Using Web Technologies to Teach Mathematics

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Abstract: The technical barriers hindering the development of on-line learning in the sciences have disappeared in 2005. It is now possible and practical to teach on-line via videoconferencing supported by systems like Festoon. Students are ready to embrace synchronous and asynchronous on-line learning at the university level. Instructors are still largely either against teaching on-line or, at best, hesitant about the effectiveness of on-line learning. This paper reports on the recent experiments at the University of Helsinki and at Florida State University. We also discuss the enhancements that the WebALT Project will bring to on-line learning. The experiences gathered at the University of Helsinki and at Florida State University show that on-line instruction is a real alternative to traditional instruction, a way to improve learning and a way to reduce costs.

Introduction

The development of e-Learning in the sciences in general and in mathematics in particular has not met the general expectations. This is rather surprising, for authorities in charge of education in various countries and leaderships of educational institutes see rather unanimously that using information technology one can improve education and decrease its cost. In fact, as a consequence of this view, national funding agencies in many countries have invested heavily in educational technology.

In some areas the use of technology in education has been quite successful, and serious improvements in the delivery of education have been achieved while the cost of the delivery of instruction has decreased. The National Center for Academic Transformation (NCAT) is non-profit organization dedicated to the effective use of information technology in higher education in the US. NCAT has reported systematic improvements in students' learning outcomes and retention rates, while the cost of the delivery of education has dramatically decreased with proper of use of information technology (NCAT).

Regardless of these very successful experiences, there have been almost no attempts to use information technology in a serious way in mathematics and sciences beyond the most elementary courses. Courses are still based on printed textbooks, traditional lectures and exercises. Information technology is being used mainly to distribute the printed material in an electronic form, usually as pdf files. Students then print these materials when preparing to traditional lectures, exercise sessions or examinations. Furthermore, there is a strong resistance among faculty to adopt the advanced methods offered by information technology.

Largely this state of affairs can be attributed to the difficulties involved with presenting scientific materials on the web in a way that, for example, makes formulae searchable using the meaning of the formula as a key word in the search. This is today possible, but requires new techniques and the writing of new materials, a daunting task. In most presentations, scientific formulae appear as pictures only (gif or pdf). Lectures and exercises are usually stored as TeX or LaTeX files, which can be used to produce high quality printed materials.

TeX and LaTeX have been great achievements. They allow any scientist to produce highest quality printed materials and have rendered manual typesetting completely obsolete. TeX and LaTeX have, in fact, been too successful, and as such have set a very high bar for the adoption of more advanced XML based systems.

In a way TeX and LaTeX have given new life to traditional printed publications. The new media is, however, different from the printed page. Even the best monitors do not offer the resolution of a high quality

printed book, journal or magazine. For an effective delivery, information has to be presented differently for the computer screen than for the printed page. So a new way to present content needs to be developed. One can even claim that the universal acceptance of TeX and of LaTeX among scientists and educators of sciences has hindered the necessary development much in the same way as Minitel (Wikipedia) hindered the advance of internet in France for many years (Barry James, 1996).

In this paper we investigate what needs to be done as we move to the next level in the use of information technology in education especially in sciences. We start with the observation that the leading current learning platforms, Blackboard and WebCT, which are now merging, offer only a limited support for scientific formulae, and offer no automatic grading of test or quiz questions in which the answer is a formula. Hence more advanced solutions need to be used for sciences. Maple T.A. is currently the only commercial learning platform which can be used to automatically grade problems in which answers are formulae. Maple T.A. can be used also from within Blackboard giving an edge to this learning platform. Moodle together with STACK/AIM is an open source solution to the same problem and gaining popularity especially in the UK.

Algorithmic problems, i.e. problems which depend on a number of random variables, can be used to test routine operations like differentiation and integration. A large portion of any typical undergraduate course in mathematics is devoted to mastering such routine calculations. Here information technology can make a dramatic difference. We will discuss this in more detail below.

Traditional classroom instruction can largely be replaced by on-line instruction based on collections of well developed presentations. These presentations can be delivered synchronously, using programs like Skype and Festoon, or asynchronously. We will discuss the requirements of the presentations that work. What does it take to deliver a science class completely on-line? We will offer our answer to this question.

Teaching Mathematics with Web Technologies

On-line Lectures: Presentation

The presentation of lectures and other on-line materials is absolutely crucial. The presentation has to be appealing, and it has to serve the educational goal as well as possible.

Electronic presentations of printed books have commonly been used. They can be produced rather easily, especially if the book in question has been typeset using LaTeX. One can enhance these files with suitable links and with live mathematical illustrations adding some value to the printed books. They can also be enhanced, as for instance (Sebastian Xambó, 2003), with links that access an advanced interface to examples propelled by an advanced remote Computer Algebra System (see (D. Marquès R. Eixarch, S. Xambó, 2002) for details about the **Wiris** system, or the site http://www.wiris.com/). Students usually then print such materials and study the printed pages rather than buying the books. This adds only little value to books, as it merely makes them more easily available. It is pity that usually the students do no take full advantage of the provided materials. In certain situations the electronic delivery of materials alone may make a decisive difference. For the majority of students in the US and in Europe, this is only a matter of convenience.

Materials for synchronous or asynchronous on-line learning have to present the content so that one screen page explains only limited amount of mathematics, usually only one fact.

Figure 1. Traditional Presentation vs. Power Point Slide Show. A typical screenshot, on the left, of the Calculus Bible by G. S. Gill (G. S. Gill), and a typical screenshot, on the right, of Single Variable Calculus by M. Seppälä (Mika Seppälä, 2004).

An electronic copy of a text book cannot be used very well as supporting materials projected to a class during a traditional lecture. Figure 1 shows, on the left, a sample screenshot of The Calculus Bible by G. S. Gill (G. S. Gill) and, on the right, a screenshot of a Power Point presentation of the same subject matter taken from Single Variable Calculus by M. Seppälä (Mika Seppälä, 2004).

The Calculus Bible of Gill (G. S. Gill) exemplifies typical current ways to use the web to deliver educational content. The presentation is a high quality electronic version of a traditional book with some enhancements. Such materials cannot very well be used in a normal classroom and projected for all the students. The font is too small for large classrooms and one printed page, presented electronically, contains too much information for a screenfull.

Slide shows are typically accompanied by a professor explaining the materials. The content of a good slide show is much more compressed than that of a book. Converting a traditional book to a collection of slide shows usually requires a complete rewriting of the materials. This is an enormous job, and most instructors will not even consider doing it. They would, however, use slide shows to support their teaching if high quality collections of slide shows were easily available to them.

The recent advances in video conferencing, with programs like Festoon, make the synchronous teaching over the web a reality. Most students have the required hardware and internet connectivity to participate in a course delivered as Festoon video conferences over the web.

Again here content needed to support live lectures delivered over the web consists of a collection of slide shows.

LaTeX Slide Shows vs. Power Point Slide Shows

Using specialized LaTeX classes, such as the Beamer, one can produce very high quality slide shows incorporating graphics and text. The graphical outlook of these is superb.

Using tools like PowerPoint together with MathType, one has more flexibility in composing pages containing graphics. A serious advantage over tools like the Beamer is that mathematical formulae embedded in PowerPoint slide shows by MathType are encoded using MathML, although, alas, only Presentation MathML for the moment. Hence simple formulae can be copied from the Slide Shows, pasted to programs like Maple, and then

computed with using Maple's mathematical power. Direct links to **Wiris** pages can also be readily included. Another important feature is that the MathType formulae in Slide Shows support aural rendering, i.e., they can be automatically read aloud by the computer. These features alone make PowerPoint together with MathType a much better tool for the creation of educational presentations than LaTeX together with the Beamer Class.

When using presentations created by others, instructors usually would like to edit the presentations. With PowerPoint this is easy, since the presentation file alone already includes all the components including graphics. With LaTeX Slide Shows one must have also all auxiliary files in order to be able to effectively edit a slide show. Even then the editing is cumbersome at best in comparison with editing PowerPoint slide shows. Almost all instructors have computers equipped with PowerPoint. Those using Linux can resort to OpenOffice.

Students' Comments

The On-Line Single Variable Calculus (Mika Seppälä, 2004) was used to teach calculus at Florida State University in fall 2004. Students were told, as is customary at FSU, that they need to have the textbook, Stewart's Early Transcendentals. An anonymous study about studying habits was conducted during weeks 10 – 12 of classes.

In spite of having the book, most students (76.2%) responding to the survey told that on-line materials are their primary source for the course.

When studying calculus what materials you are using?	%
Only Stewart's book	9.5
Mainly Stewart's book accompanied by the on-line materials	0
Mainly the on-line lecture notes accompanied by Stewart's book	14.3
On-line materials are my primary source for the course	**76.2**

PowerPoint Slide Shows and their pdf exports can easily be used by students without having to install additional software. This is an important point. Previously similar slide shows, based on IBM's techexplorer, were used in class at FSU. Lecture Notes were delivered to students using techexplorer. This required students to install that program in their computers. Even though students were provided with free professional versions of techexplorer, the usage of these materials remained low. Many students protested because of the additional work they had with installing new programs.

On-line Lectures vs. Traditional Instruction

The basic course in calculus at the University of Helsinki was offered, fall 2004, both as a traditional course and as a fully on-line course. The traditional course consisted of lectures in large sections, problems sessions in small sections and proctored examinations. Students of the on-line course did all of their studying over the web. Examinations for the on-line students were proctored and were the same as the examinations for the traditional course. The same graders graded all examinations, and they did not know which students were on-line students and which students were traditional students. The course was based on (Mika Seppälä, 2004). Both groups had the same exercises.

The results were surprising. The retention rate among the online students was 66%. For the traditional contact instruction class it was only 62%. Hence there were less drop-outs among the on-line students. The average of the results of the on-line students in the first examination was 12.74 points (out of 24), while in the traditional class the average was 11.74. Hence the on-line students fared also better than the traditional students.

When the experiment with on-line courses was started at the University of Helsinki, the course was opened with a cap of 15 students. That filled in days. Then the cap was raised first to 30 and then to 40. The size of the traditional class was about 70 students. It is clear that among students there is lot of demand for on-line courses.

On-line Lectures: Live Mathematics

The most common way to use information technology to enhance teaching materials in mathematics has been to add java illustrations letting students to experiment with mathematical statements. Some of these illustrations are very sophisticated and valuable new elements in instruction.
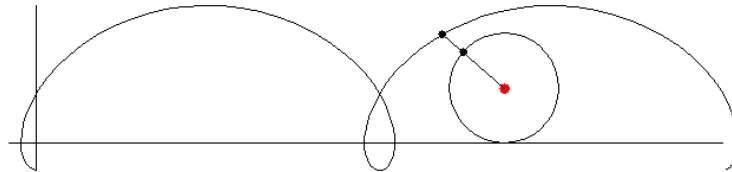

Figure 3. Screenshot from a LiveGraphics animation of a cycloid, authors Henna Komi and Simo Kivelä.

These methods can be used to provide students with web based special purpose course calculators in addition to examples illustrating specific topics. They can also be used, for example, to offer simple quizzes. Tools like MapleNET and Mathematica can easily produce such animations and course calculators. Here again an important requirement is that the materials must be usable without having to install new software manually. Solutions which install the necessary new programs and fonts automatically can be used. See Figure 4 for an other illustration (the vertices *A, B, C*, and the point *P*, can be freely dragged, thus providing an immediate rich experience on the meaning of the concepts involved).
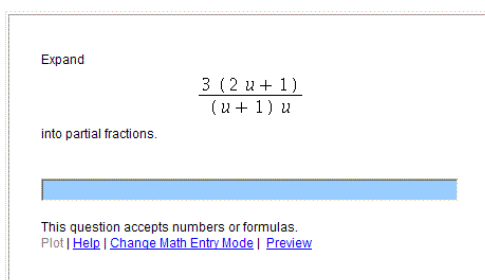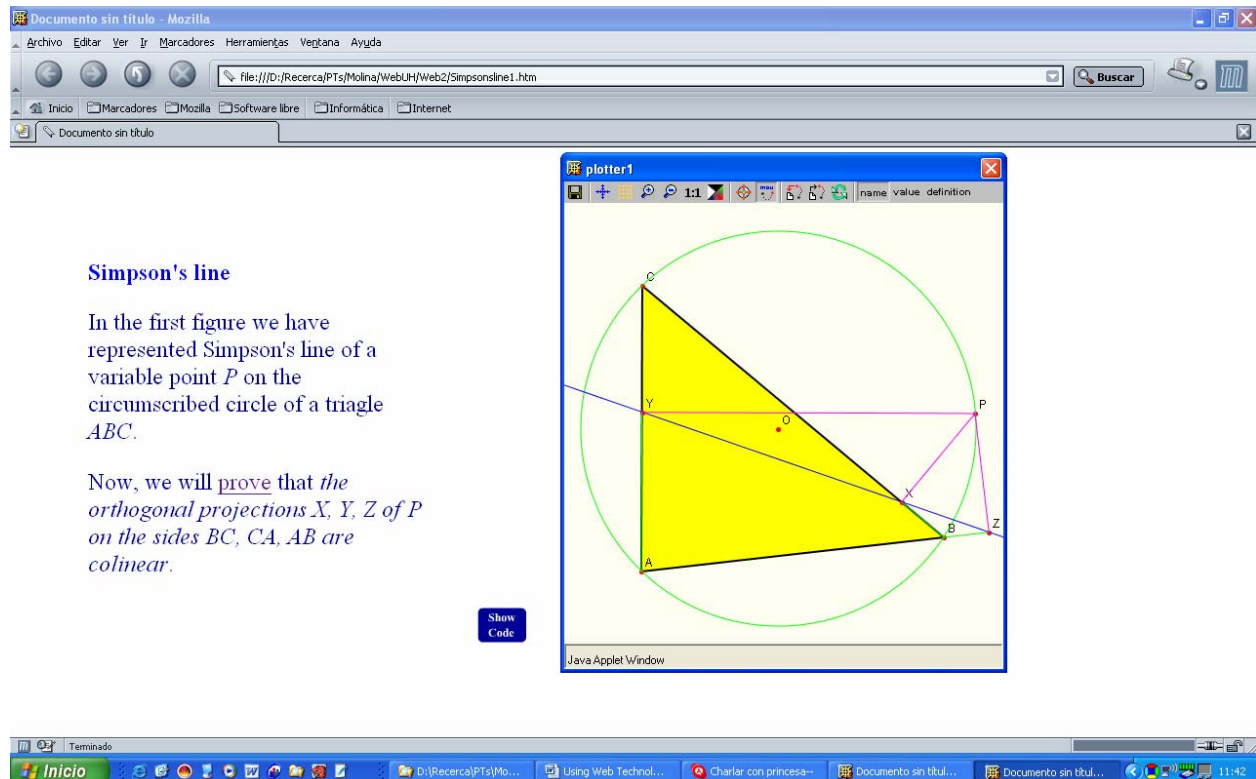

Figure 4. Screenshot of a Wiris Interactive Geometry System (S. Xambó S. Molina, 2005)

Automatic Assessment

Homework, quizzes and tests offered by a learning system providing automatic assessment is probably the most valuable component that information technology can offer to education. The impact of tools like Maple T.A. goes much beyond the impact of even the most sophisticated illustrations of mathematics. The WebALT Project (WebALT, 2005) is working to take automatic assessment to the next level by providing adaptivity and automatic localization. The automatic localization means that the content in the WebALT database is encoded in such a way that it is possible to generate versions of the problems in many languages. Initially WebALT supports English, French, Dutch, Catalan, Swedish and Finnish.

Algorithmic Problems are the most valuable problems for automatic assessment. An algorithmic problem depends usually on several random variables. Each time a student invokes an algorithmic problem he or she gets a new version of it. Such algorithmic problems have already been offered for years by systems like LON CAPA (LON-CAPA). These algorithmic problems become, however, really powerful when the computational capabilities of mathematics systems, like Maple, can be used to produce the algorithmic problems and to grade them. Maple T.A. and STACK offer such power today. WebALT System can be used to administer multilingual materials to be used with these systems.

Expand

$$\frac{3\,(2\,u+1)}{(u+1)\,u}$$

into partial fractions.

This question accepts numbers or formulas.
Plot | Help | Change Math Entry Mode | Preview

The above screen shot shows a typical algorithmic problem. The problem itself is a simple program which has single digit integer valued parameters. Every such problem leads to a simple answer and all the problems are equally difficult or easy. The program produces over 100 000 such problems. From the students' point of view, this is infinitely many problems.

Students' responses get automatically graded and grades entered into students' grade books. Programs like Maple T.A. and STACK are wonderful tools when teaching mechanical tasks to students. Thanks to the algorithmic problems, examinations can be published beforehand. Students can practice as much as they want, and when they come to the examination they still get different versions of the problems. Hence memorizing certain solutions by heart is not useful. They must learn the procedure in order to be able to pass the exam.

This way of studying suits most students very well. Monitoring students' behaviors shows that many students use these tools in a serious way, solving hundreds of problems when practicing tasks like computing limits, differentiating or integrating. These tasks form the basics of calculus. Almost all basic courses in sciences teach students certain mechanical procedures. They have to be learned, and students can study these procedures using automatic tools. This can make a dramatic difference improving the learning experience and, at same time, reducing the cost of the instruction.

Metadata for Learning Resources

At a large university there may be even 20 instructors teaching calculus courses in a large number of small sections. These instructors create their own teaching materials, slide shows, homework problems, quizzes and examinations. Each one of these instructors today use his or her own way to classify the materials. This means that sharing does not happen. It is too complicated.

Sharing becomes possible if appropriate metadata is being used to classify the materials (questions, sets of homework problems, slide shows, examinations, etc.). The WebALT Project has developed a version of the metadata needed for basic mathematics courses. This metadata will evolve. The project also offers an editor to extend the metadata definitions. Several other projects offer also metadata to classify teaching materials. The

process is evolutionary: the best and the fittest will remain and eventually get universally used. We are, however, only at the very beginning of this process. The inclusion of metadata to various files is not always easy, and some times even impossible. Hence authors do not use metadata systematically. The Mathematics Classification Index (Dave Rusin, 2000) is an example of widely used metadata to classify mathematics research papers. This classification of mathematical papers has been used for years and makes the libraries searchable in a meaningful way for mathematicians. Similar classification of educational materials is missing. This hinders the development and the use of the valuable existing materials.

Web Advanced Learning Technologies

The WebALT project (WebALT, 2005) will bring tools like Maple T.A. to the next level by adding features like multilinguality. WebALT will offer an adaptive database of multilingual exercises that can be used with a variety of platforms including Maple T.A. In addition to that WebALT will produce its own exercise player that can be used with Moodle to track students' performance.

The eContent produced by the WebALT project forms a new way to deliver mathematics instruction. The recently introduced programs, like Skype and Festoon, provide important infrastructure making it possible to offer on-line instruction that, in no way, lacks the interactivity of a class room contact lecture. This will result into significant savings in the cost of instruction world wide. Students can study from the comfort of their homes and several of the tasks of the instructor can be automated without reducing quality. Advanced portals will also offer educational institutes the possibility to export education.
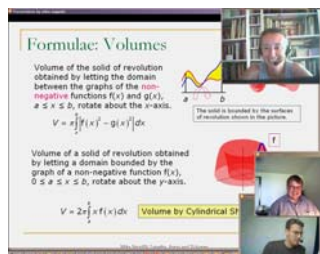
Sample face-to-face on-line teaching session

Conclusion

The general expectations regarding the impact and the use of technology in science instruction have not been fulfilled for many reasons. They include:

1. for practical reasons, authors of educational materials in sciences have been forced to present formulae as pictures only,
2. distributed computing has not been easily available to instructors,
3. students have not had sufficiently high quality broad band connections to the internet,
4. video-conferencing has not been available to the average user.

Now all these barriers have disappeared. Authors can encode mathematics in their documents using MathML, browsers can display such formulae. Instructors can write materials using the computational power of Maple or Mathematica residing in a server. Intelligent automatic testing is available. On-line learning is coming.

References

G. S. Gill (2005). Calculus Bible. http://www.math.byu.edu/Math/CalculusBible/

Barry James (1996). Beyond Minitel: France on the Internet. http://www.iht.com/articles/1996/01/08/minitel.t.php

LON-CAPA (2005). Distributed Learning Content Management and Assessment System. http://www.lon-capa.org/

NCAT (2005). National Center for Academic Transformation: Who We Are. http://www.thencat.org/whoweare.html

D. Marquès R. Eixarch, S. Xambó (2002). "An Internet Platform for the Teaching and Learning of Mathematics in Large Educational Communities." Contributions to Science **2**(3): 269-276.

Dave Rusin (2000). USING MATHEMATICS SUBJECT CLASSIFICATION. http://www.math.niu.edu/~rusin/known-math/index/

S. Xambó S. Molina (2005). Web Accessed Interactive Classical Geometry System. http://webalt.net

Mika Seppälä (2004). Single Variable Calculus. http://www.webalt.net/Calculus-2004/

WebALT (2005). Web Advanced Learning Technologies. http://www.webalt.net

Wikipedia. Minitel. http://en.wikipedia.org/wiki/Minitel

Sebastian Xambó (2003). <u>Block-Error Correcting Codes: A Computational Primer</u>, Springer-Verlag. http://www.wiris.com/links--books/articles/