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Planning of aircraft departure trajectories by using fuzzy logic and lexicographic optimization

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ABSTRACT

In this paper a strategy for planning aircraft departure trajectories for a given airport is presented. Noise annoyance produced by overflying aircraft is modeled by using fuzzy logic in function of the received noise level during the trajectory, the type and specific sensibility of the areas being overflyed and the time of the day when the aircraft departure takes place. Hence, an annoyance figure is obtained at different locations in the vicinity of the airport in function of a given trajectory. A non-linear multi-objective optimal control problem is presented in order to minimize the annoyance at all different locations and an optimal departure trajectory is obtained. The multi-objective optimization problem is solved by using a lexicographic technique where a hierarchical order among the optimization objectives is established in function of the perceived annoyance at each different location. Finally, a practical example is given where an optimal departure trajectory is obtained while minimizing the noise nuisance at some residential zones, hospitals, schools and industrial zones

1 INTRODUCTION

The noise produced by aircrafts during take-off and landing operations around airports is a very serious ecological and social problem. Aircraft noise can be very annoying for people living in the vicinity of the airports. Noise is generally defined as an unwanted sound and its effects can be appreciated physiologically but also psychologically (annoyance and disturbed well being) [1].

Annoyance is a concept that is hard to quantify because there is no underlying physically measurable scale. However, it is usually qualitatively assessed with social surveys where it has been shown that the correlation coefficient between noise exposure and average response is relatively high, implying that noise scales are useful predictors of average reactions [2].

It is clear that fuzzy techniques can help to make more accurate predictions by incorporating the vagueness and uncertainty into the modeling and reasoning process. Recently, few research papers based on fuzzy logic in noise pollution area have been reported [3-5]. In [4], annoyance is considered as a function of noise level, its duration of occurrence, and the socioeconomic status of a person and the results were applicable to the urban areas of India. In [5], a fuzzy model has been developed, on the basis of field surveys conducted by

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various researchers and reports of World Health Organization, for predicting the effects of sleep disturbance by noise on humans as a function of noise level, age and duration of its occurrence. Fuzzy set theory is a generalization of traditional set theory and provides a means for the representation of imprecision and vagueness. Zadeh [6] further developed the corresponding fuzzy logic to manipulate the fuzzy sets.

The International Civil Aviation Organization (ICAO) publishes two different Noise Abatement Departure Procedures (NADP) defined in [7]. NADP are generic procedures, which do not always fit into the specific problems that a particular airport may suffer and are far from being the optimal ones minimizing noise nuisances. This is due to several factors, such as the impossibility to define a general criterion fitting all airports necessities, the limitations of nowadays technology on-board and the constraints imposed by airport capacity or air traffic control issues. Nevertheless, research is being done on optimization for depart/approach procedures. Criteria for these optimization studies can include passenger comfort, fuel consumption; time spent during the procedure and aircraft noise considerations. For instance, in [8] a tool combining a noise model with a Geographical Information System (GIS) and a dynamic trajectory optimization algorithm is presented aimed at obtaining optimal noise depart and approach procedures. A similar methodology is proposed in [9], and an adaptive algorithm for noise abatement can be found in [10]. Another study, see [11], empathizes that most current noise abatement procedures are local adaptations of generic procedures trying to minimize the noise footprint and do not generally take into account the actual population density and distribution. In the same work, a noise performance trade-off between arrival trajectories that are optimized according to different types of noise abatement criteria is presented. Typically, these different criteria are not compatible and the variables that optimize one objective may be far from optimal for the others, pointing out the difficulty to properly identify the absolute minimal trajectory among all the local minimal ones. In order to deal with this kind of multi-objective problems in a better way, this work presents an optimization strategy which uses goal hierarchical or lexicographic techniques.

This paper is organized as follows: in Section 2, a fuzzy model of the acoustic annoyance is given. In Section 3, we present the lexicographic method to deal with our multi-objective optimization problem. Section 4 shows the results obtained for a hypothetical airport scenario containing two residential zones, a school, a hospital and an industrial zone.

2 FUZZY ANNOYANCE MODEL

The annoyance or perception of the acoustic noise describes the relation between a given acoustic situation and a given individual or set of persons affected by the noise and how cognitively or emotionally they evaluate this situation. The acoustic annoyance of the aircraft flights around an urban airport depends logically of the acoustic behavior produced in the sensible locations (using, as an example, the *Lmax* or SEL metrics) but it is not a sufficient measurement to define completely the annoyance behavior of a noise. For example, a list of non acoustic elements to take into account to define the annoyance behavior could be:

- Types of affected zones (rural zone, residential zone, industrial zone, hospitals, schools, markets,…)
- Time interval during the noise event (day, evening, night)
- Period of time between two consecutive flights
- Personal elements (emotional, apprehension to the noise, personal healthy, age,…)
- Cultural aspects (young or aged people habits, activities, holiday,…)

In conclusion, the annoyance is a subjective and a complex concept which can be studied as a qualitative form using fuzzy logic sets, as previous similar works in this area have been done [3-5], [12]. In this paper, the annoyance generated by the aircraft trajectories will be represented by fuzzy logic sets from the fuzzification of the maximum sound level (Lmax) and from the hour of day where the trajectory is supposed to be flown regarding 4 typical zones around an urban airport: a residential zone, a hospital, a school and an industrial zone.

Two sets of membership functions have been defined. A first set is related with the maximum sound level (*Lmax*) and 5 linguistic terms have been selected to define the noise: very high (VH), high (HH), medium (ME), low (LL) and very low (VL) following a similar structure of fuzzy sets proposed by [3] from the *Lmax* metrics in dB(A). A second set is related with the hour of day, establishing three linguistic terms: morning, afternoon, and night to indicate the moment of the day for a given aircraft flight. Both membership functions are shown in Figure 1.

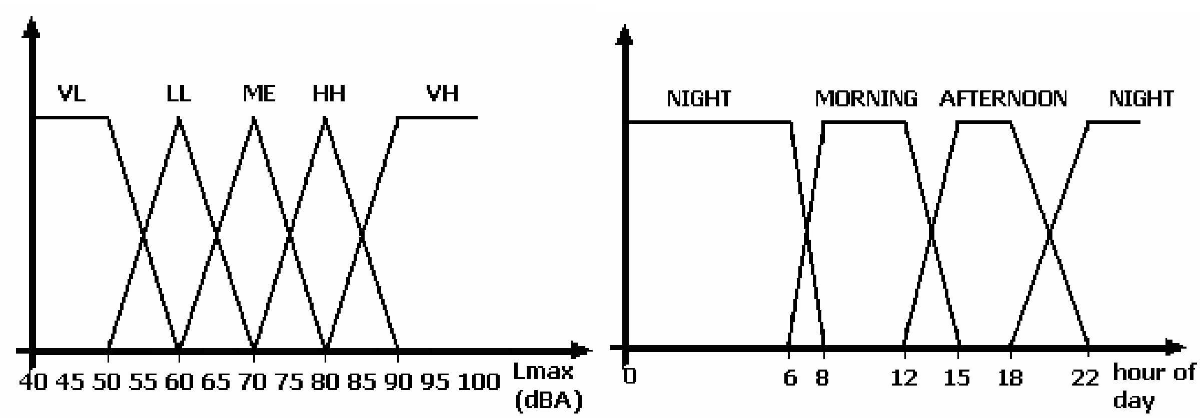

Figure 1: Membership functions of the maximum sound level and time of day fuzzy sets.

Afterwards, a rule base has been designed to represent the annoyance of an event defined by the two fuzzy logic sets for each of the 4 zones considered in this work. The annoyance concept has been represented by the following linguistic terms: extreme annoyance (EA), high annoyance (HA), moderated annoyance (MA), small annoyance (SA) and null annoyance (NA). To build the rule base, one initial selection has been considered as a "baseline" situation, consisting in considering the terms of the annoyance set (EA, HA, MA, SA and NA) equivalent to the terms of the perceived maximum sound level (VH, HH, ME, LL and VL).

The residential zone, during the evening has been chosen as the "baseline" situation for this work. On the other hand, the residential zone during the night is more sensible to noise and, consequently, the annoyance terms have been shifted one higher position in the linguistic terms of the *Lmax*. In the same way in the residential zone, during the morning, it is worth to suppose that less people are living in the zone. In addition, environmental noise is higher during the morning than in the evening or night, thence the annoyance terms have been shifted to one lower position.

Similarly, in the school the baseline situation has been considered during the lecture period of the evening, meanwhile in the morning period a higher position has been considered because usually there is more people than afternoon and because the activities of the morning need to pay more attention than these of the evening. Finally, the annoyance at the school during the night is practically non existent because there is no activity in this zone.

In the hospital a standard situation has not been considered, because hospitalized people are more sensible to the noise. Therefore, morning and afternoon annoyance have been shifted one position higher than the Standard situation, while the night period has been shifted two positions.

Finally, being industrial zones more noisy than other zones the baseline situation has been shifted three lower positions during the morning and two lower positions during the afternoon and night because the ambient noise is supposed to be lower. Based on this simple reasoning the four rule bases have been designed for this study. See Table 1 and Table 2:

Table 1: Rule base table for the annoyance in the residential zone and the school

	RESIDENTIAL ZONE			**SCHOOL**		
	Morning	***Afternoon***	***Night***	***Morning***	***Afternoon***	***Night***
Very Low noise	NA	NA	SA	SA	NA	NA
Low noise	NA	SA	MA	MA	SA	NA
Medium noise	SA	MA	HA	HA	MA	NA
High noise	MA	HA	EA	EA	HA	NA
Very High noise	HA	EA	EA	EA	EA	SA

Table 2: Rule base table for the annoyance in the hospital and industrial zone

	HOSPITAL			**INDUSTRIAL ZONE**		
	Morning	***Afternoon***	***Night***	***Morning***	***Afternoon***	***Night***
Very Low noise	SA	SA	MA	NA	NA	NA
Low noise	MA	MA	HA	NA	NA	NA
Medium noise	HA	HA	EA	NA	SA	SA
High noise	EA	EA	EA	SA	MA	MA
Very High noise	EA	EA	EA	MA	HA	HA

For the sake of simplicity, the fuzzy set of the annoyance has been defined as a crisp set to derive the normalized degree of annoyance. Extreme annoyance (EA) corresponds to a normalized annoyance value of 1, high annoyance (HA) takes 0.75 value, medium annoyance (MA) takes 0.5, small annoyance (SA) takes 0.25 and finally null annoyance (NA) corresponds to the 0 value of this normalized scale. In this work a "max-min" inference method has been applied and the common centre of gravity technique has been considered as the method for the defuzzification process.

To sum up, Figure 2 represents the relation between the normalized annoyance degree in function of the maximum sound level (*Lmax*) and the time of day for each of the four different zones that are considered in this work. This function has been obtained after the process of fuzzification and defuzzification of all possible value combinations between *Lmax* and the time of day.

3 MULTIOBJECTIVE OPTIMISATION

3.1 Multi-objective optimization

The solution of the optimization problem associated with the computation of the trajectory that minimizes noise nuisances is a multi-objective optimization problem since additionally to fuel and time minimization used in the trajectory, nuisance minimization in several zones at the same time should be considered.

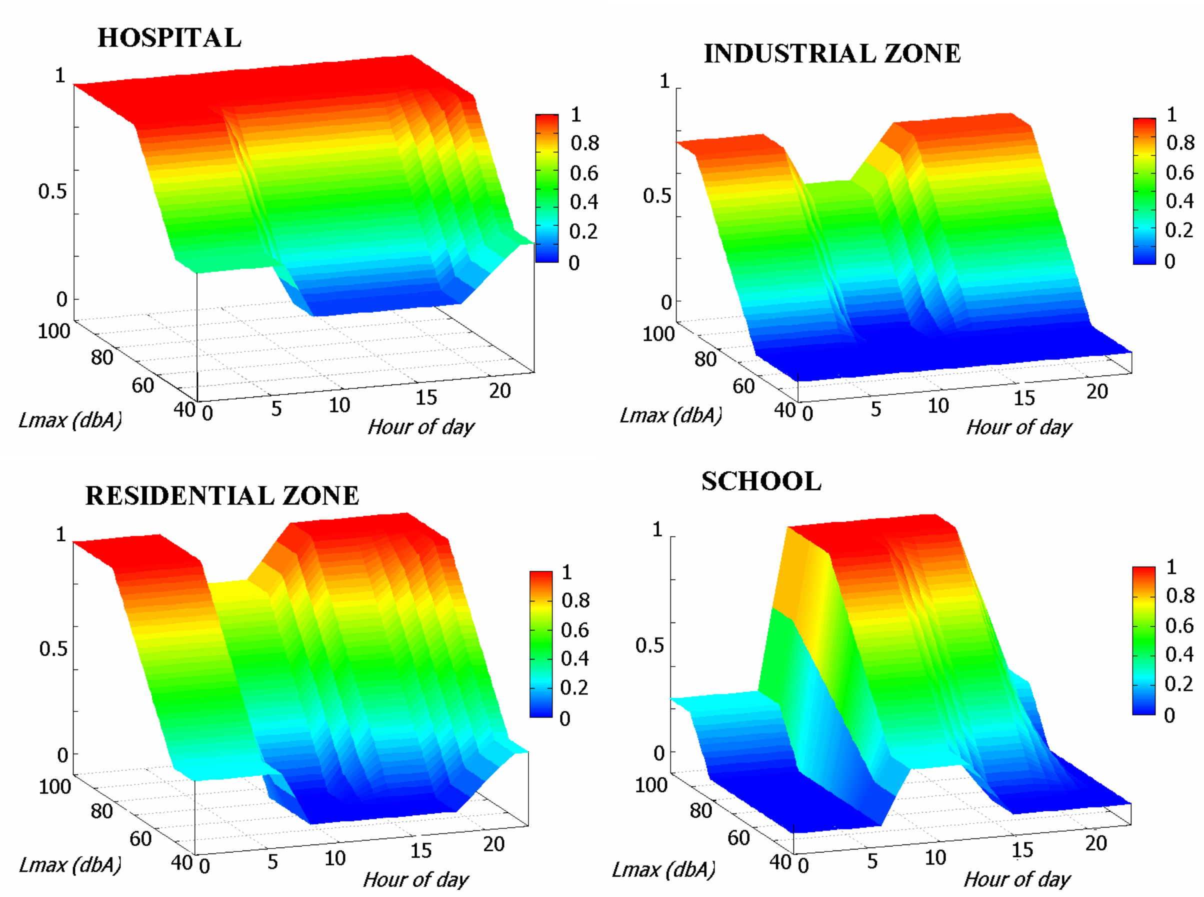

Figure 2: Normalized annoyance degree in function of *Lmax* and the hour of the day after the fuzzyfication and defuzzyfication process for the four different noise sensitive locations.

Let $\vec{x}(t)=[x(t),y(t),z(t),t]$ be an aircraft trajectory described by the three spatial components in a particular time slot during the day. The considered optimization problem takes into account 5 criteria evaluated through the corresponding objective functions that will be denoted in the following as:

$f_r(\vec{x})$	noise nuisance in residential zone
$f_h(\vec{x})$	noise nuisance in hospital zone
$f_i(\vec{x})$	noise nuisance in the industrial zone
$f_s(\vec{x})$	noise nuisance in school zone
$f_c(\vec{x})$	fuel consumed during the trajectory

The aim is to find an optimal aircraft trajectory $\vec{x}^*(t)$ within the set of all possible trajectories χ that minimizes each of the previously defined criteria by solving the following multi-optimization problem:

$$\begin{aligned} &\min_{\vec{x}} f_r, f_h, f_i, f_s, f_c \\ &subject \quad to: \vec{x} \in \chi \end{aligned} \tag{1}$$

where the domain χ is defined through a set of restrictions: $g_j(x) \leq 0, j = 1,...,m$ that comes from aircraft dynamics and operating limits in actuators and state variables.

A solution $x^* \in X$ of (1) is said to be Pareto optimal if and only if there does not exist a $x \in X$ and an i such that $f(x) \leq f(x^*)$ and $f_i(x) \leq f_i(x^*)$. In other words, a solution is (global) Pareto optimal if and only if an objective f_i can be reduced at the expense of increasing at least one of the other objectives.

There are several methods to solve multi-optimization problems [13]. In general, in these methods, multi-objective optimization problems are solved by scaling. This means converting the problem into a single or a family of single objective optimization problems with a real-valued objective function. This objective function is called the scaling function and it may be a function of some parameters. This enables the use of the theory and the methods of scalar optimization. One of the most well known multi-objective techniques is the linearly weighted sum, where the vector objective function is scaled in such a way that the value judgment of the decision making can be incorporated. The most obvious problem with weighted formulae is that, in general, the setting of the weights is ad-hoc, based either on a somewhat vague intuition of the user about the relative importance of different quality criteria or in trial and error experimentation with different weight values. Another problem with weights is that, once a formula with precise values of weights has been defined, the optimization algorithm will be effectively trying to find the best model for that particular setting of weights, missing the opportunity to find other solutions that might be actually more interesting to the user, representing a better trade-off between different quality criteria. In particular, weighted formulas involving a linear combination of different criteria have the limitation that they cannot find solutions in a non-convex region of the Pareto front. This problem is particularly serious when the weighted formula involves a summation/subtraction (rather than a multiplication/division) of terms representing different scales in their units of measurement. This problem can be dealt with by normalizing the different quality criteria so that they refer to the same scale. This approach is well-known in the literature and at first glance it is a very satisfactory approach. There is, however, a subtle problem associated with normalization that is rarely discussed in the literature. In essence, the problem is that in general there are several different ways of normalizing, and the decision about which normalization procedure should be applied tends to be ad-hoc. Finally, another subtle problem associated with the weighted approach, which is often ignored in the literature and it is related to addition/substraction of non-commensurable criteria to/from each other in the criteria does not make any sense at all, regardless of normalization. These drawbacks of the weighted approach arise when dealing with noise nuisance criteria. For example, in [11] these weighting methods are used and the obtained solution is highly dependent on the chosen weights.

3.2 Lexicographic method

If a hierarchy between objectives can be defined "a priori" according to their absolute importance, the lexicographic method can be used [14]. Let the objective functions be arranged according to the lexicographic order from the most important f_1 to the least important f_k. We can write the lexicographic problem as:

$$\begin{aligned} & lex \min_{\vec{x}} f_r, f_h, f_i, f_s, f_c \\ & subject \quad to: \vec{x} \in \chi \end{aligned} \tag{2}$$

A standard method for finding a lexicographic solution is to solve a sequential order of single objective constrained optimization problems. After ordering, the most important objective function is minimized subject to the original constraints. If this problem has a unique solution, it is the solution of the whole multi-objective optimization problem. Otherwise, the second most important objective function is minimized. Now, in addition to the original constraints, a new constraint is added. This new constraint is there to guarantee that the most important objective function preserves its optimal value. If this problem has a unique solution, it is the solution of the original problem. Otherwise, the process goes on as above.

More formally, $f^* := [f_1^*, \cdots, f_r^*]$ is the lexicographic minimum of (2) iff

$$f_1^* = \min_{x \in \chi} f_1(x) \qquad (3)$$

for all $i \in \{2, \cdots, r\}$

$$f_i^* = \min \{ f_i(x) \,|\, f_j(x) \leq f_j^*, j = 1, \cdots, i-1 \} \qquad (4)$$

On the other hand, a given $x^* \in \chi$ is a lexicographic solution of (4) iff

$$x^* \in \{ x \in \chi \,|\, f_j(x) \leq f_j^*, j = 1, \cdots, r \} \qquad (5)$$

4 NUMERICAL EXAMPLES

This section presents a practical example concerning a hypothetical scenario, where the optimum take off trajectories for a given airport should be computed in function of the time of the day. The environment of the airport contains five different noise sensitive areas: an Hospital (H), two residential zones (R1 and R2), a school (S) and an industrial zone (I) as shown in Figure 3 and Figure 4. The optimal control problem described in Section 3 is firstly transformed into a non linear programming (NLP) problem by parametrizing the control and state variables. This technique, see for instance [15] and [16], allows for an easy use of commercial optimization software which copes with these kinds of problems. In the following simulations the General Algebraic Modeling System (GAMS)[e] has been used to code the problem for the NLP solver CONOPT[f]. In this example, the aircraft is supposed to take off eastwards and, for the sake of simplicity, the origin of coordinates (0,0) is taken at the point where the initial straight trajectory of the aircraft reaches 400ft in height above the runway. From this point on, the trajectory optimization takes place under aircraft's dynamic constraints, which are described in detail in [17], and some airspace restrictions which define the usable area of airspace, as it can be seen by dashed lines inFigure 3 and Figure 4. Finally the final coordinates of the trajectory are fixed at point (20 km,10 km) where the altitude of the aircraft is enforced to be higher than 6000 ft.

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[e]GAMS: The General Algebraic Modeling System is a high-level modeling system for mathematical programming and optimization (www.gams.com).

[f]CONOPT is a solver for large-scale nonlinear optimization (NLP) based on a improved version of the GRG method and it is fully integrated within the GAMS system (www.conopt.com).

For this example, the optimization priorities have been established as:

- Priority 1: minimize annoyance in the Hospital
- Priority 2: minimize annoyance in the School
- Priority 3: minimize annoyance in the Residential Zone 1
- Priority 4: minimize annoyance in the Residential Zone 2
- Priority 5: minimize annoyance in the Industrial Zone

Figure 3 shows all intermediate steps in the lexicographic optimization process when the trajectory is supposed to be flown at 17h local time in the airport. As it can be seen, the trajectory resulting from first optimization step avoids the Hospital location and remains close to the left airspace boundary line, regardless of the annoyance produced in the other sensible locations. Second step tries to minimize the annoyance at the school, while maintaining the minimum annoyance in the hospital obtained in the first step. Therefore, the initial take-off path is modified and the annoyance in the school is significantly improved. Similar behavior is observed in third and fourth steps, where the annoyance in both residential zones is minimized. On the other hand, last step (minimization of the annoyance in the industrial zone) produces very small changes in the trajectory because the optimization has become too constrained due to all previous higher priorities and there are almost no degrees of freedom left for the last optimization. In Table 3 all annoyance values obtained after each lexicographic optimization step are shown.

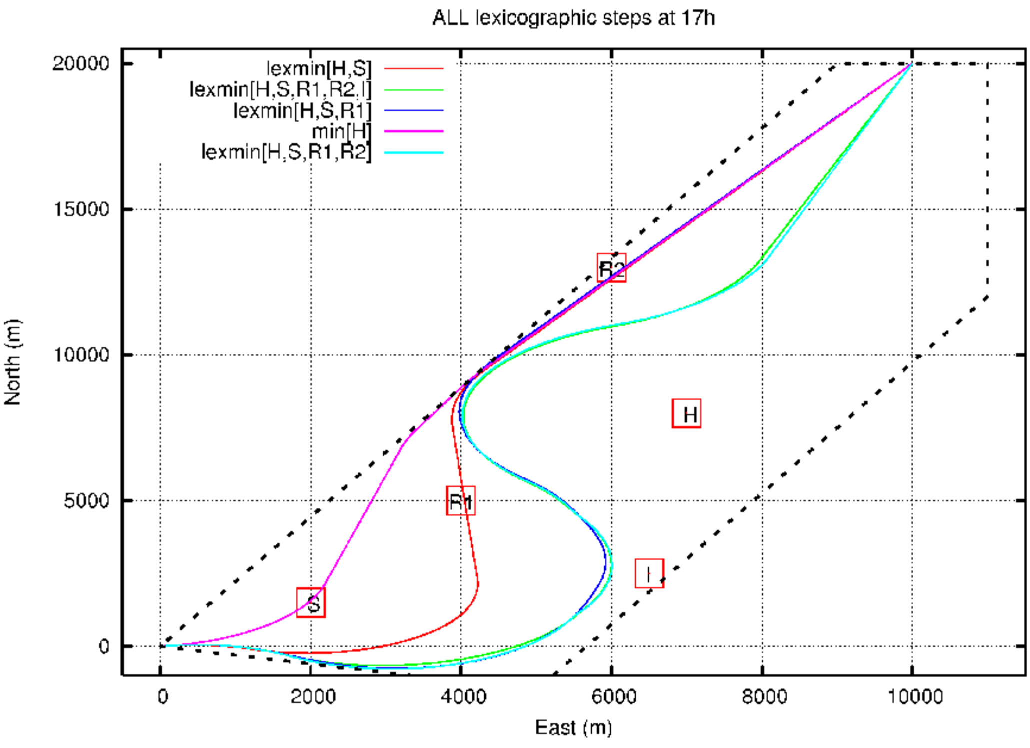

Figure 3: Trajectories obtained after each lexicographic optimization step.

Figure 4 shows the final optimal trajectories (after all the lexicographic process described above) for three different day hours when the trajectories are supposed to be flown: 4h, 7h and 17h. As it can be seen night trajectory (4h) is quite different from day trajectories (7h and 17h). This is due to the fact that during night period the annoyance produced in the school is almost zero introducing a very small constraint when minimizing the nuisance at the residence zone 1 (the following step after annoyance minimization at the school). Therefore, the aircraft can perform a left turn from the very beginning of the trajectory improving in this way the noise impact at the residential zone 1.

Table 3: Annoyance values after each lexicographic optimization step

	Annoyance values				
	Hospital	***School***	***Resident. 1***	***Resident. 2***	***Industrial***
$\min f_h$	**0.325**	1.000	0.510	0.460	0.000
$lex \min f_h, f_s$	0.325	**0.426**	0.723	0.460	0.135
$lex \min f_h, f_s, f_{r1}$	0.325	0.426	**0.351**	0.460	0.466
$lex \min f_h, f_s, f_{r1}, f_{r2}$	0.325	0.426	0.351	**0.176**	0.458
$lex \min f_h, f_s, f_{r1}, f_{r2}, f_i$	0.325	0.426	0.351	0.176	**0.457**

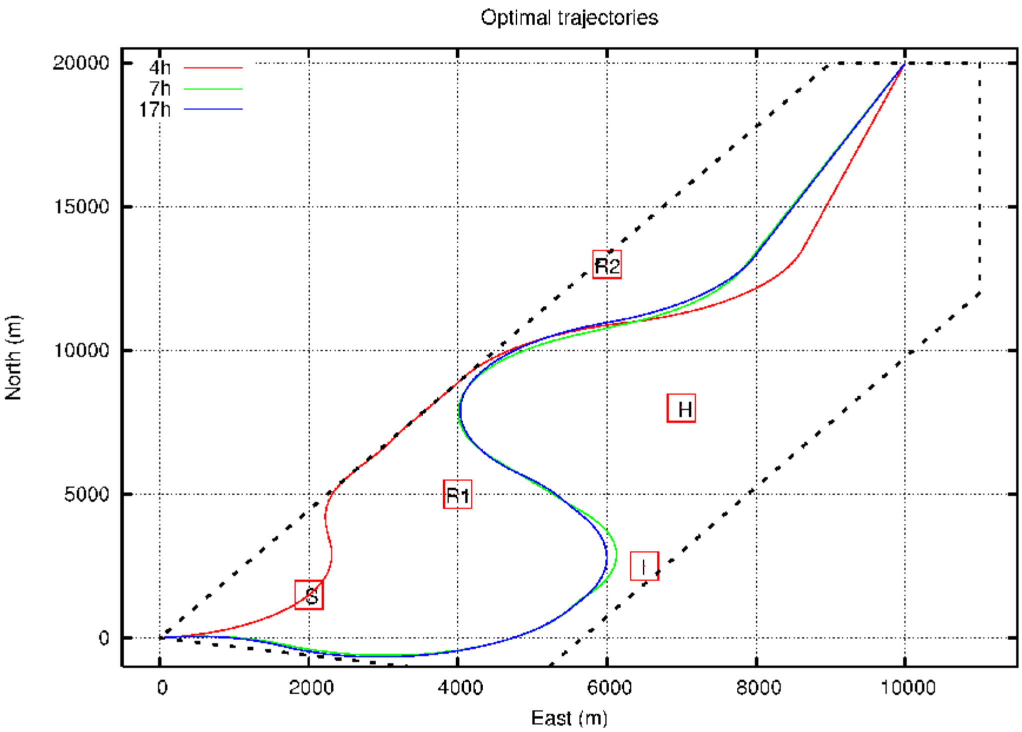

Figure 4: Optimal trajectories for different hours of day.

5 CONCLUSIONS

A strategy for planning aircraft departure trajectories is presented in this paper. Noise annoyance produced by overflying aircraft is modeled by using fuzzy logic in function of the received noise level during the trajectory, the sensibility of the areas being overflyed and the time of the day when the aircraft departure takes place. In this work, *Lmax* metric is used regarding the perceived noise level and Industrial, Residential, Hospital and School locations are considered. Nevertheless, this fuzzy model allows easily to be extended by using different metrics or different sensitive zones as far as the modeler can describe linguistically the degree of annoyance in function of the chosen input parameters. After a fuzzification and defuzzyfication process an annoyance non linear function is obtained at different locations, becoming the objective function for the minimization algorithm. A non-linear multi-objective optimal control problem is presented and the optimal departure trajectory in function of the hour of day is obtained. The multi-objective optimization problem is solved by using a lexicographic technique where a hierarchical order among the optimization objectives is established at the beginning of the optimization. Numerical examples show how the lexicographic technique permits to solve this multi-criteria optimization problem obtaining a final trajectory that minimizes all nuisances by respecting an *a-priori* fixed priority scale among all noise sensitive locations. On the other hand fuzzy modeling has an important effect in the objective functions showing that, for example, different optimal trajectories are obtained for different day hours while maintaining the same priority scale among the objectives.

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