Corresponding Author:
Amine Mahjoub
amahjoub@uob.edu.bh

Received: 18 September 2018
Accepted: 10 October 2018
Published: 15 October 2018

Publishing services provided by
Knowledge E
(c) Amine Mahjoub and Youssef Harrath. This article is distributed under the terms of the Creative Commons Attribution License, which permits unrestricted use and redistribution provided that the original author and source are credited.

Selection and Peer-review under the responsibility of the Sustainability and Resilience Conference Committee.

G OPEN ACCESS

## Conference Paper

# Scheduling in Cloud Computing Environment <br> Amine Mahjoub and Youssef Harrath <br> College of Information Technology, University of Bahrain 

## Abstract

In this work, the authors focus on the problem of scheduling in cloud computing environment. Such environment is characterized by a very strong volatility, in the sense that the resources can integrate and leave the system in a frequent and completely random way. The classical scheduling problems consider that the resources are always available. This hypothesis is not realistic especially in cloud computing environment. In this article, the authors present an efficient algorithm to solve the problem of scheduling in a simplified model of a cloud environment where the resources are subject to deterministic unavailability periods.

Keywords: Parallel machine scheduling, Resource unavailability, List scheduling; LPT.

## 1. Introduction and Motivation

Cloud computing is a recent and important topic in the Computer Science field. Considerable efforts continue to be produced to develop techniques and efficient solutions to many emerging problems in this topic. Cloud computing nowadays is an IT-related service provider that allows to get many possible IT services at one place. The most common services provided by a cloud are:

1. SaaS or Software as a service: Allow the users to temporarily get profit of available software without the need of personal license.
2. Utility Computing: This is probably the most important feature of a cloud computing where the users can access virtual storages, as well as purchase computing capacities to remotely perform super calculation tasks.
3. Platforms: This is a more advanced feature of cloud computing service where complete platforms providing whole development environment are made available for users.

In this paper, we are interested in utility computing services of a cloud, where computing resources are available to execute users' tasks. The main goal is to execute the customers' applications as early as possible with the lowest cost. The particularity of the cloud computing environment is that the resources are not continuously available. Thus, the execution of an application in the cloud can be seen as a scheduling problem of independents tasks on parallel machines with unavailability periods.

The proposed research is focused on developing an efficient solution to the scheduling problem on parallel and identical machines subject to unavailability constraints. The objective is to optimize the completion execution time of the application. Classically, an application is composed by a set of independent tasks, each has a processing time. The scheduling problem consists of determining on which processor each task has to be executed and at what time it starts so that the last executed task terminates as early as possible. In this paper we propose a new efficient linear program, and will prove that LPT-based list algorithm based provides performant solutions.

The rest of the paper is organized as follows. Section 2 will be dedicated to present an overview of the literature. In section 3 an efficient linear program will be proposed. An experimental study will be shown in section 4 . Section 5 concludes the study proposes some perspectives for future works.

## 2. Literature Review

The most parallel machine scheduling problem was proved to be NP-complete [2]. However, several heuristics based on different techniques, have been proposed in the literature. Among these, list schedules are paramount [1]. List schedules can be adapted to non-continuously available machines (due to unavailability periods or breakdowns).

A list algorithm, first establishes a priority list. At each time $t$, if some machines $M_{i}$ are available, the first non-scheduled task $j$ of the list is placed on the machine if it can be executed before the machine becomes unavailable. If $j$ cannot be entirely executed, the next task of the list is considered, and so on. Hence, contrary to the case without unavailability, a task with smaller priority can be executed before a task of larger priority. When the list of task is ordered in decreasing order of processing times, the obtained algorithm is called LPT algorithm.

For the remaining of this paper, $m$ and $n$ represent respectively the number of machines and the number of tasks. The scheduling problem of machines with unavailability constraints is widely studied under different constraints such as
resumable/non-resumable jobs, offline/online scheduling and various objective functions including minimizing the maximum completion time (makespan), total completion times, etc. ([3], [4], [5], and [6]).

Lee [7] showed that, if the unavailabilities are at the beginning of the execution period (that is, machines are not all available at time o), LPT has a relative bound of $\frac{3}{2}-\frac{1}{2 m}$. A survey on scheduling problems with machine unavailabilities may be found in [9], [10] and [8].

In this paper, we consider $n$ independent tasks to be scheduled on $m$ identical machines. Each task $j$ has a processing time $p_{j}$. The machines are subject to many unavailability periods each of them has a starting and ending times. We denote by $C_{L S}$ the makespan of the schedule obtained by any list algorithm $L S$. C* represents the optimal makespan. We denote by $P=\operatorname{Max}_{1 \leq j \leq n}\left(p_{j}\right)$ the maximum processing time of tasks.

## 3. Linear Program

We consider the case of many unavailability periods per machine. This means that every machine is subject to at least one unavailability period and there exists at least one machine, which is subject to at least two unavailability periods. For a machine $M_{i}$, we denote by $s_{i k}$ and $e_{i k}$, respectively the starting and ending times of the $k^{\text {th }}$ unavailability period, $1 \leq k \leq u_{i} 1 \leq k \leq u_{i}$, where $u_{i}$ is the number of unavailabilities in machine $M_{i}$.

The problem under consideration is equivalent to a multiple knapsack problem with different capacities. In fact, each machine has a sequence of availability periods regularly interrupted by random and predefined events. Each availability period can be seen as a knapsack with a capacity equals to the difference between its ending and starting times ( $e_{i k}$ ) and ( $s_{i k}$ ). The tasks represent the items, where the item size is the duration of a task. Since the tasks have no priorities, the items will be assigned the same value.

The objective is to assign the maximum of tasks in the most left knapsacks according to the time line. Classically, the linear programs proposed to formulate a scheduling problem, care about assigning tasks to machines as well as find the sequencing of tasks in every machine. To do so, two family of decision variables have to be used. In the proposed linear program, the machines are not anymore directly considered. Thus, the order of tasks in every machine is not any more important. As a fact, the complexity of the developed linear program is extremely reduced. We denote by $m_{k}$ the total number
of knapsacks (availability periods) and $K_{p s}$ an array of size $m_{k}$ containing the staring times of the knapsacks. Similarly, we denote by $K_{p c}$ an array of size $m_{k}$ that contains the capacities of the knapsacks. $K_{p s}$ and $K_{p c}$ are parallel arrays. Consider the following decision variables:

$$
\begin{aligned}
& x_{k j}=\left\{\begin{array}{l}
1 \text { if the task } j \text { is assigned to the knapsack } k \\
0 \text { otherwise }
\end{array}\right. \\
& y_{k}=\left\{\begin{array}{l}
1 \text { if at least on task is assigned to the knapsack } k \\
0 \text { otherwise }
\end{array}\right.
\end{aligned}
$$

Where $1 \leq j \leq n$ and $1 \leq k \leq m_{k}$
The objective function can be summarized as:

$$
\begin{equation*}
\operatorname{Min} z=\underset{1 \leq k \leq m_{k}}{\operatorname{Max}}\left(y_{k} K_{p s}[k]+\sum_{j=1}^{n} x_{k j} p_{j}\right) \tag{1}
\end{equation*}
$$

Subject to the constraints:

$$
\begin{gather*}
y_{k} \leq 1 \quad \forall 1 \leq k \leq m_{k}  \tag{2}\\
y_{k} \leq \sum_{j=1}^{n} x_{k j} \quad \forall 1 \leq k \leq m_{k}  \tag{3}\\
y_{k} \geq x_{k j} \quad \forall 1 \leq k \leq m_{k} \quad \text { and } 1 \leq j \leq n_{k}  \tag{4}\\
\sum_{j=1}^{n} x_{k j} p_{j} \leq K_{p c}[k] \quad \forall 1 \leq k \leq m_{k}  \tag{5}\\
\sum_{k=1}^{m_{k}} x_{k j} \leq 1 \quad \forall 1 \leq j \leq n \tag{6}
\end{gather*}
$$

The constraints (2), (3) and (4) guarantee that if there is at least one task assigned to a period $k$, then the corresponding decision variable $y_{k}=1$ Otherwise $y_{k}=0$.

## 4. Experimental study

The objective of this section is to test the performance of the LPT algorithm for the problem as well as the limitations of the proposed linear program. We have implemented the following LPT rule: Sort the tasks in a decreasing order of their processing
times. The first task of the list is assigned to the first available machine. If that task cannot be executed because of an unavailability period, the next task of the list is examined. If two machines are simultaneously available, one of them is chosen randomly. In the other hand, we implemented the linear program using ILOG CPLEX 12.7. To test both the LPT algorithm and the linear program, a wide range of instances of different sizes were uniformly generated as follows.

1. The processing times of the tasks are in the interval $[p, P]$. Experimentally, $p$ and $P$ were respectively set to the values 10 and 20 .
2. The durations of the availability periods are in $[P, 2 P]$.
3. The durations of the unavailability periods are in $[p, 2 P]$.
4. The number of machines is 4 for the small-sized instances where the number of tasks varies between 20 and 70 . However, for the big-sized instances, where the number of tasks varies between 500 and 1000, the number of machines is fixed to 10 . Based on a deep conducted experimental study, where we have varied the number of machines $m$, we remarked that $m$ has no significant effect on the LPT performance. For this reason, we fixed the number of machines.
5. For both, small and big-sized instances, we have varied the number of unavailabilities between 3 and 5 .

To compare the LPT solutions and the optimal ones, we define an error ratio $e_{r}=\frac{C_{L p r}}{C *}$. Table 1 and Table 2 show the different results obtained and the error ratio between LPT and the optimal solution for big and small size instances.

For both configurations, LPT performs well and has almost similar behavior toward the optimal solution provided by the linear program. In fact, the completion time $C_{L P T}$ is very close to the optimal solution. In addition, LPT seems to have slightly better performance for the big sized-instances. Furthermore, the error ratio slightly increases when the unavailabilities increase but still small.

The obtained experimental results prove the fact that although the problem is strongly NP-hard, it can be solved efficiently by a simple and polynomial LPT-based list algorithm. The experimental study showed also, that the developed linear program is very efficient. Indeed, for large instances with more than 20 machines and 2000 tasks, CPLEX returns the solutions in few seconds.

Table 1: Error ratios for big size instances.

|  | 3 Unavailabilities |  |  | 4 Unavailabilities |  |  | 5 Unavailabilities |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of tasks | LPT | Optimal | Ratio | LPT | Optimal | Ratio | LPT | Optimal | Ratio |
| 500 | 780 | 770 | 1.3 | 836 | 821 | 1.8 | 850 | 835 | 1.8 |
| 520 | 803 | 792 | 1.4 | 866 | 852 | 1.6 | 869 | 856 | 1.5 |
| 540 | 835 | 823 | 1.5 | 901 | 888 | 1.5 | 899 | 888 | 1.2 |
| 560 | 862 | 854 | 0.9 | 927 | 916 | 1.2 | 935 | 921 | 1.5 |
| 580 | 898 | 889 | 1.0 | 957 | 948 | 0.9 | 968 | 954 | 1.5 |
| 600 | 926 | 916 | 1.1 | 990 | 979 | 1.1 | 994 | 981 | 1.3 |
| 620 | 957 | 944 | 1.4 | 1018 | 1004 | 1.4 | 1024 | 1010 | 1.4 |
| 640 | 992 | 980 | 1.2 | 1050 | 1033 | 1.6 | 1048 | 1034 | 1.4 |
| 660 | 1012 | 1000 | 1.2 | 1075 | 1060 | 1.4 | 1084 | 1072 | 1.1 |
| 680 | 1036 | 1026 | 1.0 | 1100 | 1085 | 1.4 | 1119 | 1100 | 1.7 |
| 700 | 1070 | 1058 | 1.1 | 1127 | 1111 | 1.4 | 1143 | 1124 | 1.7 |
| 720 | 1096 | 1084 | 1.1 | 1151 | 1137 | 1.2 | 1170 | 1155 | 1.3 |
| 740 | 1130 | 1121 | 0.8 | 1184 | 1161 | 2.0 | 1201 | 1182 | 1.6 |
| 760 | 1147 | 1140 | 0.6 | 1210 | 1195 | 1.3 | 1218 | 1207 | 0.9 |
| 780 | 1189 | 1178 | 0.9 | 1235 | 1220 | 1.2 | 1261 | 1246 | 1.2 |
| 800 | 1222 | 1211 | 0.9 | 1259 | 1246 | 1.0 | 1292 | 1276 | 1.3 |
| 820 | 1247 | 1235 | 1.0 | 1296 | 1280 | 1.3 | 1326 | 1312 | 1.1 |
| 840 | 1281 | 1269 | 0.9 | 1318 | 1308 | 0.8 | 1356 | 1341 | 1.1 |
| 860 | 1297 | 1286 | 0.9 | 1348 | 1336 | 0.9 | 1382 | 1365 | 1.2 |
| 880 | 1342 | 1330 | 0.9 | 1368 | 1357 | 0.8 | 1404 | 1389 | 1.1 |
| 900 | 1347 | 1337 | 0.7 | 1402 | 1391 | 0.8 | 1433 | 1416 | 1.2 |
| 920 | 1397 | 1385 | 0.9 | 1441 | 1431 | 0.7 | 1466 | 1449 | 1.2 |
| 940 | 1414 | 1403 | 0.8 | 1473 | 1464 | 0.6 | 1492 | 1479 | 0.9 |
| 960 | 1453 | 1438 | 1.0 | 1499 | 1490 | 0.6 | 1530 | 1514 | 1.1 |
| 980 | 1486 | 1475 | 0.7 | 1531 | 1519 | 0.8 | 1560 | 1544 | 1.0 |
| 1000 | 1502 | 1494 | 0.5 | 1560 | 1550 | 0.6 | 1585 | 1572 | 0.8 |

## 5. Conclusion

In this paper, we have considered the scheduling problem of independents tasks on identical parallel machines. The machines are subject to many unavailability periods. The objective was to minimize the ending time of the last task or the makespan. A new linear program was designed and tested to solve variety of instances of different sizes. This program served for proving that and LPT-based algorithm can solve the problem in an efficient way. This work is a first part of a global project where the goal consists of producing efficient algorithms to schedule and manage online parallel applications in cloud computing environment.

Table 2: Error ratios for small size instances.

|  | 3 Unavailabilities |  |  | 4 Unavailabilities |  |  | 5 Unavailabilities |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of tasks | LPT | Optimal | Ratio | LPT | Optimal | Ratio | LPT | Optimal | Ratio |
| 19 | 132 | 114 | 15.8 | 130 | 130 | 0.0 | 127 | 116 | 9.5 |
| 21 | 142 | 125 | 13.6 | 146 | 138 | 5.8 | 129 | 121 | 6.6 |
| 23 | 165 | 124 | 33.1 | 157 | 155 | 1.3 | 170 | 147 | 15.6 |
| 25 | 173 | 162 | 6.8 | 163 | 154 | 5.8 | 174 | 157 | 10.8 |
| 27 | 209 | 193 | 8.3 | 191 | 180 | 6.1 | 201 | 170 | 18.2 |
| 29 | 206 | 196 | 5.1 | 203 | 183 | 10.9 | 191 | 181 | 5.5 |
| 31 | 201 | 195 | 3.1 | 224 | 195 | 14.9 | 230 | 191 | 20.4 |
| 33 | 215 | 198 | 8.6 | 233 | 216 | 7.9 | 232 | 212 | 9.4 |
| 35 | 222 | 209 | 6.2 | 242 | 223 | 8.5 | 250 | 232 | 7.8 |
| 37 | 229 | 220 | 4.1 | 235 | 225 | 4.4 | 259 | 242 | 7.0 |
| 39 | 226 | 221 | 2.3 | 247 | 230 | 7.4 | 282 | 263 | 7.2 |
| 41 | 236 | 225 | 4.9 | 253 | 235 | 7.7 | 296 | 275 | 7.6 |
| 43 | 232 | 226 | 2.7 | 254 | 244 | 4.1 | 308 | 286 | 7.7 |
| 45 | 249 | 238 | 4.6 | 272 | 253 | 7.5 | 315 | 298 | 5.7 |
| 47 | 263 | 250 | 5.2 | 277 | 262 | 5.7 | 320 | 302 | 6.0 |
| 49 | 273 | 263 | 3.8 | 291 | 272 | 7.0 | 331 | 313 | 5.8 |
| 51 | 270 | 256 | 5.5 | 284 | 268 | 6.0 | 337 | 324 | 4.0 |
| 53 | 266 | 256 | 3.9 | 299 | 282 | 6.0 | 349 | 329 | 6.1 |
| 55 | 275 | 264 | 4.2 | 303 | 294 | 3.1 | 350 | 329 | 6.4 |
| 57 | 285 | 272 | 4.8 | 321 | 306 | 4.9 | 356 | 337 | 5.6 |
| 59 | 284 | 276 | 2.9 | 329 | 314 | 4.8 | 362 | 346 | 4.6 |
| 61 | 294 | 282 | 4.3 | 329 | 318 | 3.5 | 364 | 348 | 4.6 |
| 63 | 299 | 292 | 2.4 | 337 | 316 | 6.6 | 360 | 350 | 2.9 |
| 65 | 326 | 314 | 3.8 | 332 | 317 | 4.7 | 382 | 368 | 3.8 |
| 67 | 320 | 315 | 1.6 | 345 | 333 | 3.6 | 390 | 375 | 4.0 |
| 69 | 328 | 319 | 2.8 | 347 | 336 | 3.3 | 403 | 380 | 6.1 |

## References

[1] R.L. Graham, Bounds on multiprocessing timing anomalies, SIAM Journal of Applied Mathematics, Vol. 17, No. 2, pp. 416-429, 1969.
[2] M.R. Carey and D.S. Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness, Freeman publisher, San Francisco, CA, 1979.
[3] A. Berrichi, F. Yalaoui, L. Amodeo, M. Mezghiche, Bi-Objective Ant Colony Optimization approach to optimize production and maintenance scheduling, Computers and Operations Research, Vol. 37, pp. 1584-1596, 2010.
[4] Bin Fu, Yumei Huo, Hairong Zhao, Approximation schemes for parallel machine scheduling with availability constraints, Discrete Applied Mathematics, Vol. 159, pp. 1555-1565, 2011.
[5] Ming Liu, Feifeng Zheng, Chengbin Chu, Yinfeng Xu, Optimal algorithms for online scheduling on parallel machines to minimize the makespan with a periodic availability constraint, Theoretical Computer Science, Vol. 42, pp. 5225-5231, 2011.
[6] Zhiyi, Parallel machines scheduling with machine maintenance for minsum criteria, European Journal of Operational Research, Vol. 212, pp. 287-292, 2011.
[7] Chung-Yee Lee, Parallel machines scheduling with nonsimultaneous machine available time, Discrete Applied Mathematics, V. 30, pp. 53-61, 1991.
[8] J. Kaabi, Y. Harrath, A Survey of Parallel Machine Scheduling under Availability Constraints, International Journal of Computer and Information Technology, Vol. 3, No 2, pp. 238-245, 2014
[9] Y. Ma, C. Chu, C. Zuo, A survey of scheduling with deterministic machine availability constraints, Computers and Industrial Engineering, Vol. 58, No. 2, pp. 199-211, 2010.
[10] E. Sanlaville, G. Schmidt, Machine scheduling with availability constraints, Acto Informatica, Vol. 35, pp. 795-811, 1998.

