Correspondence

Comments on "On Resistor-Induced Thermal Noise in Linear Circuits"

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In a recent paper [1], Giannetti considered the thermal noise generated by a resistor R on a generic output port in any linear network. He reasoned that, because the contribution of the thermal noise of the resistor to the output noise of the circuit vanishes for both $R \to 0$ and $R \to \infty$, there should be a resistor value whose contribution to the output noise is maximal. In order to find that value, from Fig. 1 he obtained that the thermal noise contributed by R was [1, (3)]

$$\overline{v_u^2}(R) = |H'|^2 \left| \frac{R}{R+Z} \right|^2 \frac{4kT}{R} = A \left| \frac{R}{R+Z} \right|^2 \frac{1}{R} \tag{1}$$

which is incorrect because the second and third terms of (1) do not have the dimensions of voltage spectral density. The correct analysis yields

$$\overline{v_u^2}(R) = |H'|^2 \left| \frac{RZ}{R+Z} \right|^2 \frac{4kT}{R} = A \left| \frac{RZ}{R+Z} \right|^2 \frac{1}{R}. \tag{2}$$

The maximal noise will result for that value of R that nulls the first derivative of (2), which is $R = |Z|$. This agrees with [1, (5)] because the mistake in (1) reduces to a multiplying factor (Z) that does not depend on R. The maximal output noise contributed by the thermal noise of R can then be calculated by replacing R with $|Z|$ in (2) to obtain

$$\overline{v_u^2} = \frac{2kT|Z|\,|H'|^2}{1+\cos\varphi} \tag{3}$$

which is different from that given by [1, eq. (6)] [derived from (1)], whose dimensions were not those of a voltage spectral density and included $H'(s)$ instead of $|H'|$. Nevertheless, the conclusion in [1] is correct; the output voltage spectral density will increase if the impedance Z is not purely resistive.

This interesting result has no simple explanation and deserves further elaboration because it might lead to the wrong conclusion that a reactive impedance Z would increase the output noise. According to (3), the voltage spectral density at the output of an amplifier whose input impedance includes, say, a shunting capacitance, will certainly increase when $R = |Z|$, but, because of the frequency dependence of the impedance of a capacitance, that condition will happen at a single frequency. What will then happen in a finite frequency bandwidth? If $Z = R^* + jX^*$, the output noise power from f_L to f_H will be

$$\begin{aligned} V_u^2 &= \int_{f_L}^{f_H} \overline{v_u^2}\, df \\ &= 4kTR \int_{f_L}^{f_H} \left| \frac{Z}{R+Z} \right|^2 |H'|^2\, df \\ &= 4kTR \int_{f_L}^{f_H} \frac{R^{*2} + X^{*2}}{(R+R^*)^2 + X^{*2}} |H'|^2\, df \end{aligned} \tag{4}$$

whose result is determined by the frequency dependence of R^*, X^*, and H'. For the sake of simplicity, let us assume that $|H'| = 1$. For a purely capacitive input impedance ($R^* = 0$, $1/X^* = -2\pi f C$), we would have

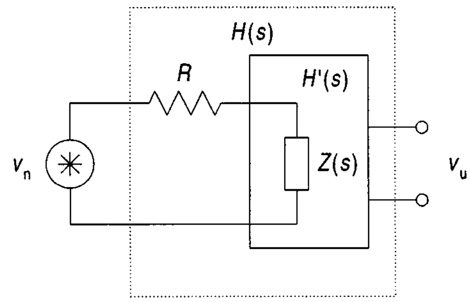

Fig. 1. Effect of the thermal noise of a resistor in a circuit can be analyzed by separating the resistor from the rest of that circuit.

$$V_u^2\Big|_{R^*=0} = \frac{kT}{C}\,\frac{2}{\pi} \arctan \frac{2\pi(f_H - f_L)RC}{1 + \left(2\pi\sqrt{f_H f_L}RC\right)^2} \tag{5}$$

whose maximal value would be

$$V_{u(\max)}^2\Big|_{R^*=0} = 4kTR\sqrt{f_L f_H} \arctan \frac{f_H - f_L}{2\sqrt{f_H f_L}} \tag{6}$$

at

$$R = \left| Z\left(j2\pi\sqrt{f_L f_H}\right) \right| = \frac{1}{2\pi\sqrt{f_L f_H}C}. \tag{7}$$

For a purely resistive input impedance ($Z = R^*$), we would have

$$V_u^2\Big|_{Z=R^*} = 4kTR\left(\frac{R^*}{R+R^*}\right)^2 (f_H - f_L) \tag{8}$$

whose maximal value would be

$$V_{u(\max)}^2\Big|_{Z=R^*} = kTR(f_H - f_L) \tag{9}$$

at $R = R^*$.

Dividing (6) by (9) yields

$$\frac{V_{u(\max)}^2\Big|_{R^*=0}}{V_{u(\max)}^2\Big|_{Z=R^*}} = \frac{4\sqrt{f_L f_H}}{f_H - f_L} \arctan \frac{f_H - f_L}{2\sqrt{f_H f_L}}. \tag{10}$$

When $f_H - f_L \ll \sqrt{f_H f_L}$, the output noise power ratio is 2. When $f_H - f_L \gg \sqrt{f_H f_L}$, the output noise power ratio is

$$\frac{V_{u(\max)}^2\Big|_{R^*=0}}{V_{u(\max)}^2\Big|_{Z=R^*}} \cong \frac{4\sqrt{f_L f_H}}{f_H - f_L}\,\frac{\pi}{2} = \frac{2\pi\sqrt{f_L f_H}}{f_H - f_L} \ll 1. \tag{11}$$

Therefore, from the point of view of output noise, when the source resistance and the input impedance are matched ($R = |Z|$), for narrowband signals it is advisable to reduce the input capacitance as much as possible, whereas for a wide band signal, the input capacitance helps in reducing the output noise power. In the usual cases of voltage or current measurement we have $R \ll |Z|$ and $R \gg |Z|$, respectively, which results in minimal output voltage noise spectral density. By the same token, the output noise contributed by the amplifier input resistance R^* will be small whenever it is unmatched to R. Thus, the common design practice to reduce voltage or current loading also reduces output noise.

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References

[1] R. Giannetti, "On resistor-induced thermal noise in linear circuits," *IEEE Trans. Instrum. Meas.*, vol. 49, pp. 87–88, Feb. 2000.