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Amaia Lusa, Rafael Pastor, Albert Corominas

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DETERMINING THE MOST APPROPPRIATE SET OF WEEKLY WORKING HOURS FOR PLANNING ANNUALISED WORKING TIME*

Amaia Lusa†, Rafael Pastor, Albert Corominas

Research Institute IOC, School of Industrial Engineering of Barcelona, Universitat Politècnica de Catalunya, Avda. Diagonal 647 p11, 08028 Barcelona (Spain)

{amaia.lusa/rafael.pastor/albert.corominas}@upc.edu

Abstract

Annualised hours (possibility of irregularly distributing working hours over a year) permit companies to adapt capacity to demand, thus reducing overtime, temporary workers and inventory costs. To avoid a significant worsening of the working conditions, many laws and agreements constraint the distribution of working time. One way to constrain solutions is by specifying a finite set of weekly working hours and bounding the annual number of weeks of each type. Even though its impact on the solution, that set is usually agreed without considering all data available (demand, costs, etc.). In this paper two MILP models are used to determine in one step the most appropriate set of weekly working hours, the annual number of weeks of each type and the annual working time planning.

Keywords: human resources, manpower planning, annualised hours

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† CORRESPONDING AUTHOR:
Amaia Lusa
Research Institute IOC, School of Industrial Engineering of Barcelona
Universitat Politècnica de Catalunya
Avda. Diagonal, 647, planta 11
08028 Barcelona (Spain)
Tel. +34 93 401 17 05
Fax +34 93 401 66 05
e-mail: amaia.lusa@upc.edu

1 INTRODUCTION

Annualising working hours (AH) consists in hiring workers for a certain number of hours per year and distributing those hours irregularly over the year in order to accommodate fluctuations in demand. It allows for a better use of potential capacity because one can adapt capacity more closely to demand over time. This flexibility in the use of human resources is especially useful in service organizations and in manufacturing organizations where products cannot be inventoried (e.g. are perishable). However, AH often implies a worsening of the staff's working conditions and the need to solve complicated working time planning problems.

There are papers that deal with production and working time planning problems in which some kind of flexibility is considered (see, for example, Wild and Schneeweiss, 1993), but, overall, annualised hours is a subject that remains largely unexplored. Actually, some authors emphasize that the concept of annualised hours is surprisingly absent from the literature on planning and scheduling; see Hung (1999a), Hung (1999b), Grabot and Letouzey (2000) and Azmat and Widmer (2004).

In Corominas et al. (2004) the characteristics of annualised working hours planning problem are discussed and a classification scheme is proposed, which gives rise to thousands of different cases. The authors have already solved some of these types of problems (see Corominas et al., 2002 and Corominas et al., 2005). In all cases, and in spite of the large size of the models, MILP has proved to be an efficient tool for solving these kinds of problems.

A way to constrain the problem and to reduce the worsening in workers' conditions is to define a finite set of weekly working hours (for example, 35, 40 and 45 hours/week). Also, it can be imposed that, in a year, each type of working week has to be assigned a certain number of times to each worker. Obviously, the set of working weeks has a great influence in the solution so the first thing a company should do in a situation like the one described is to determine the set of working weeks. In this paper two MILP models are proposed to determine the most appropriate set of weekly working hours and to plan annual hours.

The layout of the rest of this paper is as follows: Section 2 introduces the problem; Section 3 discusses the objective function; Section 4 describes the planning models; Section 5 describes the computational experiment and Section 6 contains the conclusions.

2 PROBLEM DESCRIPTION

The problem consists in determining the number of weekly working hours for each member of the staff at a service centre (assuming levels of seasonal demand) and for each nonholiday week of the planning horizon (a year, for instance), with the objective of optimising a utility function.

The conditions to be met by the solution may stem from a legal resolution or from a collective bargaining agreement between management and workers. Due to the impossibility of establishing an exhaustive list of conditions a priori, the most common conditions are considered in order to build a basic model for the problem. To adapt this model to any specific case, constraints can be added or deleted.

One way to constrain the solutions is by specifying that the number of weekly working hours must belong to a finite set (of types of working weeks). In addition the number of annual working weeks of each type can be previously established or bounded. Given some rules agreed by managers and workers, a planning procedure should also propose the most appropriate set of working weeks. In this paper two possibilities are considered, which give rise to two different models:

1. The set of working weeks has to be chosen among a finite set of types of working weeks (for example, 15 types are considered and 3 have to be chosen). It can be imposed also that a type of working week has to be chosen among a certain subset of available types. Finally, the number of weeks in a year in which every type is assigned can be lower and upper bounded.

2. The number of hours corresponding to each type of working week has to be determined. For each type of working week one can impose a lower and an upper bound to the number of hours and also to the number of weeks in which the type is assigned to each worker.

It is assumed that the number of weeks in which each type of working week is assigned has to be the same for all workers.

To avoid overburdening workers in long high demand periods an additional constraint, which is set in the French 35 hours law (see MES, 2005), is considered: the average number of weekly working hours, for any set of twelve consecutive working weeks, is upper bounded.

In some countries, collective bargaining agreements do not permit overtime. Other times tasks are too difficult to be performed by temporary workers. For the purposes of this paper, as overtime and temporary workers are not considered, a capacity shortage is possible during certain weeks. According to those assumptions the cost of the staff is the same for any feasible solution. Thus, capacity has to be distributed in order to optimise the service level. Next section contains a discussion about the most appropriate objective function.

The characteristics of the problem are summarized below:

- The number of annual working hours is upper bounded.

- The number of weekly working hours must belong to a finite set, which has to be determined. Two possibilities are considered:
  1. Each type of working week has to be chosen among an available set of working weeks. For example, 3 types of working weeks to be chosen among 9 available: one type to be chosen among {36, 37, 38}, another type among {40, 41, 42} and the last among {43, 44 and 45}.
  2. The number of hours corresponding to each type of working weeks is lower and upper bounded. For example, the lower and upper bounds corresponding to three types of working weeks could be [35, 38], [40, 43] and [43.5, 48], respectively. A solution could be 36.4, 42 and 48 hours/week.
- The number of times in a year in which each type of working week is assigned has to be the same for all workers and is lower and upper bounded.
- The average number of working hours for a group of L consecutive nonholiday weeks cannot be greater than h_L hours/week.
- Each worker has two consecutive holiday weeks in winter and four consecutive holiday weeks in summer. The holiday weeks for each worker are agreed to previously.
- Overtime is not permitted.
- Hiring temporary workers is not possible.
- A utility function is to be optimised.

3 OBJECTIVE FUNCTION

All customers are supposed to be served when they go to the service centre: demand is met during each period, but if there is a capacity shortage, then the service level will not be as good as is desirable (workers may spend too little time assisting customers). If the required capacity (which is fixed according to foreseen demand and the optimal service level, as in a queue system) is greater than the actual capacity, then the service level worsens. Of course, to suppose that demand is always covered would be unsustainable if relative capacity shortages (i.e., capacity shortage divided by the required capacity) were large.

Large relative shortages must be avoided: if a capacity shortage represented a small proportion of the required capacity—although less attention would be paid to some customers—workers, with a little extra effort, could meet the demand.

The maximum relative capacity shortage, a function that is relatively simple to minimise, is a suitable objective function. The service level during the worst period is optimised and large capacity shortages are avoided.

The main inconvenience of this objective function is that once the maximum relative capacity shortage has been minimised, it is indifferent to whether there is a capacity shortage in other weeks, provided that relative capacity shortages are not higher than the maximum (Figure 1 shows, for every week, the demand, capacity and shortage profiles obtained by solving an example using this objective function). Obviously, it is more desirable to have a smaller shortage, if possible (that is to say, if it is possible to choose between different optimal solutions).

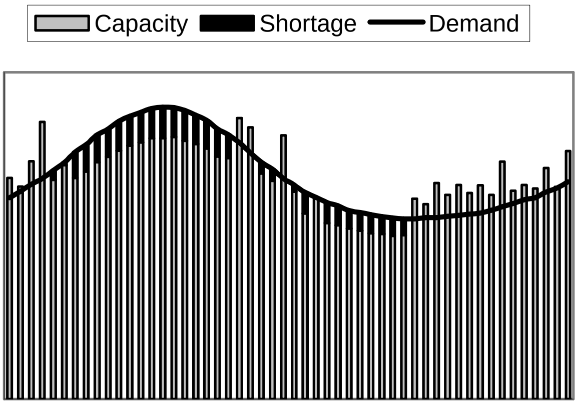

Figure 1: minimise maximum relative capacity shortage.

It is possible to break the tie between optimal solutions (in order to obtain small relative capacity shortages for every week) by considering a secondary objective function (which is weighted and then added to the first one): the sum of weekly relative capacity shortages. Following this, the objective function to minimise is the weighted sum of two terms: (i) the maximum relative capacity shortage; and (ii) the sum of the weekly relative capacity shortages. Figure 2 shows, for every week, the demand, capacity and shortage profiles obtained by considering this objective function using the same data as in Figure 1: one can see how capacity is adapted to demand, which thus improves the global service level (and the maximum relative shortage is the minimum, as in Figure 1).

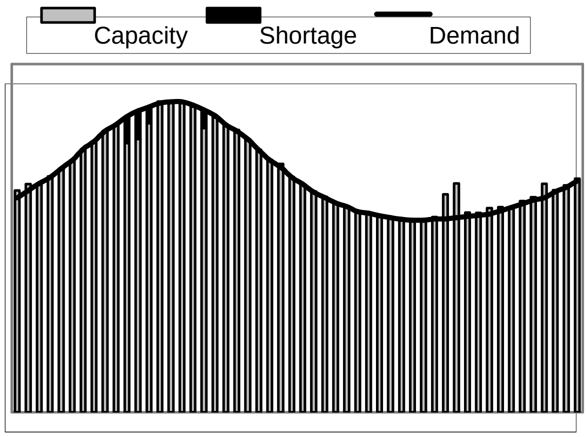

Figure 2: minimise maximum relative capacity shortage plus the sum of weekly relative capacity shortages.

4 MODELS

4.1 Model *M1*

The first model corresponds to the situation in which each type of working week has to be chosen among an available set.

We make use of the following notation.

Data

W	Set of staff members.
N	Number of staff members.
T	Weeks in the planning horizon (usually 52).
S_i	Set of available weeks for worker i (the weeks of the planning horizon excluding the worker's holiday weeks), $\forall i \in W$.
H	Upper bound for annual working hours.
K	Number of types of working weeks to determine.
J_k	Working week of type k has to be chosen among the set of types of working weeks J_k (k=1,…,K).
JT	Set of types of working weeks $\left(JT = \cup_{k=1}^{K} J_k\right)$.
j_w	w is one of the types of working weeks among which working week of type j_w ($j_w \in$ [1..K]) has to be chosen ($\forall w \in JT$).
h_w	Number of hours corresponding to working week of type w ($\forall w \in JT$).
lb_k , ub_k	Lower bound and upper bound for the number of working weeks of type k that must be performed by each worker (k=1,…,K).
L, h_L	The average number of working hours, in a group of L consecutive weeks, cannot be greater than h_L (for example, L = 12 and h_L = 44 hours).
e_t	Working hours required during week t (t=1,...,T), considering the time that a worker would need to complete the task, and fixed according to foreseen demand of task and the desired service level.
a, b	Weight given to the two different criteria considered in objective function (maximum relative shortage and sum of relative shortages).

Variables

x_{iwt}	Binary variable that indicates whether worker i performs a type w working week during week t ($\forall i \in W$; $\forall t \in S_i$; $\forall w \in JT$).
y_w	Binary variable that indicates whether working week of type w has been chosen ($\forall w \in JT$).

a_k	Integer variable that indicates the number of weeks in which the selected working week of type *k* is assigned to workers (k=1,..., *K*). Actually, this variable can be declared as a non negative real variable because, anyway, it can only take integer values (it is expressed in constraint (5) as the sum of binary variables).
d_t^-	Non negative real variable indicating the capacity shortage: that is to say, the forecasted required capacity that cannot be met by the staff in week *t* (t=1,..., *T*).
D	Non negative real variable that indicates the maximum relative shortage, being relative shortages the shortages normalised by their corresponding demands.

Model M1

$$[MIN]\, Z = a \cdot D + b \cdot \sum_{t=1}^{T} \frac{d_t^-}{e_t} \qquad (1)$$

$$D \geq \frac{d_t^-}{e_t} \qquad t = 1,...,T \qquad (2)$$

$$\sum_{\forall t \in S_i} \sum_{\forall w \in JT} h_w \cdot x_{iwt} \leq H \qquad \forall i \in W \qquad (3)$$

$$\sum_{\forall w \in JT} x_{iwt} = 1 \qquad \forall i \in W;\ \forall t \in S_i \qquad (4)$$

$$\sum_{\forall t \in S_i} \sum_{\forall w \in J_k} x_{iwt} = a_k \qquad \forall i \in W;\ k = 1,...,K \qquad (5)$$

$$lb_k \leq a_k \leq ub_k \qquad k = 1,...,K \qquad (6)$$

$$\sum_{\forall w \in J_k} y_w = 1 \qquad k = 1,...,K \qquad (7)$$

$$\sum_{\forall i \in W} \sum_{\forall t \in S_i} x_{iwt} \geq y_w \qquad \forall w \in JT \qquad (8)$$

$$\sum_{\forall i \in W} \sum_{\forall t \in S_i} x_{iwt} \leq N \cdot ub_{j_w} \cdot y_w \qquad \forall w \in JT \qquad (9)$$

$$\sum_{(\forall i \in W) | (t \in S_i)} \sum_{\forall w \in JT} h_w \cdot x_{iwt} + d_t^- \geq e_t \qquad t = 1,...,T \qquad (10)$$

$$\sum_{t=j-L+1}^{j} \sum_{\forall w \in JT} h_w \cdot x_{iwt} \leq h_L \cdot L \qquad \forall i \in W;\ j = L,...,T \mid [j-L+1,...,j] \in S_i \qquad (11)$$

Finally, variables x_{iwt} and y_w are binary and variables a_k, d_t^- and D are real and not negative.

The objective function to minimise (1) is the weighted sum of: (i) the maximum relative capacity shortage; and (ii) the sum of weekly relative capacity shortages; with α, β > 0; (2) expresses that *D* is the maximum relative capacity shortage; (3) imposes, for each worker, the upper bound for the number of annual working hours; (4) imposes, for each worker and for each available week, that one type of working week be assigned; (5) and (6) impose the lower and upper bound for the number of times that each selected working week is assigned, which must the same for all workers; (7) imposes that one type of working week has to be chosen among each available set; (8) and (9) imply that the variable y_w takes value 1 if working week of type w is assigned and 0 otherwise; (10) is the balance, for each week, between the number of required working hours, the number of working hours performed by the staff and the capacity shortage; and finally, (11) imposes an upper bound on the average of weekly working hours for any subset of L consecutive nonholiday weeks.

4.2 Model *M2*

The second model corresponds to the situation in which each type of working week has to be determined. The number of hours corresponding to each type of working week can take any real value belonging to an interval defined by a lower and an upper bound. Obviously, the optimal solution solving model *M2* cannot be worse than the optimal solution obtained when solving model *M1*. Nevertheless, considering each type of working week as a real variable gives a nonlinear model that has to be linearised and implies the use of a huge number of integer variables and constraints. For the sake of clarity, the nonlinear model is included before the linearisation.

The additional notation is defined.

Data

hm_k , hM_k Lower bound and upper bound for the hours of the working week of type k (k=1,…,K).

Variables

u_{ikt} Binary variable that indicates whether worker i performs the type of working week k during week t ($\forall i \in W$; $\forall t \in S_i$; k=1,…,K).

h_k Non negative real variable that indicates the number of working hours corresponding to working week type k (k=1,…,K).

Variables x_{iwt} and y_w are not used in model *M2*.

Nonlinear model M2

$$[MIN]\, Z = a \cdot D + b \cdot \sum_{t=1}^{T} \frac{d_t^-}{e_t} \qquad (1)$$

$$D \geq \frac{d_t^-}{e_t} \qquad t = 1,\ldots,T \qquad (2)$$

$$\sum_{k=1}^{K} h_k \cdot \left(\sum_{\forall t \in S_i} u_{ikt} \right) \leq H \qquad \forall i \in W \qquad (12)$$

$$\sum_{k=1}^{K} u_{ikt} = 1 \qquad \forall i \in W;\ \forall t \in S_i \qquad (13)$$

$$\sum_{\forall t \in S_i} u_{ikt} = a_k \qquad \forall i \in W;\ k = 1,\dots,K \qquad (14)$$

$$lb_k \leq a_k \leq ub_k \qquad k = 1,\dots,K \qquad (6)$$

$$\sum_{k=1}^{K} h_k \cdot \left(\sum_{(\forall i \in W) \mid (t \in S_i)} u_{ikt} \right) + d_t^- \geq e_t \qquad t = 1,\dots,T \qquad (15)$$

$$\sum_{k=1}^{K} h_k \cdot \left(\sum_{t=j-L+1}^{j} u_{ikt} \right) \leq h_L \cdot L \qquad \forall i \in W;\ j = L,\dots,T \mid [j-L+1,\dots,j] \in S_i \qquad (16)$$

$$hm_k \leq h_k \leq hM_k \qquad k = 1,\dots,K \qquad (17)$$

Finally, variables u_{iwt} are binary and variables a_k, h_k, d_t^- and D are real and not negative.

Previous equations are basically the same included in model *M1*, formalised in a different way. Thus, equations (12), (13), (14), (15) and (16) correspond to equations (3), (4), (5), (10) and (11) of model *M1*, respectively. Equation (17) is added, which imposes, for each type of working week, the lower and the upper bound for the number of working hours.

As it can be observed, equations (12),(15) and (16) include the sum of products of variables h_k by a sum of binary variables u_{ikt}. In general, is it possible to linearise the product of a real variable, *R*, by a sum of binary variables (which is an integer variable, *I*) in the following way:

- $R \cdot I$ is replaced by a new real variable *S*;
- *I* is expressed as $\sum_{n=nmin}^{nmax} n \cdot b_n$, being b_n binary variables and *nmin* and *nmax* the lower and upper bound for integer variable *I*;
- Constraint $\sum_{n=nmin}^{nmax} b_n = 1$ is added;
- Finally, the following equations are added:

$$S - n \cdot R \leq M \cdot (1 - b_n) \qquad n = nmin,\dots,nmax \qquad (18)$$

$$n \cdot R - S \leq M \cdot (1 - b_n) \qquad n = nmin,\dots,nmax \qquad (19)$$

That procedure has been used to linearise equations (12), (15) and (16). First of all, some new data and variables must be defined:

Data

NW_t Number of available workers in week t (t=1,…,T).

Variables

p_k Non negative real variable that indicates the annual number of hours corresponding to working week of type k that each worker does (k=1,…,K).

r_{mk} Binary variable that indicates whether working week of type k has been assigned in m weeks to each worker (m=lb_k,…,ub_k; k=1,…,K).

q_{tk} Non negative real variable that indicates the capacity of the group of workers that are been assigned a working week of type k in week t (t=1,…T; k=1,…,K).

s_{mtk} Binary variable that indicates whether working week of type k has been assigned to m workers in week t (m=0,…,NW_t; t=1,…,T; k=1,…,K).

g_{ijk} Non negative real variable that indicates the number of working hours corresponding to working weeks of type k that worker i has done in the group of L consecutive weeks that finishes in week j (∀i∈ W; k=1,…,K; j=L,…,T| [j-L+1,…,j] ∈ S_i).

v_{mijk} Binary variable that indicates whether working week of type k has been assigned m times to worker i in the group of L weeks that finishes in week j (m=0,…,L;∀i∈ W;k=1,…,K;j=L,…,T | [j-L+1,…,j]∈ S_i).

Linear model M2

Then, the following changes should be done to previous nonlinear model: equation (12) is replaced by (20); equations (21), (22), (23) and (24) must be added; equations (14) and (6) must be deleted; equation (15) is replaced by (25); equations (26), (27), (28) and (29) must be added; equation (16) is replaced by (30); and, finally, equations (31), (32), (33) and (34) must be added.

$$\sum_{k=1}^{K} p_k \leq H \tag{20}$$

$$\sum_{\forall t \in S_i} u_{ikt} = \sum_{m=lb_k}^{ub_k} m \cdot r_{mk} \qquad \forall i \in W;\ k=1,...,K \tag{21}$$

$$\sum_{m=lb_k}^{ub_k} r_{mk} = 1 \qquad k=1,...,K \tag{22}$$

$$p_k - m \cdot h_k \leq hM_k \cdot ub_k \cdot (1 - r_{mk}) \qquad m = lb_k,...,ub_k;\ k=1,...,K \tag{23}$$

$$m \cdot h_k - p_k \leq hM_k \cdot ub_k \cdot (1 - r_{mk}) \qquad m = lb_k,...,ub_k;\ k=1,...,K \tag{24}$$

$$\sum_{k=1}^{K} q_{tk} + d_t^- \geq e_t \qquad t=1,...,T \tag{25}$$

$$\sum_{(\forall i\in W)\,|\,(t\in S_i)} u_{ikt} = \sum_{m=1}^{NW_t} m \cdot s_{mtk} \qquad t=1,\dots,T;\ k=1,\dots,K \tag{26}$$

$$\sum_{m=0}^{NW_t} s_{mtk} = 1 \qquad t=1,\dots,T;\ k=1,\dots,K \tag{27}$$

$$q_{tk} - m \cdot h_k \le hM_k \cdot NT_t \cdot (1 - s_{mtk}) \qquad m=0,\dots,NT_t;\ k=1,\dots,K;\ t=1,\dots,T \tag{28}$$

$$m \cdot h_k - q_{tk} \le hM_k \cdot NT_t \cdot (1 - s_{mtk}) \qquad m=0,\dots,NT_t;\ k=1,\dots,K;\ t=1,\dots,T \tag{29}$$

$$\sum_{k=1}^{K} g_{ijk} \le h_L \cdot L \qquad \forall i \in W;\ j=L,\dots,T\ |\ [j-L+1,\dots,j] \in S_i \tag{30}$$

$$\sum_{t=j-L+1}^{j} u_{ikt} = \sum_{m=1}^{L} m \cdot v_{mijk} \qquad \forall i \in W;\ k=1,\dots,K;\ j=L,\dots,T\ |\ [j-L+1,\dots,j] \in S_i \tag{31}$$

$$\sum_{m=0}^{L} v_{mijk} = 1 \qquad \forall i \in W;\ k=1,\dots,K;\ j=L,\dots,T\ |\ [j-L+1,\dots,j] \in S_i \tag{32}$$

$$g_{ijk} - m \cdot h_k \le hM_k \cdot L \cdot (1 - v_{mijk}) \qquad m=0,\dots,L;\ \forall i \in W;\ k=1,\dots,K;\ j=L,\dots,T\ |\ [j-L+1,\dots,j] \in S_i \tag{33}$$

$$m \cdot h_k - g_{ijk} \le hM_k \cdot L \cdot (1 - v_{mijk}) \qquad m=0,\dots,L;\ \forall i \in W;\ k=1,\dots,K;\ j=L,\dots,T\ |\ [j-L+1,\dots,j] \in S_i \tag{34}$$

Finally, it must be added that variables r_{mk}, s_{mtk} and v_{mijk} are binary and variables p_k, q_{tk} and g_{ijk} are real and not negative.

Equations (21) and (22) imply that the binary variable r_{mk} takes value 1 if working week of type k has been assigned m times to each worker. When a r_{mk} takes value 1, then equations (23) and (24) force p_k to indicate the annual number of working hours corresponding to working week of type k. Equations (26) and (27) imply that the binary variable s_{mtk} takes value 1 if working week of type k has been assigned to m workers in week t. When a s_{mtk} takes value 1, then equations (28) and (29) force q_{tk} to indicate the capacity of all workers that have been assigned a working week of type k in week t. Equations (31) and (32) imply that the binary variable v_{mijk} takes value 1 if working week of type k has been assigned m times to worker i in the group of L weeks that finishes in week j. When a v_{mijk} takes value 1, then equations (33) and (34) force g_{ijk} to indicate the the number of working hours corresponding to working weeks of type k that worker i has done in the group of L weeks that finishes in week j.

5 COMPUTATIONAL EXPERIMENT

A computational experiment was performed in order to evaluate the effectiveness of the models. As it was forseeeable, whilst model *M1* can be solved in a very efficient way, model *M2* is hard to solve and for most instances is not possible to get a feasible solution in short times.

The basic data used for the experiment are as follows:

- $|W|$ = staff sizes of 10, 20, 30, 40 and 50 workers;
- T = 52 weeks;
- Each worker has two noninterrupted holiday weeks in winter and four in summer. The temporary location of holidays was fixed at random for each worker.
- There are three different patterns of required capacity during the year: (1) nonseasonal capacity pattern; (2) seasonality pattern with one peak; and (3) seasonality pattern with two peaks. In each case, total demand is equal to total capacity and a random noise is added.
- For model *M1*, three types of working weeks have to be chosen among the following 23 available: {28, 29, 30, 31, 32, 33, 34, 35}, {36, 37, 38, 39, 40, 41, 42, 43} and {44, 45, 46, 47, 48, 49, 50}.
- For model *M2*, the lower and upper bound for the hours corresponding to each one of the 3 types of working week are: [28-35], [35.1-43], and [43.1-50] hours.
- Each one of the 3 types of working weeks has to be assigned in a number of weeks between 6 and 34.
- α= 0.99 and β= 0.01/T (β is divided by T to normalise the second term, whose upper bound is equal to T).

For each combination of W and pattern of required capacity, 20 instances were generated, giving an amount of 300 instances for each model.

The experiment was performed with ILOG CPLEX 9.0 and a Pentium IV PC at 3.0 GHz with 512 Mb of RAM. Absolute and relative MIP gap tolerances were set to 0.001 and the maximum computing time was set to 3,600 seconds.

Table 1: Size of the models.

M	**|*W*|**	**Number of binary variables**	**Number of real variables**	**Number of constraints**
	10	10,603	56	829
	20	21,183	56	1,499
M1	30	31,763	56	2,169
	40	42,343	56	2,839
	50	52,923	56	3,509
	10	9,615	728	18,576
	20	19,005	1,238	36,276
M2	30	28,395	1,748	53,976
	40	37,785	2,258	71,676
	50	47,175	2,768	89,376

Table 2: Computing times (in seconds).

Model	**|*W*|**	t_{min}	$\bar{t}$	t_{max}	***%Opt***	***%5Opt***	***%>5***	***%NoFeas***
	10	7	12	61	100			
	20	11	202	2,304	94	6		
M1	30	22	320	1,410	92	8		
	40	28	656	2,548	92	8		
	50	60	1,729	3,600	80		20	
	10						24.5	75.5
	20						4.5	95.5
M2	30		3,600				6.7	93.3
	40						20	80
	50						16	84

Model size is given in Table 1. In Table 2, for each problem size, one can see the minimum (t_{min}), the average ($\bar{t}$) and the maximum (t_{max}) computing times. *%Opt* expresses the % of instances that were solved to optimality; *%5Opt* and *%>5Opt* are the % of instances whose difference with best bound is not greater than 5% and greater than 5%, respectively, when achieving the maximum computing time of 3,600 seconds; finally, *%NonFeas* shows the % of instances without feasible solution after 3,600 seconds.

Solving model *M1* gives always a feasible solution and an optimal or near optimal solution for most of the instances. Even though the efficiency of the model decreases with the number of workers, note that, as it is stated in Azmat and Widmer (2004), companies with more than 20 workers have mainly small teams of workers who are specialised in performing certain types of tasks (thus, the planning problem can be separated into sub-problems). Since working time of workers belonging to different sections or teams can be planned and programmed separately, it can be concluded that model *M1* can be solved in a very efficient way for most of real situations (note that computing times are short, considering the problem that is been solved).

The results also show that model *M1* is better than model *M2*, both in terms of computing times and quality solution. The value of the objective function of the solution given by model *M1* is always better (a 67%, on average) than the one obtained by solving model *M2*.

Computing times increase with the number of staff workers (which determines the size of the models). In Figure 3, the influence of the type of demand (*TD*) on computing times of model *M1* can be observed. It seems that computing times for second and third pattern of demand are longer than for first pattern.

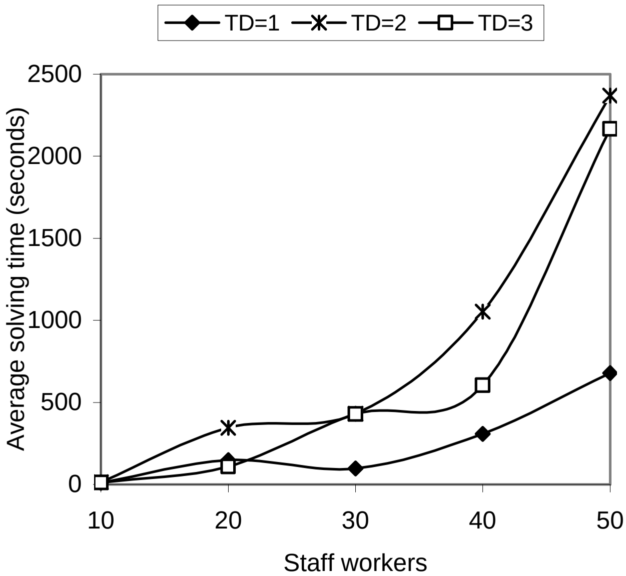

Figure 3: influence of type of demand (*TD*) on the computing times.

6 CONCLUSIONS

Annualising working hours is a good and economical way of adjusting productive capacity to seasonal demand and obtaining flexibility in the distribution of annual working hours. This paper presents a specific problem of planning working hours in a service company where the number of weekly working hours must belong to a finite set and overtime and temporary workers are not permitted. Usually, the finite set of working weeks is used to be negotiated by company and workers without taking into account that its influence in the final solution is very important. This paper presents two MILP models for solving the planning problem and also determining the most appropriate set of weekly working hours, optimising a function related to service level.

A computational experiment has shown that model *M1* (which considers an available set of working weeks among which the set to be actually used has to be chosen) is an efficient tool to solve the problem in an optimal way even for large instances. Model *M2* (determines each type of working week by means of a real variable) implies a huge number of binary variables and, mainly, constraints and need long times to be solved. Nevertheless, given the effectiveness of model *M1*, one way of getting even better solutions might be considering a greater number of available working weeks and, hence, approximating the solution to the one that would be obtained by solving model *M2*.

The proposed planning procedure can be a useful tool to negotiate the best set of types of working weeks. That set complies with the conditions that workers and company may agree and optimises an objective function related to the service level.

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