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ABSTRACT

Annualising working hours (AH) is a means of achieving flexibility in the use of human resources to face the seasonal nature of demand. Some of the existing planning procedures are able to minimise costs due to overtime and temporary workers but, due to the difficulty of solving the problem, it is normally assumed both that the holiday weeks are fixed beforehand and that workers from different categories who are able to perform a specific type of task have the same efficiency. In the present paper, those assumptions are relaxed and a more general problem is solved. The computational experiment leads to the conclusion that MILP is a technique suited to dealing with the problem.

Keywords: human resources, manpower planning, annualised hours.

1. Introduction

Annualising working hours (AH)—i.e., the possibility of irregularly distributing the total number of staff working hours over the course of a year—is a means of achieving flexibility, because AH allows production capacity to be adapted to fluctuations in demand, thus reducing costs (overtime, temporary workers and inventory costs).

AH gives rise to new problems that have hitherto been given little attention in the literature. For instance, in Hung (1999a), Hung (1999b), Grabot and Letouzey (2000) and Azmat and Widmer (2004) it is emphasised that the concept of annualised hours is surprisingly absent from the literature on planning and scheduling. A significant difficulty to be faced is that the diversity of production systems means that the problems that AH entails vary greatly; in

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Corominas et al. (2004), the characteristics of the planning problem are discussed and a classification scheme is proposed, giving rise to thousands of different cases; moreover, AH often implies the need to solve a complicated working time planning problem. Some authors deal with different versions of the problem (e.g., Hung, 1999a; Hung, 1999b; Vila and Astorino, 2001; Azmat and Widmer, 2004; Azmat et al., 2004; Corominas et al., 2002), but most papers (e.g., Lynch, 1995; MacMeeking, 1995 and Mazur, 1995) discuss AH only from a qualitative point of view.

In Corominas et al. (2002), a MILP (Mixed Integer Linear Programming) model is used to solve the problem of planning staff working hours with an annual horizon. Two hierarchical categories of workers are considered and the costs of overtime and of employing temporary workers are minimised. In the aforementioned paper, the following is assumed to ease its resolution: (i) the holiday weeks are fixed a priori; and (ii) the workers from different categories who are able to perform a specific type of task have the same efficiency.

Actually, although workers from different categories may be able to perform a specific type of task, obviously certain categories frequently require more time than others do. In addition, the allocation of holiday weeks may be a decision variable of the model with the objective both of minimising costs and helping in the bargaining process: computing the difference of costs between a situation in which holidays are fixed a priori and one in which those are decision variables, the company knows the maximum amount of money that could offer to workers in exchange of being able to fix their holidays in the best moment. Therefore, in this paper, assumptions in aforementioned paper are relaxed and a more general problem is solved.

The main aims are to approach the planning of working hours and holiday weeks over the course of a year in services that employ cross-trained workers who have different relative efficiencies, to show that MILP is an appropriate tool for this aim, and to quantify the improvement in the solution when there is the possibility of determining holiday weeks with the model. The rest of the paper is organised as follows: section 2 introduces the problem and four MILP models for planning AH over a year; section 3 includes the results of the computational experiment; section 4 shows how results could be used to help in the bargaining process; and, finally, section 5 exposes the conclusions.

2. Four MILP models to plan holidays and working time under AH

Solving the planning problem involves determining the number of weekly working hours and holiday weeks for each member of staff.

A service system that is carried out on an individual basis is considered (so working hours for each worker may be different). Different types of tasks are involved, the product is not storable and the company forecasts the demand.

The production capacity in any given week must be greater than or equal to that which is needed and, if the staff does not provide entirely this capacity, temporary workers will be hired for the number of hours required. Overtime is admitted, but its total amount is bounded; overtime hours are classified into two blocks and the cost of an hour belonging to the second block is greater than that of an hour of the first. From the outset, the objective function is the cost of overtime plus the cost of employing temporary workers; it is possible to break the tie between optimal solutions by considering the penalties associated with the assignment of different types of tasks to categories of employees (adding this function to the first one with a small weight).

Workers from different categories may frequently be able to perform a specific type of task, although certain categories may require more time than others may. Therefore, cross-trained workers are considered: certain categories can perform different types of tasks and can have different relative efficiencies associated with them (for example, a value of 0.9 means that a worker in that category needs to work 1/0.9 hours to serve a demand that a worker with a relative efficiency equal to 1 would serve in 1 hour).

The conditions to be fulfilled by the solution are the following (see Corominas et al., 2002 for more details):

i) the total of annual working hours is fixed;

ii) the weekly number of working hours must fall within an interval defined by a lower and upper bound;

iii) the average weekly working hours for any set of twelve consecutive weeks is upper bounded;

iv) if the average of weekly working hours over a specified number of consecutive weeks ("week-block") is greater than a certain value, then, over a given number of weeks immediately succeeding the week-block, the number of working hours must not be greater than a certain value;

v) if “strong” and “weak” weeks are defined as those in which the number of working hours is respectively greater or less than certain specified values, there is an upper bound for the number of strong weeks and a lower bound for the number of weak weeks.

Below, we introduce the four MILP models to be tested.

The objective function to be minimised in models *M1* and *M2* has already been specified: the cost of overtime plus the cost of employing temporary workers (the penalties associated with the assignment of types of tasks to categories are considered in order to break the tie between optimal solutions). Cross-trained workers are considered in both models. In *M1,* holiday weeks are determined by the model but, in *M2,* these are fixed a priori (in both cases, two consecutive holiday weeks in winter and four consecutive holiday weeks in summer are assumed).

As pointed out by Corominas et al. (2002), the AH models that minimise the cost usually have an infinite number of optimal solutions. In addition, in the solution provided by the optimiser, the number of weekly working hours for an employee over the course of a year and weekly working time provided by temporary workers for each week are usually very irregular. To regularise the profile of an employee’s working hours over a year and the profile of weekly working time provided by temporary workers, i.e., to obtain the most regular solution from all those that involve the minimum cost, two other models (*M3* and *M4*) are used.

The objective function to be minimised in models *M3* and *M4* is the weighted sum of: i) the sum of the discrepancies between the weekly working hours of staff members and the average weekly working hours; and (ii) the sum of the discrepancies between the working hours provided by workers who do not belong to the staff and the average of weekly working hours provided by these workers. The penalties associated with the assignment of types of tasks to categories are again considered to break the tie between optimal solutions. In both models, the minimum cost obtained by *M1* is guaranteed. The difference between *M3* and *M4* is that in *M3* the holiday weeks are determined by the model but in *M4* these are the ones obtained when solving *M1*.

We use the following notation:

Data

T	Weeks in the planning horizon
C	Set of categories of workers
F	Set of types of tasks
E	Set of members of staff
r_{jk}	Relative efficiency associated with the workers in category j in the accomplishment of tasks of type k ($\forall j \in C$; $\forall k \in F$); $0 \leq r_{jk} \leq 1$. If $r_{jk}=0$, workers in category j are not able to perform tasks of type k.
$\hat{C}_k$	Sets of categories of workers that can be assigned to tasks of type k ($\forall k \in F$)
$\hat{F}_j$	Sets of types of tasks which can be performed by employees in category j ($\forall j \in C$)
p_{jk}	Penalty associated with an hour of work in a task of type k of a staff member in category j ($\forall k \in F$; $\forall j \in \hat{C}_k$)
l	Parameter to weigh the penalties to establish the trade-off between these and the monetary costs of the solution.
$\hat{E}_j$	Set of employees in category j ($\forall j \in C$)
r_{tk}	Required capacity (in working hours) for tasks of type k in week t (t=1,..,T; $\forall k \in F$)
H_i	Stipulated ordinary annual working hours of employee i ($\forall i \in E$)
a_1, a_2	Maximum proportions, over the annual amount of ordinary working hours, of overtime corresponding to blocks 1 and 2 respectively.
$b1_i$, $b2_i$	Respectively, the cost of an hour of overtime for block 1 and block 2 for employee i ($\forall i \in E$), with $b1_i < b2_i$
hm_{it}, hM_{it}	Lower and upper bounds of the number of working hours for worker i in week t ($\forall i \in E$; t=1,..,T); $hm_{it} < hM_{it}$
L, h_L	L is the maximum number of consecutive weeks in which the average weekly working hours cannot be greater than h_L
B, b, h_B, h_b	b is the minimum number of weeks, after a week-block of B consecutive weeks with a weekly average of working hours greater than h_B, in which the number of weekly hours cannot be greater than h_b
N_S, h_S	N_S is the maximum number of “strong” weeks, i.e., weeks with a number of

working hours greater than h_S

N_W, h_W — N_W is the minimum number of “weak” weeks, i.e., weeks with a number of working hours not greater than h_W

$hw1_i$, $hw2_i$ — Number of holiday weeks in the first and second holiday periods respectively for worker i ($\forall i \in E$)

$t1_i$, $t2_i$ — First and last week respectively in which worker i can take holidays in the first holiday period ($\forall i \in E$)

$t3_i$, $t4_i$ — First and last week respectively in which worker i can take holidays in the second holiday period ($\forall i \in E$)

g_k — Cost of an hour for tasks of type k performed by a worker who is not a member of staff ($g_k > b2_i$, $\forall i \in \hat{E}_j \,\big|\, j \in \hat{C}_k$)

Variables

x_{it} — Working hours of employee i in week t $(\forall i \in E; t = 1,...,T)$.

y_{tjk} — Working hours of employees in category j dedicated to tasks of type k in week t ($\forall k \in F;\quad \forall j \in \hat{C}_k;\quad t = 1,...,T$).

d_{tk} — Working hours corresponding to tasks of type k to be supplied in week t by workers who are not members of staff ($\forall k \in F$; t=1,…,T).

$v1_i$, $v2_i$ — Overtime corresponding respectively to blocks 1 and 2 of employee i ($\forall i \in E$).

$vc1_{it} \in \{0,1\}$ — Indicates whether employee i starts the first holiday period in week t ($\forall i \in E$, t=$t1_i$,...,$t2_i$-$hw1_i$+1).

$vc2_{it} \in \{0,1\}$ — Indicates whether employee i starts the second holiday period in week t ($\forall i \in E$, t=$t3_i$,...,$t4_i$-$hw2_i$+1).

$d_{it} \in \{0,1\}$ — Indicates whether the average working hours of employee i, in a week-block of B weeks that ends with week t, is (or is not) greater than h_B hours ($\forall i \in E$; t=B,…,T-b).

$s_{it} \in \{0,1\}$ — Indicates whether employee i has a planned number of working hours greater than h_S hours for week t ($\forall i \in E$; t=1,…,T).

$w_{it} \in \{0,1\}$ — Indicates whether employee i has a planned number of working hours equal to or less than h_W hours for week t ($\forall i \in E$; t=1,…,T).

All the non-binary variables are real and non-negative.

MODEL 1 (M1)

$$[MIN]z = \sum_{i\in E} b1_i \cdot v1_i + \sum_{i\in E} b2_i \cdot v2_i + \sum_{k\in F}\sum_{t=1}^{T} g_k \cdot d_{tk} + l \cdot \sum_{t=1}^{T}\sum_{k\in F}\sum_{j\in \hat{C}_k} p_{jk} \cdot y_{tjk} \qquad (1)$$

$$\sum_{t=1}^{T} x_{it} = H_i + v1_i + v2_i \qquad \forall i \in E \qquad (2)$$

$$v1_i \le a_1 \cdot H_i \qquad \forall i \in E \qquad (3)$$

$$v2_i \le a_2 \cdot H_i \qquad \forall i \in E \qquad (4)$$

$$\sum_{i\in \hat{E}_j} x_{it} = \sum_{k\in \hat{F}_j} y_{tjk} \qquad t = 1,\dots,T;\ \forall j \in C \qquad (5)$$

$$\sum_{j\in \hat{C}_k} r_{jk} \cdot y_{tjk} + d_{tk} \ge r_{tk} \qquad t = 1,\dots,T;\ \forall k \in F \qquad (6)$$

$$\sum_{t=t-L+1}^{t} x_{it} \le L \cdot h_L \qquad t = L,\dots,T;\ \forall i \in E \qquad (7)$$

$$\sum_{t=t-B+1}^{t} x_{it} \le B \cdot h_B + \left(\sum_{t=t-B+1}^{t} hM_{it} - B \cdot h_B \right) \cdot d_{it} \qquad t = B,\dots,T-b; \forall i \in E \qquad (8)$$

$$\sum_{t=t-B+1}^{t} x_{it} \le B \cdot h_B \qquad t = T-b+1,\dots,T;\ \forall i \in E \qquad (9)$$

$$x_{i,t+l} \le hM_{i,t+l} - (hM_{i,t+l} - h_b) \cdot d_{it} \qquad \forall i \in E;\ t = B,\dots.T-b;\ l = 1,\dots,b \qquad (10)$$

$$x_{it} \le h_S + (hM_{it} - h_S) \cdot s_{it} \qquad \forall i \in E;\ t = 1,\dots T \qquad (11)$$

$$x_{it} \le hM_{it} - (hM_{it} - h_W) \cdot w_{it} \qquad \forall i \in E;\ t = 1,\dots,T \qquad (12)$$

$$\sum_{t=1}^{T} s_{it} \le N_S \qquad \forall i \in E \qquad (13)$$

$$\sum_{t=1}^{T} w_{it} \ge N_W \qquad \forall i \in E \qquad (14)$$

$$\sum_{t=t1_i}^{t2_i - hw1_i + 1} vc1_{it} = 1 \qquad \forall i \in E \qquad (15)$$

$$\sum_{t=t3_i}^{t4_i - hw2_i + 1} vc2_{it} = 1 \qquad \forall i \in E \qquad (16)$$

$$x_{it} \le hM_{it} \qquad \forall i \in E;\ t \notin \left([t1_i,\dots,t2_i] \vee [t3_i,\dots,t4_i] \right) \qquad (17)$$

$$x_{it} \ge hm_{it} \qquad \forall i \in E;\ t \notin \left([t1_i,\dots,t2_i] \vee [t3_i,\dots,t4_i] \right) \qquad (18)$$

$$x_{it} \le hM_{it} \cdot \left(1 - \sum_{t=\max(t1_i,\ t-hw1_i+1)}^{\min(t,\ t2_i - hw1_i + 1)} vc1_{it} \right) \qquad \forall i \in E;\ t = t1_i,\dots,t2_i \qquad (19)$$

$$x_{it} \geq hm_{it} \cdot \left(1 - \sum_{t=\max(t1_i,\, t-hw1_i+1)}^{\min(t,\, t2_i-hw1_i+1)} vc1_{it}\right) \qquad \forall i \in E;\ t = t1_i,\ldots,t2_i \tag{20}$$

$$x_{it} \leq hM_{it} \cdot \left(1 - \sum_{t=\max(t3_i,\, t-hw2_i+1)}^{\min(t,\, t4_i-hw2_i+1)} vc2_{it}\right) \qquad \forall i \in E;\ t = t3_i,\ldots,t4_i \tag{21}$$

$$x_{it} \geq hm_{it} \cdot \left(1 - \sum_{t=\max(t3_i,\, t-hw2_i+1)}^{\min(t,\, t4_i-hw2_i+1)} vc2_{it}\right) \qquad \forall i \in E;\ t = t3_i,\ldots,t4_i \tag{22}$$

$$d_{it} \in \{0,1\} \qquad \forall i \in E;\ t = B,\ldots,T-b \tag{23}$$

$$s_{it}, w_{it} \in \{0,1\} \qquad \forall i \in E;\ t = 1,\ldots,T \tag{24}$$

$$vc1_{it} \in \{0,1\} \qquad \forall i \in E; t = t1_i,\ldots,t2_i - hw1_i + 1 \tag{25}$$

$$vc2_{it} \in \{0,1\} \qquad \forall i \in E; t = t3_i,\ldots,t4_i - hw2_i + 1 \tag{26}$$

$$v1_i, v2_i \geq 0 \qquad \forall i \in E \tag{27}$$

$$y_{tjk} \geq 0 \qquad t = 1,\ldots,T;\ \forall k \in F;\ \forall j \in \hat{C}_k \tag{28}$$

$$d_{tk} \geq 0 \qquad t = 1,\ldots,T;\ \forall k \in F \tag{29}$$

(1) is the objective function, which includes the cost of overtime plus that of employing external workers and the (weighted) penalties associated with the assignment of tasks to the types of employees on the staff; (2) imposes that the total number of worked hours should be equal to the ordinary annual hours stipulated plus overtime, if applicable; (3) and (4) stipulates that the overtime for each of the two blocks should not exceed their respective upper bounds; (5) is the balance between the hours provided by specific types of workers of the staff and the hours assigned to different types of tasks; (6) expresses that the hours assigned to a type of task that are to be carried out by members of staff plus, if applicable, the hours provided by external workers for that same type of task must not be less than the number of hours required; (7) imposes the upper bound on the average weekly working hours for any subset of L consecutive weeks; (8) implies that variable d_{it} is equal to 1 if the average number of working hours in a week-block of B weeks is greater than h_B; (9) prevents the average hours worked from being greater than h_B in the last weeks of the year, when after the week-block of B weeks there are no longer b weeks to "compensate"; (10) implies that, if variable d_{it} is equal to 1, the upper bound of the number of working hours is h_b; (11) imposes that, if the number of working hours is greater than h_S, then variable s_{it} is equal to 1; (12) states that, if the number of working hours is greater than h_W, then variable w_{it} is equal to 0; (13) and (14) stipulate that the number of "strong" and "weak" weeks cannot be greater than N_S and less than N_W respectively; (15) and (16) establish that the worker must start holidays in one and only one week; (17) and (18) set the lower and upper bounds of the number of weekly working hours in non-holiday weeks; (19), (20), (21) and (22) set the

lower and upper bounds of the number of weekly working hours for possible holiday weeks; (23), (24), (25) and (26) express the binary character of the corresponding variables; and (27), (28) and (29) impose the non-negative character of the rest of the non-binary variables.

MODEL 2 (M2)

M2, which considers holidays as a data, can be obtained by deleting the variables $vc1_{it}$ and $vc2_{it}$ and their associated constraints (15, 16 and 19 to 22, 25 and 26) from model *M1* and making several minor and straightforward modifications.

MODEL 3 (M3)

Once model *M1* has been solved, the cost of overtime and temporary workers is stored. The formalisation of *M3* is not included but it may be easily obtained by starting from model *M1* and keeping in mind the following changes:

i) A constraint is added, which requires that the cost of the solution of *M3* cannot exceed that obtained with *M1*.

ii) Variables x_{it} are eliminated using the expression $x_{it} = \overline{x}_i + x_{it}^+ - x_{it}^-$, where $\overline{x}_i$ is the average number of weekly working hours corresponding to worker *i* and x_{it}^+ and x_{it}^- are the positive and negative deviations from the average number of working hours of worker *i* in week *t*.

iii) Variables d_{tk} are eliminated using the expression $d_{tk} = \overline{d}_k + s_{tk}^+ - s_{tk}^-$, where $\overline{d}_k$ is the average number of weekly working hours provided by temporary workers for a task of type *k* and s_{tk}^+ and s_{tk}^- are the positive and negative deviations from the average number of working hours provided by temporary workers for task *k* in week *t*.

iv) The objective function to be minimised is replaced with a new one that has three weighted components. The first is the sum of the discrepancies in the number of working hours of staff members and the second is the sum of the discrepancies in the number of working hours provided by temporary workers. The penalties associated with the assignment of tasks to categories of workers are also considered to break the possible tie between optimal solutions:

$$\sum_{i \in E} \sum_{t=1}^{T} \left(x_{it}^+ + x_{it}^- \right) + \sum_{t=1}^{T} \sum_{k \in F} \left(s_{tk}^+ + s_{tk}^- \right) + l \cdot \sum_{t=1}^{T} \sum_{k \in F} \sum_{j \in \hat{C}_k} p_{jk} \cdot y_{tjk}$$

MODEL 4 (M4)

M4 can be obtained from *M3* by fixing the holiday weeks obtained when solving *M1* (basically, variables $vc1_{it}$, $vc2_{it}$ and their associated constraints have to be deleted).

3. Computational experiment

A computational experiment was performed to evaluate the effectiveness (in terms of computing time and the quality of the solutions) of the models. Overall, the results, as it is justified below, were very satisfactory.

The basic data used for the experiment are as follows:

– Five MILP models: *M1*, *M2*, *M3*, *M4* and *M4*+*M3'* (this compound model consists in solving *M4* and, in the remaining calculation time, executing *M3'*, which is obtained when a constraint is imposed on *M3* so that the value of the solution of *M3* cannot exceed the value obtained by means of *M4*).

– 10, 40, 70, 100 and 250 staff workers.

– A time horizon of 52 weeks (46 working weeks and 6 holiday weeks).

– The holiday weeks for each worker are distributed into two uninterrupted periods, including two weeks in winter and four weeks in summer. In *M2*, the temporary allocation of holidays (for each worker) was fixed at random.

– There are three categories and three types of tasks. There are two patterns of relative efficiency (and penalty). Table 1 shows the relative efficiency (and the penalty) values for each pattern.

	Pattern 1			**Pattern 2**		
	Task 1	Task 2	Task 3	Task 1	Task 2	Task 3
Category 1	1 (1)	0.9 (2)	0	1 (1)	0	0
Category 2	0	1 (1)	0.9 (2)	0.9 (2)	1 (1)	0
Category 3	0	0	1 (1)	0.8 (2)	0	1 (1)

Table 1. Relative efficiency (and penalty) values for Pattern 1 and Pattern 2

– The capacity (in working hours) required over the year follows three different patterns. Demand Type 1 corresponds to a non-seasonal capacity pattern with a random noise of $\pm 5\%$. Demand Type 2 corresponds to a seasonality pattern with one peak, with a random noise of $\pm 5\%$. Demand Type 3 corresponds to a seasonality pattern with two peaks, with a random noise of $\pm 5\%$. In each case, the total demand is equal to the total capacity multiplied by 0.99.

For every combination of models, number of staff workers, type of demand and pattern of relative efficiency (and penalty), 20 instances were generated (varying demand noise and, in *M2*, holiday weeks at random), which gave 3,000 instances.

In spite of the dimension of the models may be considered large (the average number of variables and constraints are given in Table 2); they were solved to optimality using an ILOG CPLEX 8.1 optimiser and a Pentium IV PC at 1.8 GHz with 512 Mb of RAM. The absolute and relative MIP gap tolerances were set to 0.01. The maximum computing time for all instances was set to 1,800 seconds.

		Number of workers				
		10	**40**	**70**	**100**	**250**
MODELS	***M1***	2,817/3,915	9,387/14,715	15,957/25,515	22,527/36,315	55,377/90,315
	M2	2,310/2,567	7,357/9,319	12,405/16,072	17,452/22,822	42,689/56,572
	M3	4,169/4,592	13,859/16,952	23,549/29,312	33,239/41,672	81,689/103,472
	M4	3,664/3,658	11,835/13,191	20,004/22,718	28,172/32,230	69,035/79,946
	M4+M3'	4,170/4,594	13,860/16,954	23,550/29,314	33,240/41,674	81,690/103,474

Table 2. Average number of variables/constraints

For each model and each number of staff workers, the number of instances that do not have solutions, that have feasible solutions and that have a proven optimal solution are given in Table 3 (for the model *M4+M3'*, the number of instances in which there was not enough time to carry out *M3'* is added). Table 4 shows the minimum (t_{min}), the average ($\bar{t}$) and the maximum computing time (t_{max}) (in seconds).

			Number of workers				
			10	**40**	**70**	**100**	**250**
MODELS	***M1***	***No solution***	0	0	0	0	0
		Feasible solution	59	57	7	1	0
		Optimal solution	61	63	113	119	120
	M2	***No solution***	0	0	0	0	0
		Feasible solution	0	0	0	0	0
		Optimal solution	120	120	120	120	120
	M3	***No solution***	1	2	0	0	0
		Feasible solution	109	11	27	112	120
		Optimal solution	10	107	93	8	0
	M4	***No solution***	0	0	0	0	0
		Feasible solution	0	0	0	0	22
		Optimal solution	120	120	120	120	98
	M4+M3'	***No time for M3'***	0	0	0	0	22
		No solution of M3'	2	11	1	8	98
		Feasible solution of M3'	106	16	3	108	0
		Optimal solution of M3'	12	93	116	4	0

Table 3. Number of instances with no solution, with a feasible solution and with a proven optimal solution

			Number of workers				
			10	**40**	**70**	**100**	**250**
MODELS	***M1***	t_{min}	24.20	15.55	26.49	42.91	139.94
		$\bar{t}$	935.91	890.23	164.88	82.59	300.02
		t_{max}	1,800	1,800	1,800	1,800	1,097.71
	M2	t_{min}	7.06	7.89	8.75	9.64	16.27
		$\bar{t}$	9.53	9.41	11.21	12.26	30.58
		t_{max}	110.78	14.86	21.63	20.73	198.66
	M3	t_{min}	130.03	193.30	671.26	1,450.44	1,800
		$\bar{t}$	1,716.66	716.28	1,369.97	1,790.47	1,800
		t_{max}	1,800	1,800	1,800	1,800	1,800
	M4	t_{min}	6.22	25.52	67.25	105.91	531.20
		$\bar{t}$	9.82	36.49	119.86	265.38	1,361.07
		t_{max}	156.08	78.92	258.28	446.29	1,800
	M4+M3'	t_{min}	79.22	200.92	656.06	1,408.45	1,800
		$\bar{t}$	1,695.76	842.78	1,238.95	1,793.65	1,800
		t_{max}	1,800	1,800	1,800	1,800	1,800

Table 4. Computing times (in seconds)

The maximum computing times are very reasonable considering the problem to be solved (the aim of the models is to establish an annual plan) and its maximum size (two hundred and fifty workers, which is a large enough number, since we are supposed to be dealing with a production system of services or a part of this system). For the models in which costs were to be minimised (*M1* and *M2*), feasible solutions were always obtained and most of these were optimal solutions. Regarding the models which have regularity as objective (*M3*, *M4* and *M4+M3'*), in only one test (of *M3*) no feasible solution was obtained. The variants that were hardest to solve were *M1* and *M3* (or *M3'*), as expected, given that these variants include more constraints and binary variables than others do.

The experiments provided satisfactory results regarding the quality of the solutions of the models. Table 5 shows the minimum (a_{min}), the average ($\bar{a}$) and the maximum (a_{max}) percentage saved when *M1* is used versus *M2*. As shown, the possibility of determining holiday weeks with model *M1* provides very good solutions and savings of more than 90%. These values also show how the capacity of the staff can be adapted to demand by determining the holiday weeks of the staff (this is also due to the flexibility provided by the annualisation of working time).

		Number of workers				
		10	**40**	**70**	**100**	**250**
M1 vs. M2	a_{min}	65.24	97.17	99.02	99.69	100
	$\bar{a}$	89.53	99.49	99.96	99.99	100
	a_{max}	99.75	100	100	100	100

Table 5. Percentage saved when using *M1* versus *M2*

The way in which capacity is adapted to demand can also be seen in Figure 1, in which required capacity (demand), workers capacity and the hours to be provided by temporary workers (shortage) are represented.

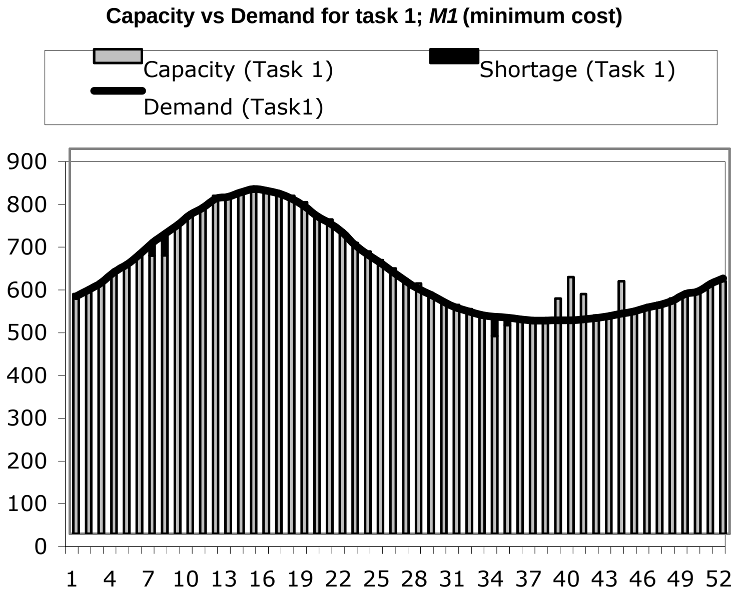

Figure 1. Capacity vs Demand for task 1, *M1*

It can be concluded that, from the point of view of the company, the quality of the solution can be considered very good. Although for the workers the solution should be quite good (given the amount and type of conditions imposed to the solution), looking at Figure 2, in which the working hours of a certain worker are represented, one can see that the distribution of the working time is significantly irregular, resulting in a solution which few workers would easily accept. Fortunately, as it can be observed in Figure 3, this problem is well solved by using the models whose objective is the regularity at minimum cost (models *M3*, *M4* and *M4+M3'*).

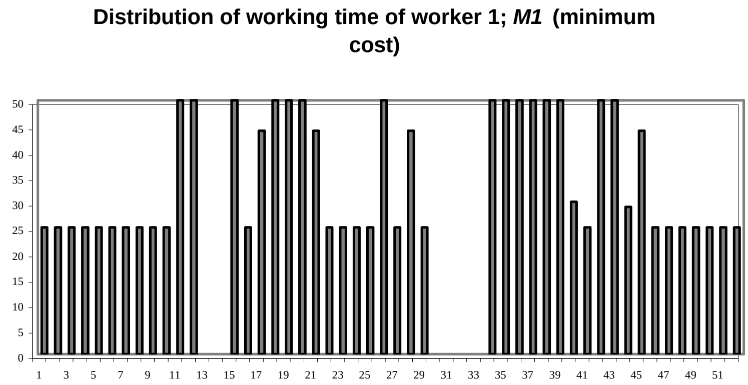

Figure 2. Distribution of working time, *M1*

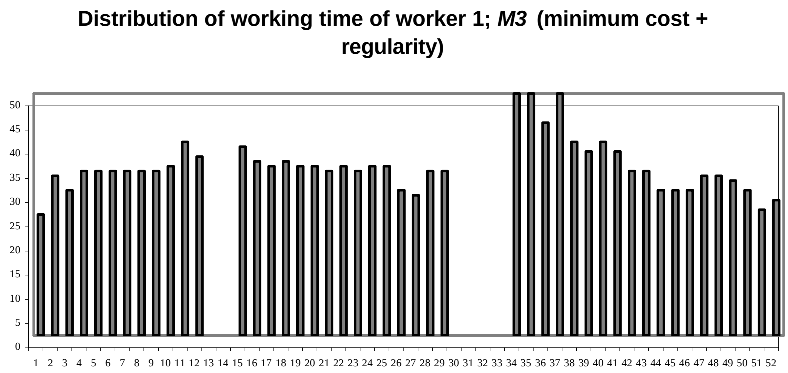

Figure 3. Distribution of working time, *M3*

Table 6 shows the minimum (mr_{min}), the average ($\overline{mr}$) and the maximum (mr_{max}) percentage of improvement of regularity when two models are compared. Models *M3*, *M4* and *M4+M3'* were very effective in regularising the workload of staff members and of temporary workers over the course of a year (the two main components in the function of regularity). In all cases, the percentage of improvement of regularity is about 50%. Moreover, if 1,800 seconds can be used, it would seem that the *M4+M3'* model is slightly better than the *M3* model.

		Number of workers				
		10	40	70	100	250
M3 vs. M1	mr_{min}	39.74	47.54	47.78	47.05	44.88
	$\overline{mr}$	46.02	50.89	51.53	50.73	49.38
	mr_{max}	58.94	59.66	59.00	59.39	58.81
M4 vs. M1	mr_{min}	35.18	44.01	45.55	45.55	45.98
	$\overline{mr}$	44.15	48.99	49.89	49.97	50.30
	mr_{max}	58.02	57.57	58.32	58.62	59.70
M4+M3' vs. M1	mr_{min}	39.77	46.60	47.68	47.00	45.98
	$\overline{mr}$	46.01	50.80	51.65	50.92	50.30
	mr_{max}	59.00	59.62	59.15	59.25	59.70
M3 vs. M4	mr_{min}	-0.11	-0.80	-2.46	-0.64	-2.10
	$\overline{mr}$	1.87	1.97	1.64	0.76	-0.92
	mr_{max}	5.49	4.31	3.73	2.32	0.56
M3 vs. M4+M3'	mr_{min}	-0.88	-1.50	-2.54	-1.35	-2.10
	$\overline{mr}$	0.00	0.13	-0.12	-0.19	-0.92
	mr_{max}	1.58	2.11	1.28	1.00	0.56
M4+M3' vs. M4	mr_{min}	0.00	0.00	0.00	0.00	0.00
	$\overline{mr}$	1.86	1.81	1.76	0.95	0.00
	mr_{max}	5.47	4.06	3.71	2.96	0.00

Table 6. Percentage of improvement of regularity when two models are compared

Finally, another computational experiment was performed with the following new data: total demand is equal to total capacity multiplied by 1.05; for each combination, 5 instances were generated (giving 750 new instances).

The results show that if the system is not adequately sized (total capacity is less than total demand), the solution is a little more difficult (and the number of optimal/feasible solutions obtained decreases); the results, nevertheless, can be considered very good (Table 7 shows the minimum, the average and the maximum percentage saved when using *M1* versus *M2*).

		Number of workers				
		10	**40**	**70**	**100**	**250**
M1* vs. *M2	a_{min}	8.61	3.78	2.14	1.13	0
	$\overline{a}$	10.84	8.81	6.55	5.42	3.54
	a_{max}	40.85	15.87	10.69	10.19	8.95

Table 7. Percentage saved when using *M1* versus *M2*

As in the first experiment, we can conclude that, if 1,800 seconds can be used, the *M4+M3'* model is slightly better than the *M3* model.

4. A tool for a bargaining process

In most countries companies cannot introduce irregular working hours if workers do not agree, so the question is whether workers will really accept an increase in flexibility (and also their holidays being planned by the company). Besides the convincing argument of conserving their jobs even in periods of low demand, companies should offer some kind of compensation that will lead workers to accept more or less flexibility. Some papers point out that one of the most difficult things of adopting an annual hours scheme is the time that is necessary to reach an agreement between the company and the workers. A planning procedure can be also a useful tool to help in the bargaining process.

Planning working time under different AH scenarios provides the company and the workers with quantitative information that can be very useful for the bargaining process in order to adopt an annual hours scheme. These scenarios may be characterised, for example, by the weekly flexibility accepted by workers, the total amount of annual working hours (the company could eventually reduce the annual working time), the maximum overtime, the conditions related to the strong and weak weeks and, of course, the possibility of, some rules provided, planning the holiday weeks. For each scenario, the model (for example, *M1* if holidays can be planned and *M2* otherwise) would give the cost of the solution and the company and the workers could agree to satisfactory conditions for both. Obviously, doing this implies solving several instances of each model. Hence, this would be possible only if solving the model requires a reasonable time, which is the case of the models presented in this paper.

Table 8 shows the results of a case in which scenarios are characterised by the total amount of annual hours (first column) and the weekly flexibility (first row). For each scenario, the first and second values correspond to the cost obtained by *M1* –holidays fixed by the model– and *M2* –holidays fixed a priori–, respectively. Note that *K* is the cost obtained in a situation without flexibility, without a reduction in working time and with holidays fixed a priori. It can be seen how the cost diminishes when flexibility is high, even when reducing working time.

Two options for reducing the cost by implementing annualised hours might be as follows: (1) by increasing weekly flexibility and reducing working time as a compensation for the workers; or (2), by increasing flexibility and not reducing working time but instead offering financial compensation to the workers. As it is shown in Table 8, in both cases the cost can be further reduced if workers' holidays are planned by the model.

MINIMUM COST		**Weekly flexibility (h/week)**				**Model**
		[40,40]	**[40, 50]**	**[30, 45]**	**[25, 50]**	
H **(annual hours)**	**1,840 (40 h/week)**	0.64·*K*	0.52·*K*	0.16·*K*	0	*M1*
		K	0.86·*K*	0.52·*K*	0.21·*K*	*M2*
	1,748 (38 h/week)	-	-	0.16·*K*	0	*M1*
		-	-	0.51·*K*	0.21·*K*	*M2*
	1,610 (35 h/week)	-	-	0.45·*K*	0	*M1*
		-	-	0.58·*K*	0.21·*K*	*M2*

Table 8. Cost of different scenarios (annual hours, weekly flexibility and planning holidays)

5. Conclusions

Annualising working hours (AH) is a means of obtaining flexibility in the use of human resources to face the seasonal nature of demand. There are few papers dealing with the problem of planning staff working hours under an annualised hours agreement; moreover, most of them include some assumptions that could be relaxed in order to solve a less restrictive problem. For example, in Corominas et al. (2002), a MILP model is used to minimise the costs of overtime plus those of temporary workers, assuming that: (i) the holiday weeks are fixed a priori; and (ii) the workers from different categories who are able to perform a specific type of task have the same efficiency.

In this paper, these assumptions are relaxed and a more general problem is solved: planning the working hours and holiday weeks of cross-trained workers who have different relative efficiencies over the course of a year in the service sector. Our computational experiment leads us to conclude that MILP is a technique suited to dealing with the problem in many real situations and, as is obvious, that better results are obtained when the holiday weeks are determined by the model. Finally, it has been shown how the MILP models could be a useful tool for helping in the bargaining process carried out before the adoption of an annual hours scheme.

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