ENERGY-BASED MODELLING AND CONTROL OF A MULTI-DOMAIN ENERGY STORAGE AND MANAGEMENT SYSTEM

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Abstract. We give an overview of part of the chapter of the Geoplex book devoted to examples, specifically the one which deals with electromechanical systems. We study a rather complex example of a port Hamiltonian system made of two subsystems, presenting each some remarkable characteristics, namely interconnection structures which depend either on the system state or on a discontinuous control variable. The first subsystem, a doubly-fed induction machine (DFIM), like most rotating electric machinery, has a complicated, geometry depending energy function, encoding in a lumped parameter description the interaction of the stator and rotor magnetic fields. After a coordinate change, this dependence can be absorbed into the interconnection structure, resulting in a model which has the additional feature of yielding itself quite easily to the formulation of sensible control problems. The second subsystem, a back-to-back (B2B) power converter made of a rectifier and an inverter, also has, this time from the beginning, an interconnection structure which varies with the topology of the system, and which can be controlled by the state of a set of switches.

We describe the detailed port Hamiltonian structure of both subsystems, their interconnection and the design of suitable IDA-PBC controllers for both of them. The associated bond-graph description is also presented, and simulations using 20sim are run.

1 Introduction

In this paper, bond graph and PCHS models of a complex system, see Figure 1, obtained from the interconnection of a doubly-fed induction machine (DFIM) and a back-to-back (B2B) power converter, are presented. Simulations in closed loop, with a control designed by means of Hamiltonian passive techniques, are also performed.

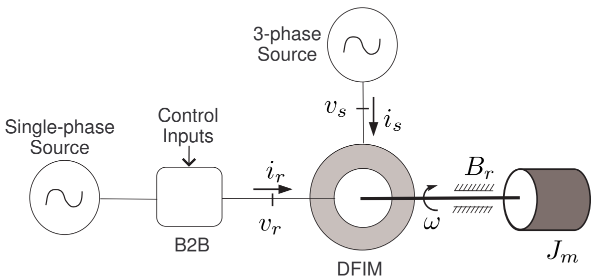


Figure 1: The system: A DFIM whose rotor is fed by a B2B converter.

A back-to-back converter, connected to an auxiliary single-phase grid, provides the desired PWM rotor voltages to the DFIM. The B2B has the nice feature that power can flow in any direction. In particular, the rotor energy of the DFIM can flow back to the converter for some operating conditions [2].

The presentation is organized as follows. In Section 2 the concepts of energy and co-energy for electromechanical systems are worked out. Expressions for the available mechanical work are also deduced.

A general port Hamiltonian form for electromechanical systems is presented and everything is exemplified with a very simple model, namely an elementary electromagnet. In Section 3 the equations of the DFIM are written as a PHS in the original coordinates and also in the transformed and reduced dq-coordinates, which bring in an interconnection matrix which depends on the state variables. Section 4 introduces electronic power converters and shows how to implement the variable structure inherent to them in the port Hamiltonian framework, using first a simple converter, the dc-dc boost, and then the back-to-back converter used to drive the DFIM rotor windings. The bond graph of the B2B is constructed, using ideal transformers to represent the switches. Section 5 states the control objectives for both the DFIM and the B2B, and illustrates how to map them into regulation problems which can be solved in the IDA-PBC framework. In fact, the two problems illustrate the two extreme ways of solving the IDA-PBC matching equation, namely the purely algebraic approach and the PDE one. For the B2B problem, a kind of truncated generalized Fourier expansion is introduced, and its properties are briefly discussed.

Section 6 puts together the DFIM and the B2B. The bond graph of the whole system is written and introduced into `20sim`, and some simulations are run. Finally, in Section 7 we summarize the main results presented and some of the advantages of the PHS modelling approach.

As this is just an overview of the book contents, only the essential results are described, omitting the detailed deductions and some of the simulations. An extended version of part of the topics and examples discussed here is available online at

`http://www-lar.deis.unibo.it/euron-geoplex-sumsch/lectures.html`.

Detailed discussions of the operation modes of the DFIM and the construction of the IDA-PBC controllers for both the DFIM and the B2B can be found in [2] and [1].

2 Electromechanical systems

We start with the energy supplied to a system by m power ports $(e_i, f_i)_{i=1,\ldots,m}$ between t_0 and t:

$$W_f(t) - W_f(t_0) = \int_{t_0}^{t} \sum_{i=1}^{m} e_i(\tau) f_i(\tau)\ \mathrm{d}\tau. \tag{1}$$

We assume that the state of the system can be described by n state variables x_i, which evolve according to a dynamics which has a dependence on the state and the efforts in the ports, $\dot{x} = G(x) + g(x)e$. Assuming that the internal dynamics has a certain skew symmetric structure, and that the flows can be computed as $f = g^T(x)\phi(x)$, as can be justified from the general PHS theory, we show that

$$W_f(t) - W_f(t_0) = \int_{t_0}^{t} \langle \dot{x}(\tau), \phi(x(\tau)) \rangle\ \mathrm{d}\tau = \int_{\gamma(x_0,x)} \phi(z)\ \mathrm{d}z \tag{2}$$

where $\gamma(x_0, x)$ is any curve connecting $x_0 = x(t_0)$ and $x = x(t)$ in state space. Next, assuming that the system is *conservative*, it is easy to show that the 1-form $\phi(x)\ \mathrm{d}x$ is *exact*, *i.e.* in components

$$\phi_i(x) = \frac{\partial \Phi(x)}{\partial x_i}, \quad i = 1, \ldots, n, \tag{3}$$

where Φ is a state space function. Actually, from (2) it follows immediately that then W_f is also a state space function and

$$W_f(x) - W_f(x_0) = \Phi(x).$$

We set $t_0 = 0$, $x_0 = 0$ and $W_f(0) = 0$ (there is no problem in doing so for time-invariant systems) and hence

$$W_f(x) = \int_{\gamma(x)} \phi(z)\ \mathrm{d}z, \tag{4}$$

where $\gamma(x)$ is now *any* curve connecting the state space origin (or any other selected point) with x. The function $W_f(x)$ is thus the energy necessary to bring the system to state x, and it is the same no matter how the power is injected through the ports. It is thus *the* energy of the system in state x; it coincides with the Hamiltonian function $H(x)$, and we use the two notations indistinctively.

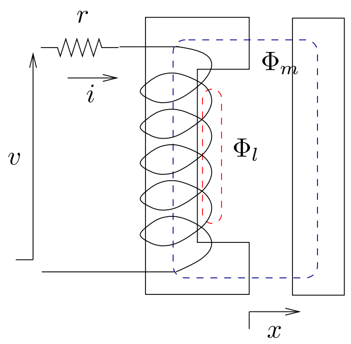


Figure 2: An elementary electromagnet: a magnetic system with a moving part.

From (3) it follows, assuming continuity of the second order derivatives of Φ, that

$$\frac{\partial \phi_i}{\partial x_j}(x) = \frac{\partial \phi_j}{\partial x_i}(x), \quad i, j = 1, \ldots, n. \tag{5}$$

These are known as *Maxwell's reciprocity relations* for the constitutive equations of a conservative field. It allows, using derivation with respect to x_i followed by integration with respect to x_j, to obtain $\phi_i(x)$ if $\phi_j(x)$ is known. Also, from (3), one obtains that all the constitutive relations can be computed if the energy is known:

$$\phi_i(x) = \frac{\partial W_f(x)}{\partial x_i}. \tag{6}$$

Several power sign conventions adopted in the literature are also discussed, and it is noticed that dissipation can be brought in by closing some of the ports with resistive elements, as in the standard PHS setting.

In electromechanical systems, some of the state variables, x_m, are geometrical in nature, and their associated power variables are generalized velocities and forces, while the other variables pertain to the electric domain, x_e (fluxes or charges). If, for instance, the electrical constitutive relations are known, use of (5) allows the computation of the generalized mechanical forces.

Figure 2 shows an elementary electromagnet, a magnetic system with a moving part the flux linkage λ through the coil depends on a geometry variable, the "air gap" x. The electrical equation of motion is $v = ri + \dot{\lambda}$, where the flux linkage can be computed from the number of turns, N, and the magnetic induction flux, Φ, as $\lambda = N\Phi$. In turn, Φ has a leakage, Φ_l, and a magnetizing, Φ_m, parts, $\Phi = \Phi_l + \Phi_m$, which can be computed in terms of the reluctances of the respective paths. The reluctance of the magnetizing path has a fixed contribution, the part of the iron path, and a variable one, the part of the air gap. Taking this into account, the relation between the current and the flux linkage can be written, under suitable simplifications, as

$$\lambda = (L_l + L_m(x))i, \quad L_m(x) = \frac{N^2 \mu_0 A}{\frac{l_i}{\mu_{ri}} + 2x} \equiv \frac{b}{c + x} \tag{7}$$

where μ_{ri} is the relative magnetic permeability of the iron core, A is the section of the magnetic path and l_i is the length of the iron path. It is this geometrical dependence of the magnetic constitutive relation which allows the transduction of energy between domains. From (7) the field energy and the mechanical force are computed as an illustration of the ideas exposed in this section.

For a wide class of electromechanical systems, the total energy can be written as

$$H(\lambda, p, \theta) = \frac{1}{2}\lambda^T L^{-1}(\theta)\lambda + \frac{1}{2}p^T J_m^{-1} p, \tag{8}$$

where λ are the generalized electrical energy variables (they may be charges or magnetic fluxes), p are the generalized mechanical momenta (linear or angular, or associated to any other generalized coordinate), and θ are the generalized geometric coordinates. One has

$$\begin{aligned} \dot{\lambda} + R_e i &= Bv, \\ \dot{p} &= -R_m\omega - T_e(\lambda, \theta) + T_m, \\ \dot{\theta} &= J_m^{-1} p \end{aligned} \tag{9}$$

where B is a matrix indicating how the input voltages are connected to the electrical devices, T_e is the electrical torque, T_m is the external applied mechanical torque and

$$i = L^{-1}(\theta)\lambda = \partial_\lambda H, \quad \omega = J_m^{-1} p = \partial_p H \tag{10}$$

are the electrical currents and mechanical velocities $\omega = \dot{\theta}$. We have, according to our discussion on electromechanical energy conversion,

$$T_e = \frac{\partial H}{\partial \theta} = -\frac{1}{2}\lambda^T L^{-1} \partial_\theta L L^{-1} \lambda, \tag{11}$$

where $\partial_\theta L^{-1} = -L^{-1}\partial_\theta L L^{-1}$ has been used. Using (10) and (11), equations (9) can be written in an explicit port Hamiltonian form as

$$\dot{x} = \begin{pmatrix} -R_e & 0 & 0 \\ 0 & -R_m & -1 \\ 0 & 1 & 0 \end{pmatrix} \partial_x H + \begin{pmatrix} B & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ T_m \end{pmatrix} \tag{12}$$

where $x = (\lambda\ p\ \theta)^T$. This general model includes many of the classical electrical machines, as well as linear motors and levitating systems. As a simple exercise, the PHS description of the elementary electromagnet is developed.

3 The doubly-fed induction machine

The doubly-fed, three-phase induction machine contains 6 energy storage elements (the 3 rotor and the 3 stator inductances) with their associated dissipations and 6 ports (the 3 stator and the 3 rotor voltages and currents).

We assume [5][10] that the machine is symmetric (all windings are equal) and that the stator-rotor cross inductances are smooth, sinusoidal functions of θ, with just the fundamental term. Then the linear magnetic constitutive relation (in $\mathbb{R}^6$) is given by the θ-dependent inductance matrix

$$\tilde{L}(\theta) = \begin{pmatrix} \tilde{L}_s & \tilde{L}_{sr}(\theta) \\ \tilde{L}_{sr}^T(\theta) & \tilde{L}_r \end{pmatrix} \quad \tilde{L}_{sr}(\theta) = L_{sr} \begin{pmatrix} \cos\theta & \cos(\theta + \frac{2}{3}\pi) & \cos(\theta - \frac{2}{3}\pi) \\ \cos(\theta - \frac{2}{3}\pi) & \cos\theta & \cos(\theta + \frac{2}{3}\pi) \\ \cos(\theta + \frac{2}{3}\pi) & \cos(\theta - \frac{2}{3}\pi) & \cos\theta \end{pmatrix} \tag{13}$$

Introducing $p = J\dot{\theta}$ and $x = (\lambda\ \theta\ p)^T$, the PCHD description of the DFIM is given by

$$\dot{x} = \begin{pmatrix} -R & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -r_m \end{pmatrix} \partial_x H + \begin{pmatrix} B & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ T_m \end{pmatrix} \tag{14}$$

with the Hamiltonian

$$H(x) = \frac{1}{2}\lambda^T L^{-1}(\theta)\lambda + \frac{1}{2J}p^2. \tag{15}$$

From the original three phase electrical variables y_{abc} (currents, voltages or magnetic fluxes) we compute transformed variables by means of

$$y = T y_{abc}, \quad \text{where} \quad T = \begin{pmatrix} \frac{\sqrt{2}}{\sqrt{3}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \tag{16}$$

Notice that, since $T^T = T^{-1}$, this is a power–preserving transformation:

$$\langle i, v \rangle = \langle i_{abc}, v_{abc} \rangle. \tag{17}$$

As it is common, from now on we work only with the two first components (the dq components) of any electrical quantity and neglect the third one (the homopolar component, which is zero for any balanced set and which, in any case, is decoupled from the remaining dynamical equations.)

The steady–state for the equations of the DFIM are periodic orbits that can be transformed into equilibrium points by means of the well–known Blondel-Park transformation [10]. This standard procedure also eliminates the dependence of the equations on θ, and consists in defining new variables f^r via

$$f = K(\theta, \delta) f^r, \quad K(\theta, \delta) = \begin{bmatrix} e^{J_2 \delta} & O_2 \\ O_2 & e^{J_2(\delta - \theta)} \end{bmatrix}, \quad J_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}. \tag{18}$$

δ is an arbitrary function of time that, for convenience, is selected as $\dot{\delta} = \omega_s$, with ω_s the line frequency, which is assumed constant. This is the so–called synchronous reference frame, for which v_s^r gets a very simple form. We drop the transformed variable superindex r from now on.

The mechanical variables are not affected by this transformation. Taking into account the tensorial character of the elements of the PHS description, the transformed objects can be computed, and one gets, with variables $z^T = (z_e^T, z_m) \in \mathbb{R}^5$, $z_e^T = (\lambda_s^T, \lambda_r^T) \in \mathbb{R}^4$, $z_m = p \in \mathbb{R}$,

$$\dot{z} = [J(z) - \mathcal{R}] (\nabla H)^\top + B_1 v_r + B_2 v_s \tag{19}$$

with total energy

$$H(z) = \frac{1}{2} z_e^\top L^{-1} z_e + \frac{1}{2 J_m} z_m^2, \quad L = \begin{bmatrix} L_s I_2 & L_{sr} I_2 \\ L_{sr} I_2 & L_r I_2 \end{bmatrix}, \tag{20}$$

interconnection and dissipation matrices given respectively by

$$J(z) = \begin{bmatrix} -\omega_s L_s J_2 & -\omega_s L_{sr} J_2 & O_{2\times 1} \\ -\omega_s L_{sr} J_2 & -(\omega_s - \omega) L_r J_2 & L_{sr} J_2 i_s \\ O_{1\times 2} & L_{sr} i_s^\top J_2 & 0 \end{bmatrix}, \ R = \begin{bmatrix} R_s I_2 & O_2 & O_{2\times 1} \\ O_2 & R_r I_2 & O_{2\times 1} \\ O_{1\times 2} & O_{1\times 2} & B_r \end{bmatrix}, \tag{21}$$

and

$$B_1 = \begin{bmatrix} O_2 \\ I_2 \\ O_{2\times 1} \end{bmatrix}, \quad B_2 = \begin{bmatrix} I_2 \\ O_2 \\ O_{2\times 1} \end{bmatrix}, \tag{22}$$

where I_2 is the 2×2 identity matrix, and the Os are matrices of suitable dimensions with all the elements equal to zero. Notice that the price paid for getting rid of the dependence on the angle is an interconnection matrix which depends on the state variables, through ω and i_s.

4 Power converters

Electronic power converters [7] are devices able to deliver electrical energy in a suitable way for the applications, *i.e.* with prescribed frequency, voltage amplitude or any other specification. They do the trick by periodically storing the energy in inductors and capacitors before releasing it in the desired form; in a given period the converter goes through a series of topological circuit changes by means of controlled switches (for instance IGBT switches).

A change in the state of a switch effectively changes the way the energy flows inside the system, *i.e.* the interconnection matrix of the system. The way to introduce this into the PHS description[8] is by considering first the switches as open ports, and then imposing their constitutive relations, which, for ideal switches, amount to the fact that the switch behaves as an ideal zero current source (when open) or as an ideal zero voltage source (when closed).

Fig. 3 shows the back-to-back converter selected for this system. It is made of a full bridge AC/DC single-phase boost-like rectifier and a 3-phase DC/AC inverter. The whole converter has an AC single-phase voltage input and its output are 3-phase PWM voltages which feed the rotor windings of the electrical machine. This system can be split into two parts: a dynamical subsystem (the full bridge rectifier, containing the storage elements), and an static subsystem (the inverter, which, from the energy point of view, acts like a transformer).

A single-phase AC voltage source $v_i(t) = E \sin(\omega_s t)$ provides the energy in the direct operation mode, L is the inductance (including the effect of any transformer in the source), C is the capacitor of the DC part, r takes into account all the resistance losses (inductor, source and switches), s_k and t_k, $k = 1, 2, 4, 5, 6$. Switch states take values in $\{-1, 1\}$ and t-switches are complementary to s-switches: $t_k = \bar{s_k} = -s_k$. Additionally, $s_2 = \bar{s_1} = -s_1$.

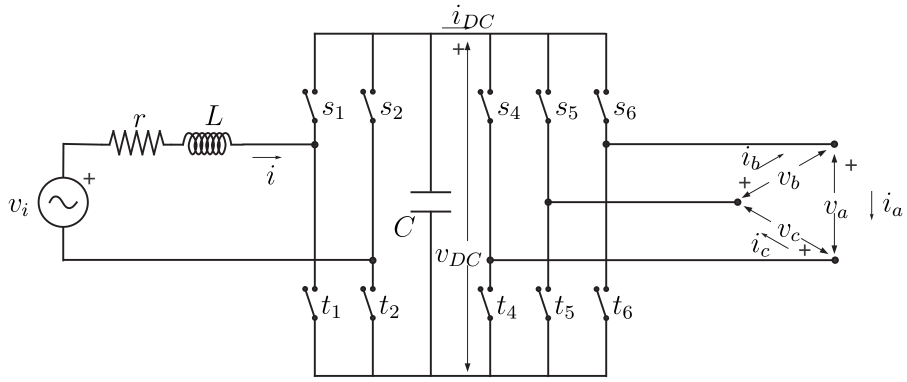


Figure 3: Back-to-back converter.

From a PHS point of view, the rectifier can be considered as a system by itself, with a port in parallel with the capacitor, while the inverter has no storage elements and can be described as an ideal transformer.

Using the techniques in [8], the PHS structure of a simpler dc-dc boost converter is written down, and then the PHS description of the rectifier is shown to be as follows. The Hamiltonian variables are $x^T = (\lambda, q) \in \mathbb{R}^2$, where λ is the inductor flux and q is the DC charge in the capacitor. The Hamiltonian function is

$$H = \frac{1}{2L}\lambda^2 + \frac{1}{2C}q^2, \tag{23}$$

while the interconnection and dissipation matrices are

$$J = \begin{pmatrix} 0 & -s_1 \\ s_1 & 0 \end{pmatrix} \in \mathbb{R}^{2\times 2} \qquad R = \begin{pmatrix} r & 0 \\ 0 & 0 \end{pmatrix} \in \mathbb{R}^{2\times 2}. \tag{24}$$

The port matrix is

$$g = \begin{pmatrix} 1 & O_{1\times 3} \\ 0 & f^T \end{pmatrix} \in \mathbb{R}^{2\times 4}, \quad f = \frac{1}{2}\begin{pmatrix} s_6 - s_4 \\ s_5 - s_6 \\ s_4 - s_5 \end{pmatrix} \in \mathbb{R}^3, \tag{25}$$

with inputs

$$u = \begin{pmatrix} v_i \\ -i_{abc} \end{pmatrix} \in \mathbb{R}^4, \tag{26}$$

where $i^T_{abc} = (i_a, i_b, i_c) \in \mathbb{R}^3$ are the three-phase currents in the inverter part. Notice that the inverter subsystem can be seen as a Dirac structure with

$$\begin{aligned} v_{abc} &= f v_{DC} \\ i_{DC} &= f^T i_{abc} \end{aligned} \tag{27}$$

where $v^T_{abc} = (i_a, i_b, i_c) \in \mathbb{R}^3$ are the three-phase voltages and $v_{DC} \in \mathbb{R}$, is the DC voltage, and $i_{DC} \in \mathbb{R}$ is the DC current supplied by the rectifier subsystem.

The bond graph model of the B2B converter is depicted in Fig. 4. In the bond graph, the switches are replaced by transformers, which have the same behavior than an ideal switch for an averaged model [6].

5 IDA-PBC control of electric machines and power converters

The central idea of the IDA-PBC technique [12][13][14] is to assign to the closed loop a desired energy function via the modification of the interconnection and dissipation matrices, still preserving the PCH structure. That is, the desired target dynamics is a PCH system of the form

$$\dot{z} = [J_d(z) - R_d(z)](\nabla H_d)^\top \tag{28}$$

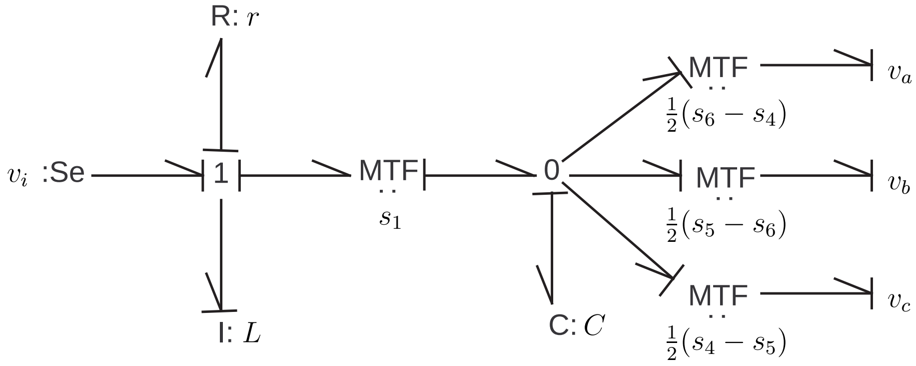


Figure 4: Bond Graph of the B2B converter.

where $H_d(z)$ is the new total energy and $J_d(z) = -J_d^\top(z),\ R_d(z) = R_d^\top(z) > 0$, are the new interconnection and damping matrices, respectively. To achieve stabilization of the desired equilibrium point one imposes

$$z^* = \arg\min H_d(z). \tag{29}$$

The election of z^* is determined by the control objectives to be achieved.

For the DFIM, the desired power transfer between the grid, the B2B and the flywheel can stated in terms of a desired equilibrium point in dq coordinates; this equilibrium point can, in turn, be parameterized in terms of the dq components of the stator current. For the B2B, the control objectives are essentially that a constant voltage is achieved in the capacitor, no matter the current flowing to or from the DFIM rotor, and that the current and voltage of the single-phase port are in phase or in opposition, depending on the direction of the power flow. In order to map this non-standard control problem to a regulation one, a kind of generalized Fourier series with time moving integration window, yielding time dependent coefficients, and known as Generalized State Space Averaging (GSSA)[4][11], must be introduced.

Averaging techniques for VSS are based on the idea that the change in a state or control variable is small over a given time length, and hence one is not interested on the fine details of the variation. Hence one constructs evolution equations for averaged quantities of the form

$$\langle x\rangle(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)\ \mathrm{d}\tau, \tag{30}$$

where $T > 0$ is chosen according to the goals of the problem.

The GSSA expansion tries to improve on this and capture the fine detail of the state evolution by considering a full Fourier series. Thus, one defines

$$\langle x\rangle_k(t) = \frac{1}{T}\int_{t-T}^{t} x(\tau)e^{-jk\omega\tau}\ \mathrm{d}\tau, \tag{31}$$

with $\omega = 2\pi/T$ and $k \in \mathbb{Z}$. The time functions $\langle x\rangle_k$ are known as index-k averages or k-phasors.

In terms of these, the original state variables can be written as a kind of "moving" Fourier series:

$$x(\tau) = \sum_{k=-\infty}^{+\infty} \langle x\rangle_k(t)e^{jk\omega\tau}, \quad \forall\tau. \tag{32}$$

If the expected steady state of the system has a finite frequency content, one may select some of the coefficients in this expansion and get a truncated GSSA expansion. The desired steady state can then be obtained from a regulation problem for which appropriate constant values of the selected coefficients are prescribed. In [3] it is shown that the system obtained from any finite truncation is again a PHS, and explicit formulae for all the elements of the truncated PHS description are provided.

It is sensible for the control objectives of the problem to use a truncated GSSA expansion with $\omega = \omega_s$, keeping only the zeroth-order average of the dc-bus voltage, q_0, and the two components of the first harmonic of the inductor current, λ_1^R and λ_1^I. Furthermore, we write $z = \frac{1}{2}q^2$ instead of q, and use the new control variable $v = -Sq$.

Now we apply an GSSA expansion to this system, and set to zero all the coefficients except for $x_1 \equiv z_0$, $x_2 \equiv \lambda_1^R$, $x_3 \equiv \lambda_1^I$, $u_1 \equiv v_1^R$ and $u_2 \equiv v_1^I$. Using that $i_l = -i_{DC}$ is assumed to be locally constant, and that the only nonzero coefficient of v_i is $v_{i1}^I = -\frac{E}{2}$, one gets

$$\dot{x} = (J(u) - R)(\nabla H)^T + g_1(x_1)i_l + g_2 E, \tag{33}$$

with

$$J = \begin{pmatrix} 0 & -u_1 & -u_2 \\ u_1 & 0 & \frac{\omega_s L}{2} \\ u_2 & -\frac{\omega_s L}{2} & 0 \end{pmatrix} \quad R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{r}{2} & 0 \\ 0 & 0 & \frac{r}{2} \end{pmatrix} \quad g_1 = \begin{pmatrix} -\sqrt{2x_1} \\ 0 \\ 0 \end{pmatrix} \quad g_2 = \begin{pmatrix} 0 \\ 0 \\ -\frac{1}{2} \end{pmatrix} \tag{34}$$

and the Hamiltonian function

$$H = \frac{1}{C}x_1 + \frac{1}{L}x_2^2 + \frac{1}{L}x_3^2. \tag{35}$$

The control objectives for this rectifier are a dc value of the output voltage $V = \frac{1}{C}q$ equal to a desired point, V_d, and the power factor of the converter equal to one, which in GSSA variables translates to $x_2^* = 0$. This allows fixing an equilibrium point for the closed loop dynamics, and the IDA-PBC machinery can then be applied.

The matching equation, *i.e.* the equation which equals (28) to the original PHS dynamics via selection of the inputs u, can be solved in several ways. For the DFIM, it is convenient to select $R_d = R$ and take as H_d a quadratic expression with the minimum at the desired fixed point; the matching equation becomes then an algebraic equation for J_d, plus an equation fixing v_r (in dq coordinates). This yields a v_r which is singular at the equilibrium point, but the problem can be solved by taking as R_d an appropriate, state space dependent matrix. The final result is

$$v_r = v_r^* - (\omega - \omega^*)(L_r J_2 i_r^* + L_{sr} J_2 i_s) - L_{sr}\omega^* J_2(i_s - i_s^*) - rI_2(i_r - i_r^*), \tag{36}$$

where starred quantities denote values at the equilibrium point.

For the B2B, it is better to fix $J_d = J$ and $R_d = R$; the matching equation becomes then a partial differential equation for H_d. The output of the these computations, after returning from the GSSA to the original variables, is

$$s_1 = \frac{2\omega_s x_3^*}{V_d}\cos(\omega_s t) - \frac{L i_l}{x_3^*}\sin(\omega_s t), \tag{37}$$

where x_3^* can be computed from the desired capacitor voltage V_d. The application of this controller to the rectifier part of the B2B yields a constant voltage in the capacitor; from this, an appropriate switching procedure for the inverter generates the control signal for the DFIM controller.

6 B2B and DFIM interconnection and simulation

Having computed controllers for both the DFIM and B2B subsystems, the two can be assembled and sensible simulations can be run. The bond graph of the whole system is shown in Fig. 5. Due to the common power port interface of both the bond graph and the PHS descriptions, some of the parts of this global bond graph could be replaced by an equivalent equation model in PHS form, without affecting the global structure and properties of the model. Notice that the DFIM bond graph presents I-type elements with differential causality. This is due to the fact that these elements have been introduced artificially to represent the cross inductance between the rotor and stator windings, and can be eliminated in terms of the independent, physical state variables. An alternative representation using I-fields [9] could also have been used.

The simulation has been performed using the `20sim` modeling and simulation software. As a first test of the model and the controllers, the desired mechanical speed changes around $\omega = 2 \cdot 50\pi$ and a desired bus voltage $v_{DC}^* = 150$ has been selected. Fig. 6 shows the desired and simulated mechanical speed, as well as the voltage v_i and the current i at the single phase source feeding the B2B, showing that they are nearly in phase. Other operating modes are also tested, showing how power can be redirected through the system.

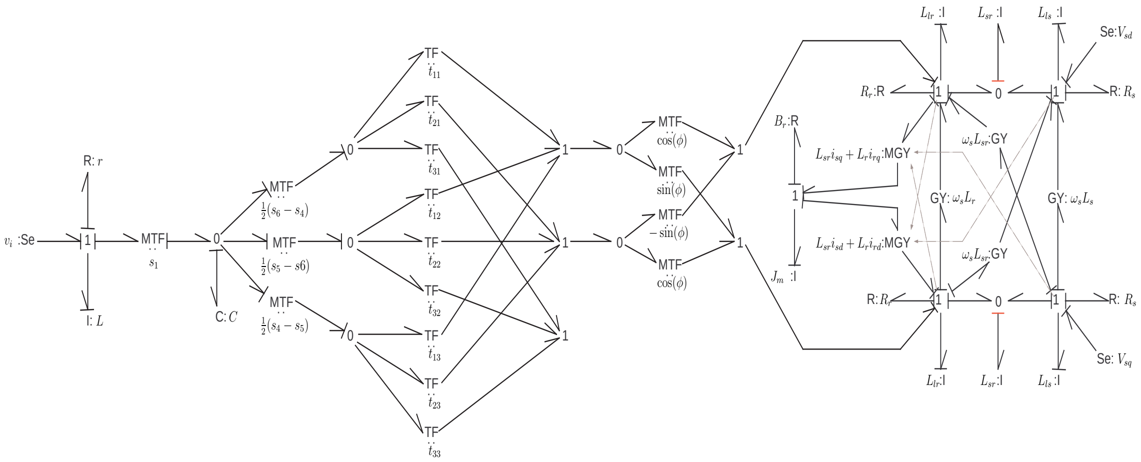
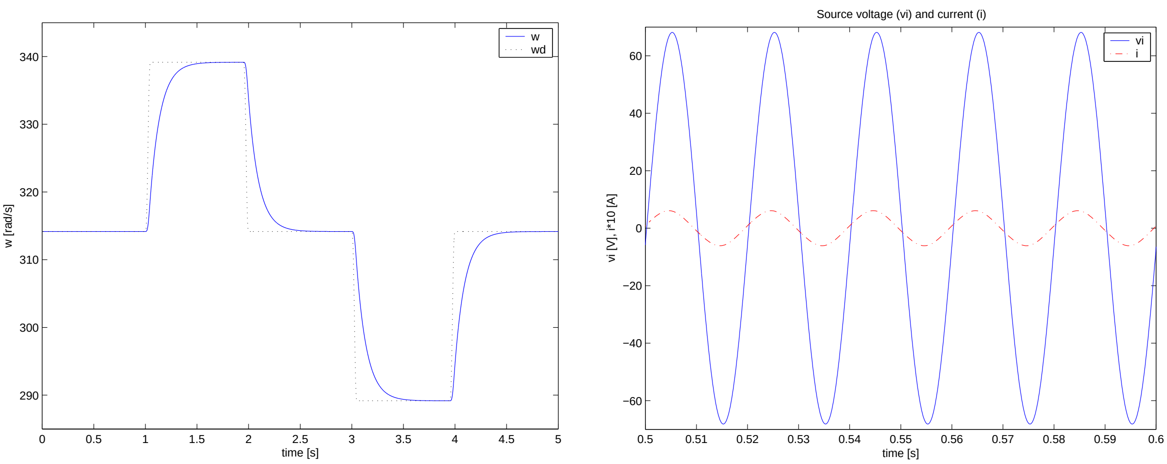


Figure 5: Bond graph representation of the whole system.



(a) Mechanical speed of the DFIM.

(b) Detail of the AC single-phase voltage and current for the B2B.

Figure 6: Testing some of the outputs of the complete B2B+DFIM system.

7 Conclusions

The whole PHS approach to modelling and control has been illustrated for a complex, although lumped parameter, system, made of subsystems from the electrical and mechanical domains. Each subsystem has some particularities which show the power and adaptability of the port Hamiltonian approach. In particular, how to deal with variable structure system in the PHS framework has been demonstrated with electronic power converters, and the apparition of state-space dependent interconnection matrices has been exemplified for a rotating induction machine in synchronous coordinates. For both subsystems, the selection of suitable coordinate transformations, which in the case of the power converter includes a generalized Fourier expansion truncated to the relevant terms, has proven useful for the design of the controllers with the IDA-PBC technique.

The PHS models of both subsystems have been coupled together, and introduced in the `20sim` modelling and simulation software. The PHS framework, with its intrinsic object orientation approach to modelling, can be illustrated with `20sim` by interchanging bond graph submodels with the corresponding PHS counterparts.

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