Error Analysis in Two-Terminal Impedance Measurements With Residual Correction

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Abstract—Residual impedance correction in impedance analyzers when using an asymmetrical test fixture needs three reference measurements, usually open circuit, short circuit, and load (meaning an impedance close to the impedance under test). This paper provides an error estimate for impedance measurements that apply a simple open/short correction in spite of using an asymmetrical test fixture. Experimental results show that the minimal error is obtained for impedance values close to the geometric mean of the short-circuit and open-circuit impedances, and that the theoretical prediction is indeed an upper limit for the actual error.

Index Terms—Error analysis, impedance measurements, residual correction.

I. INTRODUCTION

ACCURATE impedance measurements require us to consider the effects of residual impedance in test fixtures. Test fixtures can be modeled as two-port networks described by their transmission parameters (A, B, C, D) [1]. Usually, test fixtures are symmetrical (balanced), so that $A = D$ and they can be described by two ratios between transmission parameters. These ratios can be determined from two reference measurements, for example, open-circuit and short-circuit conditions (open/short correction), and the results allow us to correct for the unknown residual impedance in electric contacts and cables connecting the impedance under test to the impedance analyzer. When the test fixture is not symmetrical, residual correction needs three reference measurements because we have to determine three ratios between transmission parameters [2]. Usually, a reference impedance close to the impedance under test is measured, hence the name open/short/load correction.

However, it is sometimes difficult to obtain reference impedances close to the impedance under test, for example, when measuring electrolytes. Therefore, the question arises about the error associated to impedance measurements that use an asymmetrical test fixture but implement a simple open/short correction procedure. This paper provides an upper limit for the error in such impedance measurements and substantiates the predictions by experimental results obtained in two-wire impedance measurements.

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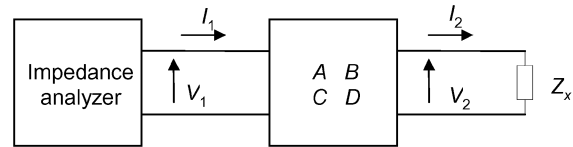
Fig. 1. Test fixture connecting the impedance under test to the impedance analyzer is modeled by a two-port network described by its four transfer parameters A, B, C, D.

II. ERROR MODEL IN TWO-TERMINAL IMPEDANCE MEASUREMENTS

Fig. 1 shows the impedance under test connected to an impedance analyzer through a test fixture modeled by its transfer parameters $A, B, C,$ and D. The voltage and current at the input of the impedance meter can be obtained from

$$\begin{pmatrix} V_1 \\ I_1 \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} V_2 \\ I_2 \end{pmatrix}. \tag{1}$$

The measured impedance, disregarding the uncertainty of the impedance meter, will be

$$Z_{xm} = \frac{V_1}{I_1} = \frac{AV_2 + BI_2}{CV_2 + DI_2} \tag{2}$$

whereas the actual impedance under test is

$$Z_x = \frac{V_2}{I_2}. \tag{3}$$

In order to estimate Z_x from Z_{xm}, we first rewrite (2) by replacing $V_2/I_2 = Z_x$ to obtain

$$Z_{xm} = \frac{AZ_x + B}{CZ_x + D}. \tag{4}$$

Solving for Z_x yields

$$Z_x = \frac{DZ_{xm} - B}{A - CZ_{xm}}. \tag{5}$$

Measuring at open-circuit condition $(I_2 = 0)$, (2) yields $Z_{om} = A/C$. Measuring at short-circuit condition $(V_2 = 0)$, (2) yields $Z_{sm} = B/D$. Therefore, (5) can be rewritten as

$$Z_x = \frac{D}{A} \frac{Z_{xm} - Z_{sm}}{1 - \frac{Z_{xm}}{Z_{om}}}. \tag{6}$$

If we measure a known impedance Z_1 and obtain a result Z_{lm}, we will have

$$Z_1 = \frac{D}{A} \frac{Z_{lm} - Z_{sm}}{1 - \frac{Z_{lm}}{Z_{om}}}. \tag{7}$$

Solving for D/A and replacing it in (6) yields

$$Z_{x\text{u}} = Z_\text{l} \frac{1 - \frac{Z_{lm}}{Z_{om}}}{Z_{lm} - Z_{sm}} \frac{Z_{xm} - Z_{sm}}{1 - \frac{Z_{xm}}{Z_{om}}} \qquad (8)$$

where the subscript "u" has been added to indicate that Z_x has been measured with an unbalanced (asymmetrical) fixture and corrected by three reference readings: Z_{om}, Z_{sm}, and Z_{lm}. If the network connecting Z_x to the impedance analyzer were balanced (symmetrical), we would have $A = D$, and (6) would reduce to

$$Z_{x\text{b}} = \frac{Z_{xm} - Z_{sm}}{1 - \frac{Z_{xm}}{Z_{om}}} \qquad (9)$$

where the "b" in the subscript stands for balanced. Equation (9) differs from (8) by a gain factor. Therefore, using only two reference measurements when the network connecting the impedance under test to the impedance meter is asymmetrical implies to consider a unity gain factor in (8), which is equivalent to use (9) to calculate the corrected impedance value from the reading Z_{xm} and the two reference measurements Z_{om} and Z_{sm}, instead of using (8). Hence, the modulus of the relative error is

$$\left| \frac{e(Z)}{Z} \right| = \left| \frac{Z_{x\text{b}} - Z_{x\text{u}}}{Z_{x\text{u}}} \right| = \left| \frac{1}{Z_\text{l}} \frac{Z_{lm} - Z_{sm}}{1 - \frac{Z_{lm}}{Z_{om}}} - 1 \right|. \qquad (10)$$

Because it is recommended to select Z_l close to Z_x, and we can assume Z_{xm} to be close to Z_x, we can approximate $Z_\text{l} \approx Z_x \approx Z_{xm}$ and $Z_{lm} \approx Z_{xm}$. Under these assumptions, (10) leads to

$$\left| \frac{e(Z)}{Z} \right| \cong \left| \frac{Z_{xm}^2 - Z_{om} Z_{sm}}{Z_{xm}(Z_{om} - Z_{xm})} \right|. \qquad (11)$$

In summary, disregarding measurement uncertainties in the impedance analyzer, if we use open/short impedance measurements to correct residual impedances for an unbalanced test fixture, we can estimate the true impedance value corresponding to an instrument reading Z_{xm} by applying (9), but the calculated result has a maximal relative error given by (11). Furthermore, because impedance values are complex numbers, the error due to an asymmetrical test fixture will affect both the amplitude and phase of the result.

From (11), when the result is close to the geometrical mean of the open circuit and short circuit measurements

$$Z_{xm}|_\text{opt} = \sqrt{Z_{om} Z_{sm}} \qquad (12)$$

the error because of the unbalanced connecting network is zero. Nevertheless, this calculated impedance value is not necessarily close to any impedance of interest.

III. EXPERIMENTAL RESULTS AND DISCUSSION

We have applied the analysis above to predict the contribution from incompletely corrected residual impedances to the error of impedance measurements performed with the HP4294A impedance analyzer when using an unbalanced (asymmetrical) test fixture. We assessed the influence of the asymmetrical test fixture on the result by measuring several reference impedances,
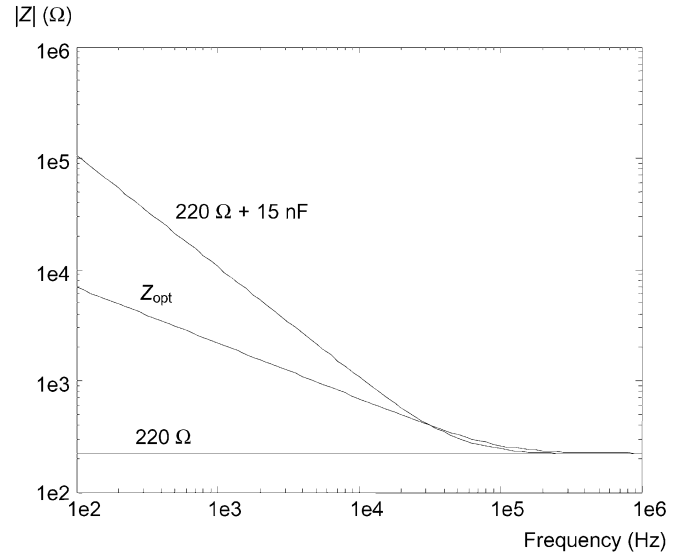

Fig. 2. Frequency dependence of the modulus of optimal impedance $Z_\text{opt} = \sqrt{(Z_{om} Z_{sm})}$ and of two actual impedances 220 Ω and 220 Ω in series with 15 nF not far from Z_opt.

first with a symmetrical test fixture provided by the manufacturer (HP16047A) and then by connecting each impedance under test to the instrument with RG214 cables 1-m long. Both measurement sets used the corresponding open/short correction. The impedance values calculated from (9) when using a symmetrical test fixture and its open/short correction were considered the "true" value $Z_{x\text{u}}$. That is, instead of applying (8) to the results obtained when using the asymmetrical test fixture and open/short/load correction, we obtained $Z_{x\text{u}}$ by using the symmetrical test fixture and open/short correction, according to (9). This alternative procedure was possible because the test impedances selected could be connected to the symmetrical test fixture.

The impedance analyzer averaged eight readings for each impedance value before transferring the results to a computer for calculation and display. Measurements were performed in the "precise mode," which implies a narrow bandwidth, and hence a long measurement time (about 5 min each ten readings). Because of this, averaging a higher number of readings could result in thermal drifts due to temperature changes in the laboratory. The impedance under test was inside a grounded Faraday cage measuring $10 \times 10 \times 12.5$ cm. Cable shields were grounded too. Because the impedance analyzer is a self-balanced bridge, the capacitance contributed by the grounded shield did not affect the measurement result [3].

Test impedance values were chosen according to the open and short-circuit measurements when using the asymmetrical test fixture. We obtained $Z_{om} = 1/(j\omega \times 2 \text{ pF})$ and $Z_{sm} = 60 \text{ m}\Omega + j\omega \times 100 \text{ nH}$. From these values, (12) yields an "optimal" impedance (Z_opt) that is impossible to implement in a broad frequency range. Z_{om} being capacitive, an inductive Z_{sm} would result in a resistive Z_opt. However, because of the resistive component of Z_{sm}, contributed by the length of the cable in the asymmetrical test fixture, Z_opt displays a -10 dB/dec slope (Fig. 2) at low frequency that cannot be obtained by common

TABLE I
VALUE OF THE TEST IMPEDANCES MEASURED WITH THE HP4294A IMPEDANCE ANALYZER USING A SYMMETRICAL (HP16047A) OR AN ASYMMETRICAL TEST
FIXTURE (RG214 CABLES 1-m LONG), WHEN USING AN OPEN/SHORT RESIDUAL COMPENSATION IN BOTH CASES

Test impedance	Measured impedance (Ω)					
	Symmetrical test fixture			Asymmetrical test fixture		
	100 Hz	10 kHz	1 MHz	100 Hz	10 kHz	1 MHz
220 Ω	216 - j0.072	216 - j0.0001	216 - j0.095	216 - j0.066	216 + j0.002	216 - j0.296
220 Ω +15 nF	540 - j121 × 10³	226 - j1.23 × 10³	216 - j12.8	547 - j121 × 10³	226 - j1.23 × 10³	216 - j13.0
10 Ω	10.00 - j0.005	10.00 - j0.002	10.00 + j0.049	10.04 - j0.009	10.05 - j0.005	10.10 + j0.596
51 Ω	49.46 - j0.017	49.46 - j0.008	49.47 + j0.024	49.50 - j0.022	49.50 - j0.0002	49.55 + j0.523
1 kΩ	974 - j0.358	974 - j0.100	974 - j1.45	974 - j0.337	974 - j0.169	973 - j13.5
5.1 kΩ	5.10 × 10³ - j2.19	5.10 × 10³ - j1.12	5.10 × 10³ - j39.1	5.10 × 10³ - j2.19	5.10 × 10³ - j4.74	5.07 × 10³ - j375

passive components. At higher frequencies, the inductive component of Z_{sm} predominates over its resistive component, and $Z_{opt} \approx (100 \text{ nH}/2 \text{ pF})^{1/2} = 100\sqrt{5} \ \Omega \approx 223 \ \Omega$. We have selected two impedances relatively close to that optimum: a 220-Ω off-the-shelf resistor (nominal value) and the series combination of that resistor and a 15-nF capacitor (nominal value). Fig. 2 shows the modulus of Z_{opt}, 220 Ω, and 220 Ω in series with 15 nF, from 100 Hz to 1 MHz. The remaining test impedances were also off-the-shelf resistors whose nominal values were 1 kΩ, 51 Ω, 5.1 kΩ, and 10 Ω. These values were selected far from the optimal impedance but such that the respective geometric mean of each pair (1 kΩ, 51 Ω), (5.1 kΩ, 10 Ω) were close to Z_{opt}.

We measured these six impedances (five resistors and one resistor in series with a capacitor) from 100 Hz to 1 MHz using both the symmetrical and the asymmetrical test fixture and the open/short correction in both cases. Table I shows the results at 100 Hz, 10 kHz, and 1 MHz. The use of the open/short correction prevents, for example, that small-value resistors display an inductive component that would be contributed by the short-circuit impedance. Nevertheless, Table I shows that actual resistors have a reactive component. In general, the results obtained with the asymmetrical test fixture different more from the nominal values than those obtained with the symmetrical test fixture, as expected.

Fig. 3 shows the maximal modulus of the relative error predicted by (11) for the six test impedances in the range from 100 Hz to 1 MHz. The theoretical maximal error is minimum for the two impedances whose value is close to the optimum calculated from (12), namely 220 Ω and 220 Ω + 15 nF, and increases with frequency for impedances far from that optimum. At high frequency, the relative error is similar for 51 Ω and 1 kΩ, and for 5.1 kΩ and 10 Ω, whose values are (geometrically) symmetrical about that given by (12). At low frequency, however, the relative error for each resistor from each pair is quite different: it is larger for 10 Ω than 51 kΩ, and larger for 51 Ω than for 1 kΩ. Furthermore, the relative error increases for impedance values
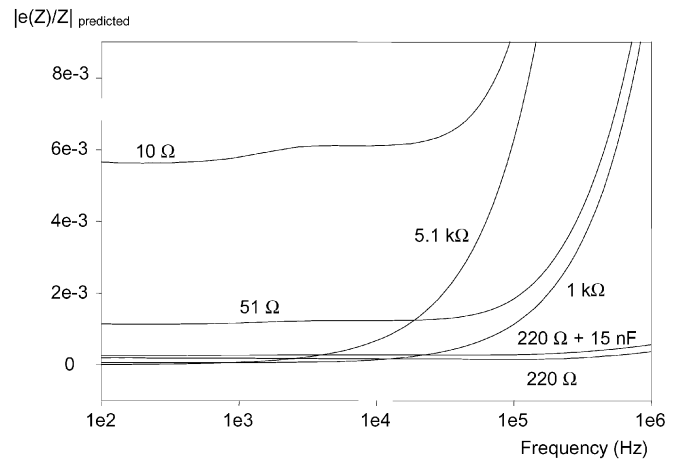

Fig. 3. Maximal modulus of the predicted relative error for the six test impedance values according to (11), from 100 Hz to 1 MHz.

that are very different from (12): it is larger for 5.1 kΩ and 10 Ω than for 1 kΩ and 51 Ω.

In order to understand why, at low frequency, resistors larger than Z_{opt} have a reduced relative error compared to resistors smaller than Z_{opt}, Fig. 4 plots (11) for different measured resistors (Z_{xm}), at five frequencies. These plots confirm the result in Fig. 3 and can be explained as follows: the denominator in (11) increases whenever Z_{xm} or $Z_{om} - Z_{xm}$ are small, which at high frequencies means a very low or a very high measured impedance, compared to Z_{opt}. At low frequency, however, $Z_{om} - Z_{xm}$ is large even for a high Z_{xm} because Z_{om} is reactive and Z_{xm} is resistive (that is, at low frequency, the reactive component of the resistor is small). Hence, for the instrument and test fixtures used, at low frequency, low resistors yield a large relative error but high resistors yield a small relative error.

Fig. 5 shows the modulus of the relative error of the impedance for the six test impedances measured in the range from 100 Hz to 1 MHz. The relative error is minimal for 220 Ω
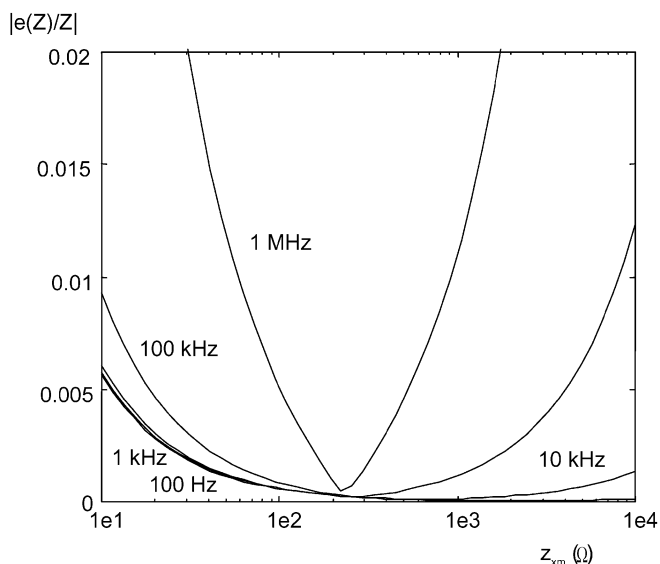
Fig. 4. Maximal modulus of the predicted relative error according to (11) for different measured impedances Z_{xm} at five different frequencies.
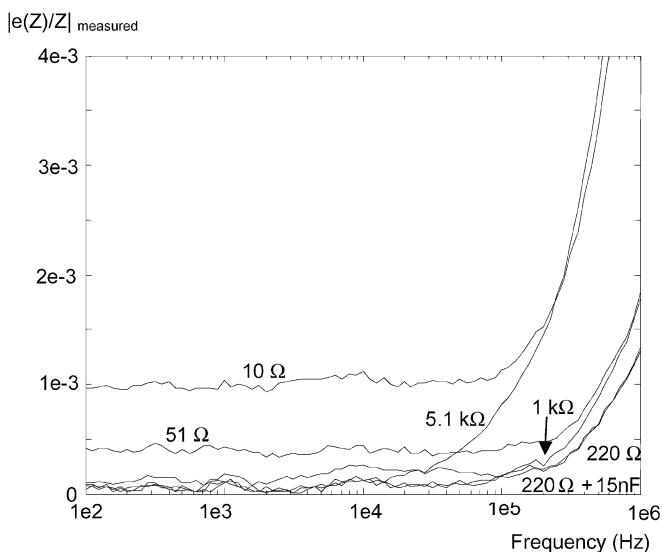

Fig. 5. Modulus of the relative error for the six test impedance values measured with the HP4294A.

and $220 \ \Omega + 15$ nF, as predicted, and is smaller than the predicted error in Fig. 3. The relative error increases from 100 kHz and up, particularly for the highest and lowest impedance values (5.1 kΩ and 10 Ω), whereas the predicted relative error increases above 10 kHz. At high frequency, the error is similar for 51 Ω and 1 kΩ, the same as in Fig. 3. Also, the relative error is smaller for 5.1 kΩ than for 10 Ω, particularly at low frequency, which is consistent with the theoretical prediction.

IV. CONCLUSION

Accurate measurements using impedance analyzers require us to compensate for impedance residuals in the test fixture. When the test fixture is symmetrical, this correction is usually performed by measuring short-circuit and open-circuit impedance and using the results to compute the actual impedance according to (9). When the text fixture is asymmetrical, we need a third reference measurement in order to compensate for impedance residuals according to (8). Using simple open/short correction instead adds gain error to that of the impedance analyzer. Equation (11) estimates the maximal relative error added, and (12) determines the impedance value having the minimal relative error. Experimental results confirm that 1) actual relative errors when performing simple open/short correction are smaller than the theoretical limit calculated from (11), 2) relative errors for impedance values close to the optimum calculated from (12) are very small, and 3) relative errors for the HP4294A impedance analyzer and a test fixture consisting of RG214 cables 1-m long, are larger for low impedance values than for high impedance values.

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