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HTS nonlinearities in microwave disk resonators

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Abstract

This article describes a procedure for the calculation of the intermodulation behavior of the TM_{010} mode in high temperature superconducting (HTS) disk resonators from a description of the local HTS nonlinearities. Successful cross-checks are performed by comparing the theoretical results with experimental measurements and simulations based on the multiport harmonic balance algorithm for a specific model of HTS nonlinearity. The application of this procedure to the determination of nonlinear material parameters from disk resonator measurements is illustrated and compared to theoretical predictions.

Keywords: Superconducting devices; Microwave resonators; Intermodulation; Nonlinearities

1. Introduction

High temperature superconducting (HTS) devices are promising candidates for use in telecommunications because they provide filters with much better performance than conventional filters of comparable size. These filters consist of many resonators, and therefore each resonator element must, in general, support large current densities. Since HTS materials are inherently nonlinear, large current densities can provoke degradation in the filter performance either by intermodulation or reduction of the quality factor of the resonators [1].

Much effort has been dedicated to the study of nonlinearities in HTS materials [1], and the results have varied considerably. To date, there exists no consensus as to the origin of the nonlinearities, even as to whether they are caused by intrinsic or extrinsic properties.

A promising method for studying nonlinearities in HTS materials is with the use of disk resonators. Disk resonators are large so that the effects of material homogeneity are less pronounced, and in addition the TM_{0m0} modes have no currents parallel to the sample edges and thereby avoid edge effects.

2. Fields and currents for disk geometry

The structures to be considered here are HTS disk resonators consisting of two circular thin films on opposite sides of a dielectric substrate of thickness h, and the smaller of the two films (or both) has radius r_d. We define standard cylindrical coordinates where the z-axis passes through the

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two films, perpendicular to their surfaces. For the purposes of this article, we will deal only with the TM_{010} mode for which the azimuthal magnetic field is

$$H_\theta(t,\rho) = H(t)\left[\frac{\mathscr{J}_1(k\rho)}{\mathscr{J}_0(kr_{\rm d})}\right], \tag{1}$$

where $H(t)$ is the time-dependent field strength, $\mathscr{J}_n$ is the Bessel function of order n, and k is the wave number determined by the condition that $H_\theta(t,\rho = r_{\rm d}) = 0$, i.e. $kr_{\rm d} = 3.8317$.

We will assume that the disk is driven with external fields at frequencies ω_1 and ω_2, both of which are very near the resonant frequency. The azimuthal magnetic field is then given by Eq. (1) with

$$H(t) = H_1\cos(\omega_1 t + \phi_1) + H_2\cos(\omega_2 t + \phi_2), \tag{2}$$

where ϕ_1 and ϕ_2 are arbitrary phases.

In a manner similar to that in [2], we will define a surface impedance,

$$Z(\omega_0, J) = Z(\omega_0, 0) + \Delta Z(\omega_0, J), \tag{3}$$

where $\Delta Z(\omega_0, J) = \Delta R(\omega_0, J) - {\rm i}\omega_0\Delta L(\omega_0, J)$ is the change in surface impedance induced by finite current at a given frequency, while $\Delta R(\omega_0, J)$ and $\Delta L(\omega_0, J)$ are the separate changes in surface resistance and inductance, respectively.

The magnetic field at the principal frequencies, given by Eq. (2), will produce a radial surface current, J. Associated with this current will be a radial electric field inside the superconductor, $\mathscr{E}$, which we will assume can be separated into linear and nonlinear parts as $\mathscr{E} = \mathscr{E}_{\rm L} + \mathscr{E}_{\rm NL}$. In general, the electric field in the superconductor can be expressed as a series expansion in powers of J and its time derivatives. As outlined in [3], we will assume that the nonlinear part can be adequately described by simple resistive and reactive terms in the form

$$\mathscr{E}_{\rm NL} = a_{\rm NL}(J)J + \frac{\partial}{\partial t}[b_{\rm NL}(J)J]. \tag{4}$$

From Eq. (4) it is evident that $a_{\rm NL}(J)$ represents the resistive response and $b_{\rm NL}(J)$ the reactive response.

In order to proceed, one must choose a particular current dependence of the nonlinear electric field in Eq. (4). A simple, yet illustrative example is to begin with current dependencies as suggested in [2,4], where the nonlinear coefficients in Eq. (4) vary as the magnitude of the surface current density,

$$a_{\rm NL}^{\rm mod}(J) \equiv \Delta R_{\rm m}|J| \tag{5a}$$

and

$$b_{\rm NL}^{\rm mod}(J) \equiv \Delta L_{\rm m}|J|, \tag{5b}$$

where $\Delta R_{\rm m}$ and $\Delta L_{\rm m}$ are constants reflecting the current-induced changes in surface resistance and inductance, respectively. Now, we substitute Eqs. (5a) and (5b) into Eq. (4) for the nonlinear electric field and then substitute for J using Eq. (1) (since the surface current density is proportional to the magnetic field, $\mathbf{J} = \hat{n}\times\mathbf{H}$) to find

$$\mathscr{E}_{\rm NL}^{\rm mod}(t,\rho) = \left[\Delta R_{\rm m}H|H| + \Delta L_{\rm m}\frac{\partial(H|H|)}{\partial t}\right] \times \left[\frac{\mathscr{J}_1(k\rho)}{\mathscr{J}_0(kr_{\rm d})}\right]^2, \tag{6}$$

where H is given by Eq. (2). Ideally, one would like to analytically calculate the terms in Eq. (6) in order to separate the various frequency components. However, because of the nonanalytic nature of the modulo, in general this cannot be done in closed-form. Nevertheless, one can numerically generate a time series of the terms in Eq. (6) for various values of the frequencies, amplitudes, and phases. One can then perform a discrete Fourier transform to the frequency domain and pick out the desired frequency components. We have carried out this procedure for various values of the parameters, and found that the nonlinear electric field can be expressed in terms of a function of the input tones H_1 and H_2, denoted by $\Upsilon(H_1, H_2, \omega)$, as

$$\mathscr{E}_{\rm NL}^{\rm mod}(\omega,\rho) = (\Delta R_{\rm m} - {\rm i}\omega\Delta L_{\rm m})\left[\frac{\mathscr{J}_1(k\rho)}{\mathscr{J}_0(kr_{\rm d})}\right]^2 \times \Upsilon(H_1, H_2, \omega). \tag{7}$$

As a concrete example of interest, for intermodulation products at frequency $\omega_{12} \equiv 2\omega_1 - \omega_2$, when the two fundamentals are identical in magnitude, the function is given by $\Upsilon(H_1, H_2, \omega_{12}) = 0.2882H_1^2$, so that from Eq. (7), the nonlinear electric field in the superconductor at frequency ω_{12} is given by

$$\mathscr{E}_{\mathrm{NL}}^{\mathrm{mod}}(\omega_{12},\rho) = 0.2882(\Delta R_{\mathrm{m}} - \mathrm{i}\omega_{12}\Delta L_{\mathrm{m}}) \times \left[\frac{\mathscr{J}_1(k\rho)}{\mathscr{J}_0(kr_{\mathrm{d}})}\right]^2 H_1^2. \quad (8)$$

Eq. (8) determines the intermodulation generated in terms of the fundamental signals. However, one would like to relate the intermodulation signals to measurable quantities. To do so, we follow a procedure similar to that in [5], but taking into account the current dependence given by Eqs. (5a) and (5b), resulting in a magnetic field at frequency ω_{12} of

$$\mathscr{H}_{12}^{\mathrm{mod}} = 0.7029 H_1^2(\Delta R_{\mathrm{m}} - \mathrm{i}\omega_{12}\Delta L_{\mathrm{m}}) \times \frac{Q_L}{\omega_{12}\mu_0 h}\left[\frac{\mathscr{J}_1(k\rho)}{\mathscr{J}_0(kr_{\mathrm{d}})}\right]. \quad (9)$$

Eq. (9) is one of the central results of this article. Note that, according to this equation, both ΔR_{m} and ΔL_{m} affect the amplitude of the intermodulation signals. If intermodulation power measurements are made, the effect of ΔR_{m}, ΔL_{m} is indistinguishable and gives identical results whenever $\Delta R_{\mathrm{m}} = \omega\Delta L_{\mathrm{m}}$.

We can corroborate Eq. (9) by comparing to the simulations described in [3]. These simulations discretize the disk and treat it as a radial transmission line, solving the resultant set of equations in an iterative manner using the multiport harmonic balance (MHB) algorithm. The simulations also incorporated the disk parameters from measurements carried out in [6] of $Q_{\mathrm{L}} = 4.01 \times 10^3$, $\omega_0 = 2\pi \times 1.987$ GHz, $h = 5.0 \times 10^{-4}$ m, and $r_{\mathrm{d}} = 1.875 \times 10^{-2}$ m. The simulations reproduce the quadratic dependence on input signal amplitude given by Eq. (9) (and as measured [6]). In addition, with the values

$$\Delta R_{\mathrm{m}} = 1.526 \times 10^{-9}\ \Omega\,\mathrm{m}\,\mathrm{A}^{-1}, \quad \Delta L_{\mathrm{m}} = 0, \quad \text{or equivalently} \quad (10a)$$

$$\Delta R_{\mathrm{m}} = 0, \quad \Delta L_{\mathrm{m}} = 1.223 \times 10^{-19}\ \mathrm{H}\,\mathrm{m}\,\mathrm{A}^{-1}, \quad (10b)$$

(since the magnitude of $\mathscr{H}_{12}^{\mathrm{mod}}$ depends only on $|\Delta R_{\mathrm{m}} - \mathrm{i}\omega_{12}\Delta L_{\mathrm{m}}|$), the simulations gave a ratio of maximum field amplitudes of $(\mathscr{H}_{12}^{\mathrm{mod}})_{\max}/(H_1^2)_{\max} = 3.796 \times 10^{-7}\ \mathrm{m}\mathrm{A}^{-1}$. By maximum field amplitudes we refer to the maximum as a function of ρ as determined by the Bessel function in Eq. (9), i.e. $[\mathscr{J}_1(k\rho)/\mathscr{J}_0(kr_{\mathrm{d}})]_{\max} = 1.4447$. Substitution of these values into Eq. (9) verifies that the calculation agrees with the simulation to within <1%.

Another case of interest is that for which the amplitudes of the two fundamental tones are unequal. In this case, no closed-form expression can be given for the function $\Upsilon(H_1, H_2, \omega)$, and it must be calculated for various values of the input parameters. As an example, we have calculated $\Upsilon(H_1, H_2, \omega)$ for $0.03 \leqslant H_1 \leqslant 30$, $H_2 = 1$, and $(\omega_2 - \omega_1)/\omega_0 \ll 1$ (it depends only on the relative frequency difference and is independent of phase if the relative frequency difference is small). The results are shown in Fig. 1, in which it can be seen that the intermodulation products have an unusual power dependence. When $H_1 \ll H_2$, the intermodulation product at ω_{12} varies as H_1^2 and crosses over to an H_1 dependence for $H_1 \gg H_2$. The intermodulation product at $\omega_{21} \equiv 2\omega_2 - \omega_1$ varies as H_1 for $H_1 \ll H_2$ and is independent of H_1 for $H_1 \gg H_2$ [7].

We emphasize that the method outlined above can be used to calculate the response of the resonator for other models for the current dependence of the surface impedance. For example, assuming a quadratic dependence of the form

$$a_{\mathrm{NL}}^{\mathrm{quad}}(J) = \Delta R_{\mathrm{q}} J^2 \quad (11a)$$

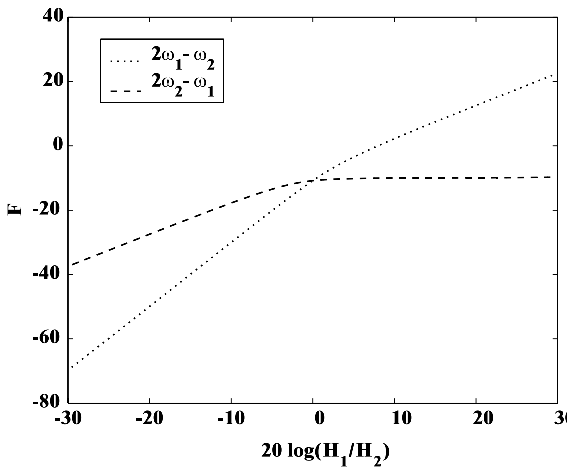

Fig. 1. $F \equiv 20\log[\Upsilon(H_1, H_2, \omega)/H_2^2]$ at frequencies of ω_{12} (- - -) and ω_{21} (···) as a function of $H_1(\omega_1)$, with fixed $H_2(\omega_2)$.

and

$$b_{NL}^{quad}(J) = \Delta L_q J^2. \quad (11b)$$

The entire process used for the modulo current dependence above can be carried out analytically, since the Fourier transform leading to the equivalent of Eq. (7) is straightforward. This results in an intermodulation product at frequency ω_{12} of

$$\mathscr{H}_{12}^{quad} = 2.325 H_1^3 (\Delta R_q - i\omega \Delta L_q) \frac{Q_L}{\omega_{12}\mu_0 h} \left[\frac{\mathscr{J}_1(k\rho)}{\mathscr{J}_0(kr_d)} \right], \quad (12)$$

which agrees with the results published in [3].

3. Connection with microscopic parameters and measurements

Now that we have made a connection between the intermodulation products and experimentally measurable parameters, it would also be useful to make a connection with microscopic parameters of the HTS material. Theories invoking intrinsic explanations for nonlinearities in HTS materials usually involve current-induced changes in the penetration depth, λ [9]. The current-dependent penetration depth can be expressed as

$$\lambda(j) = \lambda(0)[1 + 0.5 f(j)], \quad (13)$$

where $j \approx J/\lambda$ is the current density and $f(j)$ contains the current dependence which is related to the current dependent superfluid density by $n_s(j) = n_s(0)[1 - f(j)]$. As demonstrated in [8], the parameter $b_{NL}(J)$ defined in Eq. (4) can be expressed in terms of $f(j)$, for the modulo current dependence of Eq. (5b), as

$$b_{NL}(J) = \mu_0 \lambda(0) f(j)/2 = \mu_0 |J|/2 j_{IMD}, \quad (14)$$

where j_{IMD} is a characteristic current density. Relating Eq. (14) to the definition in Eq. (5b) gives

$$j_{IMD} = \frac{\mu_0}{2\Delta L_m} = 5.2 \times 10^8 \ \mathrm{A/cm^2}, \quad (15)$$

where we have used Eq. (10b) for ΔL_m and ΔR_m. We note that in the theory of [4], $j_{IMD} = j_c$, the pair-breaking critical current, and that the result of Eq. (15) is of the same order of magnitude as the estimated value for YBCO [10].

The resisitve nonlinear term a_{NL} can be related to the nonlinear surface impedance and to the function $f(j)$ that sets the nonlinearity in the HTS material [8]:

$$a_{NL} = \Delta R(\omega_0, J) \approx \mathrm{Re}[Z(\omega_0, 0)] \left[\frac{3}{2} + \frac{1}{\left(\lambda(0)/\lambda(0)|_{T=0}\right)^2 - 1} \right] \times \frac{J}{\lambda(0) j_{IMD}}. \quad (16)$$

The effect of a_{NL} on the intermodulation products is usually much smaller than that of b_{NL} and, thus Eq. (14) should be used instead of Eq. (16) to extract the value of j_{IMD}.

4. Conclusions

Microwave disk resonators are a useful tool for the study of nonlinearities in HTS materials because the TM_{0m0} modes have no current parallel to the film edges. Understanding these nonlinearities is important for the design and use of microwave filters because of the large current densities required.

We have analyzed the nonlinear behavior of microwave disk resonators and calculated the intermodulation products produced by introducing two fundamental tones near the resonant frequency of the TM_{010} mode. The method used is applicable to arbitrary nonlinearities caused by a current-dependent surface impedance. For the particular case of modulo current dependence, the calculations reproduce the results of experimental data and also have been verified with simulations using the MHB algorithm. Lastly, the characteristic current, j_{IMD}, extracted from the measurements is comparable to the intrinsic pair-breaking critical current, j_c.

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