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Independent Contact Regions for Discretized 3D Objects with Frictionless Contacts

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Abstract

This report deals with the problem of determining independent contact regions on a 3D object boundary such that a seven finger frictionless grasp with a contact point in each region assures a force-closure grasp on the object, independently of the exact position of the contact points. These regions provide robustness in front of finger positioning errors in grasp and fixturing applications. The object's surface is discretized in a cloud of points, so the procedure is applicable to objects of any arbitrary shape. The procedure finds an initial form-closure grasp that is iteratively improved through an oriented search procedure; once a locally optimum grasp has been reached, the independent contact regions are computed. The procedure has been implemented, and application examples are included in the paper.

Keywords: Grasp planning, force-closure grasps, independent contact regions.

1 Introduction

The determination of contact locations to immobilize the object despite external disturbances has been a topic of great interest in grasping, manipulation and fixturing. The contact locations are characterized by the properties of form or force-closure [1]. With form-closure, the position of the contacts ensures the object immobility; this property is mostly used when the task requires a robust grasp not relying on friction, e.g. the fixture of objects to be manufactured or inspected. On the other hand, force-closure is achieved when the forces applied in the contact points ensure the object immobility; it is specially used in grasping and manipulation of objects with a low number of frictional contacts using mechanical grippers or hands.

Several algorithms have been developed to determine grasps formed by a set of contact points on the object´s surface, with different number of fingers and satisfying the form or force-closure condition in 2D polygonal [2], non-polygonal [3] or discrete objects [4], 3D polyhedral objects [5] [6], objects with smooth curved surfaces [7] [8] or 3D discretized objects [9] [10]. These algorithms determine precision grasps, i.e. grasps formed by a set of finger contact points on the object. These grasps require a good precision in the finger placements; however, in a real execution the actual and the theoretical grasp may differ due to fingers positioning errors. Nguyen [11] introduced the concept of independent contact regions (ICR) in order to provide robustness to the grasp in front of positioning errors. ICR are regions on the object boundary such that the fingers can be positioned on them independently one from each other, but assuring a force-closure (FC) grasp with independence of the exact position of each finger.

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The determination of ICR was initially addressed by Nguyen [11] for two frictional contacts on polygonal and polyhedral objects, and with four frictionless contacts on 2D polygonal objects. The concept was extended to two fingers on curved 2D objects [12], three and n-finger grasps of polygonal objects [13] [14], and to four-finger grasps on polyhedral objects [5]. The concept of ICR has also been used to determine contact regions on 3D objects based on initial examples, although the results depend on the choice of the example [15]. Recently, the computation of ICRs for 2D discrete objects has also been addressed [16]; however, the determination of ICRs on 3D discrete objects has not been directly tackled until now.

This report deals with the problem of determining independent contact regions on a 3D object boundary for a seven finger frictionless grasp; the ICRs assure a FC grasp with a controlled minimum quality. The proposed approach has three phases. The first phase finds an initial force-closure grasp with an algorithm similar to the one proposed in [9], but using a different FC test that decreases the search complexity. The second phase improves the initial grasp through an oriented search procedure. The optimization is carried out using a quality measure equivalent to the largest perturbation wrench that the grasp can resist, with independence of the perturbation direction [17] [18]; it is one of the most popular grasp quality measures, and will be referred hereafter as the largest ball criterion. The optimization is carried out to obtain a locally optimum FC grasp. Finally, the third phase computes the ICRs from the locally optimum grasp obtained in the previous phase.

A work in this line [15] presents a procedure to compute a family of grasps for 3D objects that keep a fraction of the quality of the grasp in an initial example; the quality measure is the reciprocal of the sum of magnitudes of contact normal forces required to achieve the worst case wrench in a task set [19]. However, the selection of a good initial example remains as a critical step; this example is provided here with a procedure assuring a locally optimum grasp.

The rest of the report is organized as follows. Section 2 presents the approach to compute locally optimum frictionless FC grasps (phases one and two), and Section 3 presents the procedure to compute the independent contact regions (phase three). The algorithms have been implemented, and Section 4 shows the results of their application to several objects. Finally, Section 5 presents the conclusions of the work.

2 Locally optimum force-closure grasp

2.1 Object and contact models

To compute the independent contact regions for a frictionless grasp on an arbitrary 3D object, the following assumptions are considered:

- The external surface of the object is represented with a mesh Ω of points, described by position vectors $\boldsymbol{p}_i$ measured with respect to a reference system located in the center of mass (CM) of the object. Each point has an associated unitary normal direction $\hat{\boldsymbol{n}}_i$ aiming to the interior of the object.
- The number of points in Ω is large enough to accurately represent the surface of the object.
- Each point on the surface of the object is connected with three neighboring points.

Seven frictionless contacts are necessary and sufficient to hold a 3D object with a FC grasp, provided that the object has no rotational symmetries [20]. With frictionless contact points, the grasp forces can only be applied in the direction normal to the object surface. A force $\boldsymbol{f}_i = \alpha_i \hat{\boldsymbol{n}}_i$ applied on the object at the point $\boldsymbol{p}_i$ generates a torque $\boldsymbol{\tau}_i = \boldsymbol{p}_i \times \boldsymbol{f}_i$ with respect to CM, with α_i being a nonnegative value representing the magnitude of the grasping force. The force and the torque are grouped together in a wrench vector (also known as generalized force vector) given by

$$\tilde{\boldsymbol{\omega}}_i = \begin{pmatrix} \boldsymbol{f}_i \\ \boldsymbol{\tau}_i \end{pmatrix} = \alpha_i \begin{pmatrix} \hat{\boldsymbol{n}}_i \\ \boldsymbol{p}_i \times \hat{\boldsymbol{n}}_i \end{pmatrix} \quad (1)$$

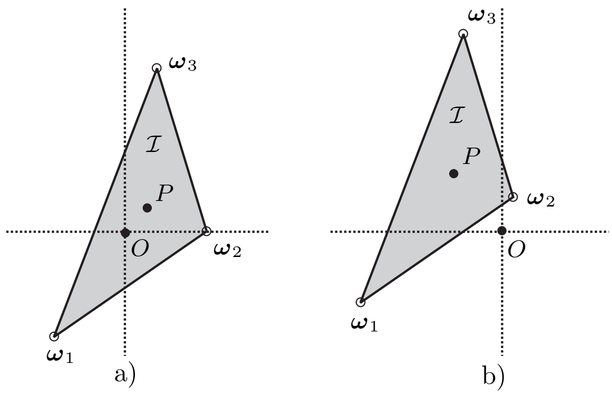


Figure 1. Illustration of the force-closure test in a two-dimensional wrench space: a) Force-closure grasp; b) Non force-closure grasp (P and O lie in different sides of $\overline{\boldsymbol{\omega}_1\boldsymbol{\omega}_2}$).

Since each physical point $\boldsymbol{p}_i$ in the set Ω has a corresponding wrench $\boldsymbol{\omega}_i$ in the wrench space, both of them will be used to indicate a grasp point. For a given grasp $G = \{\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_7\}$, the wrenches applied through the contact points on the object are grouped in a wrench set $W = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \ldots, \boldsymbol{\omega}_7\}$, where each $\boldsymbol{\omega}_i$, $i = 1, \ldots, 7$, is called a primitive contact wrench when $\alpha_i = 1$ in equation (1).

2.2 Force-closure test

Several criteria have been proposed to test the force-closure property in a particular grasp. A necessary and sufficient condition for the existence of a FC grasp is that the origin of the wrench space lies strictly inside the convex hull (CH) of the primitive contact wrenches [21]. Querying whether the origin lies inside the CH is also equivalent to a ray-shooting problem, solved as a linear programming problem [22]. The FC test used in this work is based on the following lemma.

Lemma 1: Let G be a grasp with a set W of primitive contact wrenches, $\mathcal{I}$ the set of strictly interior points of $CH(W)$, and H a boundary hyperplane of $CH(W)$ (i.e. a hyperplane containing one of the facets of $CH(W)$). The origin of the wrench space $O \in \mathcal{I}$ iff any $P \in \mathcal{I}$ and O are in the same half-space for every H of $CH(W)$.

Proof. The proof is straightforward. By definition, any hyperplane H containing a face of $CH(W)$ leaves the set $\mathcal{I}$ in one of the half spaces defined by H. In order for O to belong to $\mathcal{I}$, O must be in the same half-space than any $P \in \mathcal{I}$ for every H. □

From *Lemma 1*, checking whether a given point $P \in \mathcal{I}$ and the origin O lie in the same half-space defined by each boundary hyperplane H of $CH(W)$ is enough to prove whether O lies inside $CH(W)$, i.e. to prove whether the grasp G is FC. P is chosen as the centroid of the primitive contact wrenches, which is always an interior point of $CH(W)$. Then, the FC test verifies if the centroid P and the origin O lie on the same side for all the boundary hyperplanes of $CH(W)$; Fig. 1 illustrates the concept with a FC grasp and a non FC grasp in a hypothetical 2D wrench space (the actual wrench space is 6-dimensional).

2.3 First phase: getting one force-closure grasp

The main ideas of the algorithm used in the first phase have a close similarity to those used by Liu et al. [9]. The algorithm generates an initial grasp G^1 selecting seven random points from Ω; builds the corresponding wrench set W^1 and checks whether the points form a FC grasp. If they do, then the algorithm finishes. If G^1 is not a FC grasp, then an oriented search is performed, based on separating hyperplanes that define a subset Ω_C^1 containing candidate points to replace one of the current points in G^1. The steps in the algorithm are:

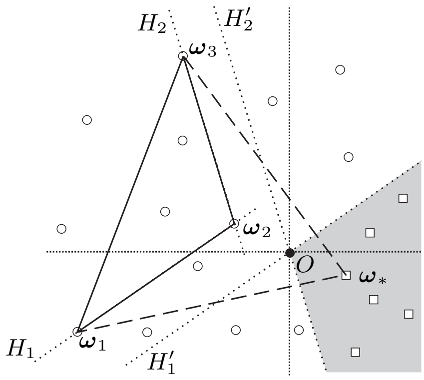


Figure 2. The grasp with wrench set $W = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \boldsymbol{\omega}_3\}$ (with CH represented in continuous lines) is non-FC. The subset of points to be replaced is $G_R^k = \{\boldsymbol{\omega}_2\}$. Wrenches in the gray zone (depicted as white squares) belong to Ω_C^k. The grasp with wrench set $W^* = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_*, \boldsymbol{\omega}_3\}$ (with CH represented in discontinuous lines) using a candidate point $\boldsymbol{\omega}_*$ is a FC grasp.

Algorithm 1: Search of a FC grasp

1. Generate a random initial grasp $G^k = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_7\}$, $k = 1$.

2. Form the corresponding wrench set W^k.

3. Check whether G^k is a FC grasp; if so, the algorithm finishes and returns G^k. If G^k is not a FC grasp, the search procedure iteratively tries to improve the grasp by changing one of the points in G^k, looking for a reduction in the distance between $CH(W)$ and the origin O, as follows in steps 4 to 6.

4. Find the subset G_R^k of grasp points in G^k that may be replaced. This subset is formed from all the wrenches in W that simultaneously belong to all the hyperplanes that produce the FC test failure (hereafter called critical hyperplanes). For instance, in Fig. 2 two hyperplanes, H_1 and H_2, produce the FC test failure, and $G_R^k = \{\boldsymbol{\omega}_2\}$.

5. Build the subset Ω_C^k with candidate points to replace one of the points in G_R^k. This subset is determined using hyperplanes passing through the origin and parallel to the critical hyperplanes; the candidate points are those than simultaneously lie in the opposite side of P with respect to those hyperplanes. In Fig. 2, wrenches that lie in the gray zone, determined by hyperplanes H_1' and H_2', belong to Ω_C^k.

6. Replace one point in G_R^k with a point from Ω_C^k. A point $\boldsymbol{\omega}_*$ is randomly picked up from Ω_C^k; then, $\boldsymbol{\omega}_*$ replaces the closest point in G_R^k. The candidate grasp G^* is formed with that replacement (in the example in Fig. 2, $G^* = \{\boldsymbol{\omega}_1, \boldsymbol{\omega}_*, \boldsymbol{\omega}_3\}$), and the centroid P^* and the distance $\overline{P^*O}$ are computed for the candidate grasp. If for any candidate G^* the relation $\overline{P^*O} < \overline{P^kO}$ is satisfied, then the best-first option is taken, and the corresponding point $\boldsymbol{\omega}_*$ is selected as the replacement point. If all the points in G_R^k have been checked out and none of them decreases the distance $\overline{P^kO}$, the replacement is done choosing the candidate G^* that gives the smaller distance $\overline{P^*O}$. Finally, the counter k is updated, the selected point is included in the new grasp G^k, and the algorithm returns to step 2.

To avoid falling in a local minimum, the generated grasps G^k are stored, and if Step 6 gives an already considered grasp, it is discarded and the next best non-visited candidate is actually taken for the replacement. This consideration allows the grasp search procedure to overcome local minima until a FC grasp is found. In this sense, the algorithm is complete in the discrete domain (as the algorithm in [9] it finds a FC grasp if there is one).

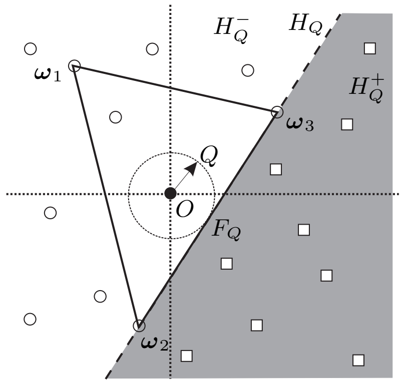


Figure 3. Selection of the subset Ω_C^k of candidate points (depicted as white squares in the gray area) that may improve the grasp quality; in this example, $F_Q = \overline{\boldsymbol{\omega}_2\boldsymbol{\omega}_3}$.

2.4 Second phase: finding a locally optimum grasp

The optimization algorithm begins with an initial FC grasp obtained through the procedure described above, and the optimization is done according to the largest ball criterion, that indicates the largest perturbation wrench that the grasp can resist with independence of its direction [17] [18]; it is one of the most popular grasp quality measures. Geometrically, that quality is equivalent to the radius of the largest ball centered at the origin of the wrench space and fully contained in $CH(W)$, or, in other words, it is also equivalent to the distance from the origin of the wrench space to the closest facet of $CH(W)$. The steps in the algorithm are:

Algorithm 2: Search of a locally optimum grasp

1. Find an initial FC grasp, $G^k = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_7\}$, $k = 1$, using Algorithm 1 presented in Subsection 2.3.

2. Determine F_Q, the facet of the convex hull $CH(W^k)$ closest to the origin. The distance from the origin O to F_Q is the current grasp quality Q^k.

3. Build the subset Ω_C^k with the candidate points that may produce an improvement in the grasp if they replace one point in F_Q, as illustrated in Fig. 3 for a hypothetical two-dimensional wrench space. The subset Ω_C^k is defined using H_Q, the hyperplane containing the facet F_Q. The candidate points are those lying in the open half-space defined by H_Q that does not contain the origin O, i.e. H_Q^+ in Fig. 3.

4. Picking one point ω_* from Ω_C^k, generate 6 candidate grasps $G_i^*, i = 1, \ldots, 6$, by replacing each one of the vertices defining the facet F_Q. Due to the selection procedure, all the wrenches $\omega^* \in \Omega_C^k$ are external points to $CH(W)$, therefore, when replacing one vertex $\boldsymbol{\omega}_i$ from the actual CH with the candidate wrench $\boldsymbol{\omega}_*$, the latter will be a vertex of the new CH. The explicit computation of the new CH is not required, as its facets are constructed from the old ones replacing $\boldsymbol{\omega}_i$ with $\boldsymbol{\omega}_*$. The candidate grasps are checked for the FC property using *Lemma 1*. For the FC candidate grasps, the expected grasp quality Q^* is computed; if for any candidate grasp $Q^* > Q^k$, then the candidate becomes the new grasp G^k. Fig. 4 illustrates three possible cases related with the candidate grasps; case (a) is a non-feasible grasp because it loses the FC property, case (b) is discarded because the grasp has a smaller quality than the previous one, and case (c) is a good grasp that actually improves the grasp quality, thus it becomes the grasp for the next iteration cycle. After this step, if the quality is improved the algorithm returns to step 2. If there is no improvement in Q^k once all the points in Ω_C^k have been considered, then a local minimum has already been reached, the algorithm finishes and returns the current grasp G.

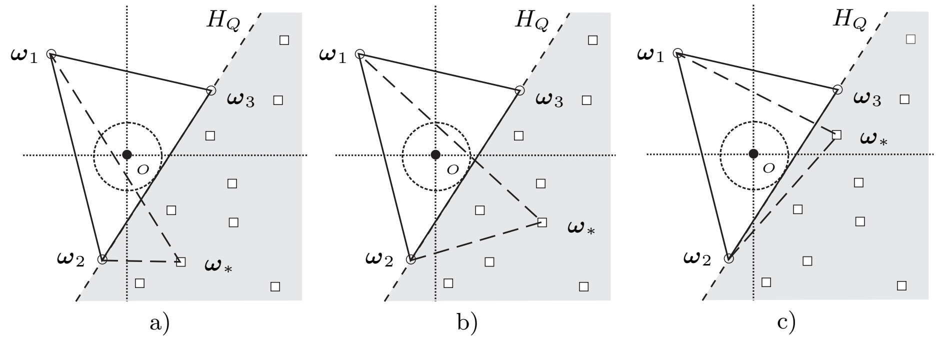


Figure 4. Possible cases for a candidate grasp in the optimization procedure: a) Non-feasible candidate grasp, b) Discarded candidate grasp, c) Feasible candidate grasp.

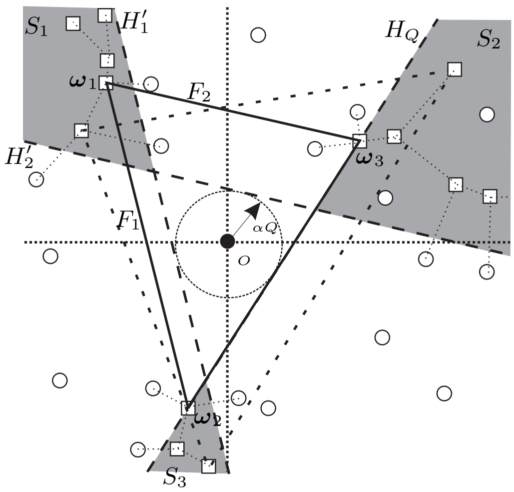
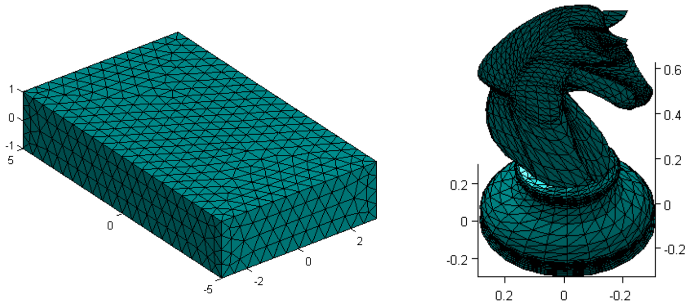


Figure 5. Search of independent contact regions. The hyperplanes H_Q, H'_1 and H'_2 define the search zones S_1, S_2 and S_3 (depicted in gray). The ICRs are the sets of neighboring wrenches falling in the search zone. Wrenches in each ICR are depicted as white squares, and an instance of a grasp with quality higher than Q ($\alpha = 1$) is also shown.

3 Independent Contact Regions

The computation of the independent contact regions (ICRs) ensuring a minimum grasp quality Q begins with a locally optimum FC grasp. Any grasp G with the corresponding wrench set W is formed with a finger contact position inside each ICR, and must satisfy $O \in CH(W)$. The proposed approach is based on this condition; Fig. 5 illustrates the geometrical concept in a hypothetical two-dimensional wrench space. For a given FC grasp, the grasp quality Q is fixed by F_Q, the facet of the convex hull closest to the origin. Six hyperplanes H' (two in the hypothetical two-dimensional wrench space), parallel to the remaining facets of the convex hull and tangent to the ball of radius $r = Q$ are then considered. These hyperplanes define the ICR search zone, S_i, in the wrench space for each wrench $\boldsymbol{\omega}_i$; S_i is the intersection of the half-spaces defined by the hyperplanes H'_j parallel to the facets of $CH(W)$ that contain the wrench $\boldsymbol{\omega}_i$. The ICR is formed by a set of neighboring points falling into the corresponding search zone S_i.

The procedure can also be applied to include contact points in the ICRs that may produce a minimum grasp quality $Q_r = \alpha Q$, being $0 < \alpha < 1$ and Q the quality of the initial grasp. This is done just considering a ball of radius Q_r instead of Q in the procedure described above. When $\alpha \to 0$, the ICRs contain FC grasps without a lower limit on the grasp quality. Actually, $\alpha = 0$ is a forbidden value, as it does not assure that $CH(W)$ will strictly contain the origin O. The algorithm used to determine the ICRs is:



a) b)

Figure 6. Objects used in the examples: a) Parallelepiped discretized with a mesh of 3422 triangles, b) Knight discretized with 4750 triangles.

Algorithm 3: Search for the independent contact regions

1. Find a locally optimum FC grasp, $G_o = \{\boldsymbol{\omega}_1, \ldots, \boldsymbol{\omega}_7\}$, with the corresponding wrench set W_o, using Algorithm 2 presented in Subsection 2.4.
2. Fix the minimum acceptable quality $Q_r = \alpha Q$.
3. Build the hyperplanes H', tangent to the ball of radius Q_r centered at the origin, that define the search space S_i, $i = 1, \ldots, 7$, for each grasp point.
4. Initialize I_i, the set of contiguous points forming the ICR for the grasp point i, as $I_i = \{\boldsymbol{\omega}_i\}$ (i.e. each ICR contains the original wrench of the set W_o). Label the points in each I_i as open.
5. For each open point $\boldsymbol{\omega}_k$ in the set I_i, check whether the neighbor points $\boldsymbol{\omega}_{kn}$ lie into the corresponding search space S_i. If $\boldsymbol{\omega}_{kn} \in S_i$ then add $\boldsymbol{\omega}_{kn}$ to I_i and label it as open; otherwise, discard the point. Label $\boldsymbol{\omega}_k$ as closed.
6. If there are open points in I_i, go back to Step 5. Otherwise, the algorithm finishes, and returns the sets of points I_i, $i = 1, \ldots, 7$, i.e. the ICRs for each finger.

4 Examples

The proposed approach to compute independent contact regions has been implemented using Matlab on a Pentium IV 3.2 GHz computer. The performance of the algorithm is illustrated using the two objects shown in Fig. 6: a parallelepiped and a knight (chess piece). The object surfaces are represented with triangular meshes (two triangles of the mesh are considered neighbors if they share an edge). The considered contact points $\boldsymbol{p}_i$ on the object surface are the centroids of the triangles in the mesh, and the corresponding surface normal directions are the directions normal to the triangles.

In the first example, the parallelepiped is described with a mesh of 3422 triangles. Fig. 7 shows an instance of the results obtained with the proposed approach. The first FC grasp, obtained with the Algorithm 1, is shown in Fig. 7a; the time elapsed to obtain this grasp was 5.1 seconds in 17 iterations. The locally optimum FC grasp, shown in Fig. 7b, was obtained with the Algorithm 2 in 24.8 seconds and 32 iterations. Fig. 7c shows the corresponding independent contact regions, obtained with Algorithm 3 in 0.25 seconds and using a minimum quality of $Q_r = 0.2168$ ($\alpha = 0.75$). Fig. 8a plots the distance $\overline{PO}$ against the iteration number in the first phase. Fig. 8b plots the grasp quality in the optimization phase, which always increases monotonically until the locally optimum grasp is found. The obtained locally optimum grasp depends on the initial grasp. In the example, the initial grasp quality is 0.0102, and the locally optimum grasp quality is 0.2891; the improvement factor, i.e. the ratio between the quality of the optimized grasp and the quality of the initial

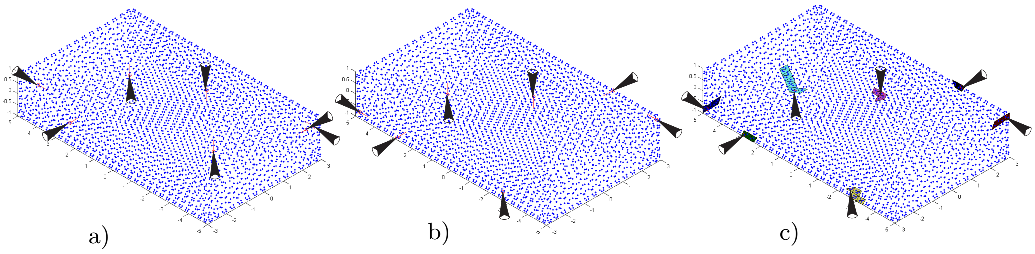


Figure 7. Example on a parallelepiped: a) Initial FC grasp, $Q = 0.0102$ (Algorithm 1), b) Locally optimum FC grasp, $Q = 0.2891$ (Algorithm 2), c) Independent contact regions for each finger, $Q_r = 0.2168$ (Algorithm 3).

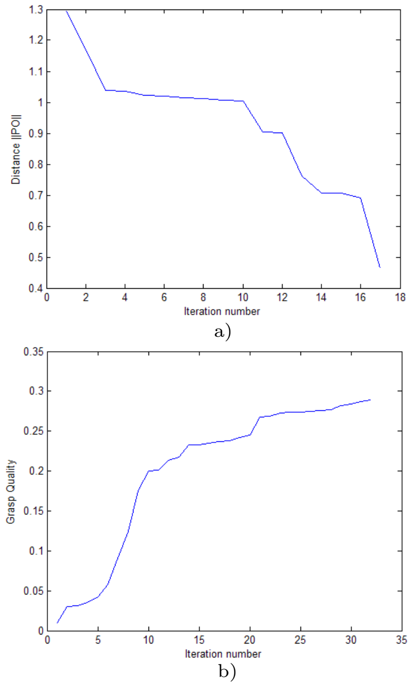


Figure 8. Performance in the search of a locally optimum FC grasp for the parallelepiped: a) Variation in the distance $\overline{PO}$, b) Increase in the grasp quality.

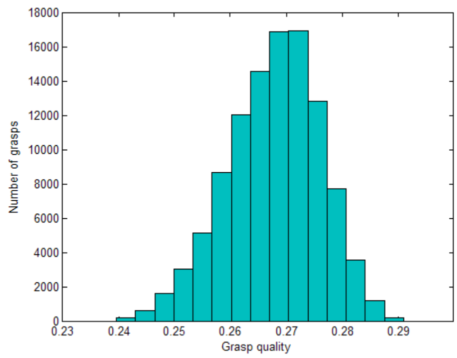


Figure 9. Histogram with the grasp quality distribution for all the possible grasps within the independent contact region on the parallelepiped for $Q_r = 0.2168$ ($\alpha = 0.75$).

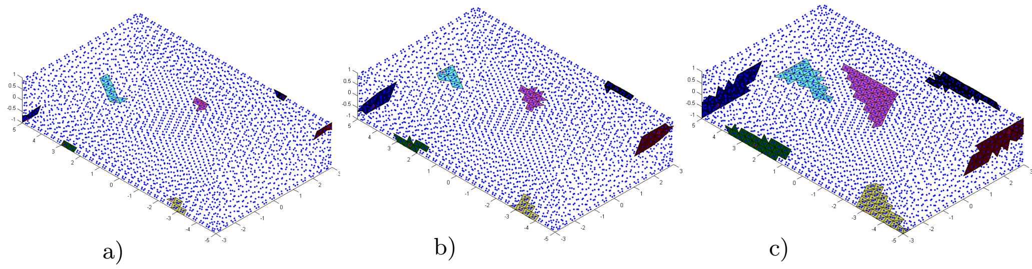


Figure 10. Independent contact regions on the parallelepiped with different minimum quality: a) $Q_r = 0.2168$ ($\alpha = 0.75$), b) $Q_r = 0.1446$ ($\alpha = 0.5$), c) $Q_r \approx 0$ ($\alpha = 10^{-5}$).

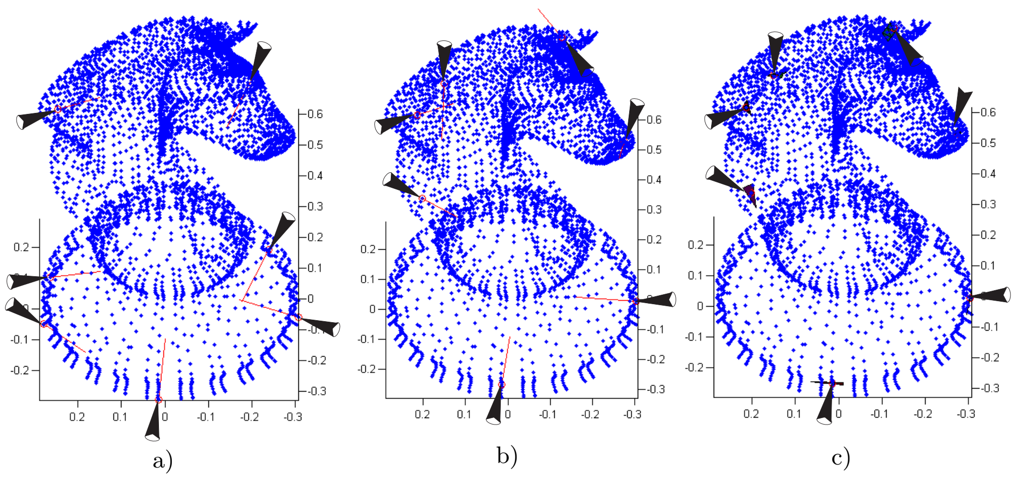


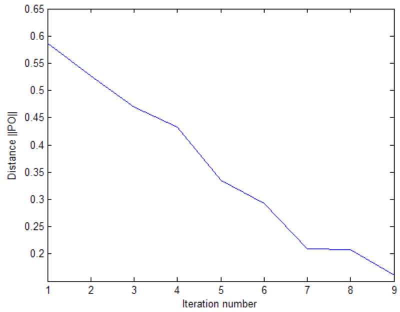
Figure 11. Example on a knight: a) Initial FC grasp, $Q = 0.0003$ (Algorithm 1), b) Locally optimum FC grasp, $Q = 0.077$ (Algorithm 2), c) Independent contact regions for each finger, $Q_r = 0.058$ (Algorithm 3).

FC grasp is 28.4. The points within the ICRs may be combined to provide 75000 different grasps; Fig. 9 shows the quality distribution for all these possible grasps. Obviously, for lower minimum grasp qualities, the ICRs grow; Fig. 10 shows the ICRs for three different minimum grasp qualities given by $\alpha = 0.75$, $\alpha = 0.5$ and $\alpha = 10^{-5} \approx 0$. In the last case, the ICRs contain points such that a finger in each region assures a FC grasp, but without a limit in the lower grasp quality.

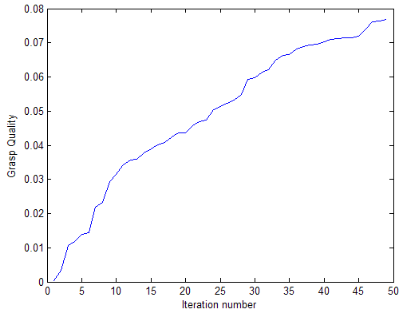
The knight used in the second example is discretized with 4750 triangles (Fig. 6b). Fig. 11 shows the results for an ICR search on the knight; the first FC grasp was found after 9 iterations in 5.4 seconds, the locally optimum grasp was obtained after 48 iterations in 47 seconds and the ICRs (with $Q_r = 0.058$, $\alpha = 0.75$) were computed in 0.17 seconds. The grasp qualities are 0.0003 and 0.077 for the initial and locally optimum FC grasps, respectively, with an improvement factor of 225.7. Fig. 12 illustrates the performance of Algorithms 1 and 2 in the search process. The points within the ICRs may be combined to provide 30 different grasps; Fig. 9 shows the quality distribution for all these possible grasps. Fig. 14 shows the ICRs for three different quality ratios: $\alpha = 0.75$, $\alpha = 0.5$ and $\alpha = 10^{-5}$.

5 Conclusions

This paper proposes an integrated approach to obtain independent contact regions on 3D discretized objects with seven frictionless contacts that assure a FC grasp with a controlled minimum quality. The procedure has three main parts: the first one looks for an initial FC grasp (its main ideas were presented in [9], although a different FC test is used here). The second part optimizes the initial FC grasp through an oriented search procedure, using as a quality measure the largest perturbation wrench that the grasp can resist, with inde-



a)



b)

Figure 12. Performance in the search of a locally optimum FC grasp for the knight: a) Variation in the distance $\overline{PO}$, b) Increase in the grasp quality.

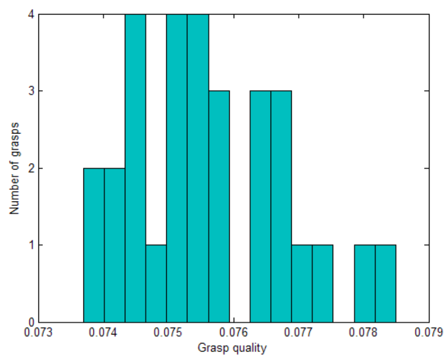
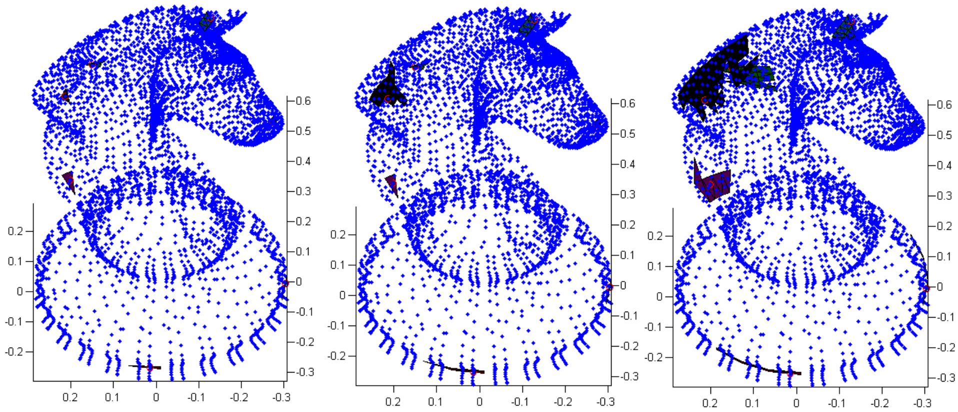


Figure 13. Histogram with the grasp quality distribution for all the possible grasps within the independent contact region on the knight for $Q_r = 0.058$ ($\alpha = 0.75$).



a) b) c)

Figure 14. Independent contact regions on the knight with different minimum quality: a) $Q_r = 0.058$ ($\alpha = 0.75$), b) $Q_r = 0.039$ ($\alpha = 0.5$), c) $Q_r \approx 0$ ($\alpha = 10^{-5}$).

pendence of the perturbation direction. The third part computes the independent contact regions around the contact locations of the locally optimum FC grasp. The algorithms were implemented and the execution results, as the examples shown in the paper, illustrate the relevance and efficiency of the approach. Although the algorithm is described just for seven frictionless fingers, it can be easily extended to determine ICRs for more fingers. In future works, the algorithm may be extended to consider frictional contacts, and the optimization of the ICRs size should be addressed.

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