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Using MILP to plan holidays and working hours under an annualised hours agreement*

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ABSTRACT

Annualising working hours (AH) is a means of achieving flexibility in the use of human resources to face the seasonal nature of demand. In Corominas et al.[1], two MILP models are used to solve the problem of planning staff working hours with an annual horizon. The costs due to overtime and to the employment of temporary workers are minimised, and the distribution of working time over the course of the year for each worker and the distribution of working time provided by temporary workers are regularised. In the aforementioned paper, the following is assumed: (i) the holiday weeks are fixed a priori and (ii) the workers from different categories who are able to perform a specific type of task have the same efficiency; moreover, the values of the binary variables (and others) in the second model are fixed to those in the first model (thus, in the second model these will intervene as constants and not as variables, resulting in an LP model). In the present paper, these assumptions are relaxed and a more general problem is solved. The computational experiment leads to the conclusion that MILP is a technique suited to dealing with the problem.

Keywords: manpower planning, annualised hours, service industry, integer programming.

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Introduction

Annualising working hours (AH)—i.e., the possibility of irregularly distributing the total number of staff working hours over the course of a year—is a means of achieving flexibility, because AH allows production capacity to be adapted to fluctuations in demand, thus reducing costs (overtime, temporary workers and inventory costs).

AH gives rise to new problems that have hitherto been given little attention in the literature. For instance, Hung[2], Grabot and Letouzey[3] and Azmat and Widmer[4] emphasise that the concept of annualised hours is surprisingly absent from the literature on planning and scheduling. A significant difficulty to be faced is that the diversity of production systems means that the problems that AH entails vary greatly; in Corominas et al.[5], the characteristics of the planning problem are discussed and a classification scheme is proposed, giving rise to thousands of different cases. Moreover, AH often implies the need to solve a complicated working time planning problem.

In Corominas et al.[1], two MILP (Mixed Integer Linear Programming) models are used to solve the problem of planning staff working hours with an annual horizon. Two hierarchical categories of workers are considered. In the first model, the costs of overtime and of employing temporary workers are minimised; in the second model, the cost of overtime is upper bounded to the minimum (obtained with the first model), and the distribution of working time for each worker over the course of a year and the distribution of working time provided by workers who are not members of staff are regularised. The computational experiment leads to the conclusion that MILP is a suitable technique for dealing with the problem. In the aforementioned paper, the following is assumed: (i) the holiday weeks are fixed a priori and (ii) the workers from different categories who are able to perform a specific type of task have the same efficiency. The values of the binary variables (and those corresponding to overtime) in the second model are fixed to those in the first model (thus, in the second model these will intervene as constants and not as variables, resulting in an LP model).

Although workers from different categories may be able to perform a specific type of task, certain categories frequently require more time than others do. In addition, the allocation of holiday weeks may be a decision variable of the model. Therefore, in this paper, the assumptions in Corominas et al.[1] are relaxed and a more general problem is solved; furthermore, the lower and upper bounds of the number of working hours per week may be different for each worker and for each week.

The main aims are to approach the planning of working hours and holiday weeks over the course of a year in services that employ cross-trained workers who have different relative efficiencies, to show that MILP is an appropriate tool for this aim, and of course to verify that the possibility of determining holiday weeks with the model provides better results. The rest of the article is organised as follows: the subsequent section introduces the problem and four MILP models for planning AH over a year; the following sections include the results of the computational experiment and the conclusions.

Four MILP models for planning working hours over the course of a year

Solving the planning problem involves determining the number of weekly working hours and holiday weeks for each member of staff.

A service system that is carried out on an individual basis is considered (so working hours for each worker may be different). Different types of tasks are involved, the product is not storable and the company forecasts the seasonal demand.

The production capacity in any given week must be greater than or equal to that which is needed and, if the staff does not provide entirely this capacity, temporary workers will be hired for the number of hours required. Overtime is admitted, but its total amount is bounded; overtime hours are classified into two blocks and the cost of an hour belonging to the second block is greater than that of an hour of the first. From the outset, the objective function is the cost of overtime plus the cost of employing temporary workers; it is possible to break the tie between optimal solutions by considering the penalties associated with the assignment of different types of tasks to categories of employees (this function is added to the first one with a small weighting).

Workers from different categories may frequently be able to perform a specific type of task, although certain categories may require more time than others may. Therefore, cross-trained workers are considered: certain categories can perform different types of tasks and can have different relative efficiencies associated with them (for example, a value of 0.9 means that a worker in that category needs to work 1/0.9 hours to serve a demand that a worker with a relative efficiency equal to 1 would serve in 1 hour).

The conditions to be fulfilled by the solution are the following (see Corominas et al.[1] for more details): i) the total of annual working hours is fixed; ii) the weekly number of working hours must fall within an interval defined by a lower and upper bound; iii) the average weekly working hours for any set of twelve consecutive weeks is upper bounded; iv) if the average weekly working hours over a specified number of consecutive working weeks ("week-block") is greater than a certain value, then over a given number of weeks immediately succeeding the week-block, the number of working hours must not be greater than a certain value; and v) if "strong" and "weak" weeks are defined as those in which the number of working hours is respectively greater or less than certain specified values, there is an upper bound for the number of strong weeks and a lower bound for the number of weak weeks.

Below, we introduce the four models to be tested.

The objective function to be minimised in models *M1* and *M2* has already been specified: the cost of overtime plus the cost of employing temporary workers (the penalties associated with the assignment of types of tasks to categories are considered in order to break the tie between optimal solutions). Cross-trained workers are considered in both models. In *M1,* holiday weeks are determined by the model but, in *M2,* these are fixed a priori (in both cases, two consecutive holiday weeks in winter and four consecutive holiday weeks in summer are assumed).

As pointed out by Corominas et al.[1], the AH models that minimise the cost usually have an infinite number of optimal solutions. In addition, in the model provided by the optimiser, the number of weekly working hours for an employee over the course of a year and weekly working time provided by temporary workers for each week are usually very irregular. To regularise the profile of an employee's working hours over a year and the profile of weekly working time provided by temporary workers, i.e., to obtain the most regular solution from all those that involve the minimum cost, two other models (*M3* and *M4*) are used.

The objective function to be minimised in models *M3* and *M4* is the weighted sum of: i) the sum of the discrepancies between the weekly working hours of staff members and the average weekly working hours; and (ii) the sum of the discrepancies between the working hours provided by workers who do not belong to the staff and the average of weekly working hours provided by these workers. The penalties associated with the assignment of types of tasks to categories are again considered to break the tie between optimal solutions. In both models, the minimum cost obtained by *M1* is guaranteed and any other variable obtained by solving *M1* or *M2* is considered (week-blocks, strong and weak weeks, etc.). The difference between *M3* and *M4* is that in *M3* the holiday weeks are determined by the model but in *M4* these are obtained with *M1*.

We use the following notation:

Data

T	Weeks in the planning horizon (in general, 52)
C	Set of categories of workers
F	Set of types of tasks
E	Set of members of staff
ρ_{jk}	Relative efficiency associated with the workers in category j in the accomplishment of tasks of type k (j=1,...,$\lvert C\rvert$; k=1,...,$\lvert F\rvert$); $0\leq \rho_{jk} \leq 1$. If $\rho_{jk}=0$, workers in category j are not able to perform tasks of type k.
$\hat{C}_k$	Sets of categories of workers that can be assigned to tasks of type k ($\hat{C}_k = j \in C \mid \rho_{jk} > 0$)
$\hat{F}_j$	Sets of types of tasks which can be performed by employees in category j ($\hat{F}_j = k \in F \mid \rho_{jk} > 0$)
p_{jk}	Penalty associated with an hour of work in a task of type k of a staff member in category j ($\forall k \in F$; $\forall j \in \hat{C}_k$)
λ	Parameter to weigh the penalties to establish the trade-off between these and the monetary costs or the regularity of the solution.

$\hat{E}_j$	Set of employees in category j (j=1,…,|C|)
r_{tk}	Required working hours for tasks of type k in week t (t=1,..,T; k=1,…,|F|)
H_i	Stipulated ordinary annual working hours of employee i ($\forall i \in E$)
α_1, α_2	Maximum proportions, over the annual amount of ordinary working hours, of overtime corresponding to blocks 1 and 2 respectively
$\beta 1_i$, $\beta 2_i$	Respectively, the cost of an hour of overtime for block 1 and block 2 for employee i ($\forall i \in E$), with $\beta 1_i < \beta 2_i$
hm_{it}, hM_{it}	Lower and upper bounds of the number of working hours for worker i in week t ($\forall i \in E$; t=1,..,T); $hM_{it} > hm_{it}$
L, h_L	L is the maximum number of consecutive weeks in which the average weekly working hours cannot be greater than h_L
B, b, h_B, h_b	b is the minimum number of weeks, after a week-block of B consecutive weeks with a weekly average of working hours greater than h_B, in which the number of weekly hours cannot be greater than h_b
N_S, h_S	N_S is the maximum number of "strong" weeks, i.e., weeks with a number of working hours greater than h_S
N_W, h_W	N_W is the minimum number of "weak" weeks, i.e., weeks with a number of working hours less than h_W
$hw1_i$, $hw2_i$	Number of holiday weeks in the first and second holiday periods respectively for worker i ($\forall i \in E$)
$t1_i$, $t2_i$	First and last week respectively in which worker i can take holidays in the first holiday period ($\forall i \in E$)
$t3_i$, $t4_i$	First and last week respectively in which worker i can take holidays in the second holiday period ($\forall i \in E$)
γ_k	Cost of an hour for tasks of type k performed by a worker who is not a member of staff ($\gamma_k > \beta 2_i$, $\forall i \in \hat{C}_k$)

Variables

x_{it}	Working hours of employee i in week t ($\forall i \in E$).
y_{tjk}	Working hours of employees in category j dedicated to tasks of type k in week t ($\forall k \in F$; $\forall j \in \hat{C}_k$; $t = 1,...,T$).

d_{tk}	Working hours corresponding to tasks of type k to be supplied in week t by workers who are not members of staff ($\forall k \in F$; $t=1,\ldots,T$).
$v1_i$, $v2_i$	Overtime corresponding respectively to blocks 1 and 2 of employee i ($\forall i \in E$).
$vc1_{it} \in \{0,1\}$	Indicates whether employee i starts his or her first holiday period in week t ($\forall i \in E$, $t=t1_i,\ldots,t2_i-hw1_i+1$).
$vc2_{it} \in \{0,1\}$	Indicates whether employee i starts his or her second holiday period in week t ($\forall i \in E$, $t=t3_i,\ldots,t4_i-hw2_i+1$).
$\delta_{i\tau} \in \{0,1\}$	Indicates whether the average working hours of employee i, in a week-block of B weeks that ends with week τ, is (or is not) greater than h_B hours ($\forall i \in E$; $t=B,\ldots,T-b$).
$s_{it} \in \{0,1\}$	Indicates whether employee i has a planned number of working hours equal to or greater than h_S hours for week t ($\forall i \in E$; $t=1,\ldots,T$).
$w_{it} \in \{0,1\}$	Indicates whether employee i has a planned number of working hours equal to or less than h_W hours for week t ($\forall i \in E$; $t=1,\ldots,T$).

All the non-binary variables are real and non-negative.

Now, the four models can be formalised.

MODEL 1 (M1)

$$[\text{MIN}]\ z = \sum_{i \in E} \beta 1_i \cdot v1_i + \sum_{i \in E} \beta 2_i \cdot v2_i + \sum_{k \in F} \gamma_k \sum_{t=1}^{T} d_{tk} + \lambda \cdot \sum_{t=1}^{T} \sum_{k \in F} \sum_{j \in \hat{C}_k} p_{jk} \cdot y_{tjk} \qquad (1)$$

$$\sum_{t=1}^{T} x_{it} = H_i + v1_i + v2_i \qquad \forall i \in E \qquad (2)$$

$$v1_i \le \alpha_1 \cdot H_i \qquad \forall i \in E \qquad (3)$$

$$v2_i \le \alpha_2 \cdot H_i \qquad \forall i \in E \qquad (4)$$

$$\sum_{i \in \hat{E}_j} x_{it} = \sum_{k \in \hat{F}_j} y_{tjk} \qquad t = 1,\ldots,T;\ \forall j \in C \qquad (5)$$

$$\sum_{j \in \hat{C}_k} \rho_{jk} \cdot y_{tjk} + d_{tk} \ge r_{tk} \qquad t = 1,\ldots,T;\ \forall k \in F \qquad (6)$$

$$\sum_{t=\tau-L+1}^{\tau} x_{it} \leq L \cdot h_L \qquad \tau = L,...,T; \ \forall i \in E \qquad (7)$$

$$\sum_{t=\tau-B+1}^{\tau} x_{it} \leq B \cdot h_B + B \cdot (h_M - h_B) \cdot \delta_{i\tau} \qquad \tau = B,...,T-b; \forall i \in E \qquad (8)$$

$$\sum_{t=\tau-B+1}^{\tau} x_{it} \leq B \cdot h_B \qquad \tau = T-b+1,...,T; \ \forall i \in E \qquad (9)$$

$$x_{i,\tau+l} \leq hM_{it} - (hM_{it} - h_b) \cdot \delta_{i\tau} \qquad \forall i \in E; \ \tau = B,....T-b; \ l = 1,...,b \qquad (10)$$

$$x_{it} \leq h_S + (hM_{it} - h_S) \cdot s_{it} \qquad \forall i \in E; \ t = 1,...T \qquad (11)$$

$$x_{it} \leq hM_{it} - (hM_{it} - h_W) \cdot w_{it} \qquad \forall i \in E; \ t = 1,...,T \qquad (12)$$

$$\sum_{t=1}^{T} s_{it} \leq N_S \qquad \forall i \in E \qquad (13)$$

$$\sum_{t=1}^{T} w_{it} \geq N_W \qquad \forall i \in E \qquad (14)$$

$$\sum_{t=t1_i}^{t2_i - hw1_i + 1} vc1_{it} = 1 \qquad \forall i \in E \qquad (15)$$

$$\sum_{t=t3_i}^{t4_i - hw2_i + 1} vc2_{it} = 1 \qquad \forall i \in E \qquad (16)$$

$$x_{it} \leq hM_{it} \qquad \forall i \in E; \ t \notin \left(\left[t1_i,...,t2_i \right] \vee \left[t3_i,...,t4_i \right] \right) \qquad (17)$$

$$x_{it} \geq hm_{it} \qquad \forall i \in E; \ t \notin \left(\left[t1_i,...,t2_i \right] \vee \left[t3_i,...,t4_i \right] \right) \qquad (18)$$

$$x_{it} \leq hM_{it} \cdot \left(1 - \sum_{\tau=\max(t1_i,\ t-hw1_i+1)}^{\min(t,\ t2_i - hw1_i + 1)} vc1_{i\tau} \right) \qquad \forall i \in E; \ t = t1_i,...,t2_i \qquad (19)$$

$$x_{it} \geq hm_{it} \cdot \left(1 - \sum_{\tau=\max(t1_i,\ t-hw1_i+1)}^{\min(t,\ t2_i - hw1_i + 1)} vc1_{i\tau} \right) \qquad \forall i \in E; \ t = t1_i,...,t2_i \qquad (20)$$

$$x_{it} \leq hM_{it} \cdot \left(1 - \sum_{\tau=\max(t3_i,\ t-hw2_i+1)}^{\min(t,\ t4_i - hw2_i + 1)} vc2_{i\tau} \right) \qquad \forall i \in E; \ t = t3_i,...,t4_i \qquad (21)$$

$$x_{it} \geq hm_{it} \cdot \left(1 - \sum_{\tau=\max(t3_i,\ t-hw2_i+1)}^{\min(t,\ t4_i - hw2_i + 1)} vc2_{i\tau} \right) \qquad \forall i \in E; \ t = t3_i,...,t4_i \qquad (22)$$

$$\delta_{i\tau} \in \{0,1\} \qquad \forall i \in E; \ \tau = T,...,B-b \qquad (23)$$

$$s_{it}, w_{it} \in \{0,1\} \qquad \forall i \in E; \ t = 1,...,T \qquad (24)$$

$$vc1_{it} \in \{0,1\} \qquad \forall i \in E; t = t1_i,...,t2_i - hw1_i + 1 \qquad (25)$$

$$vc2_{it} \in \{0,1\} \qquad \forall i \in E; t = t3_i,...,t4_i - hw2_i + 1 \qquad (26)$$

$$v1_i, v2_i \geq 0 \qquad \forall i \in E \qquad (27)$$

$$y_{tjk} \geq 0 \qquad t = 1,...,T;\ \forall j \in C;\ \forall k \in \hat{F}_j \qquad (28)$$

$$d_{tk} \geq 0 \qquad t = 1,...,T;\ \forall k \in F \qquad (29)$$

(1) is the objective function, which includes the cost of overtime plus that of employing external workers and the (weighted) penalties associated with the assignment of tasks to the types of employees on the staff; (2) imposes that the total number of worked hours should be equal to the ordinary annual hours stipulated plus overtime, if applicable; (3) and (4) stipulates that the overtime for each of the two blocks should not exceed their respective upper bounds; (5) is the balance between the hours provided by specific types of workers of the staff and the hours assigned to different types of tasks; (6) expresses that the hours assigned to a type of task that are to be carried out by members of staff plus, if applicable, the hours provided by external workers for that same type of task must not be less than the number of hours required; (7) imposes the upper bound on the average weekly working hours for any subset of L consecutive weeks; (8) implies that variable $\delta_{i\tau}$ is equal to 1 if the average number of working hours in a week-block of B weeks is greater than h_B; (9) prevents the average hours worked from being greater than h_B in the last weeks of the year, when after the week-block of B weeks there are no longer b weeks to "compensate"; (10) implies that, if variable $\delta_{i\tau}$ is equal to 1, the upper bound of the number of working hours is h_b; (11) imposes that, if the number of working hours is greater than h_S, then variable s_{it} is equal to 1; (12) states that, if the number of working hours is greater than h_W, then variable w_{it} is equal to 0; (13) and (14) stipulate that the number of "strong" and "weak" weeks cannot be greater than N_S and less than N_W respectively; (15) and (16) establish that the worker must start his or her holidays in a given week; (17) and (18) set the lower and upper bounds of the number of weekly working hours in non-holiday weeks; (19), (20), (21) and (22) set the lower and upper bounds of the number of weekly working hours for possible holiday weeks; (23), (24), (25) and (26) express the binary character of the corresponding variables; and (27), (28) and (29) show the non-negative character of the rest of the non-binary variables.

MODEL 2 (M2)

M2 is very similar to the model that is presented in Corominas et al.[1] (*M2* considers cross-trained workers who have different relative efficiencies) and it can be obtained by deleting the variables $vc1_{it}$ and $vc2_{it}$ and their associated constraints (15, 16 and 19 to 22, 25 and 26) from model *M1* (and making several minor and straightforward modifications).

MODEL 3 (M3)

Once model *M1* has been solved, the cost of overtime and temporary workers is stored. The formalisation of *M3* is not included but it may easily be obtained by starting from model *M1* and keeping in mind the following changes:

(i) A constraint is added, which requires that the cost of the solution of *M3* cannot exceed that obtained with *M1*.

(ii) Variables x_{it} are eliminated using the expression $x_{it} = \overline{x}_i + x_{it}^{+} - x_{it}^{-}$, where $\overline{x}_i$ is the average number of weekly working hours corresponding to worker i and x_{it}^{+} and x_{it}^{-} are the positive and negative deviations from the average number of working hours of worker i in week t.

(iii) Variables d_{tk} are eliminated using the expression $d_{tk} = \overline{d_k} + \sigma_{tk}^{+} - \sigma_{tk}^{-}$, where $\overline{d_k}$ is the average number of weekly working hours provided by temporary workers for a task of type k and σ_{tk}^{+} and σ_{tk}^{-} are the positive and negative deviations from the average number of working hours provided by temporary workers for task k in week t.

(iv) The objective function to be minimised is substituted for a new one that has two weighted components. The first is the sum of the discrepancies in the number of working hours of staff members and the second is the sum of the discrepancies in the number of working hours provided by temporary workers. The penalties associated with the assignment of tasks to categories of workers are also considered to break the possible tie between optimal solutions.

MODEL 4 (M4)

M4 can be obtained from *M3* by fixing the holiday weeks obtained when solving *M1* (basically, variables $vc1_{it}$, vc2it and their associated constraints have to be deleted). *M4* is rather similar to the model that is presented in Corominas et al.[1] but *M4* considers cross-trained workers who have different relative efficiencies and the value of the binary variables are not fixed to the ones obtained with model *M2*.

Computational experiment

A large-scale computational experiment was performed to evaluate the effectiveness (in terms of computing time and the quality of the solutions) of the models. Overall, the results were very satisfactory.

The basic data used for the experiment are as follows:

– Five MILP models: *M1*, *M2*, *M3*, *M4* and *M4+M3'* (this compound model consists in carrying out *M4* and, in the remaining calculation time, executing *M3'*, which is obtained when a constraint is imposed on *M3* so that the value of the solution of *M3* cannot exceed the value obtained by means of *M4*).
– 10, 40, 70, 100 and 250 staff workers.
– A time horizon of 52 weeks (46 working weeks and 6 holiday weeks).
– The holiday weeks for each worker are distributed into two uninterrupted periods, including two weeks in winter and four weeks in summer. In *M2,* the temporary allocation of holidays was fixed for each worker at random.
– There are three categories and three types of tasks. There are two patterns of relative efficiency (and penalty). Table 1 and Table 2 show the relative efficiency (and the penalty) values for each pattern.

[Table 1. Relative efficiency (and penalty) values for Pattern 1]

]Table 2. Relative efficiency (and penalty) values for Pattern 2]

- The capacity (in working hours) required over the year follows three different patterns. Demand Type 1 corresponds to a non-seasonal capacity pattern with noise. Demand Type 2 corresponds to a seasonality pattern with one peak, with noise. Demand Type 3 corresponds to a seasonality pattern with two peaks, with noise. In each case, the total demand is equal to the total capacity multiplied by 0.99.

For every combination of models, number of staff workers, type of demand and pattern of relative efficiency (and penalty), 20 instances were generated (varying demand noise and, in *M2*, holiday weeks at random), which gave 3,000 instances.

In spite of the dimension of the models may be considered large (the average number of variables and constraints are given in Table 3); they were solved to optimality using an ILOG CPLEX 8.1 optimiser and a Pentium IV PC at 1.8 GHz with 512 Mb of RAM. The absolute and relative MIP gap tolerances were set to 0.01. The maximum computing time for all instances was set to 1,800 seconds.

[Table 3. Average number of variables/constraints]

For each model and each number of staff workers, the number of instances that do not have solutions, that have feasible solutions and that have a proven optimal solution are given in Table 4 (for the model *M4+M3'*, the number of instances in which there was not enough time to carry out *M3'* is added). Table 5 shows the minimum (t_{min}), the average ($\bar{t}$) and the maximum computing time (t_{max}) (in seconds).

[Table 4. Number of instances with no solution, with a feasible solution and with a proven optimal solution]

[Table 5. Computing times (in seconds)]

The maximum computing times are very reasonable considering the problem to be solved (the aim of the models is to establish an annual plan) and its maximum size (two hundred and fifty workers, which is a large enough number, since we are supposed to be dealing with a production system of services or a part of this system). For the models in which costs were to be minimised (*M1* and *M2*), feasible solutions were always obtained and most of these were optimal solutions. Regarding the models which have regularity as objective (*M3*, *M4* and *M4+M3'*), in only one test (of *M3*) no feasible solution was obtained. The variants that were hardest to solve were *M1* and *M3* (or *M3'*), as expected, given that these variants include more constraints and binary variables than others do.

The experiments provided satisfactory results regarding the quality of the solutions of the models. Table 6 shows the minimum (a_{min}), the average ($\bar{a}$) and the maximum (a_{max}) percentage saved when *M1* is used versus *M2*. As shown, the possibility of determining holiday weeks with model *M1*, whilst observing a set of legal constraints or constraints imposed by a collective bargaining agreement between the management and the workers (two uninterrupted weeks in winter and four in summer in this case), provides very good solutions and savings of more than 90%. These values also show how the capacity of the staff can be adapted to demand by determining the holiday weeks of the staff (this is also due to the flexibility provided by the annualisation of working time).

[Table 6. Percentage saved when using *M1* versus *M2]*

Table 7 shows the minimum (mr_{min}), the average ($\overline{mr}$) and the maximum (mr_{max}) percentage of improvement of regularity when two models are compared. Models *M3*, *M4* and *M4+M3'* were very effective in regularising the workload of staff members and of temporary workers over the course of a year (the two main components in the function of regularity). In all cases, the percentage of improvement of regularity is about 50%. Moreover, if 1,800 seconds can be used, it would seem that the *M4+M3'* model is slightly better than the *M3* model.

[Table 7. Percentage of improvement of regularity when two models are compared]

Another computational experiment was performed with the following new data: total demand is equal to total capacity multiplied by 1.05; for each combination, 5 instances were generated (giving 750 new instances).

The results show that if the system is not adequately sized (total capacity is less than total demand), the solution is a little more difficult (and the number of optimal/feasible solutions obtained decreases); the results, nevertheless, can be considered very good (Table 8 shows the minimum, the average and the maximum percentage saved when using *M1* versus *M2*).

[Table 8. Percentage saved when using *M1* versus *M2*]

As in the first experiment, we can conclude that, if 1,800 seconds can be used, the *M4+M3'* model is slightly better than the *M3* model.

Conclusions

Annualising working hours is a means of obtaining flexibility in the use of human resources to face the seasonal nature of demand. In Corominas et al.[1], two MILP models are used to solve the problem of planning staff working hours over a year. The costs of overtime and employing temporary workers are minimised and the distribution of the working time for each worker over the year and the distribution of the working time provided by temporary workers are regularised. To facilitate the solving of these models, however, the following is assumed: (i) the holiday weeks are fixed a priori; (ii) the workers from different categories who are able to perform a specific type of task have the same efficiency; and (iii) the value of the binary variables (and others) in the second model are fixed and equal to the ones obtained in the first model.

In this paper, these assumptions are relaxed and a more general problem is solved: planning the working hours and holiday weeks of cross-trained workers who have different relative efficiencies over the course of a year in the service sector. Our computational experiment leads us to conclude that MILP is a technique suited to dealing with the problem in many real situations and, as is obvious, that better results are obtained when the holiday weeks are determined by the model.

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	Task 1	Task 2	Task 3
Category 1	1 (1)	0.9 (2)	0
Category 2	0	1 (1)	0.9 (2)
Category 3	0	0	1 (1)

Table 1. Relative efficiency (and penalty) values for Pattern 1

	Task 1	Task 2	Task 3
Category 1	1 (1)	0	0
Category 2	0.9 (2)	1 (1)	0
Category 3	0.8 (2)	0	1 (1)

Table 2. Relative efficiency (and penalty) values for Pattern 2

		Number of workers				
		10	40	70	100	250
MODELS	***M1***	2,817/3,915	9,387/14,715	15,957/25,515	22,527/36,315	55,377/90,315
	M2	2,310/2,567	7,357/9,319	12,405/16,072	17,452/22,822	42,689/56,572
	M3	4,169/4,592	13,859/16,952	23,549/29,312	33,239/41,672	81,689/103,472
	M4	3,664/3,658	11,835/13,191	20,004/22,718	28,172/32,230	69,035/79,946
	M4+M3'	4,170/4,594	13,860/16,954	23,550/29,314	33,240/41,674	81,690/103,474

Table 3. Average number of variables/constraints

			Number of workers				
			10	**40**	**70**	**100**	**250**
MODELS	***M1***	***No solution***	0	0	0	0	0
		Feasible solution	59	57	7	1	0
		Optimal solution	61	63	113	119	120
	M2	***No solution***	0	0	0	0	0
		Feasible solution	0	0	0	0	0
		Optimal solution	120	120	120	120	120
	M3	***No solution***	1	2	0	0	0
		Feasible solution	109	11	27	112	120
		Optimal solution	10	107	93	8	0
	M4	***No solution***	0	0	0	0	0
		Feasible solution	0	0	0	0	22
		Optimal solution	120	120	120	120	98
	M4+M3'	***No time for M3'***	0	0	0	0	22
		No solution of M3'	2	11	1	8	98
		Feasible solution of M3'	106	16	3	108	0
		Optimal solution of M3'	12	93	116	4	0

Table 4. Number of instances with no solution, with a feasible solution and with a proven optimal solution

			Number of workers				
			10	**40**	**70**	**100**	**250**
MODELS	***M1***	t_{min}	24.20	15.55	26.49	42.91	139.94
		$\bar{t}$	935.91	890.23	164.88	82.59	300.02
		t_{max}	1,800	1,800	1,800	1,800	1,097.71
	M2	t_{min}	7.06	7.89	8.75	9.64	16.27
		$\bar{t}$	9.53	9.41	11.21	12.26	30.58
		t_{max}	110.78	14.86	21.63	20.73	198.66
	M3	t_{min}	130.03	193.30	671.26	1,450.44	1,800
		$\bar{t}$	1,716.66	716.28	1,369.97	1,790.47	1,800
		t_{max}	1,800	1,800	1,800	1,800	1,800
	M4	t_{min}	6.22	25.52	67.25	105.91	531.20
		$\bar{t}$	9.82	36.49	119.86	265.38	1,361.07
		t_{max}	156.08	78.92	258.28	446.29	1,800
	M4+M3'	t_{min}	79.22	200.92	656.06	1,408.45	1,800
		$\bar{t}$	1,695.76	842.78	1,238.95	1,793.65	1,800
		t_{max}	1,800	1,800	1,800	1,800	1,800

Table 5. Computing times (in seconds)

		Number of workers				
		10	**40**	**70**	**100**	**250**
M1 vs. M2	a_{min}	65.24	97.17	99.02	99.69	100
	$\bar{a}$	89.53	99.49	99.96	99.99	100
	a_{max}	99.75	100	100	100	100

Table 6. Percentage saved when using *M1* versus *M2*

		Number of workers				
		10	**40**	**70**	**100**	**250**
M3 vs. M1	mr_{min}	39.74	47.54	47.78	47.05	44.88
	$\overline{mr}$	46.02	50.89	51.53	50.73	49.38
	mr_{max}	58.94	59.66	59.00	59.39	58.81
M4 vs. M1	mr_{min}	35.18	44.01	45.55	45.55	45.98
	$\overline{mr}$	44.15	48.99	49.89	49.97	50.30
	mr_{max}	58.02	57.57	58.32	58.62	59.70
M4+M3' vs. M1	mr_{min}	39.77	46.60	47.68	47.00	45.98
	$\overline{mr}$	46.01	50.80	51.65	50.92	50.30
	mr_{max}	59.00	59.62	59.15	59.25	59.70
M3 vs. M4	mr_{min}	-0.11	-0.80	-2.46	-0.64	-2.10
	$\overline{mr}$	1.87	1.97	1.64	0.76	-0.92
	mr_{max}	5.49	4.31	3.73	2.32	0.56
M3 vs. M4+M3'	mr_{min}	-0.88	-1.50	-2.54	-1.35	-2.10
	$\overline{mr}$	0.00	0.13	-0.12	-0.19	-0.92
	mr_{max}	1.58	2.11	1.28	1.00	0.56
M4+M3' vs. M4	mr_{min}	0.00	0.00	0.00	0.00	0.00
	$\overline{mr}$	1.86	1.81	1.76	0.95	0.00
	mr_{max}	5.47	4.06	3.71	2.96	0.00

Table 7. Percentage of improvement of regularity when two models are compared

		Number of workers				
		10	**40**	**70**	**100**	**250**
M1 vs. M2	a_{min}	8.61	3.78	2.14	1.13	0
	$\bar{a}$	10.84	8.81	6.55	5.42	3.54
	a_{max}	40.85	15.87	10.69	10.19	8.95

Table 8. Percentage saved when using *M1* versus *M2*

Table 1. Relative efficiency (and penalty) values for Pattern 1

Table 2. Relative efficiency (and penalty) values for Pattern 2

Table 3. Average number of variables/constraints

Table 4. Number of instances with no solution, with a feasible solution and with a proven optimal solution

Table 5. Computing times (in seconds)

Table 6. Percentage saved when using *M1* versus *M2*

Table 7. Percentage of improvement of regularity when two models are compared

Table 8. Percentage saved when using *M1* versus *M2*