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A SCENARIO OPTIMISATION PROCEDURE TO PLAN ANNUALISED WORKING HOURS UNDER DEMAND UNCERTAINTY[*]

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Abstract

Annualising working hours (i.e., the possibility of irregularly distributing the total number of working hours over the course of a year) enables to adapt production capacity to fluctuations in demand. The demand, which is an essential data for an optimal planning of working time, usually depends on several and complex factors. Often, it is not possible to obtain a reliable prediction of the demand or it is not realistic to consider that can be adjusted to a probability distribution. In some cases, it is possible to determine a set of demand scenarios, each one with a related probability. In this work we present a multistage stochastic optimisation model which provides a robust solution (i.e., feasible for any possible scenario) and minimises the expected total capacity shortage.

Keywords: planning, uncertainty, optimisation, scenario analysis

1. Introduction

To plan annualising working hours (i.e., the possibility of irregularly distributing the total number of working hours over the course of a year) it is essential to consider the demand of the products or services provided by the company (Corominas et al., 2004). However, the demand depends usually on several and complex factors and, often, it is not possible to obtain a reliable prediction of the demand or it is not realistic to consider that can be adjusted to a probability distribution. In some cases it is possible to determine a set of demand scenarios, each one with a related probability.

The majority of the procedures used for planning and programming are based on deterministic data sets or on average values. It is clear that the error made by not considering the demand as a random variable can result in significant costs, due to lack or excess of capacity when the reality does not meet the prevision. Furthermore, considering demand as a deterministic data one can even get a solution that may be unfeasible depending on the real values of the demand. The stochastic optimisation and, in particular, the optimisation by means of scenarios, is one of the appropriate tools to deal with this uncertainty. The objective can be the optimisation of the expected value for a certain utility function (e.g., the cost) and it is also possible to include constraints which guarantee that the solution will be feasible for any possible scenario.

This work presents a multistage stochastic optimisation model via scenarios in order to take into consideration the uncertainty of the demand when planning working time under annualised hours (AH).

The layout of the rest of this paper is as follows: Section 2 introduces the multistage stochastic optimisation via scenarios as a procedure to face uncertainty; Section 3 describes the stochastic optimisation model; Section 4 describes the computational experiment and Section 5 presents the conclusions.

2.   Multistage stochastic optimisation via scenarios

The optimisation via scenarios has been successfully used in numerous cases when dealing with decision making while taking into consideration future possible scenarios (Ramos, 1992; Morton, 1996; Pallotino et al., 2002; Li et al., 2003; Mulvey et al., 2004). When considering the problem of planning working under annualised hours, the scenarios consist in different possible demands.

The most studied and applied stochastic models by means of scenarios are the multistage stochastic linear programs (see, for example, Beraldi et al., 2005), even though most papers deal with two-stage programs (see Sen, 2003 and the references listed at http://mally.eco.rug.nl/spbib.html, for an updated survey on stochastic programming).

In a multistage problem, at the beginning of each stage a decision is made based on an uncertain future; when reaching the end of this stage, some of the uncertain aspects of the future are revealed, thus reducing the number of possible scenarios. Figure 1 represents this process. In the example (Figure 1), three stages are considered and, at the beginning of the first one (state *E0*), it is totally unknown which one of the 12 possible scenarios will be met. At the end of this stage, thus at the beginning of the second stage (state *E1*, *E2* o *E3*) additional information is available (i.e. the actual demand of the periods corresponding to the first stage), which allow us to reduce the range of possible scenarios. For example, let us assume that in the scenarios from 6 to 12 the demand is low for all the periods corresponding to the first stage. If the demand has been large, it can be concluded that we are in state *E1* and all those scenarios will be discarded. An example of the demand (or required capacity) of each scenario can be seen in Figure 2.
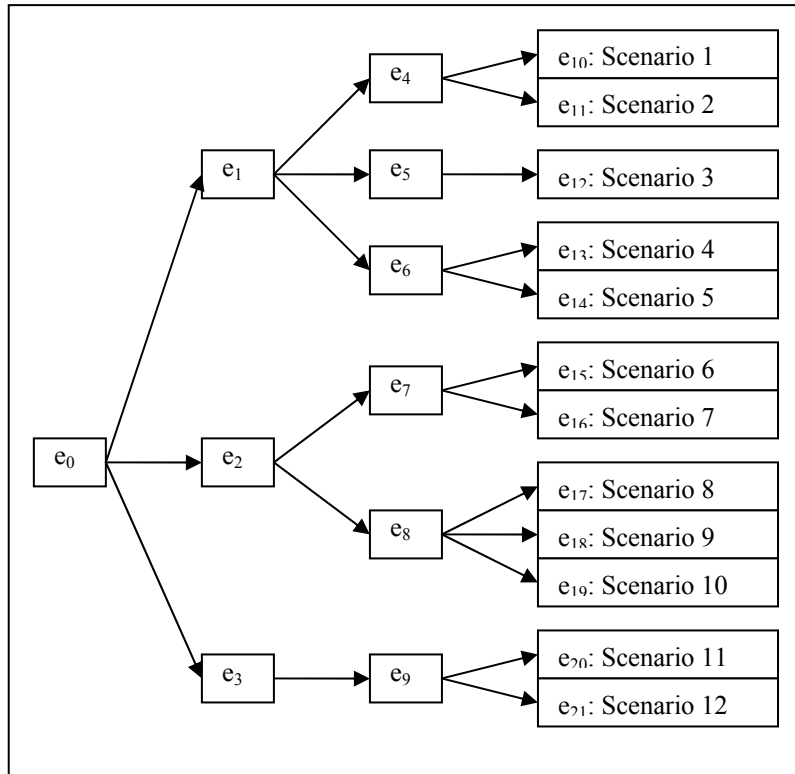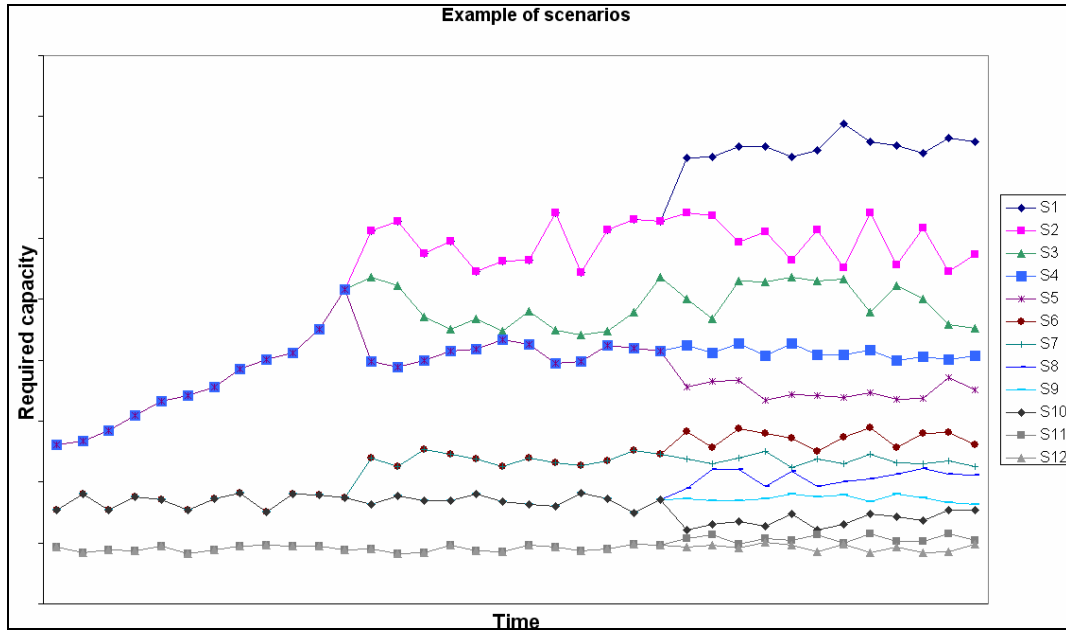
Figure 1. Scenarios tree

Figure 2. Scenarios required capacity

The planning model must provide the decision to be made for each possible state. Thus, at the beginning, in state e_0, the planning must provide all the decisions to be made in each one of the all following states. The planning is updated after each stage, taking into account the

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additional information available and the reality occurred in the previous periods, as long as this is feasible since in some cases all or part of the decisions already taken cannot be modified.

The number of stages (or their length) may depend on several factors. In the case of planning working time, the agreement between company and workers (or the collective bargaining agreement) must be considered.

For example, the agreement may establish a *frozen period* for the working hours so that the workers can plan their free time without being subjected to continuous changes of agenda. Thus, if the frozen period is one month long it will not be possible to consider stages shorter than that, as in this case it would be impossible to maintain the working hours of a full month without changes: notice that the number of working hours to be performed by the workers in a certain week of a stage cannot be known until it is known in which one of the possible states starting that stage we are.

Moreover, there can be a *communication period*. This means that the working hours to be done from a certain stage (e.g., from states e_2, e_3 o e_4) must be announced to the workers in advance. Note that this thought about the communication period affects neither the scenarios tree (Figure 1) nor the model presented in the following section. Nonetheless, it must be considered that the information to be taken into account to determine a new planning to be performed from the beginning of a stage (e.g., states e_2, e_3 o e_4) is actually the information available some time (communication period) before the beginning of that stage.

Figure 3 represents an example of the planning process. The length of the stages is equal to the frozen period. In the time t_{P2} a new planning for the weeks corresponding to Stage 2 and the following ones is determined and told to the workers; in t_{P3}, if new information is available, a new planning for the weeks corresponding to Stage 3 and the following ones is determined and told to the workers; and so forth.
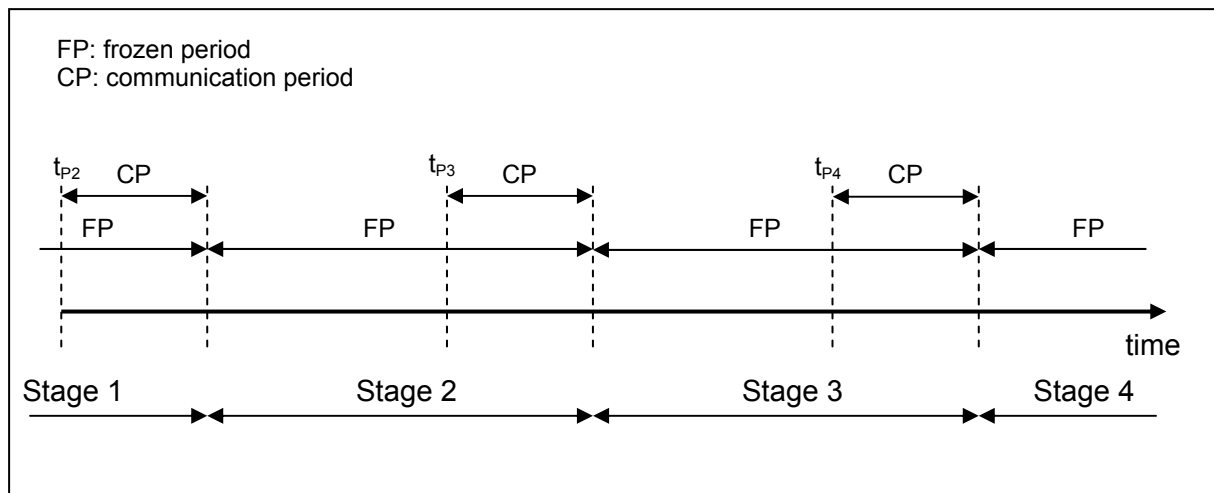
Figure 3. Planning process

On the other hand, it is meaningless to consider states for which there is no additional information. In Figure 1, this is the case of state e_9: in state e_3 the available information allow us to know for certain that the real scenario is the number 11 or 12; in state e_9 it is still not possible to distinguish between scenarios 11 and 12. Thus, all the planning done starting from state e_9 for the periods of the last stage is going to be the same as the one that can be done at the beginning of the second stage (in state e_3). This means that the number of stages between two consecutive states is not constant. In fact, the tree of Figure 1 could be considered as a general graph in which the arches do not represent period of time but connection between states only. In this case, it must be known which period corresponds to each state.

3.   Multistage optimisation model via scenarios

Following the optimisation scheme by means of scenarios a linear program model was developed. The solution, which will be feasible for all scenarios, provides the decisions that must be taken at each possible state and minimise the expected value for the total capacity shortage.

The notation used in this paper is defined in the following:

Data:

E set of states

e_0 first state

Γ_e^+ set of states reachable starting from state e ($\forall e \in E$)

τ_e time period (beginning of the week) corresponding to state e ($\forall e \in E$). Note that
$$\tau_i = \tau_j \quad \forall i, j \in \Gamma_e^+.$$

a_e state immediately precedent of state e ($\forall e \in E \backslash e_0$).

p_e probability of reaching state e from the previous state, a_e (i.e., is the probability that, being in state a_e, the future demand corresponds to any of the scenarios that are possible from state e, $\forall e \in E \backslash e_0$).

TF_e set of weeks belonging to the stage that finishes at the state e. That is to say, is the set of weeks between the precedent state of e (a_e) and the state e:
$$TF_e = \left\{ \tau_{a_e}, \ldots, \tau_e - 1 \right\}, \forall e \in E \backslash e_0$$

TI_e set of weeks belonging to the stage that starts at the state e. That is to say, is the set of weeks between state e and its successors (Γ_e^+):
$$TI_e = \left\{ \tau_e \ldots \tau_c - 1 \right\} \text{ with } a_c = e, \ \forall e \in E \mid \Gamma_e^+ \neq \varnothing.$$

$PTHI$ set of paths of the scenario tree. Each one starts at state e_0 and finishes at a state corresponding to the beginning of the last stage (i.e., the final states do not belong to a path). In the example of Figure 1, there are six possible paths: $PTHI=\{e_0\text{-}e_1\text{-}e_4, e_0\text{-}e_1\text{-}e_5, e_0\text{-}e_1\text{-}e_6, e_0\text{-}e_2\text{-}e_7, e_0\text{-}e_2\text{-}e_8, e_0\text{-}e_3\}$; note that according to previous explanations, state e_9 has been deleted.

$PTHF$ set of paths of the scenario tree, finishing each one in a final state (i.e., a scenario) and not including first state e_0. In the example of Figure 1, there are twelve possible paths: $PTHF=\{e_1\text{-}e_4\text{-}e_{10}, e_1\text{-}e_4\text{-}e_{11}, e_1\text{-}e_5\text{-}e_{12}, e_1\text{-}e_6\text{-}e_{13}, e_1\text{-}e_6\text{-}e_{14}, e_2\text{-}e_7\text{-}e_{15}, e_2\text{-}e_7\text{-}e_{16}, e_2\text{-}e_8\text{-}e_{17}, e_2\text{-}e_8\text{-}e_{18}, e_2\text{-}e_8\text{-}e_{19}, e_3\text{-}e_{20}, e_3\text{-}e_{21}\}$.

q_{pth} probability that the actual demand is the one of the scenario corresponding to the last state of the path pth ($\forall pth \in PTHF$). This probability is calculated as follows:
$$q_{pth} = \prod_{\forall e \in pth} p_e$$

C_{te} demand or required capacity (in working hours) for the scenarios that are possible from state e, and for the week t corresponding to the stage that finishes in that state ($\forall e \in E \backslash e_0; \ \forall t \in TF_e$). Note that in the weeks before state e, the required capacity (demand) is the same for all the scenarios that are possible from this state.

W set of workers.

S_i set of working weeks for worker i, $\forall i \in W$.

H_i number of annual working hours for worker i, $\forall i \in W$.

hm_{it} lower bound for the number of working hours of worker i in week t ($\forall i \in W; \forall t \in S_i$)

hM_{it} upper bound ($> hm_{it}$) for the number of working hours of worker i in week t ($\forall i \in W; \forall t \in S_i$).

L, h_L for every worker, the average number of working hours in a group of L consecutive weeks cannot be larger than h_L (with $L = 12$ and $h_L = 44$ hours).

Variables:

d_{te} capacity shortage (in working hours) for the week t belonging to the stage that finishes at state e ($\forall e \in E \setminus e_0;\ \forall t \in TF_e$), when the required capacity corresponds to any of the scenarios that are possible from state e. Remember that in the weeks before state e, the required capacity of the scenarios that are possible from this state is the same.

x_{ite} number of working hours of worker i in week t, which belongs to the stage starting from state e ($\forall i \in W;\ \forall e \in E \mid \Gamma_e^+ \neq \varnothing;\ \forall t \in \{TI_e \cap S_i\}$)

Model:

$$[MIN]\, z = \sum_{\forall pth \in PTHF} q_{pth} \cdot \left(\sum_{\forall e \in pth} \sum_{\forall t \in TF_e} d_{te} \right) \tag{1}$$

$$\sum_{\forall e \in pth} \sum_{\forall t \in \left(TI_e \cap S_i \right)} x_{ite} = H_i \qquad \forall i \in W; \forall pth \in PTHI \tag{2}$$

$$\left(\sum_{\forall i \in W \mid t \in S_i} x_{it a_e} \right) + d_{te} \geq C_{te} \qquad \forall e \in E \setminus e_0;\ \forall t \in TF_e \tag{3}$$

$$\left(\sum_{t \in [j-L+1..j]} x_{itE0} \right) \leq h_L \cdot L \quad \forall i \in W; j = L,...,T \mid \left[j-L+1,...,j \right] \in \left(S_i \cap TI_{e0} \right) \tag{4}$$

$$\left(\sum_{t\in[j-L+1..j]\,\mid\,t\in TI_{a_e}} x_{it_{a_e}}\right)+\left(\sum_{t\in[j-L+1..j]\,\mid\,t\in TI_e} x_{ite}\right)\le h_L\cdot L \tag{5}$$

$$\forall i\in W;\ \left(j=L,...,T\mid[j-L+1,...,j]\in S_i\right);\ \left(\forall e\in E\backslash e_0\mid\Gamma_e^+\ne\varnothing\wedge j\in TI_e\right)$$

$$hm_{it}\le x_{ite}\le hM_{it}\qquad \forall i\in W;\ \forall e\in E\mid\Gamma_e^+\ne\varnothing;\ \forall t\in\left(TI_e\cap S_i\right) \tag{6}$$

The objective function to minimise (1) corresponds to the expected value for the total capacity shortage; (2) expresses, for each worker, the annual balance of working hours. The balance is set for each possible path of the graph, thus guaranteeing that the solution will be feasible for all the possible scenarios; (3) imposes, for each state (except the first, e_0) and week belonging to the stage that finishes at this state, that the sum of the planned capacity and the capacity shortage must be larger than or equal to the required capacity; (4) and (5) impose, for each worker and group of L consecutive working weeks, the upper bound for the average number of working hours: Equation (4) refers to the weeks corresponding only to the first stage and equation (5) includes the possibility that a block of L weeks is distributed on two consecutive stages. In the case of short stages, this equation can be easily modified in order to include the possibility that the block of L weeks comprise the necessary number of stages; lastly, (6) imposes the lower and the upper bound for the number of working hours for each worker and week.


4.   Computational experiments

A computational experiment was performed in order to evaluate the effectiveness of the stochastic model. Overall, the results can be considered very satisfactory. Two were the main objectives of the experiments: (1) to check if the model can be solved in short times for realistic size instances (i.e., involving a large number of scenarios and states); and (2) to quantify the benefit of applying a scenario optimisation model instead of considering the required capacity as a deterministic value. For this purpose, the average required capacity was token as a data of the deterministic model and the comparing criterion was the expected value for the total capacity shortage.

A total amount of 360 instances were generated combining the different data, which is described below.

‒ Number of workers: 10, 50, 100

‒ Planning horizon: 52 weeks (one year)

‒ For each path and from each state, a required capacity is generated for each one of the weeks belonging to the stage starting in that state. The required capacity is set at random between $20 \cdot N$ and $50 \cdot N$.

‒ Number and length of the stages: since the planning horizon is fixed, the number of stages depends on the length. As it has been said before, the length of the stages belonging to a scenario tree is not constant. From a state, the next stage can have a length of S, $2 \cdot S$, $3 \cdot S$ or $4 \cdot S$ with probabilities 0.8, 0.1, 0.05 and 0.05. Two values have been considered for the basic stage length S: 6 and 12 weeks.

‒ Three types of scenario tree were generated as follows (in Figure 4 a scenario tree of each type is given):

  ▪ Type 1 (regular diversification): The number of states reachable from any state is 2 or 3, at random.

  ▪ Type 2 (diversification at the beginning): The number of states reachable from the states belonging to the first three stages is set at random between 3 and 6. For the following stages, the number of successor states is 1 or 2 with probabilities 0.75 and 0.25 respectively. This way of generating the number of states gives a tree that expands a lot at the beginning but not so much at the end.

  ▪ Type 3 (diversification at the end): The number of states reachable from the states belonging to the first stages is set to 1 or 2 with probabilities 0.75 and 0.25, respectively. For the last three stages, the number of successor states is set at

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random between 3 and 6. This way of generating the number of states gives a tree with a great expansion at the end.
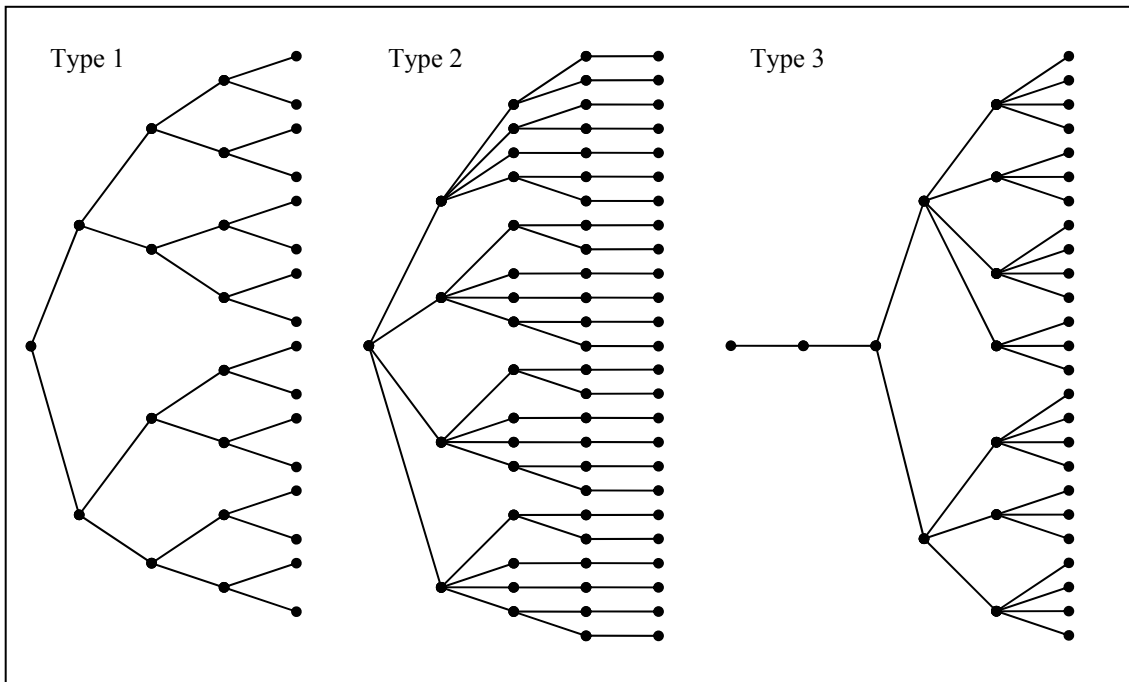

Figure 4. Types of scenario tree

This way of generating the scenario trees gives some instances with a very large number of states and scenarios, which may not look very realistic but are appropriate to test the effectiveness of the optimisation model.

The experiment was performed with ILOG CPLEX 8.1 and a Pentium IV at 3.4 GHz with 512 MB of RAM. The maximum computing time was set to 3,600 seconds. 42 of the 360 instances were too large to be solved (the computer ran out of memory). The number of workers, scenarios and states and the number of variables and constraints of the solved instances (i.e., the size of solved instances) can be seen in Figure 5 and 6, respectively.
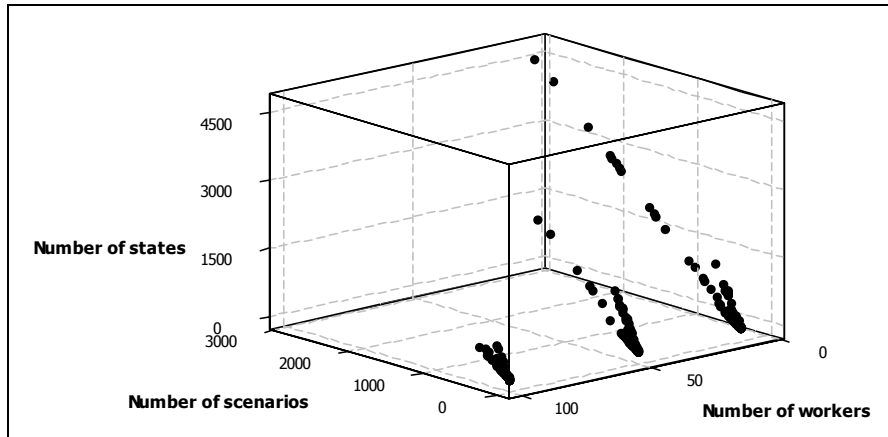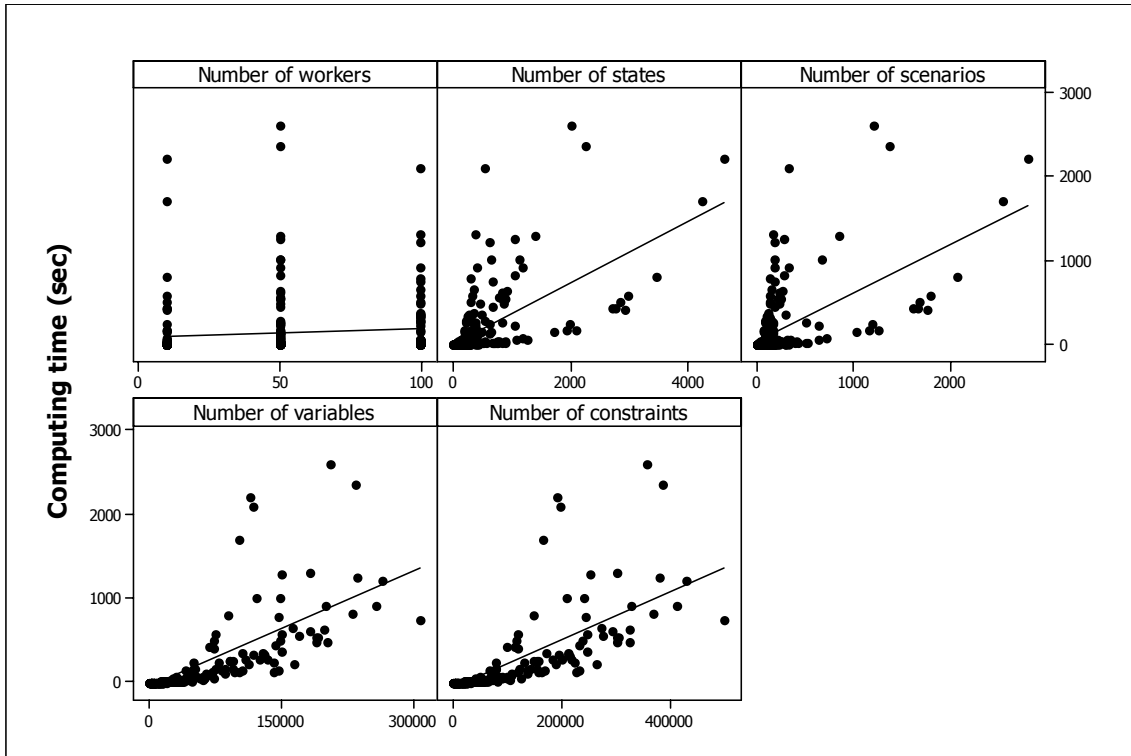
Figure 5. Number of workers, scenarios and states for each instance
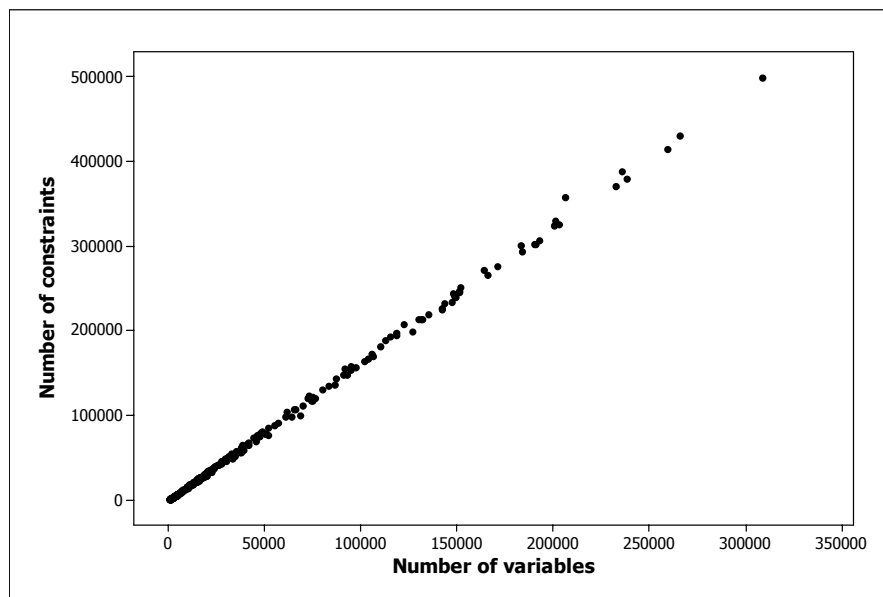
Figure 6. Number of variables and constraints for each instance

The main results of the computational experiments are given in Figure 7 and Table 1. The Figure show the computing times for the scenario model by number of workers, number of scenarios, number of states, number of variables and number of constraints. It can be observed that the influence of the number of scenarios, states, variables and constraints on the computing time is more clear than the influence of the number of workers. Overall, it is also shown that the proposed model can be solved in relatively short times for very large instances (probably larger than an actual instance would be).

Figure 7. Computing times (in seconds) by number of workers, scenarios, states, variables and constraints

In Table 1 the quality of the deterministic and stochastic models solutions is compared: the minimum, the average and the maximum saving (i.e., the % of decrease obtained in the expected value for the total capacity shortage when solving the stochastic model instead of the deterministic one) is given. As expected, considering uncertainty by means of scenario analysis results in a significance reduction of the capacity shortage and, therefore, of the corresponding costs.

Table 1. Minimum, average and maximum saving

Min Saving* (%)	Avg Saving* (%)	Max Saving* (%)
0.001	17.45	100

$* \text{ Saving} = \dfrac{\overline{D}_{\text{deterministic model}} - \overline{D}_{\text{scenario model}}}{\overline{D}_{\text{deterministic model}}} \cdot 100$; $\overline{D}$: expected value for the total capacity shortage

5.   Conclusions

Annualising working hours allows companies to adapt capacity to demand. Mixed and Linear Programming, which have been tested in previous works, are appropriate and efficient tools to plan working time optimising the use of the productive resources (e.g., minimising costs due to the lack of capacity) whilst observing a set of conditions.

The demand or required capacity, which is an essential data to plan working time, depends usually on several and complex factors and, often, it is not possible to obtain a reliable prediction of it or it is not realistic to consider that can be adjusted to a probability distribution. In some cases, however, it is possible to determine a set of demand scenarios, each one with a related probability.

This work proposes the use of an scenario optimisation approach to face the uncertainty of the demand and presents a Linear Programming mathematical model that gives a robust solution (i.e., that will be feasible under any scenario) whilst optimising the expected value for the total capacity shortage. The results of a wide computational experiment allow us to conclude that the model can be efficiently solved to optimality even for very large instances.

Finally, the expected value for the total capacity shortage obtained with the scenario model has been compared to the one that is obtained using the average demand with a deterministic model. The difference between the solutions of the two models gives and average saving of 17.45%, which lead us to conclude that the proposed model is an appropriate and useful tool to deal with the uncertainty of the demand when planning working time under annualised hours.

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