KnE Engineering

ICEUBI2019
International Congress on Engineering — Engineering for Evolution
Volume 2020

Knowledge E
enriching | engaging | empowering

Conference Paper

Short-term Hydro-thermal Coordination By Lagrangian Relaxation: A New Algorithm for the Solution of the Dual Problem

Pedro Bento[1], Filipe Pina[2], Sílvio Mariano[1], and Maria do Rosario Calado[1]

[1]Instituto de Telecomunicacoes and University of Beira Interior
[2]University of Beira Interior

Abstract

For decades, researchers have been studying the unit commitment problem in electrical power generation. To solve this complex, large scale and constrained optimization (primal) problem in a direct manner is not a feasible approach, which is where Lagrangian relaxation comes in as the answer. The dual Lagrangian problem translates a relaxed problem approach, that indirectly leads to solutions of the original (primal) problem. In the coordination problem, a decomposition takes place where the global solution is achieved by coordinating between the respective subproblems solutions. This dual problem is solved iteratively, and Lagrange multipliers are updated between each iteration using subgradient methods. To tackle, time-consuming tuning tasks or user related experience, a new adaptative algorithm, is proposed to better adjust the step-size used to update Lagrange multipliers, i.e., avoid the need to pre-select a set of parameters. A results comparison against a traditionally employed step-size update mechanism, showed that the adaptive algorithm manages to obtain improved performances in terms of the targeted primal problem.

Keywords: Hydro-Thermal coordination, Lagrangian relaxation, Lagrangian dual problem, Lagrange multipliers, Subgradient methods

Corresponding Author:
Pedro Bento
pedro.bento@lx.it.pt

Received: 26 November 2019
Accepted: 13 May 2020
Published: 2 June 2020

Publishing services provided by Knowledge E

Selection and Peer-review under the responsibility of the ICEUBI2019 Conference Committee.

1. Introduction

The objective of short-term hydro-thermal scheduling is the optimization electricity generation [1], obtaining an optimal generation dispatch or close to ideal of all the thermal and hydro units available in a system, so that the total operation cost is minimized within horizons ranging from one day to one week (168h), taking into account the entire system and its individual constraints [2]–[5], the planning period (discrete time-step) is set from hour to hour [5]. Therefore, we can see why Unit Commitment (UC) problem is very important for electricity producers and one of the most difficult to solve for power system engineering [6]. It results in a complex mathematical programming

problem [1] that has been the subject of intense research in the area of operation and optimization of electric power systems over the last decades [1], [2], [7], [8]. The UC problem decides the dispatch policy of the thermal units in such a way that the total cost (operating cost, starting cost and shut down cost) is minimal over a pre-defined time-horizon. In addition to achieving the minimum production cost, a series of operational constraints need to be fulfilled, thus reducing the freedom of choice to turn a thermal unit on or off. With this regard we are primarily speaking about the load balance constraint, i.e., ensure that our load demand is satisfied, spinning reserve constraints, minimum connected time, minimum time off, generation capacity limits, group restrictions, water restrictions, etc. [9].

To solve the optimal short-term UC, a broad spectrum of methods has been tried, and they can be generally divided into two categories: heuristic methods (trial and error) and deterministic methods [5], [9]. In the realm of conventional approaches we can highlight Benders Decomposition [10], Lagrangian Relaxation [1], [2], [11], Dynamic Programming (PD), Nonlinear Programming [12], Augmented Lagrangian [13], mixed integer linear programming [14], nonlinear programming [15], among others.

A consolidated trend has seen a growing application of evolutionary methods and methods of artificial intelligence, in addition to new deterministic heuristics. Hence, we can mention: neural networks [16], Cuckoo Search [17], Differential Evolution [18], [19], Improved Bacterial Foraging Algorithm [20], among many others. However, in general these population-based methods require a significant computational effort to solve the problem for an hourly discretized weeklong time-horizon, i.e., for large scale-problems (with a high number of dimensions) its effectiveness drops. In addition, it can frequently end up finding only suboptimal solutions [4], [15]. However, these metaheuristic search methods are based on a population to find an optimal solution, turning them into large-scale problems (many dimensions and population agents), and moreover they need to be performed several times to find an optimal solution, as premature stagnation or convergence may occur.

Due to the presence of multiple sets of constraints decomposition techniques appear as natural techniques to consider to solve this problem [1], [8]. Consequently, Lagrangian Relaxation (LR) based on decomposition techniques appears as a natural and preferred manner to tackle the short-term hydro-thermal coordination problem. The fundamentals behind LR, is to use Lagrange multipliers to relax system constraints such as demand and reserve requirements. The primal problem is then converted into a two-level structure. Given a set of multipliers, all subproblems are resolved at the low level, one for each unit, and the multipliers are updated at the high level. Multipliers are obtained by solving

the dual problem and the feasibility of solving the primal problem is usually obtained based on the dual solution [2], [3].

To update Lagrange multipliers a common approach is to apply subgradient methods, where the step-size update procedure represents a sensible decision, and often depends on ad-hoc testing. For example, in [21], [22] evolutionary programming with a Gaussian mutation is used to update the multipliers and in both, parameters are chosen considering convergence criteria.

In this sense, to avoid user dependence concerning parameters choice, an adaptative algorithm is proposed in this work. As its name suggests, the step value is dynamically updated, and afterwards the Lagrange multipliers so that the dual function converges to its optimum in a pre-arranged number of iterations. Moreover, in order to validate this algorithm, the same is tested against a traditional step-size approach in a scenario where only unit costs are considered.

The rest of this paper is organized as follows. Section 2 provides a brief description of the primal problem. Section 3 explains the Lagrangian dual problem. Section 4 introduces subgradient methods and the motivation for the proposed algorithm, introduced in Section 5. Results and discussion are provided in Section 6, and finally, Section 7 presents the conclusion of this work.

2. Primal Problem

The optimal unit commitment problem is a deterministic problem, that can be understood as the task of establishing a map of feasible operations for each generation unit available in an electrical power system at the lowest cost for a predefined time horizon, in order to satisfy the expected load demand and a set of other system restraints. Typically, the time horizon considered is from one to seven days, and the discrete time-step (in which decisions are made) is a one-hour period. This problem is treated as deterministic and whenever it is necessary to include stochastic quantities such as load diagram and reservoir inflows, their expected values are used.

In this manner, a primal problem (P) is non-convex and non-linear and can be mathematically formulated as shown in equations (1)-(4). The total operating cost for all resources(units) and over the entire considered period, K, is defined in equation (1), and is the problem's objective function, i.e., evaluates the performance of each admissible solution. Where the cost function, $C_{ik}(x_{i,k-1}, p_{ik}, u_{ik})$, is a measure that evaluates the decision made in each state, since there is an operating cost associated with the state transition (from $x_{i,k-1}$ to x_{ik}), which delivers the power p_{ik}, determined by the control

decision u_{ik}, for each unit i at time k. Secondly, equation (2) translates the (global) demand–supply balance restraint. In turn, equation (3) represents the state equation of each resource i at a time k. This equation allows us to obtain the state of each resource x_{ik} and its contribution to satisfy demand p_{ik}, whatever the decision u_{ik}. Last of all, in (4) the domain of admissible values for the control variables, as well as for the initial and final state are defined, for each individual resource i.

$$p_u^{\text{Min}} \sum_{k=1}^{k} \sum_{i=1}^{I} C_{ik}(x_{i,k-1}, p_{ik}, u_{ik}) \tag{1}$$

Subject to:

$$\sum_{i=1}^{I} p_{ik} = D_k \qquad k = 1, \ldots, K \tag{2}$$

and wherein:

$$(x_{ik}, p_{ik}) = A_{ik}(x_{i,k-1}, u_{ik}) \quad i = 1, \ldots, I \; e \; k = 1, \ldots, K \tag{3}$$

$$u_{ik} \in U_{ik} \quad x_{i0} \in X_i^0 \; x_{ik} \in X_i^K$$

$$i = 1, \ldots, I \; e \; k = 1, \ldots, K \tag{4}$$

Formulation nomenclature:

K: total number of hours

I: total number of resources

C_{ik}: cost function associated with resource allocation i at time k

x_{ik}: resource state i at time k

p_{ik}: power output by resource i at time k

u_{ik}: control (decision) variable for resource i at time k

D_k: load demand at time k

A_{ik}: state function associated with each resource i at time k

$\mathcal{U}_{ik}$: control variables (decision) universe for resource i at time k

X_i^0: resource i initial state

X_i^K: resource i final state

Although the objective function is a separable function in resources and hours, this problem, by its formulation and due to collective constraints, does not allow this separability, providing extreme complexity to this minimization problem. In other words, the optimum value cannot be found by the sum of the various sub-optimal (separately)

results from each resource. Thus, we are facing a problem of unrestrainable dimension, for which a direct approach is not viable.

The primal problem defined in this study, approaches the short-term hydro-thermal coordination considering the generation resources available to the electric utilities company, EDP, and the national load that needs to be served during a weekly time-period.

3. Lagrangian Relaxation

As discussed, the primal problem is difficult to solve using conventional nonlinear optimization techniques. A preferable path is to decompose the problem constraints, and transfer them to the objective function, i.e., to solve the dual problem, rather than solving the primal problem directly. Knowing beforehand that the optimal solution of the relaxed problem is a lower bound (good estimate) of the optimal solution of the initial problem [2], [8], [23].

This is achieved by relaxing the constraints, i.e., weakening of the problem (P), that in practical terms means open the possibility to breach the imposed constraints. However, relaxed restrictions are not completely ignored since its violations are linearly penalized in the Lagrange function (an added cost associated with violating each constraint).

So, we can write the Lagrange function ($\mathscr{L}$) for problem (P), by relaxing its load-balance constraint as expressed in equation (5), where λ is the Lagrange multiplier vector associated with the load-balance constraint, equation (2). Lagrange multipliers are expressed in units of cost per unit of the parameters of their associated constraint, in this case ($/GW).

$$\mathscr{L}(x_{i,k-1}, p_{ik}, u_{ik}, \lambda) = \sum_{k=1}^{K}\sum_{i=1}^{I} C_{ik}(x_{i,k-1}, p_{ik}, u_{ik}) + \sum_{k=1}^{K} \lambda_k \left(D_k - \sum_{i=u}^{I} p_{ik} \right) \tag{5}$$

That is, to now solve the unit commitment $\mathscr{L}$ is minimized, $((L)_u^{\text{Min}}\, \mathscr{L}(x_{i,k-1}, p_{ik}, u_{ik}, \lambda)$, subject to system constraints (3)-(4).

3.1. Lagrangian dual problem

The Lagrangian dual problem is obtained by forming ($\mathscr{L}$), and its solution provides the primal variables as functions of the Lagrange multipliers, which are labeled dual variables. Hence, the new problem is to maximize the objective function with respect to the multipliers under the derived constraints on the dual variables. Implying by decomposition, that each resource becomes a single entity and is individually optimized,

rather than a combined optimal resource allocation. Therefore, the dual function is defined in equation (6), also subject to constraints (3)-(4), presenting concave and sub-differentiable traits (resulting in inferiorly limited variables).

$$q(\lambda) = \text{Min}_u \mathscr{L}(x_{i,k-1}, p_{ik}, u_{ik}, \lambda) \tag{6}$$

Given that Lagrange's dual function is a concave function, a local optimum is also the function global optimum. Lagrange dual-function subgradients play an important role in maximizing the dual function - the easily derived constraint-deviations vector is a sub-gradient of the Lagrange dual function at a point defined by the values of λ. Consequently, we can define the subgradient of the dual function, g as follows:

$$g = D_k - \sum_{i=u}^{I} p_{ik} \tag{7}$$

Moreover, by the weak duality theorem, the optimal value of the Lagrange dual problem $q(\lambda^*)$) and the optimal value of the primal minimization problem $p(\lambda^*)$) are related by $q(\lambda^*) \leq p(\lambda^*)$), and difference between the values is called, duality gap. Implying that the dual problem offers a good indirect root to solve the primal one.

For all the reasons above, this new approach to the problem is extremely advantageous, since it lessens the computational burden of the primal problem.

4. Subgradient Methods

As we saw earlier, obtaining the Lagrange dual function optimal value is hand-in-hand with the Lagrange multiplies choice/update method, i.e., at the outset this choice determines how close we are to the solution of the dual problem and, ultimately, how close are we from reaching the primal problem best solution. To perform this task several methods are described in literature [5], however, in particular regarding our problem subgradient methods prevail as the most fitting solution by achieving higher accuracies. Further benefits include, their simplicity, as well as the computational ease with which the Lagrange dual function subgradient (solution deviation from the imposed constraints) is calculated.

These methods update the multipliers according to the subgradient direction and in a proportional manner to the violation of the corresponding constraints. Besides, a distinctive trademark of these methods concerns the step-size update heuristic, where again several approaches have been followed [5]. However, a downsize of these conventional updating heuristics is that a long-winded trial and error procedure, as well

as a highly specialized operator is frequently required. The simplest and most common subgradient method formulation is given below by:

$$\lambda^{\upsilon+1} = \left[\lambda^{\upsilon} + s^{\upsilon}\frac{g^{\upsilon}}{\|g^{\upsilon}\|}\right]^{+} \tag{8}$$

where g^{υ} is the subgradient $g(p_{\lambda}\upsilon)$, s^{υ} is a positive scalar that defines the step-size at the current iteration υ, and lastly, $[.]^{+}$, represents the projection in the set of feasible values Λ. Nonetheless, there is no guarantee that after iteration υ + 1, independently from the chosen step-size, the dual function value will actually improve (walk towards the optimal dual function value), meaning that in some occasions we will have:

$$q\left(\left[\lambda^{\upsilon} + s^{\upsilon}\frac{g^{\upsilon}}{\|g^{\upsilon}\|}\right]^{+}\right) < q(\lambda^{\upsilon}), \quad \forall\ s > 0 \tag{9}$$

Though, if the step value is sufficiently small, the distance between the obtained point in the current iteration and the optimum solution can always be reduced. The next proposition offers an estimate for the step-size domain (range):

P1 If λ^{υ} does not lead to the optimum value of the dual function then, for λ^{*}, which corresponds to the dual function optimum value, the inequality, $\|\lambda^{\upsilon+1} - \lambda^{*}\| < \|\lambda^{\upsilon} - \lambda^{*}\|$, is valid for all step-sizes, $s^{\upsilon} \in]0, \frac{(q(\lambda^{*}) - q(\lambda^{\upsilon}))}{\|g^{\upsilon}\|}[$. Therefore, suggesting a step-size equal to the middle value of the inequality range, i.e., $s^{\upsilon} = \frac{(q(\lambda^{*}) - q(\lambda^{\upsilon}))}{\|g^{\upsilon}\|}$.

But since it requires knowledge of the dual function optimal value $q(\lambda^{*})$, which is exactly the unknown we want to find, this approach is unviable in our case and we resort to heuristics that determine the step-size. In this regard, a popular choice fells over decreasing step-size rulebased approaches, much due to its simplicity and effectiveness.

So, considering a decrease on step-size, s^{υ}, towards zero, meaning that $\lim_{\upsilon\to\infty} s^{\upsilon} = 0 \wedge s^{\upsilon} > 0$, while at the same time satisfying $\sum_{\upsilon=1}^{\infty} s^{\upsilon} = \infty$, i.e., the method can ”travel” as far as possible (up to infinity) in order to converge to the optimal dual function value. Thus, under these assumptions, we can infer a 2nd proposition (P2), from which we can conclude that it is possible, by appropriately updating the step-size, to reach the dual function maximum value [15].

P2 For the sequence of all multiplier’s values $\{\lambda^{\upsilon}\}$ we have: $\lim_{\upsilon\to\infty} Max\ q(\lambda^{\upsilon}) = q^{*}$.

However, this analysis does not lead to a finite procedure, pointing an initial value of the step, as well as a mechanism for decrementing it to zero. As such, for comparison purposes with the proposed heuristic in this work, a widely used expression is introduced in equation (3), to update the step-size at each iteration υ.

$$s^{\upsilon} = \frac{a_1}{1 + \upsilon \times a_2} \tag{10}$$

Where, a_1 and a_2 are control parameters of the heuristic process. Moreover, the chosen initial step is a highly sensitive matter, since small initial steps can prevent the method from reaching the desired optimum value in a reasonable number of iterations. Whereas, using a large initial step may cause the method to oscillate erratically in the early phase, leading to poor convergence. As a result, although the obtained value is stabilized, it can still be improved by running further iterations. In addition, selecting the values to assign to parameters a_1 and a_2 is also a difficult task, with a direct influence on the obtained results.

Therefore, we can conclude that it is an intrinsically lengthy (experimentation-based) heuristic process that is highly dependent on the user's experience. Precisely to mitigate this scenario, a new algorithm will be proposed next.

5. Proposed Adaptative Algorithm

Motivated by the previously exposed shortcomings on the classical subgradient optimization, an adaptative heuristic is proposed, in order to automatically update the Lagrange multipliers, thereby removing the need to rely on user past experiences or time-consuming trial and error tasks. This means that the step-size, s^v, is automatically determined (avoiding the time costly trial and error procedures) by the adaptative algorithm when solving the dual problem with a subgradient method. The different stages of the algorithm and the rationale behind them are detailed below:

1. Define the initial step-size, $s^0 = 1$, and choose an initial value for the dual variable vector, λ_0;

   Then compute the initial dual function and subgradient values, $q^0(\lambda^0)$ and $g^0(p_{\lambda^0})$, respectively;

2. Update Lagrange multipliers according to equation (8)

3. Determine the new step-size:

   If $q^v(\lambda^v) > q^{v-1}(\lambda^{v-1})$ then

   $\alpha \in v_\delta^+(1)$

   Else

   $\alpha \in v_\delta^-(1)$

   End

   where,

$$v_{\delta}^{+}(1) = \{\alpha_1 : 1 < \alpha < 1 + \delta\}$$

$$v_{\delta}^{-}(1) = \{\alpha_2 : 1 - \delta < \alpha < 1\}$$

And the step-size is given by:

$$s^{v} = \alpha s^{v-1}$$

4. Compute the current (iteration) dual function and subgradient values, $q^{v}(\lambda^{v})$ and $g^{v}(p_{\lambda}v)$;

   If the termination criterion is met:

   Terminate the algorithm;

5. Proceed to the next iteration, $v = v + 1$;

   Return to (2);

Regarding the (above) adaptative algorithm, the following clarifications are made:

In (1) the initial dual variable vector positioning only impacts the convergence speed of the subgradient method, thereby it can be considered arbitrary. On the contrary, for the step-size update expressed by equation (10), this initial positioning needs to be nearby an optimum value (derived from past experiences), in order to maintain the method's performance, translating an important advantage of the proposed strategy.

In turn, stage (3) depicts the original step-size update mechanism, where the rationale behind it is to dynamically update the step based on the dual function value, i.e., if this value improves then the step should be augmented, in contrast if this value does not improve then the step should be diminished. Moreover, to prevent that a large step-size increases the distance between the new point and the optimum solution, this value should be increased smoothly, this fact is less sensitive when reducing step-size. Additionally, it was found that the optimal domain for variables α_1 and α_2 are [1.01, 1.05] $\Longrightarrow\delta$ = [0.01, 0.05] and [0.83, 0.95] $\Longrightarrow\delta$ = [0.05, 0.17], respectively.

Lastly, the stop criterion mentioned in (4) is traditionally to run a specific number of iterations, which was also the case in this work.

6. Numerical Results

The behavior of the subgradient method is analyzed in this section, where the step value is updated according to the adaptive algorithm proposed in section 5, and then benchmarked against a classical approach, where the step-size is updated using equation (10), and consequently the Lagrange multipliers. In the selected case study,

unit costs are the sole result from the associated generation costs, with no other parallel costs (e.g. associated with some predefined strategy).

As previously mentioned, the unit commitment (primal problem) corresponding to the solution of the Lagrange dual problem does not always lead to a feasible solution. As such, the average subgradient norm, $\|g(p_\lambda)\|/K$, is defined as quantitative metric of how a solution is accurate in terms of the primal problem. Meaning that, the lower the value, the closer we will be to a good solution, and where a value in the order of 0.5% of the peak load typically means that a good solution to the primal problem was found.

The first pair of figures will display the evolution of the dual function $q(\lambda)$) value and the second pair, the evolution of the average subgradient norm, $\|g(p_\lambda)\|/K$ over the course of iterations. Figure 1 a) and 2 a) illustrate the behavior of the Subgradient method using a classical step update expression, and Figure 1 b) and 2 b) using the new adaptive algorithm.

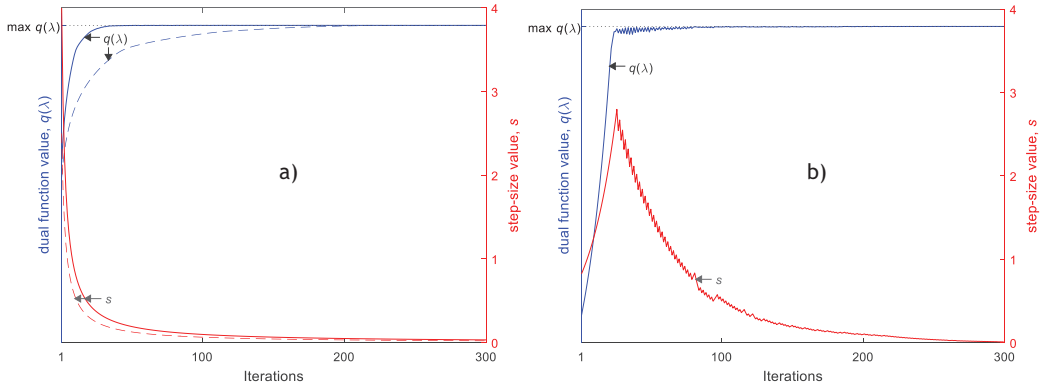

Figure 1: a) Evolution of the dual function value, $q(\lambda)$, and its step value, using equation (10), with the following parameter values: solid line, $a_1 = 20, a_2 = 2$ and with a line dashed, $a_1 = 10, a_2 = 1.5$. b) Evolution of the dual function value, $q(\lambda)$, and its step value, using the adaptative algorithm, with the following parameter values: $\alpha_1 = 1.05, \alpha_2 = 1.10$.

Regarding Figure 1 a) we can observe: (1) Achieving convergence in more or less iterations depends on the choice of the different parameters; (2) Using a smaller initial step-size increased the number of iterations need to achieve convergence (dashed line); (3) The use of a slightly larger initial step leads to some oscillation, still, without compromising convergence, represented by the solid lines; (4) The step-size evolution is strictly decreasing, and the rate of descent depends on the considered parameters.

With respect to the adaptive algorithm, the evolution of the dual function increases as the value of the step increases, as shown in Figure 1 b), until a value is reached in the vicinity of the maximum dual function value. From this point onwards, the step value decreases towards zero, but then again having a slight increment whenever the dual function value doesn't improve when compared with the previous iteration.

This dynamically adjusted (based on the dual function current value) step-size, clearly contrasts with the monotone evolution that occurs with the traditional step update formulation.

We can verify that in both cases, the dual function maximum value was reached, with regard, and difference resides in the number of iterations necessary to the convergence (which was relatively similar).

By looking at Figures 2 a) and b), and with respect to the obtained minimum average subgradient norm, we can see that both processes lead to similar results, with a value close to 23 MW by the classical step-size update approach and 21 MW using the proposed adaptative algorithm. These values represent ~0.53% of the peak load which, as mentioned, usually leads to a good solution to the primal problem. Besides, note that, once the dual function maximum value or its proximities are reached, the average subgradient norm has not yet reached its minimum value and continues to oscillate between several values over the next iterations.

This can be explained since small variations in the multipliers can cause large variations in the solutions in terms of the primal problem. Thus, even obtaining the maximum value of the dual function we may not end up with the best solution in terms of the primal problem.

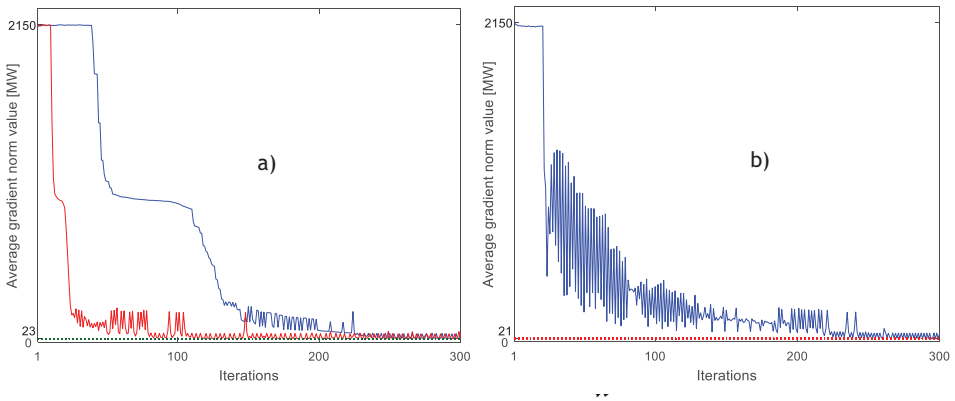

Figure 2: a) Evolution of the average subgradient norm, $\|g(p_\lambda)\|/K$, corresponding to the values of the dual function represented in Figure 1 a) (classical approach). b) Evolution of the average subgradient norm, $\|g(p_\lambda)\|/K$, corresponding to the values of the dual function represented in Figure 1 b) (proposed adaptative algorithm).

Lastly, regarding the solution in terms of the primal problem (Figure 3), corresponding to the solution of the dual problem for the lowest average subgradient norm value. The same was obtained using the adaptive algorithm, since is easy to understand from previous figures, that all primary solutions would be similar, so their presentation is unnecessary. The algorithm used in solving the primal problem based on Lagrangian relaxation, as we saw earlier does not lead to an optimal solution. The obtained primal

solution reveals the existence of a Lagrangian Duality. That is, we can say that good results were obtained for the generation profile (solid line in Figure 3), but this profile does not match the given load profile (dashed line almost coincident with the solid line of Figure 3). After solving the dual problem, several methods have been used to look for feasibility [5]. However, if we succeed when solving the dual problem then we can get, in terms of the primal problem, also a good solution. In fact, in some cases it is enough to carry out an economic dispatch of thermal units to obtain a (close) feasible strategy, which is exactly what happens in the presented case (the generation profile obtained coincides with the given load profile).

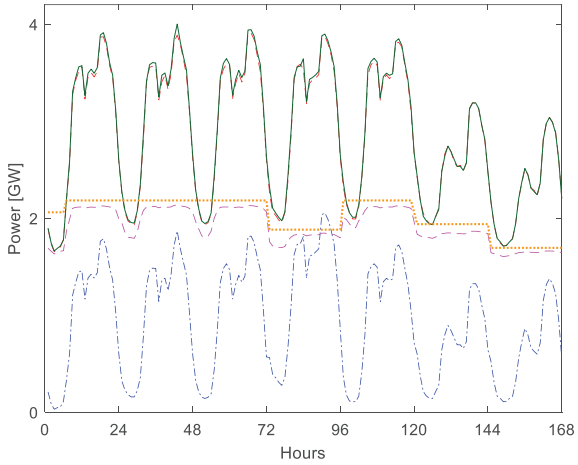

Figure 3: Solution in terms of the primal problem. In the upper portion an almost coincidently solid green and dashed red plots: the obtained generation profile and the load demand, respectively. Dotted orange line: Maximum generation capacity of the affected thermal units. Dashed magenta plot: thermal units generation profile. Dash-dot blue plot: hydro units generation profile.

7. Conclusion

The proper choice of the Lagrange multipliers has an important associated cost. This cost may not be constant due to the dynamic characteristics of the problem (load restrictions). To update the multipliers, subgradient methods are amongst the trendiest solutions. The step-size update mechanism is a vital factor on these methods functioning, and classic approaches are heavily dependent upon user's experience and fine-tuning procedures (choosing the appropriate parameters). With these limitations in mind, a novel adaptative algorithm is proposed with an important advantage of not requiring parameter choices based on experimentation. Subsequently, a classic update mechanism was compared against the proposed adaptative algorithm, where results showed an improved performance over classic formulas. This fact is justified by the algorithm's ability, to dynamically adapt the step value according to the dual function

value, i.e., we can see that during the opening iterations step value is incremented until we approach the vicinities of the dual function maximum, from then onwards the step evolves dynamically and adapted to the current dual function value, allowing convergence to the optimal dual function value and to average subgradient norm that translate a feasible primal solution. Additional test scenarios may be considered for future works.

Acknowledgments

This work is funded by FCT/MEC through national funds and when applicable co-funded by FEDER–PT2020 partnership agreement under the project UID/EEA/50008/2019.

P.M.R. Bento gives his special thanks to the Fundação para a Ciência e a Tecnologia (FCT), Portugal, for the Ph.D. Grant (SFRH/BD/140371/2018).

References

[1] F. Y. K. Takigawa, E. C. Finardi, and E. L. da Silva, “A decomposition strategy to solve the Short-Term Hydrothermal Scheduling based on Lagrangian Relaxation,” in 2010 IEEE/PES Transmission and Distribution Conference and Exposition: Latin America (T&D-LA), 2010, pp. 681–688.

[2] H. Ye, Q. Zhai, Y. Ge, and H. Wu, “A Revised Subgradient Method for Solving the Dual Problem of Hydrothermal Scheduling,” in 2011 Asia-Pacific Power and Energy Engineering Conference, 2011, pp. 1–6.

[3] Q. Zhai and H. Wu, “Several notes on Lagrangian Relaxation for unit commitment,” in Proceedings of the 29th Chinese Control Conference, 2010, pp. 1848–1853.

[4] T. T. Nguyen and D. N. Vo, “Multi-Objective Short-Term Fixed Head Hydrothermal Scheduling Using Augmented Lagrange Hopfield Network,” J. Electr. Eng. Technol., vol. 9, no. 6, pp. 1882–1890, Nov. 2014.

[5] L. A. F. M. Ferreira *et al.*, “Short-term resource scheduling in multi-area hydrothermal power systems,” *Int. J. Electr. Power Energy Syst.*, vol. 11, no. 3, pp. 200–212, Jul. 1989.

[6] Ü. Başaran Filik and M. Kurban, “Solving Unit Commitment Problem Using Modified Subgradient Method Combined with Simulated Annealing Algorithm,” *Math. Probl. Eng.*, vol. 2010, pp. 1–15, Jul. 2010.

[7] J. A. Marmolejo-Saucedo and R. Rodríguez-Aguilar, “A proposed method for design of test cases for economic analysis in power systems,” *J. Appl. Res. Technol.*, vol. 13,

no. 3, pp. 428–434, 2015.

[8] A. L. Diniz, C. Sagastizabal, and M. E. P. Maceira, “Assessment of Lagrangian Relaxation with Variable Splitting for Hydrothermal Scheduling,” in *2007 IEEE Power Engineering Society General Meeting*, 2007, pp. 1–8.

[9] P. K. Singhal, “Generation scheduling methodology for thermal units using lagrangian Relaxation,” in *2011 Nirma University International Conference on Engineering*, 2011, pp. 1–6.

[10] C. J. Lopez-Salgado, O. Ano, and D. M. Ojeda-Esteybar, “Hydrothermal scheduling with variable head hydroelectric plants: Proposed strategies using benders decomposition and outer approximation,” in *2016 IEEE Power and Energy Conference at Illinois (PECI)*, 2016, pp. 1–8.

[11] R. N. Rodrigues, E. L. da Silva, E. C. Finardi, and F. Y. K. Takigawa, “Solving the Short-Term Scheduling Problem of Hydrothermal Systems via Lagrangian Relaxation and Augmented Lagrangian,” *Math. Probl. Eng.*, vol. 2012, pp. 1–18, Feb. 2012.

[12] R. Kumar, V. Garg, and B. Lal, “A Review Paper on Hydro-Thermal Scheduling,” *Int. J. Emerg. Technol. Comput. Appl. Sci.*, vol. 5, no. 5, pp. 522–526, 2013.

[13] C. Beltran and F. J. Heredia, “Short-Term Hydrothermal Coordination by Augmented Lagrangean Relaxation: a new Multiplier Updating,” in *IX Congreso Latino-Iberoamericano de Investigación Operativa*, 1998, pp. 1–6.

[14] E. Gil and J. Araya, “Short-term Hydrothermal Generation Scheduling Using a Parallelized Stochastic Mixed-integer Linear Programming Algorithm,” *Energy Procedia*, vol. 87, pp. 77–84, 2016.

[15] S. J. P. S. Mariano, J. P. S. Catalão, V. M. F. Mendes, and L. A. F. M. Ferreira, “Optimising power generation efficiency for head-sensitive cascaded reservoirs in a competitive electricity market,” *Int. J. Electr. Power Energy Syst.*, vol. 30, no. 2, pp. 125–133, 2008.

[16] M. Basu, “Hopfield neural networks for optimal scheduling of fixed head hydrothermal power systems,” *Electr. Power Syst. Res.*, vol. 64, no. 1, pp. 11–15, Jan. 2003.

[17] T. T. Nguyen and D. N. Vo, “Modified cuckoo search algorithm for short-term hydrothermal scheduling,” *Int. J. Electr. Power Energy Syst.*, vol. 65, pp. 271–281, 2015.

[18] M. Basu, “Improved differential evolution for short-term hydrothermal scheduling,” *Int. J. Electr. Power Energy Syst.*, vol. 58, pp. 91–100, Jun. 2014.

[19] H. Zhang, J. Zhou, Y. Zhang, Y. Lu, and Y. Wang, “Culture belief based multi-objective hybrid differential evolutionary algorithm in short term hydrothermal scheduling,” *Energy Convers. Manag.*, vol. 65, pp. 173–184, 2013.

[20] I. A. Farhat and M. E. El-Hawary, “Short-term hydro-thermal scheduling using an improved bacterial foraging algorithm,” in *2009 IEEE Electrical Power & Energy* Conference (EPEC), 2009, pp. 1–5.

[21] S. Padmini, R. Jegatheesan, and D. F. Thayyil, “A Novel Method for Solving Multi-Objective Hydrothermal Unit Commitment and Sheduling for GENCO Using Hybrid LREP Technique,” Procedia Comput. Sci., vol. 57, pp. 258–268, 2015.

[22] C. Nayak and C. C. A. Rajan, “An evolutionary programming embedded Tabu search method for hydro-thermal scheduling with cooling banking constraints,” J. Eng. Technol. Res., vol. 5, no. 2, pp. 21–32, Feb. 2013.

[23] S. Ruiic, “Optimal Distance Method for Lagrangian Mulitpliers Updating in Short-Term Hydro-Thermal Coordination - Power Systems, IEEE Transactions on,” vol. 13, no. 4, pp. 1439–1444, 1998.