A framework for RNAV trajectory generation minimizing noise nuisances

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Abstract— In this work it is presented a framework for a global optimization tool that will take into account aircraft dynamics and performances, noise nuisances and RNAV radionavigation requirements in order to assess an optimum flight depart or approach procedure. This strategy would be used as an optimization process performed by the corresponding authority in charge of the air traffic management of the involved airport or by an on-board optimization algorithm integrated in the Flight Management and Guidance System (FMGS). In both cases, the optimization framework is the same and the differences reside in the specific implementation of the optimization algorithms and the availability of the data in real time. In addition, aircraft's dynamic equations are developed in order to compute the flight trajectory from a set of flight guidance control variables and a first glance into a noise optimization criterion is given. Finally, the global optimization problem is properly formulated and the proposed solving utilities are presented.

I. INTRODUCTION

Despite the substantial reduction of emited noise of recent aeronautic engines, the sustained growth of air traffic still makes noise reduction in the vicinity of the airports one of the main issues that airport authorities, air traffic service providers and aircraft operators may deal with. In this context, international and national regulations regarding noise exposure have been established by civil aviation authorities in order to cope with this problem but incurring, on the other hand, in higher operations costs for airlines.

Present noise abatement procedures around airports are based on avoiding overfly densely populated areas. This can be accomplished by either going around the concerned area (i.e. modifying the horizontal flight path) or assuring enough vertical distance between the aircraft and the populated area (i.e modifying the vertical flight path). Concerning horizontal flight path management, new Area Navigation (RNAV) concepts are about to offer great advantages in defining more flexible procedures avoiding noise sensitive areas and reducing as well flight path dispersion (due to the better accuracy of RNAV systems regarding conventional ones). In Europe, the RNAV introduction is settled as an objective for all phases of flight, and in this context, EUROCONTROL (the European organization for the safety of air navigation [1]) has defined RNAV concept and satellite navigation systems as the key enablers for future improvements in terms of safety, efficiency and/or economy of flight, provided that their implementation is based on a fully co-ordinated, harmonized, evolutionary and flexible planning process [2], [3].

It is also possible to define noise abatement procedures acting in the vertical domain of a given flight trajectory. In this case, the methodology consist in the definition of optimal climb profiles, acting on the climb speeds and thrust configurations in such a way to increase the vertical distance of the aircraft and a sensitive area located in a certain region under the flight path. For instance, the most widely used noise abatement procedures for take-off are the so called *ICAO NADP departure procedures* defined in [4]. The NADP-1 procedure is designed to protect areas located *close* to the airport, while the NADP-2 procedure is designed to protect *distant* areas to the airport. Each procedure specifies the airspeed profile that should be maintained during the initial climb as well as the points (altitudes) where thrust/power reduction may be done. The difference between NADP-1 and NADP-2 procedures resides in the fact that NADP-1 gives more importance to climb as fast as possible and then accelerate and gain airspeed while NADP-2 tries to accelerate first and then climb. However, the main problem of these kind of procedures are that they are generic procedures and not always fit into the specific problems or environment that a certain airport may suffer.

In SOURDINE I project [5] an effort to improve take off procedures was done and some simulations with specific aircraft types were carried out. In this study optimal take-off procedures were obtained involving a progressive increase in thrust (which is not feasible with present technology) and low airspeeds during the whole departure which could be a problem regarding airport capacity. Those procedures were derived from former NADP-1 and NADP-2 procedures and optimization involved changes in values for engine cut-off, acceleration and climb points (altitudes). Further work performed in SOURDINE II project [6] dealt with refined take-off procedures selecting a grid of speed/thrust combinations and altitudes where thrust cut-off was performed. In this case, more than 30 different simulations were carried out, but not global optimization was done.

Concerning the approach procedure, there exist very simple procedures such as to intercept the Instrument Landing System (ILS) glide-slope at higher altitude, higher ILS glide-

slope angles or Low Drag-Low Power Approach (LPLD) procedures. More complex and efficient approach procedures deal with Continuous Descent Approaches (CDA) which, in turn, are being tested in some airports. During a CDA the aircraft performs a thrust-idle flight until a point before ILS-Localizer interception, reducing considerably the emitted noise. CDA procedures reduce significantly noise levels during the approach, but have an important impact on air traffic control operations and airport capacity being useful only in certain circumstances while keeping airport capacity with acceptable levels [7]. Clarke et al. proposed an improvement of conventional CDA approaches by defining the Three Degree Decelerating Approach (TDDA) showing improvements on better noise abatement, while maintaining acceptable capacity levels [8].

As it is seen, present noise abatement procedures are far from being the optimal ones minimizing noise nuisances. This is due to several factors like the impossibility to define a general criterion fitting all airports necessities, the limitations of nowadays technology on-board, and the constraints imposed by airport capacity or air traffic control issues. Nevertheless, some research in theoretical optimum trajectories minimizing the noise impact in depart/approach procedures is also found in the literature. For instance, Visser et al. show in [9] and [10] a technique to obtain optimal noise depart and approach procedures respectively. This tool combines the noise computations of the FAA's Integrated Noise Model (INM) [11] a Geographical Information System (GIS) and a dynamic trajectory optimization algorithm. Similar methodology is proposed by Clarke et al. in [12], and an adaptative algorithm for noise abatement can be found in [13]. An other study (see [14]) empathizes that most current noise abatement procedures (like those explained above) are local adaptations of generic procedures aimed at optimizing aircraft noise footprint and do not generally take into account the actual population density and distribution at a specific airport site. Thence, a noise performance trade-off is presented between arrival trajectories that are optimized according to different types of noise abatement criteria. Typically, these different criteria are not compatible and the variables that optimize one objective may be far from optimal for the others, pointing out the difficulty to properly identify the absolute minimal trajectory among all the local minimal ones.

This paper firstly presents the framework for a global optimization tool, developing the different components that are involved in the process. Then, a flight guidance model is presented in order to describe the dynamic constraints that will apply to the flight trajectory. Fourth section of this paper is devoted to define the optimization criterion, which will take into account not only the noise nuisances but also some airlines considerations such as time and fuel consumptions and finally the global optimization problem is formally formulated.

II. FRAMEWORK OF THE OPTIMIZATION STRATEGY

The framework proposed to optimize depart or approach trajectories is presented in this section and it is summarized in figure 1. The involved airport, with its surrounding cartography, geography and meteorological data, will define a *scenario* which will be used to compute a given *noise nuisance* in function of the emited aircraft noise along its trajectory. This value, jointly with some fuel/time economy considerations, will define a global *optimization criterion*. Then, an *optimization algorithm* will compute the best departing or approaching trajectory which minimizes the *optimization criterion* and satisfies a set of *trajectory constraints* which, in turn, will depend on the dynamics of the aircraft, RNAV design constraints and the airspace configuration.

This strategy would be used as an optimization process performed by the corresponding authority in charge of the air traffic management of the involved airport or by an on-board optimization algorithm integrated in the Flight Management and Guidance System (FMGS). In both cases, the optimization framework is the same and the differences reside in the specific implementation of the optimization algorithms, which in the case of being integrated on-board will require some real time performance as well as the possibility of the on-board system to obtain the scenario, airspace and meteorological data. In a further study it is proposed to assess the specific requirements of both implementations.

The following sections give more details of each component involved in the trajectory optimization.

A. Input data

Different kind of input data are required for the proper model of each component. As shown i figure 1 the input data sources are:

- **Airspace**: containing the airspace characteristics of the studied area, restricted areas, airspace structure organization as well as the the departing or arriving points that will define the procedure final or initial conditions respectively.
- **RNAV Navigation**: containing the RNAV design procedure criteria that specify the constraints that should satisfy any procedure in order to meet the required level of safety.
- **Cartography**: containing cartographic data, including terrain elevations and obstacle identification, that will be needed to safely define the flight procedure according with the RNAV procedure design criteria. In addition, this information will be used by the noise model.
- **Aircraft performances**: including the aerodynamic and power plant related data of the studied aircraft that will be needed to build up the aircraft dynamic model as well as the noise model.
- **Meteorology**: containing meteorological data that will affect available runway configuration, the noise propagation model and the aircraft ground trajectory.
- **Airport**: containing the location of the airport, the type of procedure, available runway configuration etc.
- **Geography**: containing geographic data of airport's surrounding areas such as the location and characteristics of the inhabited areas.
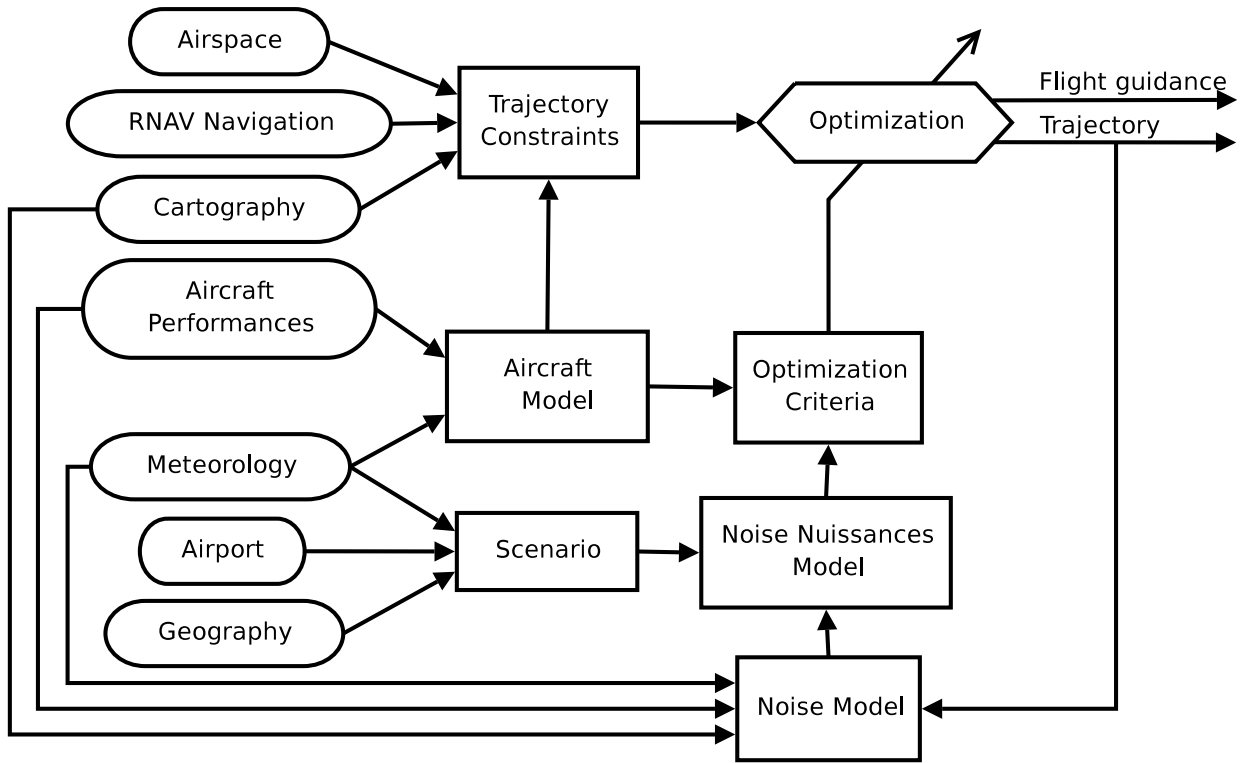
Fig. 1.   Noise nuisances optimization framework

B. Trajectory constraints

This component will define a set of rules that will restrict the amount of possible trajectories into a valid domain where the optimization will take place.

First of all, airspace organization will be taken into account, regarding prohibited, dangerous and restricted areas as well as and particular airspace sectorization focusing on the compatibility with other existing flight procedures in the same airspace. This analysis will finally identify a set of *usable airspace* portions where the obtained trajectories should be contained.

In addition, this component will take into account procedure safety issues. As it is well known, ICAO Document 8168: *Procedures for Air Navigation Services - Aircraft Operations (PANS-OPS) - Volume II, Construction of Visual and Instrument Flight Procedures* [15], contains all the rules and methodology for designing flight procedures. All these information will be transformed into the form of trajectory constraints, restricting even more the *usable airspace* defined above (for instance, in order to take into account obstacle clearance etc.) and bounding the trajectory design variables (for instance, in order to consider maximum climb gradients etc.).

Finally, trajectory boundary conditions will be also specified. If a departing procedure is studied, the final departure point (or points) location and altitude will be included as the trajectory final boundary condition. On the other hand, if an arriving procedure is studied, the initial arriving point (or points) location and altitude will be included as the trajectory

initial boundary condition. It should be noted that it is not necessary to define fixed (known) boundary points since the optimization algorithm will be able to deal with not fixed boundary conditions.

C. Aircraft Model

The dynamic equations of the aircraft trajectory and flight guidance are contained in this component, which uses the required parameters from an aircraft dependent performance data base. These equations will set the relations between the actual aircraft trajectory and the variables required to control the aircraft's trajectory. In addition, a cost function expression regarding the airplane operator objectives, will be modeled in order to be taken into account into the global optimization criterion giving a specific weight to the time and fuel consumed during the whole procedure.

D. Scenario definition

The airport configuration, the present meteorology and the geographic data around the airport will define the scenario where the trajectory should be optimized. Thence, this scenario will be used to determine the noise nuisances caused by the noise generated by the departing/approaching aircraft along its trajectory. Therefore it should provide information about:

- the runway used and the type of procedure (depart or approach)
- the time of the year and hour of the day that the procedure will be flown

- the procedure's intended frequency of use.
- the population density and distribution of the inhabited areas surrounding the airport.
- the type of activities developed in the inhabited areas (industrial zone, residential zone...)
- the location and characteristics of other existing noisy areas surrounding the airport, such as motorways, harbors...
- the location of possible sensitive areas such as environmental protected zones.

E. Noise model

This component will contain a noise model of propagation and will compute the perceived noise level at each point of the trajectory. Therefore, information about the cartography and the meteorology will be used in order to compute the noise propagation and attenuation as well as some aircraft data which will enable the model of the emited aerodynamic and propulsive noise.

F. Noise nuisances model

The aircraft perceived noise in a given point of the trajectory will be important in assessing the optimum trajectory. Nevertheless, what is intended to be minimized is the noise nuisances and not only the perceived noise. For instance, the same noise is more penalizing if it is heard in an inhabited residential zone at midnight than in an industrial zone at noon. Therefore, this component will compute a given value of nuisance in function of the perceived noise and in function of the scenario characteristics. A set of fuzzy inference rules will be defined in order to give a weight to all factors that will play a significant role in the noise nuisance build up.

G. Optimization criterion

As it has been commented before, this component will define a global optimization criterion that will take into account the noise nuisances of the flight trajectory and the aircraft operator considerations. This criterion, will be used as performance factor in the optimization process.

H. Optimization algorithms

Optimal control techniques will be applied here in order to find the optimum trajectory (and flight guidance parameters) that minimizes the global criterion of nuisance and aircraft operator economy under a set of constraints.

III. FLIGHT GUIDANCE MODEL

Starting from a dynamic and cinematic analysis of an aircraft's motion, the goal of this section is to obtain a state representation of the flight guidance equations that are needed to describe the trajectory of the aircraft. This state representation deduction will assume some initial hypotheses and will define the dynamic relationship between the state variables and a set of flight control variables. This kind of representation will enable the use of future optimization methods for dynamic systems.

A. Reference frames definition

Three different reference frames are needed to describe the aircraft's equations of motion. A *Ground* reference frame which will be used as inertial frame, an *Air* reference frame where the aerodynamic forces are easily expressed, and finally a *Body* reference frame used as an intermediate frame to convert *Air* magnitudes to *Ground* magnitudes. These three reference frames are defined as:

- **Ground** $G = [O; n, e, d]$ **reference frame**: North-East-Down conventional right handed frame on the surface of the Earth with a given origin O. The d axis points downwards following the local vertical direction (i.e the direction of the local gravity vector, $\boldsymbol{g}$) and the n-e plane is tangent to the Earth's surface at O. The e axis points to the East and therefore the n axis points to the North.
- **Body** $B = [P; x, y, z]$ **reference frame**: Conventional right handed set of body fixed axes with origin P at the center of mass of the airplane. The x axis is forward aligned, y axis starboard aligned and z axis down in the aircraft.
- **Air** $A = [P; x_A, y_A, z_A]$ **reference frame**: Conventional right handed frame with origin P at the center of mass of the airplane. The x_A axis is always aligned with the relative velocity vector between the air and the plane.

Three consecutively rotations are defined to describe the instantaneous attitude of the aircraft (Body reference frame) with respect to the Ground reference frame. Starting from G these rotations are:

- First rotation about the d axis, nose right (*yaw* angle ψ)
- Second rotation about the new e axis, nose up (*pitch* angle θ)
- Third rotation about the new n axis, right wing down (*roll* angle ϕ)

In the same way, two consecutively rotations define the frame A from the frame B:

- First rotation about the y axis, downwards (*angle of attack* angle α)
- Second rotation about the new z axis, rightwards (*sideslip* angle β)

If $\boldsymbol{a}_G$ and $\boldsymbol{a}_B$ are the position vectors of a given point in frames G and B respectively the coordinate transformation which relates both vectors is:

$$\boldsymbol{a}_G = \mathcal{R}_{GB}\boldsymbol{a}_B \tag{1}$$

with:

$$\mathcal{R}_{GB} = \begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi s\theta c\phi \\ s\psi c\theta & c\psi c\phi + s\psi s\theta s\phi & -c\psi s\phi + s\psi s\theta c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{2}$$

Where, for the sake of simplicity, c(.) and s(.) represent cos(.) and sin(.) respectively. Similarly, if $\boldsymbol{a}_A$ is the position vector of a given point in frame A, the coordinate transformation yields:

$$\boldsymbol{a}_B = \boldsymbol{\mathcal{R}}_{BA}\boldsymbol{a}_A \tag{3}$$

with:

$$\boldsymbol{\mathcal{R}}_{BA} = \begin{bmatrix} c\alpha c\beta & -c\alpha s\beta & -s\alpha \\ s\beta & c\beta & 0 \\ s\alpha c\beta & -s\alpha s\beta & c\alpha \end{bmatrix} \tag{4}$$

It is also possible to define three new angular rotations which led us from *Ground* reference frame directly to *Air* reference frame. In this case, starting from G, these rotations are:

- First rotation about the d axis, nose right (aerodynamic *yaw* angle χ)
- Second rotation about the new e axis, nose up (aerodynamic *pitch* angle γ)
- Third rotation about the new n axis, right wing down (aerodynamic *roll* angle μ)

In terms of coordinate transformations:

$$\boldsymbol{a}_G = \boldsymbol{\mathcal{R}}_{GA}\boldsymbol{a}_A \tag{5}$$

with:

$$\boldsymbol{\mathcal{R}}_{GA} = \begin{bmatrix} c\chi c\gamma & -s\chi c\mu + c\chi s\gamma s\mu & s\chi s\mu + c\chi s\gamma c\mu \\ s\chi c\gamma & c\chi c\mu + s\chi s\gamma s\mu & -c\chi s\mu + s\chi s\gamma c\mu \\ -s\gamma & c\gamma s\mu & c\gamma c\mu \end{bmatrix} \tag{6}$$

B. Dynamic analysis

At this point Newton's Second Law will be applied to a given flying airplane. First of all, the adopted notation and some basic hypothesis that will ease our study will be introduced.

1) Basic hypothesis and notation: Let $\boldsymbol{a}$ be a given vector, then $\boldsymbol{a}_X$ will be vector $\boldsymbol{a}$ expressed in X reference frame coordinates. On the other hand, if $\boldsymbol{b}$ is a velocity vector, it will be noted $\boldsymbol{b}_X^Y$ as the velocity $\boldsymbol{b}$ seen from reference frame Y, expressed in X reference frame coordinates.

Finally, the operator $\frac{d}{dt_X}(.)$ stands for the time derivative as seen from reference frame X.

HYPOTHESIS 1 : The wind components are constant
It is worth to assume that the wind velocity vector is constant over a region much larger than the size of the aircraft, so wind shearing effects and torques will be neglected.

HYPOTHESIS 2 : The total mass of the airplane remains constant with time
This problem will consider the take-off or approach maneuvers of a conventional commercial airplane. For example, a typical airplane of 180 passengers will consume around $1000\,Kg$ during a climb to cruise altitude, being its total mass of about $70000\,Kg$. Therefore the mass change over the considered time period will be about 1.5% which is considered negligeable [16].

HYPOTHESIS 3 : The mass distribution is also constant with time

Passenger movements, fuel sloshing and shifting payloads effects are neglected and therefore the center of gravity of the airplane will be supposed to stay in the same place during the time period of consideration.

HYPOTHESIS 4 : The reference frame G is supposed to be an *inertial* frame
In fact, Ground *reference frame is both accelerating and rotating, however the accelerations associated with the Earth's movement are negligible compared to the accelerations that will produced by a maneuvering aircraft.*

2) Dynamic equations: Taking into account all hypothesis and considerations above, Newton's Second Law can be written in the *Ground* reference frame as:

$$\frac{1}{m}\sum \boldsymbol{F}_G = \frac{d}{dt_G}\boldsymbol{v}_G^G = \frac{d}{dt_G}\left(\boldsymbol{v}_G^A + \boldsymbol{w}_G^G\right) = \frac{d}{dt_G}\boldsymbol{v}_G^A \tag{7}$$

were $\sum \boldsymbol{F}$ is the sum of all external applied forces, m is the total mass of the aircraft, $\boldsymbol{v}$ is the velocity of the aircraft's center of mass and $\boldsymbol{w}$ is the local wind velocity.

As it will be seen later, aerodynamic forces are much simpler if represented in the *Air* reference frame. Therefore, it is interesting to rewrite equation 7 and express all magnitudes in A frame [17]:

$$\frac{1}{m}\sum \boldsymbol{F}_A = \frac{d}{dt_G}\boldsymbol{v}_A^A = \frac{d}{dt_A}\boldsymbol{v}_A^A + \boldsymbol{\omega}_A^{GA} \times \boldsymbol{v}_A^A \tag{8}$$

where $\boldsymbol{\omega}^{GA}$, is the angular velocity vector of frame A relative to frame G.

The sum of all external applied forces will be decomposed as:

$$\sum \boldsymbol{F}_A = \boldsymbol{F}_A^a + \boldsymbol{F}_A^p + m\boldsymbol{\mathcal{R}}_{AG}\boldsymbol{g}_G \tag{9}$$

were $\boldsymbol{F}^a$ represents the sum of all aerodynamic forces, $\boldsymbol{F}^p$ the sum of all propulsive forces and $\boldsymbol{g}$ is the local gravity vector.

Lets expand now all vectors used in last equations. As commented before, in the *Air* reference frame aerodynamic forces can easily written as [16]:

$$\boldsymbol{F}_A^a = \begin{bmatrix} -D \\ Y \\ -L \end{bmatrix} \tag{10}$$

Where D and L are the *Drag* and *Lift* aerodynamic forces and Y the aerodynamic sideforce component along the *Air* y axis.

Concerning the propulsive forces, it is assumed:

HYPOTHESIS 5 : The sum of all propulsive forces is a vector directed as the body x axis
In modern jet aircraft, with the engines under the main wings, there exist typically a small thrust component in the vertical z axis that will be neglected in this study. In addition it is assumed that in our study all aircraft's engines are operative leading to a symmetrical thrust force regarding x-z plane.

Thus:

$$\boldsymbol{F}_A^p = \boldsymbol{\mathcal{R}}_{AB}\boldsymbol{F}_B^a = \begin{bmatrix} T\cos\alpha\cos\beta \\ -T\cos\alpha\sin\beta \\ -T\sin\alpha \end{bmatrix} \qquad (11)$$

where T is the total thrust forces developed by all aircraft's engines.

The local gravity vector $\boldsymbol{g}$ in G reference frame is simply:

$$\boldsymbol{g}_G = \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} \quad \text{with } g \simeq 9.81 \; ms^{-2} \qquad (12)$$

and, by definition, $\boldsymbol{v}^A$ in A reference frame is written as:

$$\boldsymbol{v}_A^A = \begin{bmatrix} v \\ 0 \\ 0 \end{bmatrix} \qquad (13)$$

where v is the module of the relative air to aircraft velocity, also known as the *True Airspeed* (TAS).

Finally, vector $\boldsymbol{\omega}_A^{GA}$ components are defined as:

$$\boldsymbol{\omega}_A^{GA} = \begin{bmatrix} p_A \\ q_A \\ r_A \end{bmatrix} \qquad (14)$$

In our study it will be perfectly reasonable to assume:

HYPOTHESIS 6 : The sideslip angle β is considered to be zero

This hypothesis assumes that the flight is always symmetrical and turns are always coordinated, which is perfectly reasonable in civil transport aircraft when all engines are operative.

Last hypothesis leads to $Y = 0$ [16].

With all considerations above, equations 8 and 9 can be finally expanded as:

$$\begin{aligned} \dot{v} &= \tfrac{1}{m}T\cos\alpha - \tfrac{1}{m}D - g\sin\gamma \\ 0 &= vr_A - g(\cos\gamma\sin\mu) \\ 0 &= -vq_A + \tfrac{1}{m}L + \tfrac{1}{m}T\sin\alpha - g\cos\gamma\cos\mu \end{aligned} \qquad (15)$$

C. Cinematic analysis

The determination of the flight path of the airplane relative to the *Ground* reference system will be done by numerical integration of the airplane's *Ground* coordinates, which in turn, are expressed in function of the velocity of the aircraft's center of mass as:

$$\dot{\boldsymbol{p}}_G = \boldsymbol{v}_G^G = \boldsymbol{\mathcal{R}}_{GA}\boldsymbol{v}_A^A + \boldsymbol{w}_G^G \qquad (16)$$

if it is assumed that the local wind has north, east and down velocity components as:

$$\boldsymbol{w}_G^G = \begin{bmatrix} w_n \\ w_e \\ w_d \end{bmatrix} \qquad (17)$$

the expansion of equation 16 leads to:

$$\begin{bmatrix} \dot{n} \\ \dot{e} \\ \dot{d} \end{bmatrix} = v \begin{bmatrix} \cos\chi\cos\gamma \\ \sin\chi\cos\gamma \\ -\sin\gamma \end{bmatrix} + \begin{bmatrix} w_n \\ w_e \\ w_d \end{bmatrix} \qquad (18)$$

On the other hand, in order to perform this integration, the time evolution of the angles $\boldsymbol{\Psi} = [\chi \; \gamma \; \mu]^T$ must be known. It can be easily shown that the angle rates $\dot{\boldsymbol{\Psi}}$ can be expressed in function of the angular velocities $\boldsymbol{\omega}_A^{GA} = [p_A \; q_A \; r_A]^T$ as [17]:

$$\begin{bmatrix} p_A \\ q_A \\ r_A \end{bmatrix} = \begin{bmatrix} \dot{\mu} - \dot{\chi}\sin\gamma \\ \dot{\gamma}\cos\mu + \dot{\chi}\cos\gamma\sin\mu \\ \dot{\chi}\cos\gamma\cos\mu - \dot{\gamma}\sin\mu \end{bmatrix} \qquad (19)$$

D. Space state representation

By substituting equations 19 into 15 yields:

$$\begin{aligned} \dot{v} &= \tfrac{1}{m}T\cos\alpha - \tfrac{1}{m}D - g\sin\gamma \\ \dot{\chi} &= \tfrac{1}{mv\cos\gamma}L\sin\mu + \tfrac{1}{mv\cos\gamma}T\sin\mu\sin\alpha \\ \dot{\gamma} &= \tfrac{1}{mv}L\cos\mu + \tfrac{1}{mv}T\cos\mu\sin\alpha - \tfrac{g\cos\gamma}{v} \end{aligned} \qquad (20)$$

Aerodynamic forces L and D can be modeled as:

$$\begin{aligned} L &= \tfrac{1}{2}\rho(d)Sv^2C_L \\ D &= \tfrac{1}{2}\rho(d)Sv^2C_D \end{aligned} \qquad (21)$$

where $\rho(d)$ is the air density, which can be considered only altitude $(-d)$ dependant and S is the total surface of the airplane's wings. Aerodynamic coefficients C_L and C_D are usually modeled as:

$$\begin{aligned} C_L &= C_{L_0} + C_{L_\alpha}\alpha \\ C_D &= C_{D_0} + \tfrac{1}{\pi Ae}C_L^2 \end{aligned} \qquad (22)$$

where C_{L_0}, C_{L_α} and C_{D_0} are known aerodynamic parameters, A is the aspect ratio of the wing $A = \frac{b}{c} = \frac{b^2}{S}$ (being b the total wing span and c the mean chord of the wing's airfoils) and e the *Oswald factor* of the wing.

Concerning the propulsive force T, considering a turbojet engine type, in a first approximation this force can be modeled as:

$$T = n_e\rho(d)S_ev(V_e - v) \qquad (23)$$

where n_e is the number of engines, S_e is the effective engine inlet area and V_e is the exhaust gas velocity. It can be shown that V_e can be fitted in a linear form depending on the engine's low pressure rotor speed $N1$ provided that $50\% \lesssim N1 \lesssim 100\%$:

$$V_e = V_{e_0} + \lambda(N1 - N1_0) \qquad (24)$$

Therefore, in a more generic way, aerodynamic and thrust forces can be written as:

$$\begin{aligned} L &= L(d, v, \alpha) \\ D &= D(d, v, \alpha) \\ T &= T(d, v, \alpha, N1) \end{aligned} \qquad (25)$$

Finally, merging equations 20 and 18 a state representation of the flight guidance equations is obtained:

$$
\begin{bmatrix} \dot{v} \\ \dot{\chi} \\ \dot{\gamma} \\ \dot{n} \\ \dot{e} \\ \dot{d} \end{bmatrix} = \begin{bmatrix} -g\sin\gamma \\ 0 \\ -\frac{g}{v}\cos\gamma \\ v\cos\chi\cos\gamma + w_n \\ v\sin\chi\cos\gamma + w_e \\ -v\sin\gamma + w_d \end{bmatrix} +
$$
$$
+ \begin{bmatrix} \frac{1}{m}\left[-D(d,v,\alpha) + T(d,v,\alpha,N1)\cos\alpha\right] \\ \frac{\sin\mu}{mv\cos\gamma}\left[L(d,v,\alpha) + T(d,v,\alpha,N1)\sin\alpha\right] \\ \frac{\cos\mu}{mv}\left[L(d,v,\alpha) + T(d,v,\alpha,N1)\sin\alpha\right] \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$
$$(26)$$

the state $\boldsymbol{x}$ and control $\boldsymbol{u}$ vectors are identified as:

$$
\begin{aligned}
\boldsymbol{x} &= [v\ \chi\ \gamma\ n\ e\ d]^T \\
\boldsymbol{u} &= [\alpha\ \mu\ N1]^T
\end{aligned}
\qquad (27)
$$

and the flight guidance equations can be summarized in the following expression:

$$
\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u}) \qquad (28)
$$

E. The final guidance model

As it has been shown, the guidance parameters adopted in last model are $\boldsymbol{u} = [\alpha\ \mu\ N1]^T$. Engine's parameter $N1$ is often the main control variable of the *auto/thrust* system but, on the other hand, the typical variables for attitude guidance are pitch and roll angles $[\theta, \phi]$ or even, in recent fly-by-wire auto-flight systems, the roll rate and the vertical load factor $[p, n_z]$. However, the use of $[\chi, \gamma, \mu]$ angles instead of $[\psi, \theta, \phi]$ angles, and the possibility of expressing all magnitudes into the *Air* reference frame simplifies considerably the state equations given in 26, playing an important role for easing further optimization algorithms. Thus, the optimization process will yield to the required guidance control variables $\boldsymbol{u}(t) = [\alpha(t)\ \mu(t)\ N1(t)]^T$ needed to achieve the optimum trajectory. From equations 1, 3 and 5 it appears that θ and ϕ angles can be obtained, through a complex trigonometric expression, from α, μ and γ angles. This expression is here summarized by:

$$
[\theta, \phi] = \boldsymbol{\Gamma}(\alpha, \mu, \gamma) \qquad (29)
$$

Therefore, the optimum values of angles $[\theta, \phi]$ can be computed in real time using equation 29 and supplied as reference values for the basic longitudinal and lateral modes of a classical autopilot (see figure 2). Nevertheless, it should be noted that thanks to the wide use of high performance inertial reference systems and integrated auto-pilots architecture, it is worth to suppose that in a future an autopilot may deal directly with $[\alpha, \mu]$ control guidance inputs.

IV. DEFINITION OF THE OPTIMIZATION CRITERION

The main cost components, related with the evolution of an aircraft during a time interval $[t_0, t_f]$ and considering either a climb, cruise or descent flight phases, are the fuel cost, the delay cost and the noise effect.

A. Flight costs model

Fuel cost C_f can be computed as:

$$
C_f = \int_{t_0}^{t_f} \pi_c FF(t)dt \qquad (30)
$$

where π_c is the fuel price and FF is the fuel flow which can be estimated as a smooth function of aircraft's speed and engine's thrust:

$$
FF = f_f(v, T) \qquad (31)
$$

For instance, the BADA aircraft performance database from EUROCONTROL [18] provides the formula:

$$
FF = c_{f1}(1 + c_{f2}v)T \qquad (32)
$$

where c_{f1} and c_{f2} are parameters whose values are characteristic of the aircraft type.

The delay cost represents the different constant rate costs associated with aircraft operations (insurances, traffic control fees, crew salaries, etc). Total delay cost is given by:

$$
C_d = \pi_d(t_f - t_0) \qquad (33)
$$

where π_d is the cost attached to one unit of time of delay. Here the value of π_d could be a source for controversy, so this cost will be merged in a global noise index, NI, representative of the traffic management policy with respect to noise.

Aircraft noise is mainly composed of an omnidirectional source related with the aerodynamic noise and a directional source related with the engine noise. The level of noise P received at point $\boldsymbol{p}_G = [p_n, p_e, p_d]$ can be modeled as [19]:

$$
\begin{aligned}
P(p_n, p_e, p_d) = (P_a(d, a) + P_e(d, V_e))\cdot \\
\cdot\varpi_d(n - p_n, e - p_e, d - p_d, \gamma, \psi))\cdot \\
\cdot\varpi_m(n - p_n, e - p_e, d - p_d, w_n, w_e, w_d)
\end{aligned}
\qquad (34)
$$

where P_a is the power of aerodynamic noise, P_e is the power of engine noise, ϖ_d is related with the directional effect of jet noise and ϖ_m is related with the noise distortion resulting from the wind. The impact of instant noise over an individual location $\boldsymbol{p}_G = [p_n, p_e, p_d]$ is then given by:

$$
\frac{\Omega(P(p_n, p_e, p_d))}{((n - p_n)^2 + (e - p_e)^2 + (d - p_d)^2))} \qquad (35)
$$

where Ω is a logarithmic function.

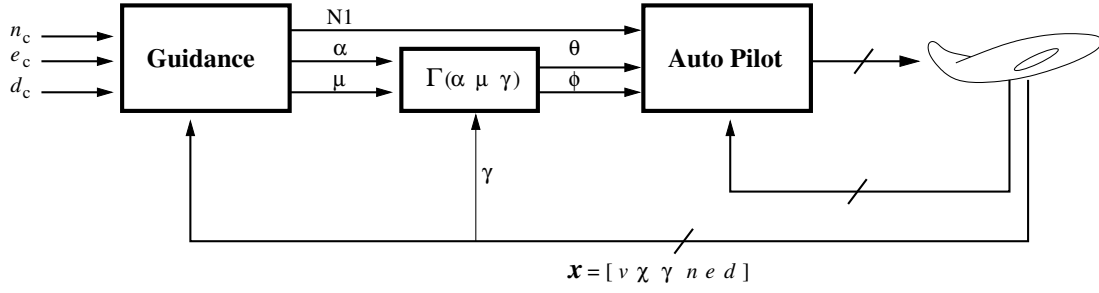Then an aggregated measure of the noise nuisances over the ground area surrounding the aircraft can be computed by:

Fig. 2. Flight guidance schema

$$\Phi_n(n, e, d) = \int_{A_n} \int_{A_e} \int_{A_d} \sigma(p_n, p_e, p_d) \cdot$$
$$\cdot \frac{\Omega(P(p_n, p_e, p_d))}{((n - p_n)^2 + (e - p_e)^2 + (d - p_d)^2))} dn de dd \qquad (36)$$

where σ is a density function which takes into account all noise nuisance factors and $A_n \times A_e \times A_d$ is the surrounding area. Considering Φ_n a penalty, a proportional cost (π_n) to this measure could be applied to the operator of the aircraft. Then, the total cost associated with noise is given by:

$$C_n = \pi_n \int_{t_0}^{t_f} \Phi_n dt \qquad (37)$$

And finally, the total cost for the operator of the aircraft is:

$$C = \int_{t_0}^{t_f} [\pi_c FF + \pi_d + \pi_n \Phi_n] dt \qquad (38)$$

B. Cost Index and Noise Index

In order to optimize aircraft trajectories over space and time, a cost index is provided to the flight management system of modern aircraft. The cost index relates the cost of delay to the price of the fuel:

$$CI = \frac{\pi_d}{\pi_c} \qquad (39)$$

Economy flights are associated with low values of the cost index while more direct and faster flights are associated with high values of the cost index.

In the same way, a noise index NI can be also introduced:

$$NI = \frac{\pi_n}{\pi_c} \qquad (40)$$

Therefore, trajectories with higher Noise Index will minimize noise impact over surrounding populations, while lower Noise Index will give priority to fuel and/or delay considerations.

V. TRAJECTORY OPTIMIZATION

A canonical formulation of the optimization problem associated with the minimum of noise nuisance flight trajectories is presented in this section and some very preliminary optimization results are also shown.

A. Optimization problem formulation and assessment

Let $\mathbf{x} \in \mathbb{R}^{\mathbf{N_x}}$ and $\mathbf{u} \in \mathbb{R}^{\mathbf{N_u}}$ be the state and control variables respectively as defined in relation 27. The optimal value of control and state variables would be determined by solving the following dynamic optimization problem in the time interval $[t_0, t_f]$:

$$\text{minimize} \qquad J(\boldsymbol{x}(t), \boldsymbol{u}(t), t_0, t_f) =$$
$$= E(\boldsymbol{x}(t_0), \boldsymbol{x}(t_f), t_0, t_f) + \int_{t_0}^{t_f} [F(\boldsymbol{x}(t), \boldsymbol{u}(t), t)] dt \qquad (41)$$

subject to:
- the dynamic constraints derived in section III:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}) \qquad (42)$$

- end point or event constraints (i.e. the initial and final boundary conditions):

$$\boldsymbol{e}_L \leq \boldsymbol{e}(\boldsymbol{x}(t_0), \boldsymbol{x}(t_f), t_0, t_f) \leq \boldsymbol{e}_U \qquad (43)$$

- mixed state-control path constraints:

$$\boldsymbol{h}_L \leq \boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), t) \leq \boldsymbol{h}_U \qquad (44)$$

- box constraint on the state and control variables:

$$\boldsymbol{x}_L \leq \boldsymbol{x}(t) \leq \boldsymbol{x}_U$$
$$\boldsymbol{u}_L \leq \boldsymbol{u}(t) \leq \boldsymbol{u}_U \qquad (45)$$

Function E can optionally model an extra cost, depending on the initial and final values of the trajectory, that would be associated to airspace capacity and efficiency criteria. On the other hand, function F contains the minimization criterion (as outlined in section IV) as:

$$F(\boldsymbol{x}(t), \boldsymbol{u}(t), t) = FF + CI + NI\Phi_n \qquad (46)$$

The vectorial functions $\boldsymbol{e}$ and $\boldsymbol{h}$ will define the event and path constraints respectively and vectors $\boldsymbol{e}_L$, $\boldsymbol{e}_U$, $\boldsymbol{h}_L$, $\boldsymbol{h}_U$, $\boldsymbol{x}_L$, $\boldsymbol{x}_U$, $\boldsymbol{u}_L$ and $\boldsymbol{u}_U$ are respectively the *Lower* and *Upper* values which will bound all constraints. All these functions and variables will be defined in the trajectory constraints component, as explained in section II.

In this problem $\boldsymbol{u} = [\alpha, \mu, N1]^T$ is the input (control) vector and drive the space-time aircraft trajectory. Of course the above problem is far from being trivial and cannot today be solved with accuracy by current on-board computers. Dynamic programming (see, for instance [20]) is one of the possible optimization techniques able to cope with this general optimization problem, but this implies the discretization, either over time or over space, of the whole problem.

As a first step, the optimization problem associated to the presented framework will be solved by using DIDO, a MATLAB application package. DIDO provides tools for solving a broad class of *Smooth and Non-smooth Hybrid Optimal Control* problems defined over a time interval $[t_0, t_f]$ that may be fixed or free. The basic idea behind the solution method is an adaptive algorithm based on a pseudospectral approximation theory [21]. The pseudospectral approach is significantly different from prior methods used to solve such problems [22] and hence the code is a realization of a fundamentally different way [23] of rapidly solving dynamic optimization problems. Currently, DIDO implements approximations of state and control functions in Hilbert spaces and employs the NLP solver SNOPT [24], through TOMLAB [25]. In particular, the type of optimal control problems that can be solved with this package, corresponds with the formulation presented above in equations 41 to 45.

B. Preliminary optimization results

A extremely basic problem has firstly used in order to test the optimization techniques presented above. A hypothetic straight take-off of a four engine aircraft was considered. The trajectory optimization starts at a point where the aircraft reaches V_2 safety operational speed and it is supposed to be at $400\,ft$ above the departing runway elevation. Only the final altitude is fixed to $3000\,ft$, leaving all the other variables free (final position, speed, flight path angle...). Finally some obvious constraints are also implemented in order to restrict for example speed and flight path angle to realistic values.

Aircraft performance data was obtained from BADA data base ([18]) provided by EUROCONTROL

In this first simulation no noise abatement constraints were considered and a mixed fuel-time minimum trajectory was obtained. Figure 3 show the vertical flight profile and figure 4 the speed profile. As it can be easily seen, the obtained optimal trajectory consist in to perform a initial horizontal segment allowing the speed to increase to a optimal value. Then a constant speed climb is performed until the final trajectory altitude is achieved. This result is in perfect accordance with the typical climb profiles performed by airline operators.

VI. CONCLUSION

A framework for a global optimization tool that will take into account aircraft dynamics and performances, noise nuisances and RNAV radionavigation requirements in order to assess an optimum flight depart or approach procedure is presented in this work. The involved scenario defines a given amount of noise nuisance, in function of the emited aircraft
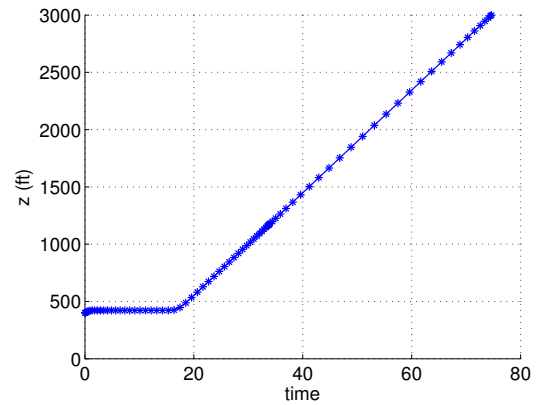

Fig. 3. Vertical flight profile for minimum fuel-time optimization
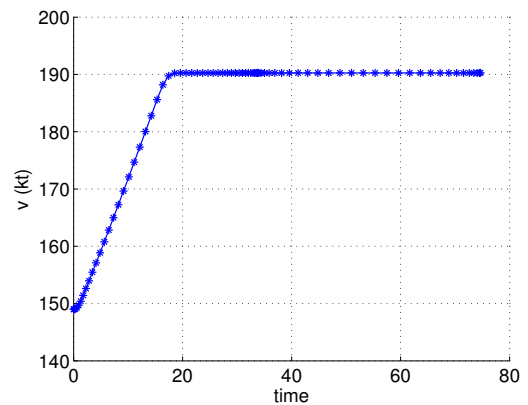

Fig. 4. Speed profile for minimum fuel-time optimization

noise along its trajectory and geographical and sociological considerations. This value, jointly with some fuel/time economy considerations, conduces to a global optimization criterion that will be minimized obtaining the best flight trajectory satisfying the procedure constraints. In addition, a complete guidance model is obtained, in a state representation form in order to be integrated into the envisaged optimization methods for dynamic systems. The formulation of the problem is formally presented as an optimal control problem, appearing to be a complex issue and requiring specific aerodynamics, propulsive and noise emission models. Further work may deal with the specific noise nuisances model and with the viability of solving the problem by using DIDO MATLAB package and the assessment of the potential use of another additional optimization techniques. First attempts to solve a simplified version of the initial problem are also presented, encouraging further work using the presented methodology.

REFERENCES

[1] "http://www.eurocontrol.int."
[2] "http://www.ecacnav.com/navigation_strategy.htm."
[3] "http://www.eurocontrol.int/corporate/public/standard_page/cb_atm_strate%gy.html."

[4] *Procedures for Air Navigation Services - Aircraft Operations (PANS-OPS) - Volume I, Flight Procedures*, 4th ed., International Civil Aviation Organisation (ICAO), Montreal (Canada), 1993, doc. 8168-OPS/611.

[5] S. I. Consortium, "Establishment of noise abatement solutions," Report D3, Tech. Rep., 2000.

[6] ——, "Definition of new noise abatement procedures," Work Package 3, Tech. Rep., Mar. 2003.

[7] V. F. Gomez Comendador, "Determinacin de condiciones de compatibilidad en un TMA de tamao medio de procedimientos ACDA de empuje mnimo, y perfil vertical variabla o perfil de velocidad veriable, con procedimeintos convencionales, miniizando su impacto sobre la capacidad," Ph.D. dissertation, Escuela Tcnica Superior de Ingenieros Aeronuticos. Universidad Politcica de Madrid, Madrid (Spain), Feb. 2004.

[8] L. Ren, J.-P. Clarke, and N. T. Ho, "Achieving low approach noise without sacrificing capacity," in *22nd Digital Avionics Systems Conference*, Oct. 2003.

[9] R. Wijnen and H. Visser, "Optimal departure trajectories with respect to sleep disturbance," *Aerospace Science and Technology*, vol. 7, pp. 81–91, 2003.

[10] H. Visser and R. Wijnen, "Optimisation of noise abatement arrival trajectories," *The aeronautical journal*, vol. 107, no. 1076, pp. 607–615, 2003.

[11] *Integrated Noise Model (INM) Version 6.0 Technical Manual*, Office of Environment and Energy, Washington, DC. USA, Jan. 2002, fAA-AEE-02-01.

[12] J.-P. Clarke and R. J. Hansman, "A systems analysis methodology for developing single event noise abatement procedures," MIT Aeronautical Systems Laboratory, Cambridge, Massachusetts (USA), Tech. Rep., 1997, report No ASL-97-1.

[13] K. Feng Zou and J.-P. Clarke, "Adaptive real-time optimization algorithm for noise abatement approach procedures," in *AIAA's 3rd Annual Aviation Technology, Integration, and Operations (ATIO) Technology Conference*, vol. 1, Nov. 2003, AIAA paper No 2003-6771.

[14] H. Visser, "Generic and site spacific criteria in the optimization of noise abatement procedures," *Transportation Research Part D: Transportation and Environment*, vol. 10, pp. 405–419, Sept. 2005.

[15] *Procedures for Air Navigation Services - Aircraft Operations (PANS-OPS) - Volume II, Construction of Visual and Instrument Flight Procedures*, 4th ed., International Civil Aviation Organisation (ICAO), Montreal (Canada), 1993, doc. 8168-OPS/611.

[16] J. Roskam, *Airplane flight dynamics and automatic flight controls*, 3rd ed. Lawrence, Kansas (USA): DARcorporation, 2001, vol. 1.

[17] B. L. Stevens and F. L. Lewis, *Aircraft control and simulation*. London, UK: John Wiley and Sons, INC., 1992.

[18] *User manual for the base of aircraft data (BADA). Revision 3.6*, EUROCONTROL Experimental Centre, Bretigny (France), Sept. 2004, aCE-C-E2.

[19] F. Nejjari, X. Prats, V. Puig, J. Quevedo, M. Polit, B. Ouattara, K. Achaibou, and F. Mora-Camino, "Containing aircraft noise levels at take-off: a mathematical programming approach," in *The 2005 Congress and Exposition on Noise Control Engineering*, Rio de Janeiro (Brazil), Aug. 2005.

[20] R. Bellman, *Dynamic Programming*. Courier Dover Publications, 2003.

[21] I. Ross and F. Fahroo, "A direct method for solving nonsmooth optimal control problems," in *Proceedings of the 2002 IFAC World Congress*, Barcelona (Spain), July 2002.

[22] J. T. Betts, "Survey of numerical methods for trajectory optimization," *Journal of Guidance, Control and Dynamics*, vol. 21, no. 2, 1998, 193-226207.

[23] J. Strizzi, I. Ross, and F. Fahroo, "Towards real-time computation of optimal controls for nonlinear systems," in *Proceedings of the AIAA Guidance, Navigation and Control Conference*, Monterey, California (USA), Aug. 2002, invited Paper no. AIAA 2002-4945.

[24] P. Gill, W. Murray, and M. Saunders, "Snopt: An sqp algorithm for large-scale constrained optimization," *SIAM Journal of Optimization*, vol. 12, no. 4, pp. 979–1006, 2002.

[25] K. Holmstrom, A. Goran, and M. M. Edvall, *User's guide for TOMLAB 4.0.6*, Tomlab Optimization, Sweden, Aug. 2003.