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Robust Hamiltonian passive control for higher relative degree outputs*

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Abstract

We present an improvement of the well-know Interconnection and Damping Assignment–Passivity-based Control (IDA-PBC) technique. The IDA-PBC method is based on the Port-controlled Hamiltonian System description and requires the knowledge of the full energy (or Hamiltonian) function. This is a problem because, in general, the equilibrium point which is to be regulated depends on uncertain parameters. In this paper, we show how select the target port-Hamiltonian structure so that this dependence is reduced. This new approach allows to improve the robustness for higher relative degree outputs, and, for illustration purposes, is applied to a simple academic nonlinear system.
Keywords: *port Hamiltonian systems, passivity-based control, robust control.*

1 Introduction

Interconnection and Damping Assignment–Passivity-based Control (IDA-PBC) [9] is a technique which applies passivity based control to dissipative port-Hamiltonian systems (PHDS) [2].

As discussed in [2][5][6] (and references therein) a large class of physical systems of interest in control applications can be modeled in PHDS framework. A general PHDS in explicit form is described by

$$\dot{x} = [J(x) - R(x)]\partial_x H + g(x)u \tag{1}$$

where x is the state in Hamiltonian variables, $H(x)$ is the total energy of the system[1], $J(x) = -J^T(x)$ is the interconnection matrix and $R(x) = R^T(x) \geq 0$ the dissipation

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[1] Gradients of functions, as in $\partial_x H$, are taken as column vectors to simplify the notation.

matrix. It is easy to see that PHDS are passive with port variables $(u, g^T(x)\partial_x H)$, and storage function the total energy H.

The central idea of the IDA-PBC technique [9] is to, still preserving the PHDS structure, assign to the closed loop a desired energy function via the modification of the interconnection and dissipation matrices. That is, the desired target dynamics is a PHDS of the form

$$\dot{x} = [J_d(x) - R_d(x)]\partial_x H_d \tag{2}$$

where $H_d(x)$ is the closed-loop total energy and $J_d(x) = -J_d^T(x)$, $R_d(x) = R_d^T(x) \geq 0$, are the closed-loop interconnection and damping matrices, respectively. To achieve stabilization of the desired equilibrium point we impose

$$x^* = \arg\min H_d(x).$$

Equaling the open-loop and target closed-loop systems one gets

$$[J_d(x) - R_d(x)]\partial_x H_d = [J(x) - R(x)]\partial_x H + gu. \tag{3}$$

IDA-PBC techniques have been applied to a large class of physical systems, including electromechanical [1][10], mechanical underactuated [8], power electronics [4][11] and power system [7] models. The standard way to solve (3) is to fix the matrices $J_d(x)$ and $R_d(x)$—hence the name IDA—and then solve the PDE for $H_d(x)$. In general, solving the PDE is a very complicated task, which can be somehow eased by a judicious chose of J_d and R_d. Alternatively, one may try to fix H_d and solve the resulting algebraic equation for J_d and H_d. In any case, once the target system is obtained, asymptotic stability of x^* follows from $R_d \geq 0$ and the application of LaSalle's theorem to H_d, since the interconnection matrix does not contribute to the total energy variation and thus

$$\dot{H}_d = -(\partial_x H_d)^T R_d \partial_x H_d \leq 0,$$

and x^* is an invariant set of the closed loop dynamics.

Robustness of passive controllers for a relative degree one output is easily solved using an integral term by means a dynamical extension, see Section 2. But the problem remains open for higher relative degree outputs, because the dynamical extension breaks the skew-symmetry property of the J_d matrix. In [3] a generalized canonical transformation of port Hamiltonian systems is used to obtain an integral control system.

In this paper we present a new strategy for the design of robust controllers using the IDA-PBC approach. Robustness is achieved removing the unknown parameter dependence from the control law. A nice feature is that the desired dynamics is obtained fixing a part of the Hamiltonian function and then solving a simpler PDE equation. This trick was also applied for a complex system solving the IDA-PBC matching equation in [7]. We illustrate the main idea for a class of simple systems, although generalizations to a wider class of systems will be presented elsewhere.

The paper is organized as follows. Section 2 discusses the main limitation of integral action for higher relative degree outputs. Section 3 presents the main contribution of this work, a general result for designing robust controllers based on the IDA-PBC technique. In Section 4 the method is applied to a nonlinear example. Section 5 shows some numerical simulations of the obtained controller. Further dynamical studies of the closed-loop system are presented in Section 6, and finally Section 7 states our conclusions.

2 Discussion of the integral extension

In this section we explain why the integral term can be used in a PHDS framework for relative degree one outputs. To expose the basic idea, consider a fully actuated control system of the form

$$\begin{cases} \dot{x}_1 = f_1(x_1, x_2) \\ \dot{x}_2 = f_2(x_1, x_2) + g(x_1, x_2)u \end{cases} \tag{4}$$

where $x_1 \in \mathbb{R}^n$, $x_2 \in \mathbb{R}^m$ and $u \in \mathbb{R}^m$, and g is full rank. Assume the IDA-PBC technique can be applied to (4) so that in closed-loop the system becomes[2]

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} J_d - R_d \end{pmatrix} \begin{pmatrix} \partial_1 H_d \\ \partial_2 H_d \end{pmatrix}$$

with control law

$$u = g^{-1}\left((O_{m\times n} \quad I_{m\times m})(J_d - R_d)\partial H_d - f_2\right).$$

Under the stated assumptions, the x_2 are relative degree one outputs. We can easily add a dynamical extension to them by means of

$$\dot{z} = -a\partial_2 H_d, \tag{5}$$

where $a \in \mathbb{R}^{m\times m}$, see [3][12]. The whole closed loop system can be rewritten in Hamiltonian form as

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \begin{matrix} J_d - R_d \end{matrix} & \begin{matrix} 0 \\ a^T \end{matrix} \\ \begin{matrix} 0 & -a \end{matrix} & 0 \end{pmatrix} \begin{pmatrix} \partial_1 H_{dz} \\ \partial_2 H_{dz} \\ \partial_z H_{dz} \end{pmatrix}$$

with a new Hamiltonian function

$$H_{dz} = H_d + \frac{k}{2} z^T z.$$

The new controller is

$$v = u + g^{-1}ka^T z = u - g^{-1}ka^T a \int \partial_2 H_{dz}.$$

Notice that (5) forces $x_2 = x_2^*$ to remain a fixed point of the extended system.

The same procedure, when applied to the higher relative degree output x_1, requires a closed loop system of the form

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{z} \end{pmatrix} = \begin{pmatrix} \begin{matrix} J_d - R_d \end{matrix} & \begin{matrix} 0 \\ b \end{matrix} \\ \begin{matrix} -a & 0 \end{matrix} & 0 \end{pmatrix} \begin{pmatrix} \partial_1 H_{dz} \\ \partial_2 H_{dz} \\ \partial_z H_{dz} \end{pmatrix}$$

where now $a \in \mathbb{R}^{n\times n}$ and $b \in \mathbb{R}^{m\times n}$. The a term is used to force the equilibrium point x_1^* of the output, while b is necessary to put the integral action into the control law. In this

[2] To simplify the notation ∂_{x_s} is written as ∂_s, where s is the subindex of x.

case stability cannot be proved using the PHDS properties, since the a, b terms break the semi-definite positiveness of the dissipation matrix:

$$R_{dz} = \left(\begin{array}{ccc} \multicolumn{2}{c}{\multirow{2}{*}{R_d}} & a^T/2 \\ & & -b/2 \\ a/2 & -b^T/2 & 0 \end{array} \right).$$

Indeed, consider a matrix of the form

$$M = \left(\begin{array}{cc} A & B^T \\ B & D \end{array} \right).$$

A simple application of Schur's complement shows that if $D = 0$, then $B \neq 0$ implies $M < 0$. In our case, this would mean $a = 0$ and $b = 0$, which makes no sense.

3 Robustness discussion

As discussed in the previous Section, it is not clear how to generalize the integral extension for higher relative degree outputs in the PHDS framework. We present here a different approach, which can be applied to a larger class of systems. Examples include the DC motor, the electrical part of a doubly-fed induction machine or the buck power converter.

Consider a dynamical system of the form

$$\left\{ \begin{array}{l} \dot{x}_o = f_o(x_o, x_u, \xi) \\ \dot{x}_u = f_u(x_o, x_u) + g(x_o, x_u)u \end{array} \right. \tag{6}$$

where $x_o \in \mathbb{R}^o$ are higher order relative degree outputs, $x_u \in \mathbb{R}^u$, $u \in \mathbb{R}^p$ are the controls and ξ is an uncertain parameter. To simplify the presentation we consider $p = u = o$ and that g is full rank.

As a control target we fix a desired x_o^d, which implies that the fixed point value of x_u is given by the following equation

$$f_o(x_o^d, x_u^*, \xi) = 0, \tag{7}$$

and depends thus on the uncertain parameter ξ.

Applying the IDA-PBC technique, we match the system to the desired port Hamiltonian structure, where $(J_d - R_d)$ is partitioned as

$$\left(\begin{array}{c} \dot{x}_o \\ \dot{x}_u \end{array} \right) = \left(\begin{array}{cc} J_{doo} - R_{doo} & -J_{duo}^T - R_{duo}^T \\ J_{duo} - R_{duo} & J_{duu} - R_{duu} \end{array} \right) \left(\begin{array}{c} \partial_o H_d \\ \partial_u H_d \end{array} \right).$$

Each $J_{d\cdot\cdot}$ and $R_{d\cdot\cdot}$ represents the interconnection and dissipative terms of the J_d and R_d matrices, respectively. This implies that J_{doo} and J_{duu} must be skew-symmetric and similarly $R_{doo} = R_{doo}^T \geq 0$ and $R_{duu} = R_{duu}^T \geq 0$. Hence, the desired interconnection and damping matrices are

$$J_d = \left(\begin{array}{cc} J_{doo} & -J_{duo}^T \\ J_{duo} & J_{duu} \end{array} \right), \quad R_d = \left(\begin{array}{cc} R_{doo} & R_{duo}^T \\ R_{duo} & R_{duu} \end{array} \right).$$

Notice that we need a H_d such that $\partial H_d|_{x=x^*} = 0$ to obtain an equilibrium point in $x^* = (x_o^d, x_u^*)$. Equaling the u rows of the IDA-PBC matching equation (3) the control law yields

$$u = g^{-1}\left[(J_{duo} - R_{duo})\partial_o H_d + (J_{duu} - R_{duu})\partial_u H_d - f_u\right].$$

Since H_d is a free function, it is chosen so that $\partial_o H_d$ does not depend on ξ (Notice that x_u^* depends on it, equation (7)). In the same way, ξ can appear in $\partial_u H_d$ through x_u^*, which can be removed from the control law setting

$$J_{duu} - R_{duu} = 0,$$

and the robustified IDA-PBC control law is

$$u = g^{-1}\left[(J_{duo} - R_{duo})\partial_o H_d - f_u\right].$$

As we set $R_{duu} = 0$, again Schur's complement shows that in order to keep the semi-positiveness of R_d, we are forced to $R_{duo} = 0$, and consequently

$$u = g^{-1}\left[J_{duo}\partial_o H_d - f_u\right]. \tag{8}$$

From the o rows of the IDA-PBC matching equation (3) the following equation must be satisfied, were we fixed $R_{duo} = 0$,

$$f_o = (J_{doo} - R_{doo})\partial_o H_d - J_{duo}^T \partial_u H_d$$

Selecting J_{duo} full rank,

$$\partial_u H_d = -(J_{duo}^T)^{-1}\left[f_o - (J_{doo} - R_{doo})\partial_o H_d\right]. \tag{9}$$

Rewriting f_o as

$$f_o = A(x)\partial_o H_d + B(x_u),$$

and choosing $(J_{doo} - R_{doo})$ so that

$$A(x) = J_{doo} - R_{doo},$$

the PDE (9) simplifies to

$$\partial_u H_d = -(J_{duo}^T)^{-1} B(x_u). \tag{10}$$

Notice that J_{duo} must be a function of x_u only, $J_{duo} = J_{duo}(x_u)$. Fixing a part of the Hamiltonian and then finding the rest of H_d solving the PDE was also proposed in [7]. Stability can be discussed, using LaSalle's theorem. Dissipativity is assured if

$$R_{doo} = R_{doo}^T > 0.$$

This is equivalent to

$$A(x) + A(x)^T < 0.$$

Notice that this condition depends only on f_o, irrespectively of u. Convergence to the equilibrium point, defined by $\partial_u H_d|_{x=x^*} = 0$, follows from the condition

$$\partial^2 H_d\Big|_{x=x^*} > 0,$$

or, in other words,

$$\partial_u\left(-(J_{duo}^T)^{-1} B(x_u)\right)\Big|_{x=x^*} > 0.$$

We can summarize this Section in the following Proposition.

Proposition 1 *Consider a dynamical system given by (6), so that f_o can be expressed as,*

$$f_o = A(x)\partial_o H_d + B(x_u) \tag{11}$$

where $\partial_o H_d$ is a design function of x_o such that

$$\partial_o H_d(x_o)\big|_{x_o = x_o^d} = 0$$

and

$$\partial_o^2 H_d(x_o)\big|_{x_o = x_o^d} > 0. \tag{12}$$

Then the control law

$$u = g^{-1}\left[J_{duo}(x_u)\partial_o H_d - f_u\right] \tag{13}$$

where $J_{duo}(x_u)$ is another design function of x_u, is robustly stable in front of variations of ξ as long as

$$A(x) + A^T(x) < 0, \tag{14}$$

$$\left(-(J_{duo}^T)^{-1}B(x_u)\right)\big|_{x=x^*} = 0, \tag{15}$$

and

$$\partial_u\left(-(J_{duo}^T)^{-1}B(x_u)\right)\big|_{x=x^*} > 0. \tag{16}$$

Notice that condition (14) implies that the dynamics of the output variables x_o is dissipative, and this is the only dissipation of the closed-loop system (due to $R_{duu} = R_{duo} = 0$).

4 Toy model example

Consider the toy model

$$\left\{\begin{array}{l} \dot{x}_1 = -x_1 + \xi x_2^2 \\ \dot{x}_2 = -x_1 x_2 + u \end{array}\right. \tag{17}$$

where $\xi > 0$ is an uncertain parameter and the desired output is fixed by x_1^d. Notice that x_1 is a relative degree two output.

The system (17) can be cast into PHDS form (1) with

$$J(x) = \begin{pmatrix} 0 & x_2 \\ -x_2 & 0 \end{pmatrix}, \quad R = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad g = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$H(x) = \frac{1}{2}x_1^2 + \frac{1}{2}\xi x_2^2.$$

The control objective is to regulate, x_1 to a desired value x_1^d. The equilibrium of (17) corresponding to this is given by

$$x_2^* = \sqrt{\frac{1}{\xi}x_1^d}, \quad u^* = \sqrt{\frac{1}{\xi}(x_1^d)^3}. \tag{18}$$

Using the IDA-PBC technique, also with the algebraic approach, we solve (3) with

$$J_d(x) = \begin{pmatrix} 0 & \alpha(x) \\ -\alpha(x) & 0 \end{pmatrix}, \quad R_d = \begin{pmatrix} 1 & 0 \\ 0 & r \end{pmatrix},$$

and

$$H_d(x) = \frac{1}{2}(x_1 - x_1^*)^2 + \frac{1}{2\gamma}(x_2 - x_2^d)^2,$$

where $\alpha(x_1, x_2)$ is a function to be determined by the matching procedure and $\gamma > 0$, $r > 0$ are adjustable parameters.

From the first row of the matching equation (3) one gets

$$-x_1 + \xi x_2^2 = -(x_1 - x_1^d) + \frac{\alpha}{\gamma}(x_2 - x_2^*),$$

from which

$$\alpha(x_1, x_2) = \frac{\gamma}{x_2 - x_2^*}(\xi x_2^2 - x_1^d) = \gamma\xi(x_2 + x_2^*).$$

Substituting this into the second row of the matching equation

$$-x_1 x_2 + u = -\alpha(x_1 - x_1^d) - \frac{r}{\gamma}(x_2 - x_2^*),$$

yields the feedback control law

$$u = x_1 x_2 - \gamma\xi(x_1 - x_1^d)(x_2 + x_2^*) - \frac{r}{\gamma}(x_2 - x_2^*). \tag{19}$$

This control law yields a closed-loop system with (J_d, R_d, H_d), and which has (x_1^d, x_2^*) as a globally asymptotically stable equilibrium point.

Notice that the control law u (19) depends on x_1^d and x_2^*, where x_2^* is a function of ξ (18) and hence the control law is not robust with respect to an uncertain $\hat{\xi}$.

Let us apply the discussion of Section 3 to this example. In this case the x_o output variable is x_1 and the x_u variable is x_2. First we fix $\partial_o H_d$ as

$$\partial_o H_d = x_1 - x_1^d \tag{20}$$

which ensures conditions (12) and (13). Then from (11), $A(x)$ and $B(x_u)$ must be

$$A(x) = -1 \qquad B(x_u) = \xi(x_2^2 - x_2^{*2}).$$

Notice that condition (14) is achieved.

The easiest choice for J_{duo} is a free constant, for instance $k > 0$, but for this nonlinear example it is necessary to add a more complicated $a(x_2)$ function[3]. Then the final choice is

$$J_{duo} = J_{duo}^T = -a(x_2)k$$

[3]The $a(x_2)$ function is included to avoid stability restrictions on the space-state. The same procedure with $a = 1$ implies

$$\partial^2 H_d|_{x=x^*} = \begin{pmatrix} 1 & 0 \\ 0 & 2\frac{\xi}{k}x_2^* \end{pmatrix}$$

which is negative for $x_2^* < 0$ and thus globally asymptotically stability is not achieved.

with $k > 0$ and

$$a(x_2) = \begin{cases} 1, & x_2 > 0 \\ b, & x_2 = 0 \\ -1, & x_2 < 0 \end{cases} \tag{21}$$

where $b \in [-1, 1]$ is a parameter that should be used to choose the equilibrium point of x_2, see Section 6. This selection ensures conditions (15)

$$\left.\frac{1}{a(x_2)k}\xi(x_2^2 - x_2^{*2})\right|_{x_2=x_2^*} = 0$$

and (16),

$$\left.\left(\frac{1}{a(x_2)k}\xi(x_2^2 - x_2^{*2})\right)\right|_{x_2=x_2^*} > 0$$

Finally the controller obtained from (8) yields

$$u = x_1x_2 - ak(x_1 - x_1^d). \tag{22}$$

We can find the Hamiltonian function integrating $\partial_x H_d$ (equations (10) and (20)) to obtain

$$H_d = a(x_2)\frac{\xi}{k}x_2\left(\frac{1}{3}x_2^2 - x_2^{*2}\right) + \frac{1}{2}(x_1 - x_1^d)^2$$

which has two local minimum both in x_1^d. H_d is depicted in Figure 1, using the parameter values of Section 5. Notice that two equilibrium points appear, namely

$$x_2^* = \pm\sqrt{\frac{1}{\xi}x_1^d},$$

and these points yield the same value for x_1^d. In the *classical* controller (19) this ambiguity did not appear, because we were fixing the *desired* value of x_2^*, while in the robust controller both fixed points of x_2 are possible.

5 Simulations

Figures 2 and 3 show simulation results testing both controllers, the robust method presented above and the classic IDA-PBC. The parameters were $\xi = 2$ (actual), $\hat{\xi} = 1$ (estimated), with initial conditions $x(0) = (0, -1.5)$ and the desired output is $x_1^d = 2$. The control parameter for the robust control law was $k = 10$ and for the classical IDA-PBC $r = 1$ and $\gamma = 1$ were selected. The robust controller achieves the desired value of x_1 even with a wrong parameter estimation, while the classical IDA-PBC controller is sensible to the ξ variations. Notice that the variations of $\hat{\xi}$ change the x_2^* equilibrium point.

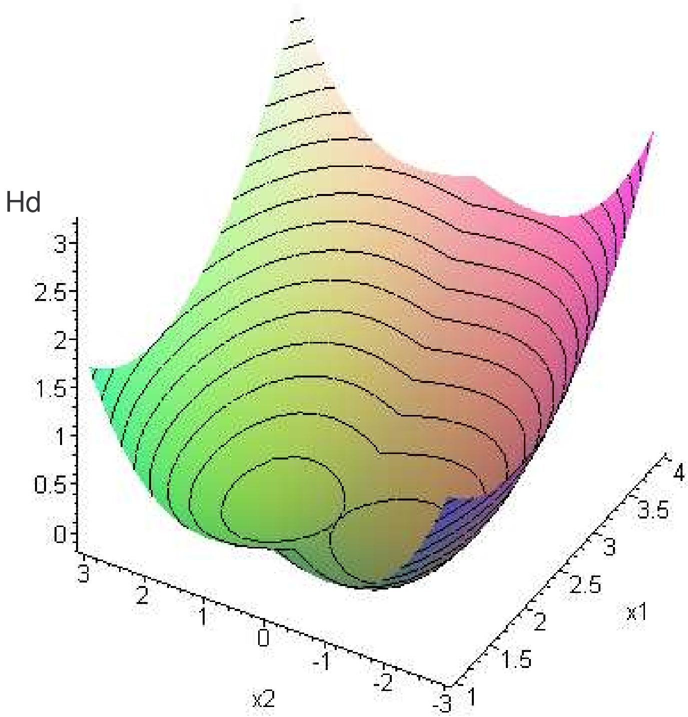

Figure 1: Desired Hamiltonian function, H_d.

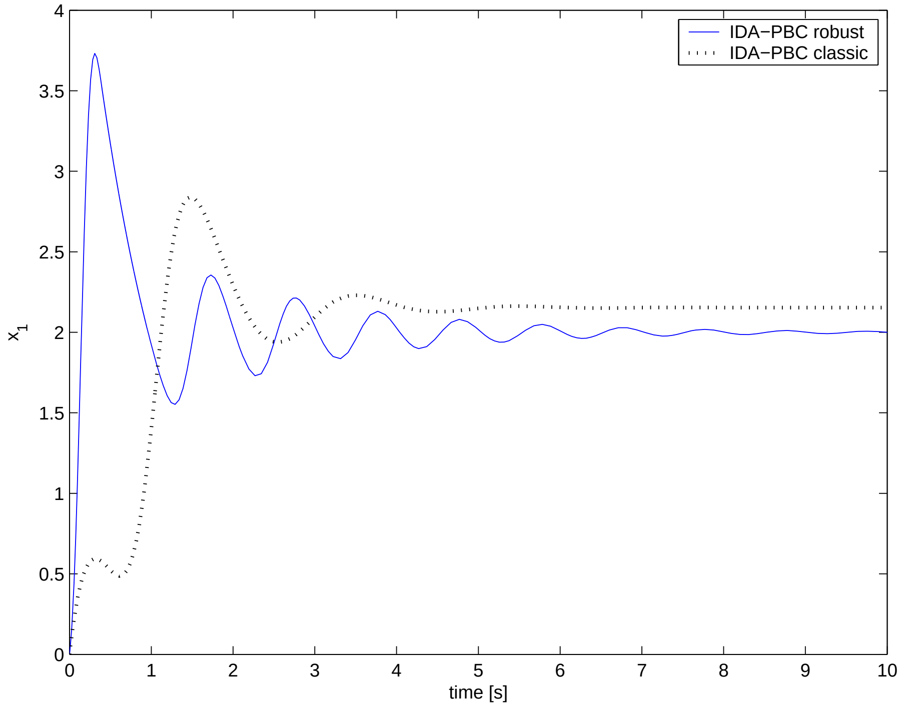

Figure 2: Comparison between the *robust method* and the *classic* IDA-PBC, behaviour of x_1.

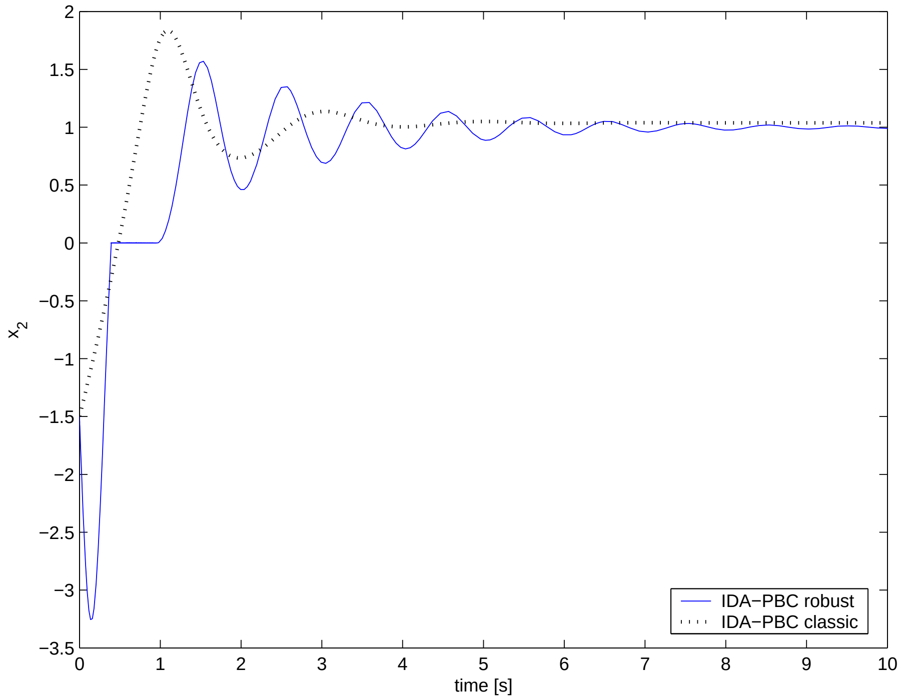

Figure 3: Comparison between the *robust method* and the *classic* IDA-PBC, behaviour of x_2.

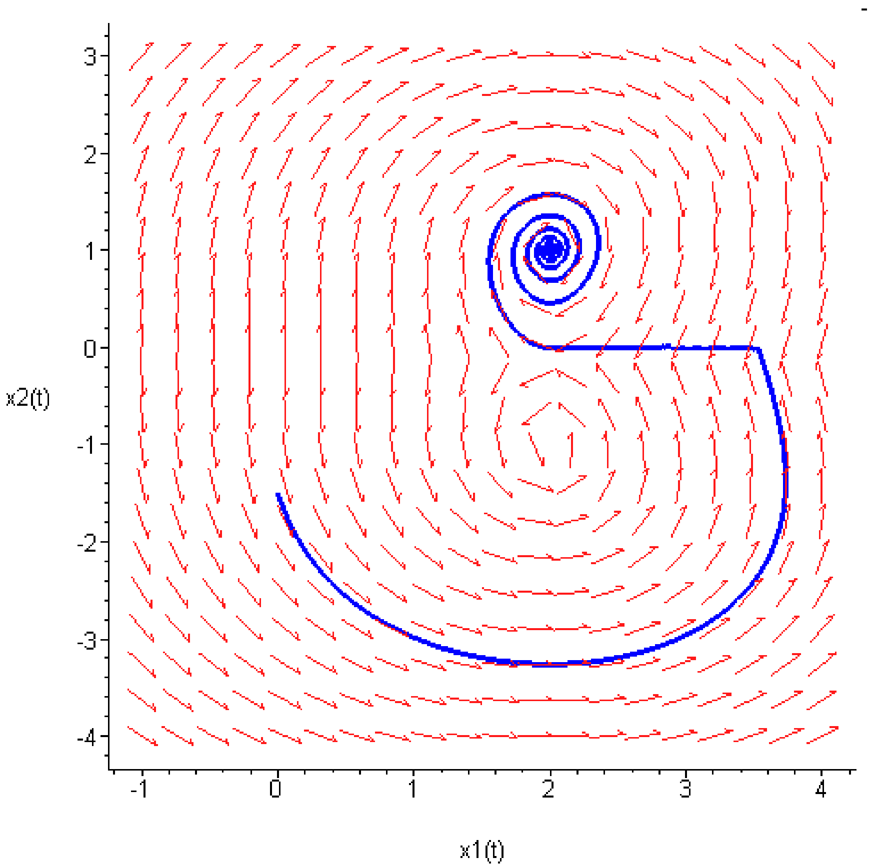

Figure 4: State space; trajectory and vector field.

6 Study of the closed-loop dynamics

This subsection is focussed on the study of the dynamical behaviour of the controller designed above. Fig. 4 shows the phase portrait of the closed–loop system (parameters are taken from the simulations, Section 5). Two stable fixed points, $x^* = (2, \pm 1)$, are present. To select x_2^* let us to write the system (17) with the feedback control law (22),

$$\begin{cases} \dot{x}_1 = -x_1 + \xi x_2^2 \\ \dot{x}_2 = -ak(x_1 - x_1^d) \end{cases}$$

The dynamics after reaching $x_2 = 0$ there is described by

$$\dot{x}_1 = -x_1,$$

so x_1 tends to $x_1 = 0$, and simultaneously the x_2 dynamics is

$$\dot{x}_2 = -ak(x_1 - x_1^d)$$

where $k > 0$ and $a(x_2) = b$ with $b \in [-1, 1]$. Notice that for $x_1 > x_1^*$, $x_2 = 0$ is an attractor set. For $x_1 < x_1^d$ and $x_2 = 0$ the dynamics of x_2 or $b = 1$ is increasing, while if $b = -1$ the

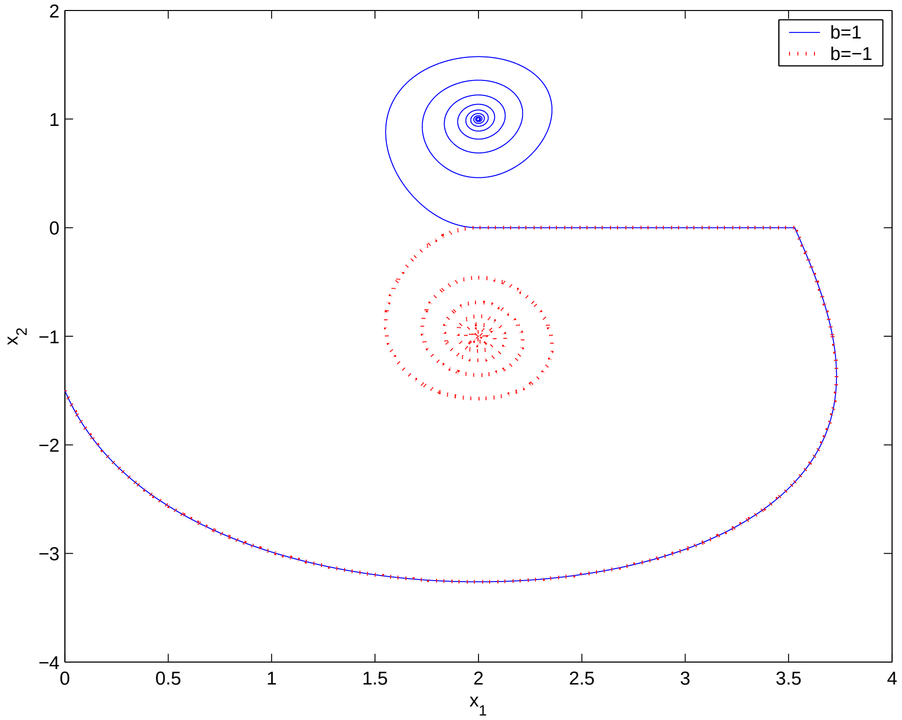

Figure 5: Phase portrait of x for two different b values. $b = 1$ with a continuous line and $b = -1$ with a dotted line.

dynamics of x_2 decreases. In other words, for $b = 1$

$$\lim_{t\to\infty} x_2 = +\sqrt{\frac{1}{\xi}x_1^d}$$

and for $b = -1$

$$\lim_{t\to\infty} x_2 = -\sqrt{\frac{1}{\xi}x_1^d}.$$

Fig. 5 shows a phase portrait of two different simulations, for $b = 1$ with a continuous line and $b = -1$ with a dotted line. The behaviour is as expected in the discussion above: for $b = 1$, x_2 tends to $+\sqrt{\frac{1}{\xi}x_1^d}$ while for $b = -1$ x_2 tends to $-\sqrt{\frac{1}{\xi}x_1^d}$. In Fig. 6 the same simulations are depicted in function of time. For numerical simulations, we modify $a(x)$ (21) as

$$a(x) = \begin{cases} 1, & x_2 > \epsilon \\ b, & -\epsilon < x_2 < \epsilon, \\ -1, & x_2 < \epsilon \end{cases}$$

where $\epsilon > 0$ is a constant, so that numerical errors do not bring the trajectory to the wrong fixed point.

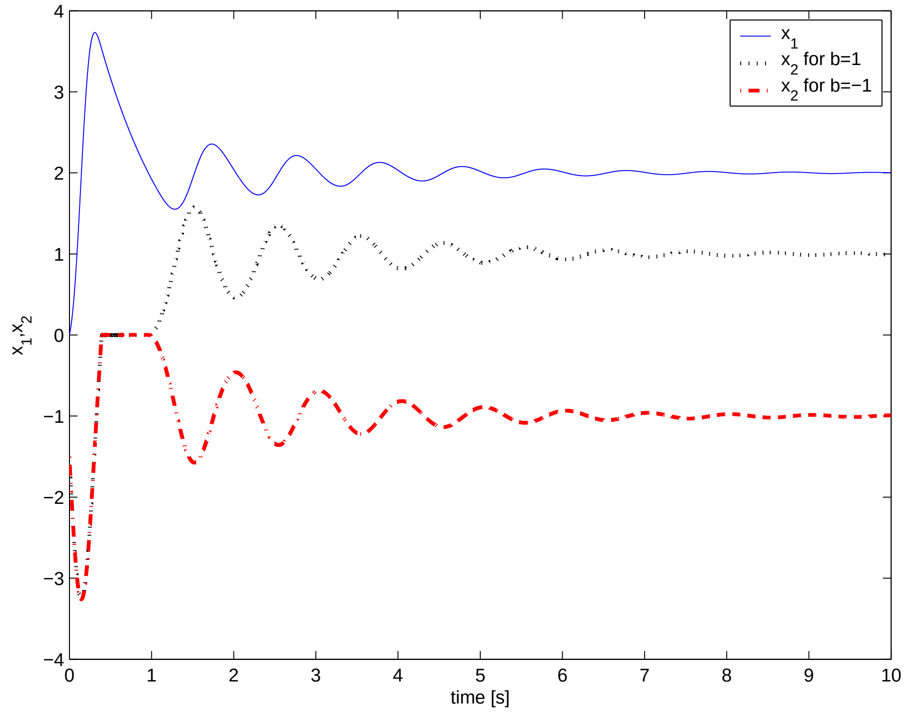

Figure 6: Simulations for two different b values.

7 Conclusions

A modification of the IDA-PBC method which makes the controller robust with respect to uncertain parameters appearing in the components of the vector field corresponding to higher order relative degree outputs has been proposed. Carefully selection of the interconnection and damping matrices allows the dependence of the controller in uncertain parameters to be removed. This new approach has been applied to a simple toy model, for which a robust controller has been designed and simulated.

Extending this idea, namely to improve robustness of IDA-PBC controllers via structure modification, to underactuated systems and to relax the conditions under which this technique can be applied will be the object of future research.

The use of integral controllers in the IDA-PBC framework for higher order relative degree outputs as a means of improving robustness is also under study.

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