On the Common Mode Response of Fully Differential Circuits

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Abstract

Differential circuits are often described by their differential gain and common mode rejection ratio (CMRR). This approach, however, neglects the effect that the common mode signal has on the transient response and stability of the circuit. This work shows that the actual behavior of differential circuits in front of common mode voltages is completely described by the common-to-differential mode gain and the common-mode gain. The CMRR is useful to assess common mode errors in the frequency domain, but some circuits that achieve a large CMRR have long transients or are unstable.

1. Introduction

Differential circuits are suited to process signals from sensor bridges and differential sensors. Often, a differential amplifier yields a single-ended voltage that undergoes further processing (demodulation, filtering) before sampling and analog-to-digital conversion. However, it has been shown that the later the differential to single-ended conversion in the measurement chain, the higher the common mode rejection ratio (CMRR) [1][2]. Moreover, because differential signals have increased immunity to interference, it is convenient not to convert them to single-ended signals. The increasing availability of differential analog-to-digital converters (ADC) enables this approach.

Fully differential circuits (differential input and output) are described by four transfer functions (Figure 1). The differential and common mode outputs are [3]

$$V_{oD} = G_{DD}V_{iD} + G_{DC}V_{iC}$$
$$V_{oC} = G_{CD}V_{iD} + G_{CC}V_{iC} \tag{1}$$

where V_{iD} and V_{iC} are, respectively, the (transforms of the)

differential and common mode input voltages. Because the information is embedded in the differential voltage, we ideally wish $G_{DC} = 0$. Having $G_{CD} = 0$ is also convenient, but it is less important if the following stage rejects common mode signals. In practice, however, $G_{DC} \neq 0$ and we define the CMRR as

$$\text{CMRR} = \frac{G_{DD}}{G_{DC}} \tag{2}$$

The differential output voltage is then [3][4]

$$V_{oD} = G_{DD}\left(V_{iD} + \frac{V_{iC}}{\text{CMRR}}\right) \tag{3}$$

which describes the effect of the common mode voltage as an additive error, provided the common and differential mode voltages are independent from each other. To further simplify matters, although the CMRR is frequency dependent, usually the error described by (3) is calculated by using the CMRR value in the frequency band of interest, namely that of V_{iD}. Equation (3) also applies to circuits with differential input and single-ended output, in which case $G_{DD} = G_D$, $G_{DC} = G_C$.
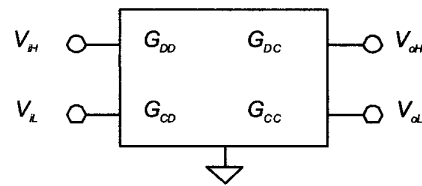

Figure 1. Fully differential circuits are described by four transfer functions.

Although eqn. (3) is extensively used, it does not describe the actual circuit behavior to the common mode input voltage v_{iC}. A large CMRR, for example, suggests a

small error due to v_{iC}, but if G_{DC} is unstable the circuit will not work [5][6]. Also, if G_{DC} has a long transient response, it cannot be explained from G_{DD} and eq. (3).

This work shows that the CMRR does not fully describe the common mode response of differential circuits and that G_{DC} and G_{CC} must be designed to achieve the desired behavior to common mode signals.

2. Theoretical analysis

2.1. Long transient response due to G_{DC}

We have investigated the actual influence of G_{DC} in several fully differential circuits. Figure 2, for example, shows a fully differential high-pass filter. $R_c = 0\ \Omega$ yields an uncoupled filter. Adding $R_c \gg R_1$ improves the low-frequency CMRR by $2R_c/R_1$ [7]. An infinite R_c would yield infinite CMRR, but R_c must be finite to bias the following amplifier. Considering component tolerances in a worst case situation,

$$R_1 = R(1+\alpha) \quad C_1 = C(1+\beta)$$
$$R_1' = R(1-\alpha) \quad C_1' = C(1-\beta) \tag{4}$$

yields

$$G_{DC} \approx \frac{2(\alpha+\beta)RCs}{(1+RCs)(1+(R+2R_c)Cs)} \tag{5}$$

where we have assumed $R_c \gg R$. From (5), the differential mode output when applying a common mode step voltage of amplitude A is

$$V_0 \approx A(\alpha+\beta)\frac{R}{R_c}\left[e^{-t/(R+2R_c)C} - e^{-t/RC}\right] \tag{6}$$

If $R_c = 0$, circuit analysis yields

$$V_0 = A\left(e^{-t/R_1C_1} - e^{-t/R_1'C_1'}\right) \tag{7}$$

Figure 3 is a normalized plot of (6) and (7) showing that the output voltage does not correspond to the step response of a high-pass filter (G_{DD}) but to the bandpass filter described by (5). The theoretical peak amplitude and the time needed to reach it are

$$A_p \approx 2A(\alpha+\beta)e^{-1}$$
$$t_p \approx RC \tag{8}$$

for the uncoupled filter and

$$A_p \approx A(\alpha+\beta)\frac{R}{R_c}$$
$$t_p \approx RC\ln\left(\frac{2R_c}{R}\right) \tag{9}$$

for the coupled filter when $R_c \gg R$. Increasing R_c reduces A_p and increases CMRR, both of which are desirable, but also lengthens t_p and the decaying exponential tail, which are undesirable. For example, $R_1 = 100\ k\Omega$, $C_1 = 1\ \mu F$ ($f_c = 1.59\ Hz$), $A = 1\ V$, $\alpha = 5\ \%$ and $\beta = 10\ \%$ yield $A_p = 110\ mV$ and $t_p = 100\ ms$ for the uncoupled filter. However, at $t = 1\ s$ the output voltage is only 163 µV. If $R_c = 1\ M\Omega$ the CMRR increases by 26 dB and A_p decreases to about 15 mV but t_p increases to 300 ms. Worse yet, since the exponential tail is determined by $2R_cC$, the output voltage after 1 s is 9.3 mV, hence 57 times larger than that of the uncoupled filter. The output decreases to 163 µV at $t = 9.5\ s$ when $R_c = 1\ M\Omega$, and at $t = 45\ s$ when $R_c = 10\ M\Omega$. Therefore, the CMRR has improved at the cost of a slower transient response.
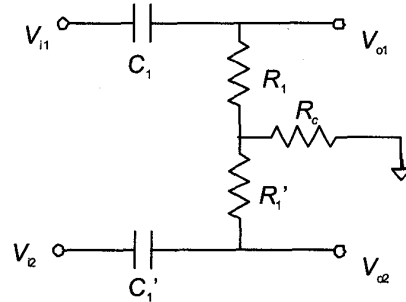
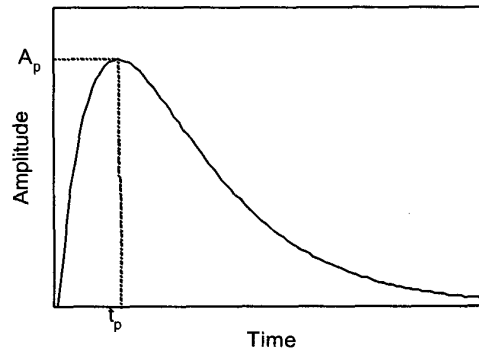**Figure 2. Fully differential high-pass filter. R_c = 0 Ω yields an uncoupled filter.**

Figure 3. Common mode step response of the fully differential filter in Figure 2.

Figure 4 shows a differential filter that does not need any grounded resistor to provide a bias path for the following amplifier. The input common mode voltage yields an output common mode voltage with the same amplitude (G_{CC} = 1) and cannot yield any differential output voltage. Hence, G_{DC} = 0 and CMRR = ∞. The frequency response for the differential voltage can be designed to be the same as in Figure 2. Therefore, circuits with the same differential response can have quite different common mode response.
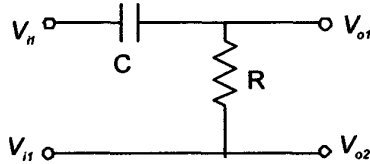

Figure 4. Asymmetrical high-pass filter.

2.2. Circuit instability due to G_{CC}

Some circuits with high CMRR are unstable. Da Cunha [5] built a fully differential, second-order, low-pass filter based on a common Sallen and Key circuit that revealed to be oscillation-prone. We have analyzed that circuit and traced its instability to G_{CC}. The CMRR would not explain that oscillation. Splitting capacitors and adding a small capacitance to ground (C_c in Figure 5) stabilizes the circuit at the cost of a reduced CMRR. If capacitors other than C_c have nominal value C and tolerance β, and resistors have nominal value R and tolerance α, when $C_c \ll C$ the CMRR is

$$\text{CMRR} \approx -\frac{1}{2RC_c(\alpha+\beta)}\frac{1+RC_c s+ R^2 CC_c s^2}{(1+RCs)s} \qquad (10)$$

and G_{CC} is

$$G_{CC} \approx \frac{1}{1+RC_c s+R^2 C\dfrac{C_c}{2}s^2} \qquad (11)$$

which has resonance with amplitude $M_r \approx \sqrt{C/(2C_c)}$ at $\omega_1 \approx 1/R\sqrt{(C C_c/2)}$. If C_c is close to zero, M_r tends to infinite, thus rendering the circuit unstable. Increasing C_c reduces M_r (and ω_1) but also the CMRR at low frequencies.

White [6] analyzed oscillation in the three-op-amp instrumentation amplifier, traced it to the common mode gain (here termed G_{CC}) and solved it by a technique similar to that in Figure 5.
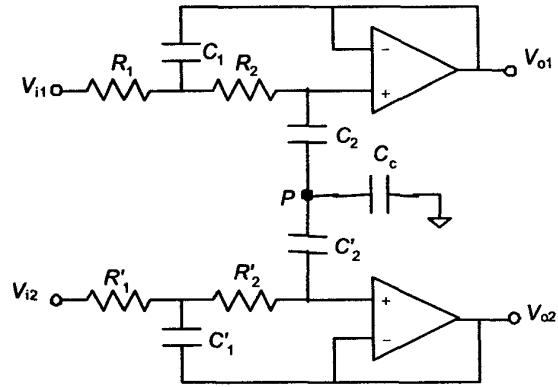

Figure 5. Fully differential, second-order, low-pass filter.

3. Experimental results and discussion

We have built the circuit in Figure 2 with $R_1 = R_1' = 100$ kΩ and $C_1 = C_1' = 1$ μF. R_c was 0 Ω (uncoupled filter), 1 MΩ, and 10 MΩ. The circuit output was connected to an instrumentation amplifier (INA114) that provided gain (100), low output impedance, and a single-ended output. Figure 6 shows the amplifier output voltage when applying a 1 V common-mode step voltage and R_c = 0 Ω and 1 MΩ. The uncoupled filter yields A_p = 3.56 V and t_p = 100 ms, which, according to (8), implies $\alpha+\beta$ = 4.8 %. When R_c = 1 MΩ, A_p decreases to 0.35 V, t_p increases to 320 ms, and the exponential decay is far slower. From t = 500 ms on, the output voltage is larger than that of the uncoupled filter. At t = 1 s, the output is still 0.27 V. The filter in Figure 4 yields a zero output, as expected.
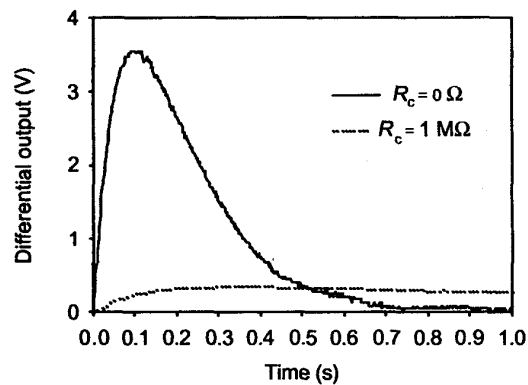

Figure 6. Output voltage for a passive differential filter (Fig. 2) followed by an INA114 instrumentation amplifier (G = 100), when

applying a 1 V common mode input voltage.

Figure 7 shows the CMRR for the filters in Figures 2 and 4 when connected to the INA114 ($G = 100$). The CMRR for the circuit in Figure 2 improves when R_c is large. The CMRR measured at high frequencies is that of the INA114, because it is smaller than that of the preceding filter. The filter in Figure 4 yields the best CMRR, as predicted.
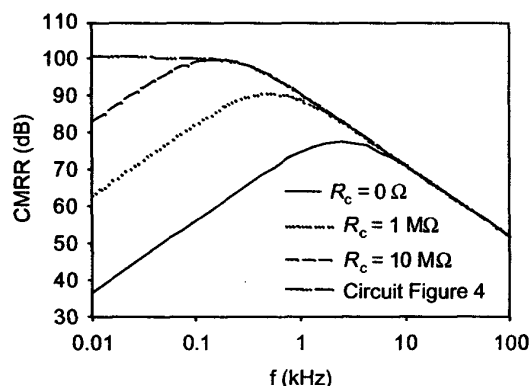
Figure 7. CMRR of the filters in Figures 2 and 4.

We have implemented the filter in Figure 5 by using TL072 op amps and designing $R_1 = R_1' = R_2 = R_2' = 1.6 \text{ k}\Omega$, $C_1 = C_1' = C_2 = C_2' = 1 \text{ }\mu\text{F}$, thus resulting in a -3dB corner frequency of 1 kHz. Grounding P yields an uncoupled filter. Figure 8 shows the experimental G_{CC} obtained for the uncoupled filter and for the coupled filter with $C_c = 100$ nF, 10 nF, and 100 pF. The resonance peak and frequency decrease for large C_c. However, a large C_c reduces the CMRR (Figure 9). Hence, there is a trade off between stability and CMRR, as anticipated by the theoretical analysis.
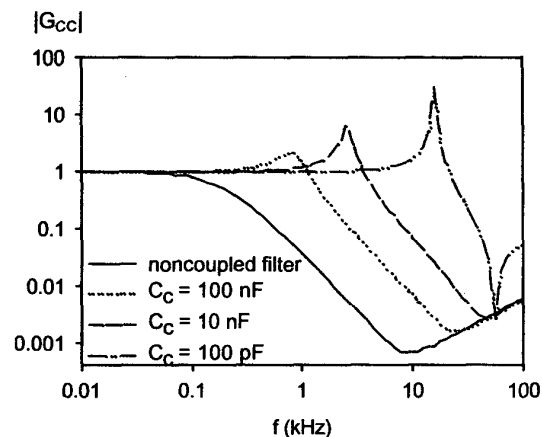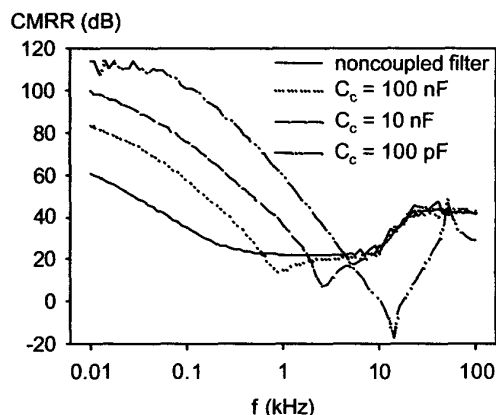
Figure 8. G_{CC} for the filter in Figure 5.

Figure 9. CMRR for the filter in Figure 5.

4. Conclusion

The common mode response of differential circuits is not fully described by the CMRR but by the common-to-differential mode gain (G_{DC}) and the common mode gain (G_{CC}). Their analysis reveals shortcomings such as long transient response and potential instability. Equation (3) is still useful to assess common mode errors in the frequency domain, but some circuits that achieve a large CMRR have long transients or are unstable.

Acknowledgement

References

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