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# Noise nuisance model for optimizing flight trajectories around airports

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#### **ABSTRACT**

This work presents a noise nuisances model that takes into account not only the perceived noise but also the sensibility of the zone where this noise is produced. Several factors are considered, like the activity of the zone (residential, industrial, medical...), the time of the day and day of the week, the environmental noise already existent in the zone, the periodicity of the noise for several sequential trajectories, etc. By means of a noise propagation model giving the Sound Exposure Level (SEL) or the *Lmax* values at certain locations, a noise nuisance model will be developed in order to supply different optimization criteria. Then, multi-objective optimization is presented and specifically, lexicographic optimization is identified as the most suitable technique that can be used to compute minimum nuisance flight trajectories. Some preliminary results are finally given showing the feasibility and suitability of this particular multi-objective optimization technique.

#### INTRODUCTION

The continuing growth in air traffic demand and the rising level of urbanization around many important airports makes noise reduction over sensible areas in the vicinity of the airports one of the main issues that airport authorities, air traffic service providers and aircraft operators may deal with. In this context, international and national regulations, regarding noise exposure, have been established by civil aviation authorities in order to cope with this problem but incurring, on the other hand, in higher operations costs for airlines.

At present, existing noise abatement procedures are far from being the optimal ones minimizing noise nuisances and/or airliners costs. This is due to several factors like the impossibility to define a general criterion fitting all airports necessities, the limitations of nowadays technology on-board, and the constraints imposed by airport capacity or air traffic control issues. However, several recent research contributions [1-8] combine trajectory constraints, aircraft dynamic models and noise models to generate, by dynamic optimization, those trajectories that minimize the noise impact of depart or approach procedures. In general, the noise model associates a numerical performance measure to the noise physical impact generated during a given trajectory. Usually this measure is expressed in terms of Sound Exposure Level (SEL), even though some authors [4-5] consider alternative measures, such as sleep disturbances, which in fact are functions of the SEL measurement.

First section of this paper briefly describes the main noise measurements used world-wide and proposes a set of noise nuisance models. Second section is devoted to multi-objective optimization basic background theory and introduces weighting and lexicographic optimization techniques. Finally, last section shows some preliminary results by using lexicographic optimization in minimizing aircraft's noise nuisances.

#### 1. ASSESSMENT ON PERCEIVED NUISANCES OF AIRCRAFT NOISE

The aircraft noise is an unwanted sound in the vicinity of the airport that disturbs our routine activities or peace and quiet and causes a feeling of annoyance or perceived nuisance. In order to minimize the effect of aircraft noise when designing optimal departure trajectories, two tasks may be considered. First of all, the emitted noise shall be measured and then, and not less important, we should quantify people's reactions against noises which disrupt their routine activities or tranquility.

Levels of annoyance or disturbance are influenced by the nature of the activities being undertaken at the moment of the noise and are related to the time of day and day of the week. The quality of life of many people living under approach or departure flight paths can be affected by aircraft noise. These effects arise from the effect of noise on concentration or sleep and from feelings of anger, frustration and powerlessness to control the noise [9].

#### 1. Models on noise measurement

Environmental noise is measured with reference to the A-weighted decibel scale, dBA. This reflects the fact that the human ear does not detect all frequencies of sound equally. To quantify sound levels, which vary with time, equivalent continuous sound level or *Leq* is calculated. This indicates the average sound level over a particular time period. Other measures of noise are also available, that relate to different measurement periods, such as the instantaneous maximum noise level (*Lmax*), or the average over certain periods, such as evening or night (such as the *Lden*).

#### 1.1.1. Maximum A-weighted Sound Level (Lmax)

One possible acoustic metric used to describe the sound environment is to measure the maximum sound level of a single sound event (like a flyover). This metric is one important component of the noise measurement but not solely, the time elapsed for the sound event is also an important component to be measured.

## 1.1.2. Sound Exposure Level (SEL)

The Sound Exposure Level measures the total perceived sound energy (in dBA) in a single event (flyover) and presents it as though it took place in one second. This metric allows comparing events that vary in duration.

### 1.1.3. Day-Night Average Sound Level (DNL)

The Day-Night average sound level metric is an accumulative measure of the perceived sound exposure (in dBA) during a 24-hour day. Generally, a 10 dBA penalty [3] is added to noise events occurring between 10:00 pm and 7:00 a.m. to take into account their greater intrusiveness and potential for disturbing sleep. The DNL is useful to take into account for the daily air traffic planning in the airport, but it is not a sense to measure the noise impact in the vicinity of the airport for a given individual aircraft trajectory.

### 2. Noise nuisances model

Aircraft noise disturbs the normal activities of airport neighbours, their conversation, sleep, and relaxation, and degrades their quality of life. Depending on the use of land contiguous to an airport, noise may also affect education, health services, and other public activities.

Each airport is unique with respect to runway configuration, layout of airspace, fleet composition, aircraft movement patterns, traffic distribution, land use planning, and commercial and environmental conditions. Thus, the impact of aircraft noise differs between airports and developing appropriate actions for aircraft noise abatement is a local issue for each airport.

Noise nuisance or annoyance describes a relation between an acoustic situation and a person or a set of persons who are forced by noise to do, who cognitively and emotionally evaluates this situation [9]. The perception of flight noise nuisance will largely depend on the acoustic characteristic (i.e. SEL metric) but this is not the only factor that completely represents the behavior of the noise nuisance. Several studies of acoustic and non acoustic factors (see for instance [10] or [11]) have been made to define quantitative or qualitative (based on fuzzy or neural networks) noise nuisance models.

Examples of non acoustic factors which can affect to noise nuisance on the vicinity of an airport are: degree of urbanization (city, residence, country), population density, household size, age of population, number of children by family, type of activities (agriculture lands, industries, commerce), type of services (schools, hospitals, markets).

In this work, we present the analysis of noise nuisance perception for four cases of study, which are very common in the vicinity of airports: a hospital, a school, a market and a residence area. Each case will be analyzed according to its own peculiarities and will be used separately in a further optimization process aimed at minimizing the noise nuisances in each case (Figure 1).

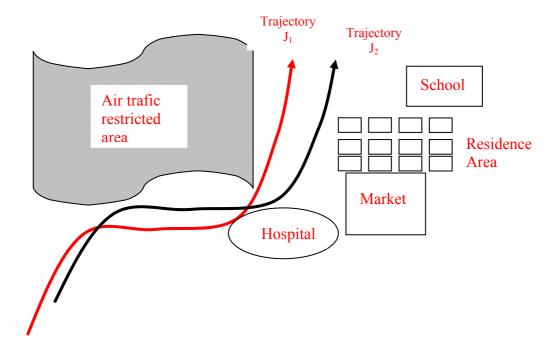


Figure 1: Scheme example of the vicinity of an airport

## 1.2.1. Nuisance model for a hospital

The perception of a noise by a patient in a hospital is higher than by a normal healthy person. For instance, we propose to double the perception of the noise, which is equivalent to add 3 dBA to the SEL value, and to consider the nuisance constant during all the day (from 7 a.m. to 10 p.m.). Following the recommendations of other authors [3], an additional penalty of 10 dBA is added to noise events occurring in night time (between 10:00 pm and 7:00 a.m.). Finally, the accumulated nuisance for a given hospital depends of the number of people concerned: Phospital. Let be *Ji* an aircraft trajectory occurred at time *Ti*. Then, the nuisance in the hospital is defined by:

$$N_{hospital}(Ji, Ti) = (SEL + 3 + 10 * NightPeriod) * P_{hospital}$$
(1)

## 1.2.2. Nuisance model for a school

Environmental aircraft noise may affect children's school performance. A study in Munich reported poor reading performance and long term memory among pupils near the airport [12]. Study performance improved after the airport closed, but test scores of children living near the new replacement airport fell. Similarly, a study in New York found reading impairments in schoolchildren exposed to over 65 dB(A) Leq [9]. On the other hand, the specific aim of the study presented in [13] confirms that chronic high levels of aircraft noise exposure in children are associated with cognitive impairments (in reading, memory and attention); and stress responses (catecholamine secretion, noise annoyance and self reported stress).

The accumulated noise nuisance on school's performance depends on the lecture period. For a given trajectory Ji occurred at time Ti, the noise nuisance is defined by:

$$N_{school}(Ji, Ti) = (SEL + 3 * Lecture Period - SEL * Nolecture Period) * P_{school}$$
 (2)

Where P<sub>school</sub> is the number of people in the school.

### 1.2.3. Nuisance model for a market

Aircraft noise may disturb the normal activity in a market localized in the neighborhood of the airport. The accumulated noise nuisance on the market activity is defined, for a given trajectory Ji at time Ti, by:

$$N_{market}(Ji, Ti) = (SEL - SEL * NoMarketTime) * P_{market}$$
(3)

Where P<sub>market</sub> is the estimated number of people present in the market.

### 1.2.4. Nuisance model for a residential area

People living near airports are concerned about health effects of aircraft related pollution and safety. These concerns are substantiated by findings that aircraft noise may have adverse health effects such as annoyance, sleep disturbance, and cardiovascular diseases. People

living directly underneath a flight path are more annoyed than people living outside the path. The fear of crashes in the neighbourhood is an important factor for generating annoyance with aircraft noise [9]. The perception of annoyance is affected by both emotional and physical attributes such as time of day (a 10 dB night time "penalty" is incorporated into the model), season (noise is considered more disturbing in the summer than in the winter since windows may be open during the summer), length of the time of exposure to noise and state of activities. For a given trajectory *Ji* occurred at time *Ti*, the accumulated noise nuisance model is given by:

$$N_{residence}(Ji, Ti) = (SEL - 3 * DayPeriod + 10 * NightPeriod) * P_{residence}$$
 (4)

Where P<sub>residence</sub> is the number of people living in the residential area.

To sum up, the global nuisance model, in the vicinity of the airport, will be the addition of each particular noise nuisance as presented above:

$$N(Ji, Ti) = N_{hospital}(Ji, Ti) + N_{school}(Ji, Ti) + N_{market}(Ji, Ti) + N_{residence}(Ji, Ti)$$
(5)

## 3. Optimization criterion

Classical optimization for aircraft trajectories took into account fuel and time consumption, which are the two parameters that aircraft operators are interested in minimizing. Minimum fuel trajectories are not necessarily the same as the minimum time ones and, in fact, both criteria are usually opposed and a trade-off must be defined. The usual formulation in current Flight Management Systems on board airplanes consists in minimizing a mixed cost criterion built up from fuel and time consumption contributions:

$$Cost = \int_0^T (FF(t) + CI)dt \tag{6}$$

being T the total time of the studied trajectory or maneuver, FF(t) the fuel flow (i.e. the time-rate of fuel consumed) and CI the "Cost Index", which serves to increase the weight of the consumed time in the global optimization criterion.

Depending on numerous variable factors (fuel price, predicted flight delays, aircraft operator's policy, etc...) a different CI value is used before each flight in order to focus on minimizing fuel or time related costs.

Nowadays, some authors suggest the introduction of a "Noise Index" (NI) into the optimization process in order to take into account noise nuisances as well and model them as additional operating costs [1]. Therefore, a formulation like the following may be proposed:

$$Cost = \int_0^T (FF(t) + CI + NI\Phi(t))dt$$
 (7)

where  $\Phi(t)$  is a noise measure of the studied trajectory, or even more, a noise nuisance measure, which can be in turn formed by several nuisance contributions, as presented in above sections.

The optimization criteria presented in equations (6) and (7) take into account different individual criteria which must be conveniently weighed in order to give them the desired importance. This weighing process is not always easy to perform, and optimal results may differ significantly in function of the chosen weights. In addition noise nuisance measurements  $\Phi(t)$  can also be formed by a set of different contributions (as presented in above sections) needing more weighting and not easy or straightforward decision making. In [14] it is presented a noise performance trade-off between arrival trajectories that are optimized according to different types of noise abatement criteria. Typically, these different criteria are not compatible and the variables that optimize one objective may be far from optimal for the others, pointing out the difficulty to properly identify the absolute minimal trajectory among all the local minimal ones. But, not only weighing methods can be used in such multi-objective problems, goal hierarchical or lexicographic approaches may provide a different point of view and will be explained in next section.

#### 2. MULTIOBJECTIVE OPTIMIZATION

#### 1. Introduction

The solution of the optimization problem associated with the computation of the trajectory that minimizes noise (or noise nuisances) is a multi-objective optimization problem as stated in previous section. In [15] a recent survey in this field can be found.

In general, a multi-optimization problem can be formulated in the following way:

$$\min_{x} f_1(x), \dots, f_r(x) 
subject to: x \in \chi$$
(8)

where the set  $\chi$  is given by a set of constraints:  $g_{i}(x) \le 0, j = 1,...,m$ 

A solution  $x^* \in \chi$  is said to be Pareto optimal if and only if there does not exist a  $x \in \chi$  and an i such that  $f(x) \le f(x^*)$  and  $f_i(x) \le f_i(x^*)$ . In other words, a solution is (global) Pareto optimal if and only if an objective  $f_i$  can be reduced at the expense of increasing at least one the other objectives. A global Pareto solution can only be guaranteed if the multi-objective function is convex. In general, there may be many Pareto optimal solutions. Generating Pareto optimal solutions plays an important role in multi-objective optimization. Mathematically, the problem is considered to be solved when the Pareto optimal set is found. However, this is not always enough providing that we usually want to obtain only one solution. This means that we must find a way to put the Pareto optimal solutions in a complete order and the final decision is made among them taking the total balance over all criteria into account. This is a problem of value judgment of decision making (DM). The totally balancing over criteria is usually called trade-off the objective i and can be reduced at the expense of increasing at least one of the other objectives.

In general, multi-objective optimization problems are solved by scaling. This means converting the problem into a single or a family of single objective optimization problems with a real-valued objective function. This objective function is called the scaling function and it may be a function of some parameters. This enables the use of the theory and the methods of scalar optimization. An important fact to keep in mind is that standard optimization algorithms for single optimization problems can only find local optima. This is why only locally Pareto optimal solution are usually obtained and handled when dealing with scaling the function. Global Pareto optimality can be guaranteed if the objective functions and the feasible region are convex, as already noticed. In case convexity is not satisfied, global Pareto optima only can be guarantee using global optimization algorithms.

In [16] there is a classification of multi-objective optimization methods according to the participation of the decision maker in the solution process:

- *No-preference* methods where no articulation of preference information is used.
- A posteriori methods where a posteriori articulation of preference information is used
- A priori methods where a priori articulation of preference information is used
- *Interactive methods* where progressive articulation of preference information is used.

## 2. Weighting method

One of the most well known multi-objective techniques is the linearly weighted sum, where the vector objective function is scaled in such a way that the value judgment of the decision making can be incorporated:

$$\min_{x} \sum_{i=1}^{r} w_{i} f_{i}(x) 
subject to: x \in \chi$$
(9)

where  $w_i$  are the different scaling weights.

This is a posteriori method. However, it can be used so that the decision maker specifies a weighting vector representing his preference information. When used in such a way, this method can be considered to belong to the class of a priori methods. Although this type of scaling is widely used in many practical problems, there is a serious drawback in it [15]. Namely, it can not provide a solution among sunken parts of Pareto surface due to a duality gap for non-convex cases. For convex cases, for example linear cases, even if we want to get a point in the middle of line segment between two vertices, we merely get a vertex of Pareto surface, as long as the well known simplex method is used. This implies that depending on the structure of problem, the linearly weighted sum can not necessarily provide a solution as the decision maker desires.

### 3. Lexicographic method

In many applications (as in the case of the present paper), a hierarchy between objectives can be defined "a priori" according to their absolute importance. Let the objective functions

be arranged according to the lexicographic order from the most important  $f_1$  to the least important  $f_k$ . We can write the lexicographic problem as:

$$lex \min_{x} f_1(x), ..., f_r(x) 
subject to: x \in \chi$$
(10)

This hierarchy defines an order on the objective function establishing that a more important objective is infinitely more important that a less important objective and this makes possible to automatically establish a trade-off between the conflicting objectives.

A given  $x^* \in \chi$  is a lexicographic solution of (10) if and only if there does not exist a  $x \in \chi$  and  $i^*$  satisfying:  $\min_i \{i \mid f_i(x) \neq f_i(x^*)\}$  such that  $f_i^*(x) \leq f_i^*(x^*)$ .

An interpretation of the above definition is that a solution is a lexicographic solution if and only if an objective  $f_i$  can be reduced only at the expense of increasing at least one of the higher-priorized objectives  $\{f_1,...,f_{i-1}\}$ . Hence, a lexicographic solution is a special type of Pareto-optimal solution that takes into account the order of the objectives.

A standard method for finding a lexicographic solution is to solve a hierarchical of single objective constrained optimization problems. After ordering, the most important objective function is minimized subject to the original constraints. If this problem has a unique solution, it is the solution of the whole multi-objective optimization problem. Otherwise, the second most important objective function is minimized. Now, in addition to the original constraints, a new constraint is added. This new constraint is there to guarantee that the most important objective function preserves its optimal value. In this problem has a unique solution, it is the solution of the original problem. Otherwise, the process goes on as above. More formally,  $f^* := [f_1^*, ..., f_r^*]$  is the lexicographic minimum of (10) if and only if

$$f_1^* = \min_{x \in \chi} f_1(x) \tag{11}$$

and for all  $i \in \{2,...,r\}$ 

$$f_i^* = \min \left\{ f_i(x) \, \middle| \, f_j(x) \le f_j^*, \, j = 1, ..., i - 1 \right\}$$
 (12)

x\* is the lexicographic minimizer of problem (10) if and only if

$$x^* \in \left\{ x \in \chi \middle| f_j(x) \le f_j^*, j = 1, ..., r \right\}$$
 (13)

#### 3. LEXICOGRAPHIC OPTIMIZATION FOR NOISE ABATEMENT PROCEDURES

In order to test the lexicographic optimization approach applied to noise abatement procedures few preliminary simulations have been performed. A simplified straight take-off for a four engine aircraft has been adopted as the case study. The trajectory of the aircraft is supposed to start at 400ft above ground level with an initial airspeed of 1.2Vs, where Vs is the stall speed and depends of the aerodynamics and weight of the aircraft. The take-off phase going from ground-level to 400ft is not considered in the optimization process since the standard operational regulations almost restrict all degrees of freedom during this particular phase. However, once 1.2Vs speed and 400ft height are achieved, operational regulations become less restrictive and some trajectory optimization can be conducted. In addition to last two initial conditions, just the final height of the trajectory is fixed to 3000ft, leaving "free" all the other remaining variables in the simulations performed in this study.

The lexicographic multi-objective criterion is adopted to be:

$$lex \min(f_n, f_f)$$

$$with \quad f_f = \int_0^T FF(t)dt \quad and \quad f_n = \int_0^T \phi(t)dt$$
(14)

where T is the final time of the trajectory,  $f_f$  is the fuel associated cost, being FF(t) the fuel flow, and  $f_n$  the noise nuisance associated cost.

In this preliminary simulation  $\phi(t)$  has been considered as the *Lmax* value received at a single noise measurement point placed 2000 m after the trajectory initial point.

The optimization must be carried out under the initial and final conditions described in last paragraph and considering the following simplified differential model for the aircraft dynamics:

$$\dot{x} = v \cos \gamma 
\dot{z} = v \sin \gamma 
\dot{v} = \frac{1}{m} [T_{\text{max}} - D_{\text{max}}(v, \gamma)] - g \sin \gamma$$
(15)

where the state vector (x,z,v) is the aircraft's horizontal track position, altitude and speed, respectively, while  $\gamma$  is the flight path angle. m is the mass of the aircraft, g is the gravity acceleration, D the aerodynamics drag force and  $T_{max}$  the maximum thrust at take-off.

Aerodynamic force can be modeled in function of aircraft's speed and flight path angle and a set of constant parameters which were obtained from the BADA data base [17]. In the same way, aircraft thrust is considered constant during the take-off phase concerned by this study.

Lexicographic optimization was carried out, considering  $f_n$  the most important criterion, followed by  $f_f$ . Next table summarizes the fuel consumed and the noise perceived at the

measuring point for three different trajectories: minimum noise trajectory, minimum fuel trajectory and lexicographic optimization trajectory.

Table 1. Preliminary optimization results

$f_n$ noise minimization	Fuel = 1610  kg	Noise $Lmax = 58.46 dBA$
$f_f$ fuel minimization	Fuel = $857 \text{ kg}$	Noise Lmax = $69.88 \text{ dBA}$
$f_n$ , $f_f$ lexicographic minimization	Fuel = 1440  kg	Noise Lmax = 58.46 dBA

As it can be seen, lexicographic optimum gives the best noise trajectory and saves approximately a 12% in fuel consumption that if only the optimum noise trajectory is considered. Figure 1 shows the optimum vertical profile of the previous three trajectories, and Figure 2 shows the corresponding flight path angle and speed profiles needed to achieve these trajectories.

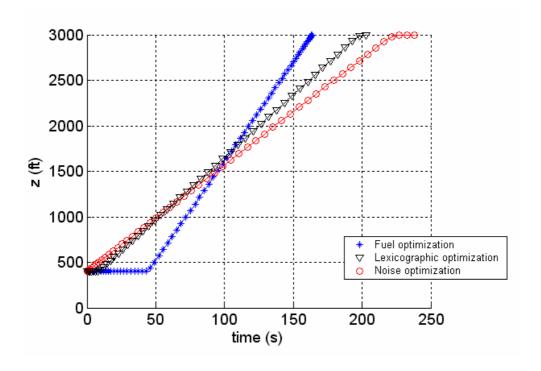


Figure 2. Vertical profile for the three different optimized trajectories.

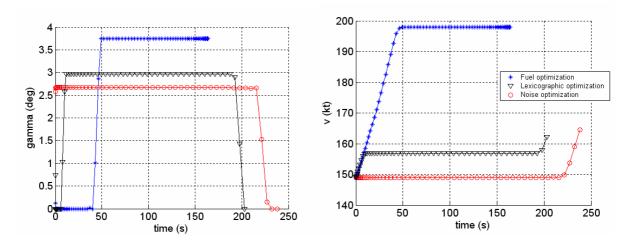


Figure 3. Flight path angle (gamma) and speed of the aircraft for the three different optimized trajectories.

#### **CONCLUSIONS**

A noise nuisance model that takes into account not only the perceived noise but also the sensibility of the zone where this noise is produced is presented in this work. Hospital, residential, market and school nuisances are conveniently modeled. This work empathizes the complexity of the decision making when all individual criteria must be put together in order to form a final global criterion. Then, two different multi-objective optimization techniques are briefly explained and compared: weighting optimization and lexicographic optimization, which in turn, is identified as the most suitable technique that can be used to compute minimum nuisance flight trajectories. Some preliminary results are finally given, considering a very simple trade-off between simplified measured noise and fuel consumption, but showing encouraging results proving the feasibility and suitability of this particular multi-objective optimization technique. Further work may deal with the improvement of the aircraft dynamical model, noise-propagation model and the lexicographic technique will be applied using all the defined noise nuisance criteria considered in the first part of this paper.

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