and steady-state behavior.

II. PROBLEM FORMULATION

A. Physical model of the boost converter

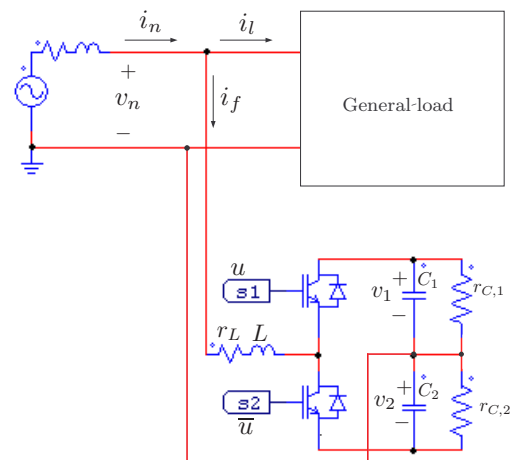


Fig. 1. Single-phase shunt filter connected to the network-load system.

Fig. 1 presents the system architecture. A load is connected to the power source, in parallel an active filter is applied in order to fulfill the required performance, i.e. to guarantee unity power factor in the network side. A boost converter with the ac neutral wire connected directly to the midpoint of the dc bus is used as active filter. The averaged (at the switching frequency) model of the boost converter is given by

$$L \frac{di_f}{dt} = -r_L i_f - v_1 d - v_2 (d-1) + v_n \quad (1)$$

$$C_1 \frac{dv_1}{dt} = -\frac{v_1}{r_{C,1}} + i_f d \quad (2)$$

$$C_2 \frac{dv_2}{dt} = -\frac{v_2}{r_{C,2}} + i_f (d-1) \quad (3)$$

where d is the duty ratio, i_f is the inductor current and v_1, v_2 are the DC capacitor voltages, respectively; $v_n = V_n \sqrt{2} \sin(\omega_n t)$ is the voltage source that represent the ac-line source; L is the converter inductor, r_L is the inductor parasitic resistances, C_1, C_2 are the converter capacitors and $r_{C,1}, r_{C,2}$ are the parasitic resistances of the capacitors. The control variable, d , takes its values in the set $[0, 1]$ and represents the averaged value of the PWM (pulse-width modulated) control signal injected to the actual plant.

B. Load Description

Due to the nature of the voltage source, the load current, in steady-state, is usually a periodic signal with only odd-harmonics in its Fourier series expansion, so the current can be written as:

$$i_l = \sum_{n=0}^{\infty} a_n \sin(\omega_n (2 \cdot n + 1) t) + b_n \cos(\omega_n (2 \cdot n + 1) t) \quad (4)$$

where $a_n, b_n \in \mathbb{R}$ are the real Fourier series coefficients of the load current. Hence a zero dc component of i_l is presumed.

C. Control objectives

The active filter goal is to assure that the load is seen as a resistive one. This goal can be stated ¹ as $i_n^* = I_d^* \sin(\omega_n t)$, i.e. the source current must have a sinusoidal shape in phase with the network voltage. Another collateral goal, necessary for a correct operation of the converter, is to assure constant average value of the dc bus voltage ², i.e. $\langle v_1 + v_2 \rangle_0^* = 2v_d$, where v_d must fulfill the boost condition ($v_d > \sqrt{2}v_s$). It would also be desirable that this voltage would be almost equally distributed among both capacitors ($v_1 \approx v_2$).

These two objectives define a non-standard control problem: the second one is a regulation objective for the mean value of $v_1 + v_2$, while the first one is not a tracking specification because only a shape and not a signal is defined, that is I_d^* is not known a priori and it must take the appropriate value to maintain the power balance of the whole system. This special form of the problem specifications implies the particular structure of the controller loops described in the next section.

¹ x^* represents the steady-state value of signal $x(t)$.

² $\langle x \rangle_0$ means the dc value, or mean value, of signal $x(t)$

D. Rewriting the Equations

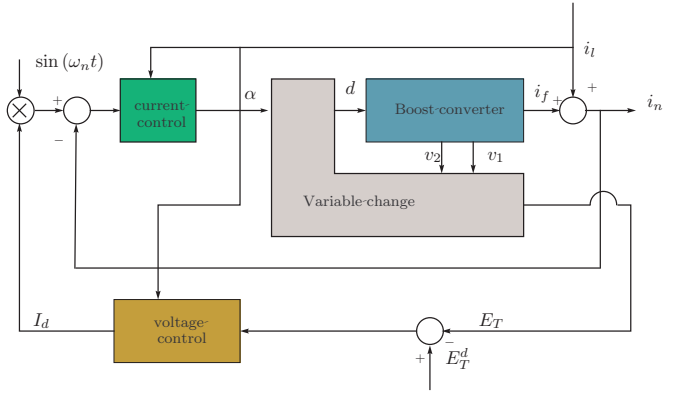


Fig. 2. Block diagram of the controller showing the current control loop (inner) and the voltage (or energy) control loop (outer).

It is standard for this kind of systems to linearize the current dynamics by the partial state feedback $\alpha = v_1 \cdot d + v_2 \cdot (d-1)$. Moreover, the change of variables $i_f = i_f$, $E_C = \frac{1}{2} (C_1 v_1^2 + C_2 v_2^2)$, $D = C_1 v_1 - C_2 v_2$ makes appear two more meaningful variables. Namely, E_C , the energy stored in the converter capacitors and D , the charge unbalance between them. Assuming that the two dc bus capacitors are equal ($C = C_1 = C_2$, $r_C = r_{C,1} = r_{C,2}$) the system dynamics in the new variables results in:

$$L \frac{di_f}{dt} = -r_L i_f + v_n - \alpha \quad (5)$$

$$\frac{dE_C}{dt} = -\frac{2E_C}{r_C \cdot C} + i_f \cdot \alpha \quad (6)$$

$$\frac{dD}{dt} = -\frac{1}{r_C \cdot C} D + i_f \quad (7)$$

It is important to note that the state feedback together with the change of variables results in a state and input diffeomorphism so the linearization is complete and formally correct [22]. In addition, eq. (7) is linear too.

This new system (5)-(7) needs a controller to fulfill the desired performance. This controller will be defined using a two step approach, first of all a current controller which forces the sine wave shape, afterwards the sine wave amplitude will be defined by an outer control loop to fulfill the appropriate active power balance for the whole system. This balance is achieved if the energy ³ stored in the active filter capacitors, E_C , is equal to a reference value, E_C^d .

The full control scheme for the system is depicted in Fig. 2. The specific controller designs will be presented in sections III-A and III-B.

III. CONTROL DESIGN

A. Current Loop

Taking benefit from the fact that current equation (5) is linear, a linear controller is designed to force a sinusoidal

³A similar reasoning can be done with the dc bus capacitor voltages as it is implied by the change of variables used.

shape in the network current. This controller will be designed in two parts:

- A feedforward controller which fixes the desired steady state :

$$i_n = I_d \sin(\omega_n t) \quad (8)$$

The control action related with this action is computed by inverting the plant dynamics in steady state and forcing the output to be the desired one.

- A feedback controller which compensates uncertainties and assures closed-loop stability. This feedback controller is designed applying the odd-harmonic repetitive control technique [23], [21].

This control technique allows to obtain perfect steady-state tracking/rejection of a certain periodic signal and all its odd-harmonics.

The complete control action is obtained by adding both control actions. Under this control action the output is the desired one also in the case of uncertainties and disturbances.

B. Energy Shaping (Voltage Loop)

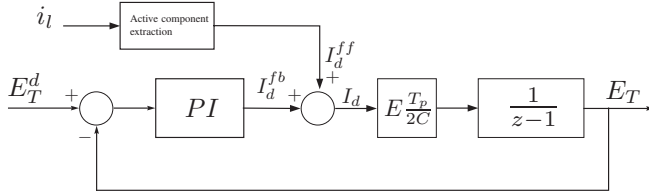


Fig. 3. Simplified 50Hz model with PI Controller

As the source voltage is assumed to be $v_n = V_n \sqrt{2} \sin(\omega_n t)$ the desired network current is $I_d \sin(\omega_n t)$, with I_d locally constant. As a consequence the desired power flow seen from the net is:

$$p_n(t) \triangleq v_n(t) \cdot i_n(t) \approx I_d \sqrt{2} V_n \sin^2(\omega_n t) \quad (9)$$

Additionally, the active filter goal is not consuming power so ideally the following relationship is desired:

$$p_n(t) = I_d \sqrt{2} V_n \sin^2(\omega_n t) \approx v_n i_l \triangleq p_l(t) + p_f(t) \quad (10)$$

where $p_l(t)$ and $p_f(t)$ are the instantaneous power of the load and the filter, respectively. However, as I_d is designed locally constant the power requirements can not be fulfilled instantaneously. What can be achieved is an energy compensation within one period

$$\int_{t-T_p}^t p_n \approx \int_{t-T_p}^t p_l + p_f, \quad (11)$$

that yields the I_d ideal value. From the power flow point of view the active filter redistributes the power flow within one period in order to assure the stated power balance:

$$\int_{t-T_p}^t p_f \approx 0 \quad (12)$$

Hence, the total energy stored in the converter (E_T) should not suffer variations within a period, i.e.

$$\int_{t-T_p}^t \dot{E}_T = 0 \quad (13)$$

The stored energy in the converter can be decomposed in the energy stored in the inductors ($E_L = \frac{1}{2} L (i_f)^2$), and the energy stored in the capacitors ($E_c = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2$). Additionally, it is important to note that some energy is lost in the parasitic resistors of the inductors, the capacitors and the switches.

Noting that $i_f \approx I_d \sin(\omega_n t) - i_l$ is an odd-harmonic signal and without taking into account the parasitic resistance of the inductors and capacitors, it can be easily proven that independently of the value of I_d and the load currents the variation of energy in the inductors on one period is zero. Thus,

$$\int_{t-T_p}^t \dot{E}_T = \int_{t-T_p}^t \dot{E}_c \quad (14)$$

In case $r_L \approx 0$ the next relationship can be stated

$$\int_{t-T_p}^t \dot{E}_T = E_T(t) - E_T(t - T_p) \approx \frac{\sqrt{2} V_s \pi (I_d - a_0)}{\omega}.$$

This energy balance can be seen as a linear discrete-time system (with sampling time T_p) with an input I_d and a constant disturbance a_0 . So, applying the z -transform

$$E_T(z) = \frac{1}{z-1} \frac{\sqrt{2} V_s \pi}{\omega_n} (I_d(z) - a_0 \cdot U_s(z)) \quad (15)$$

where $U_s(z)$ is the z -transform of the step signal. The value of a_0 corresponds to the active component of i_l (the load current). So, in order to assure the desired energy balance ($E_T \approx 0$), a closed-loop system is proposed. The control action will be composed of two main parts:

- A feedforward term : $I_d^{ff} = a_0$.
- A feedback term which is in charge of compensating the dissipative terms effects and the uncertainties in the system. Thus, a classical PI controller will regulate E_T to the desired value E_T^d without steady-state error, i.e.

$$I_d^{fb}(z) = \left(k_p + k_i \frac{z+1}{z-1} \right) (E_T^d(z) - E_T(z)) \quad (16)$$

Fig. 3 shows the complete closed-loop scheme.

IV. EXPERIMENTAL SETUP AND IMPLEMENTATION ISSUES

The experimental setup used to test the designed controller has the following parts:

- Active filter: half-bridge boost converter (split-capacitor dc bus) with IGBT switches (nominal current 100 A) and the following parameters: $r = 0.3 \Omega$, $L = 0.8 \text{ mH}$, $C_1 = C_2 = 9900 \mu\text{F}$ and $r_C = 8200 \Omega$. The switching frequency of the converter is 20 kHz and a synchronous (regular) centered-pulse single-update mode pulse-width modulation strategy is used to map the controller's output to the IGBT gate signals (see Figure 4).
- Rectifier (non-linear load): Full-wave diode rectifier with a filter capacitor $C = 4500 \mu\text{F}$. The active power with

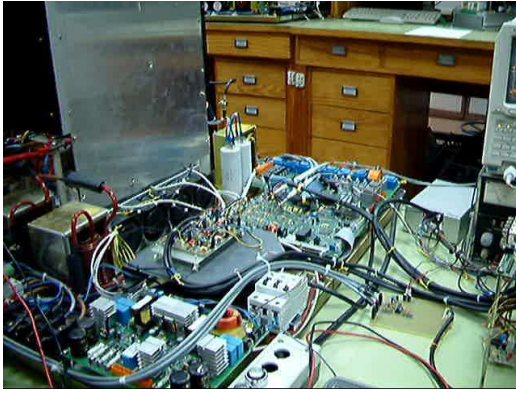


Fig. 4. Power Converter Picture

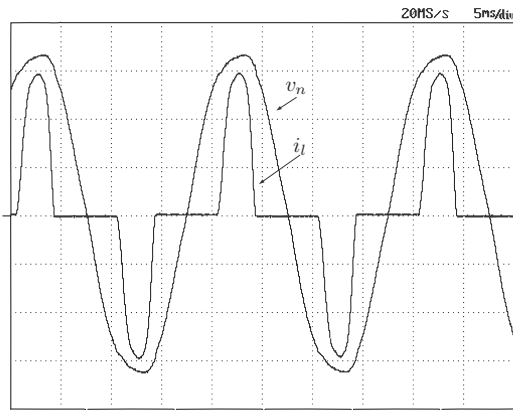


Fig. 5. Nonlinear load: voltage and load current (92 V/div, 19.2 A/div).

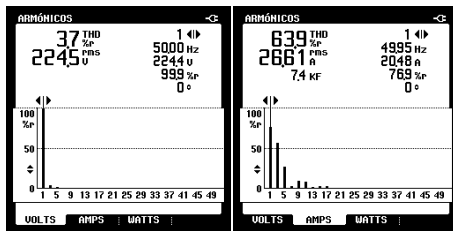


Fig. 6. Nonlinear load: voltage and load current (RMS and THD).

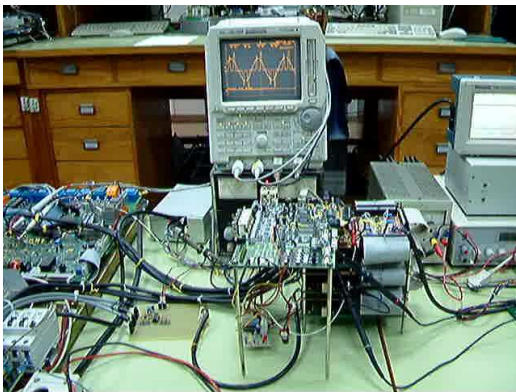


Fig. 7. Control Setup Picture

nominal dc resistor is $P = 4.56 \text{ kW}$ and its reactive power is approximately zero. Fig. 5 shows the shape of the ac mains voltage and current and Fig. 6 the harmonic content of the voltage and the current for the rectifier with the nominal dc resistor. It is worth to remark that the total harmonic distortion⁴ (THD) of this current is about 63.9% and its maximum derivative is about 70kA/s.

- Analog circuitry of feedback channels: the ac mains voltage, the ac mains current and the dc bus voltages are sensed with a voltage transformer, a hall-effect sensor and two isolation amplifiers, respectively. All the signals from the sensors pass through the corresponding gain conditioning stages to adapt their values to A/D converter input taking advantage of their full dynamic range. In addition, all the feedback channels include a first order low-pass filter with unity dc gain and 4.3 kHz cutoff frequency⁵.
- Control hardware and DSP implementation: the control board has been internally developed and is based on an ADSP-21161 floating-point DSP processor with an ADSP-21990 fixed-point mixed-signal DSP processor that acts as coprocessor, both from Analog Devices. The ADSP-21161 and the ADSP-21990 communicate each other using a high-speed synchronous serial channel in DMA mode. The ADSP-21990 deals with the PWM generation and the A/D conversions with its integrated 14 bits eight high-speed A/D channels (Figure 7). The controller has been implemented running at the IGBT switching frequency. Technologically this switching frequency is limited to 20 kHz so this frequency has been selected as the sampling one.
- The nominal voltage of the ac mains is $V_n = 230 \text{ V RMS}$ and its nominal frequency is 50 Hz.

V. EXPERIMENTAL RESULTS

This section shows some of the experimental results obtained for the active filter operation with the designed control system. The results are presented by means of oscilloscope and power analyzer screen dumps of the ac mains electrical variables and the active filter semi-bus dc voltages when necessary.

Apart from the selected experiments collected in this section, a lot of numerical simulations, including mainly capacitive or inductive loads, have been carried out showing the same good performance as it will be shown below. Also, it is worth noting that several numerical simulations including loads that work as generators at some time periods⁶ (thus imposing a negative active power flow to the source) have been carried out without problems. The voltage loop of the overall controller assures the active power balance and, after a transient, in steady state the input to the AM modulator

⁴In this work the THD figures and the harmonic content are always taken with respect to the fundamental harmonic (50 Hz) and they have been obtained using a Power Quality Analyzer Fluke 43 instrument.

⁵The oscilloscope screens in the figures of this section and the following show the voltages and currents after the corresponding analog low-pass filters.

⁶This problem was established as a hard one by Depenbrock and Staudt [24].

is negative giving a current reference shifted π rad from the network voltage that the current loop tracks without difficulty.

A. Active filter operation with no load

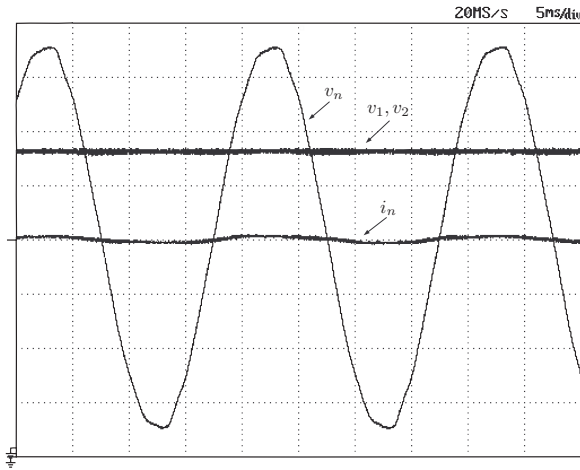


Fig. 8. Active filter with the no load: voltage, current and semi-bus dc voltages (92 V/div, 19.2 A/div and 74.5 V/div, respectively).

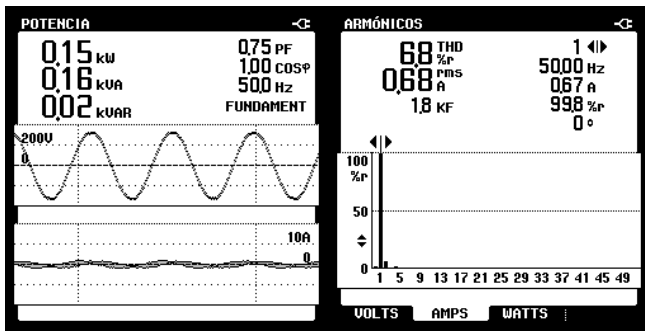


Fig. 9. Active filter with no load: RMS and THD values of current $i_n(t)$ and P , Q , $\cos \phi$ and PF.

This subsection presents some results of the no load operation of the active filter. Fig. 8 shows the network voltage and current and the semi-bus dc voltages. The RMS value of the current is about 0.68 A and its THD value is 6.8%. Then, the resulting active power consumed by the filter to cover its losses without compensating any load is about 0.15 kW. It is worth to note that the fundamental component of the current is in phase with the voltage ($\cos \phi = 1$), see Fig. 9. So, almost no reactive power is consumed by the filter. The low power factor (PF) is due to the high value of the switching ripple with respect to the fundamental component of the current.

B. Active filter operation with the nonlinear load

In this experiment the diode rectifier previously described is connected to the network. This nonlinear load has not reactive power at the fundamental frequency, however the active filter must work to compensate all the generated higher

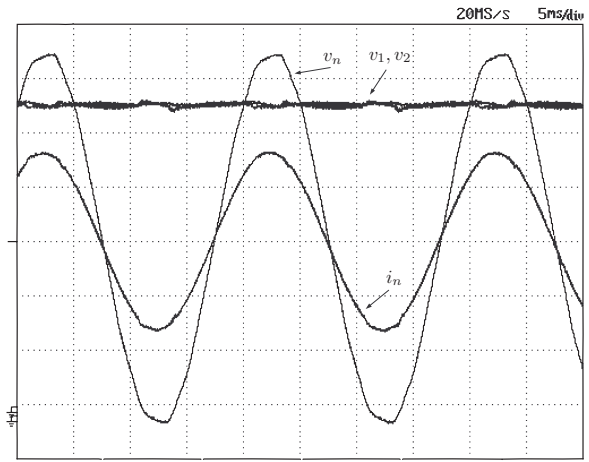


Fig. 10. Active filter with the nonlinear load: voltage, current and semi-bus dc voltages (92 V/div, 19.2 A/div and 74.5 V/div, respectively).

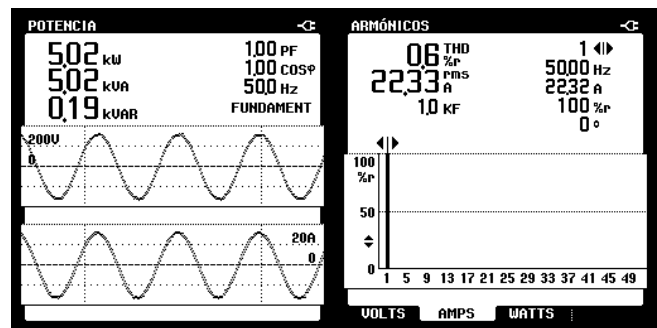


Fig. 11. Active filter with the nonlinear Load : RMS and THD %f values of mains current i_n and $\cos \phi$, PF.

order harmonics. Fig. 10 shows the current that appears with a good sinusoidal shape and in phase with the grid voltage. This figure also shows the values of each semi-bus of the active filter dc bus. As it can be seen in Fig. 11 the THD of the current is very low (0.6 %) and the power factor is 1.

C. Active filter transient response

This section presents the results for the following experiments:

- 1) the full nonlinear load is applied to the network with the active filter in operation (Fig. 12);
- 2) the full nonlinear load is disconnected from the ac mains with the active filter in operation (Fig. 13).

In each case, the overshoot in the dc bus voltage is almost imperceptible. Therefore, there is no problem with the maximum load variations expected in the system.

VI. CONCLUSIONS

The paper shows the design and implementation of a controller for a current active filter. The controller consists of a current control loop and an outer dc bus voltage control loop. The current references for the inner control loop is

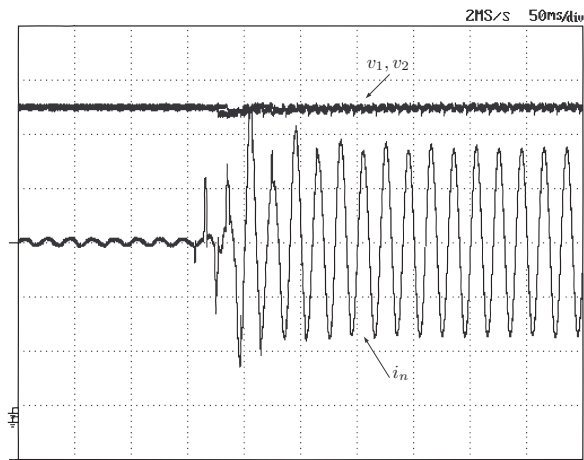


Fig. 12. Source current (i_n) and semi-bus dc voltages (v_1, v_2): from no-load to full nonlinear load (19.2 A/div and 74.5 V/div, respectively).

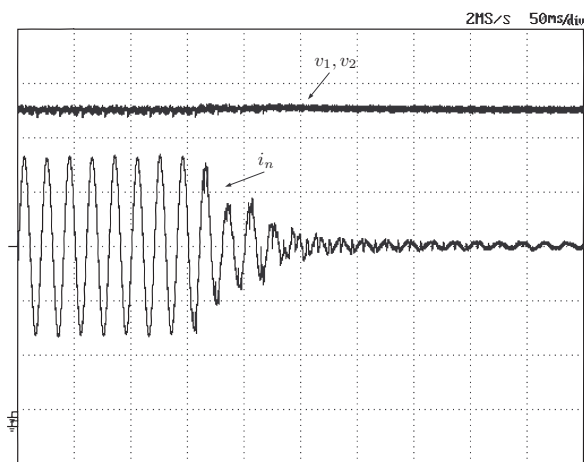


Fig. 13. Source current (i_n) and semi-bus dc voltages (v_1, v_2): from full nonlinear load to no-load (19.2 A/div and 74.5 V/div, respectively).

created passing the output of the voltage controller through an AM modulator that uses as a carrier a filtered version of the network voltage. Both, the current and the outer controllers are based on the combination effect of a feedforward and a feedback control law. This combination allows to obtain almost perfect response both in transient and steady-state operation.

As a conclusion the proposed control scheme constitutes a step forward in the active filter control. Both the transient and steady-state behaviour are very good in the network current shape and the active filter semi-bus dc voltages.

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