Conference Paper

Modeling of Metallurgical Process of Copper Fire Refining

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Abstract

The refining of blister copper is based on the partial removal of impurities that have an increased affinity for oxygen. The most interesting is the process of centralized copper refining at one plant. This is because blister copper from the producer plants has a different chemical composition. Obviously, the batch of each loading also has a variable chemical composition. Therefore, for a constantly changing average-weighted composition of a liquid metal, a different amount of oxygen is required to oxidize and slag the impurities. The aim of the work is the method of creating a mathematical model for solving the single-criterion and multicriteria task of fire refining of copper. Algorithms of the model based on the passive experiment are presented, with the chosen assumptions and limitations. Mathematical models are developed using correlation regression analysis. The resultant variable in the models is the concentration of oxygen in the melt. The objective function is determined by the main variables of the refining process. The results of mathematical modeling allow us to quickly calculate the concentration of oxygen supplied in the air composition into the melt of a batch of different chemical composition for the oxidation of impurities. The models are consistent with the general theory of anode melting, and can be used to control and predict the process.

Keywords: fire refining, blister copper, impurity oxidation, mathematical model, linear regression

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1. Introduction

Recently there has been a trend of a rapidly changing scientific and technological environment, thanks to the rapid and comprehensive development and dissemination of information technologies penetrating all spheres of modern society. This circumstance promotes cognition of the properties of the studied processes and objects based on mathematical modeling.

The methodological approach to creating a model is based on classical principles, and is well described in the literature [1–8].

The structure of the created model, in its content and description, can be differentiated. Differentiation is determined by several levels of complexity from reproductive through constructive and creative. The varying degree of complexity of the model depends on the level of training and the requirements for the model.

At each level, private mathematical models are constructed to solve specific problems.

A private model is a logically completed cycle of modeling the stage of the technological process and can be analytical (formula, equation) or imitation (programming languages).

In this case, the most interesting are dynamic models that combine the properties of empirical, stochastic and linear models. In this article, we propose a technique for creating analytical models based on the example of the metallurgical process of fire refining of copper.

2. General Provisions of the Metallurgical Process of Copper Fire Refining

As is known, fire refining of copper consists of the following operations: (1) batch loading, (2) melting of copper with melt heating, (3) oxidation of impurities, (4) slag removal, (5) deoxidation (restoring) copper, (6) copper spill. The main ones that determine the refining process are [9–13]:

- oxidation of impurities – is based on differences in the affinity for oxygen of copper and its impurities, which can be expressed in terms of the Gibbs energy. In the series elements, included in the blister copper, the affinity for oxygen decreases in the direction from *Al* to *Au*.

The oxidation stage begins with blowing the copper melt by air.

Due to the high concentration of copper in the melt, it is oxidized first, and then the impurities are oxidized according to the reactions:

$$4[Cu] + \{O_2\} = 2(Cu_2O) \quad (1)$$

$$(Cu_2O) = 2\,[Cu] + [O] \quad (2)$$

$$[Me] + [O] = (MeO) \quad (3)$$

$$[Me] + (Cu_2O) = (MeO) + 2\,[Cu] \quad (4)$$

From the chemical equations (1–4) it can be seen that the equilibrium between Cu_2O и Me, is established by concentration O_2 и Me.

To maximize the removal of impurities, it is necessary that the conditions: the greatest elasticity of dissociation Cu_2O, with minimal dissociation strength of the impurity oxide. Elasticity of dissociation Cu_2O increases with increasing concentration O_2 in the melt of copper. But significantly increase the concentration of oxygen in the copper melt, it is not advisable because excessive Cu_2O in copper it does not dissolve and forms an independent solid or liquid phase on the surface of the melt (Figure 1).

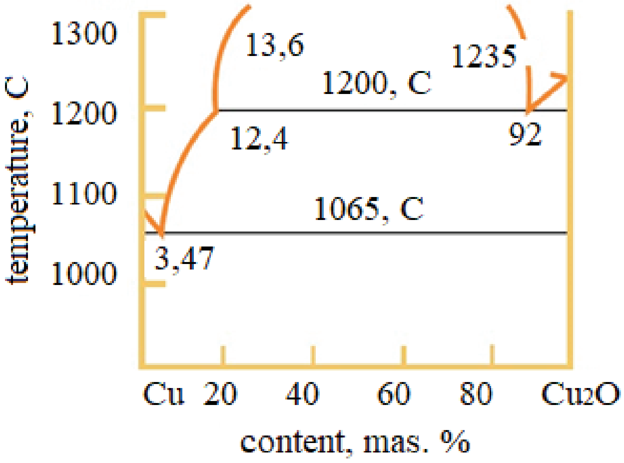

Figure 1: Status diagram Cu–O (station Cu–Cu_2O).

– deoxidation (restoring) copper – removal of excess oxygen and gas bubbles. Deoxidation is carried out by wood, fuel oil or natural gas.

Reconstructive substances decompose to form hydrogen, carbon oxide and hydrocarbons, which are released, cause mixing of the melt and removal of dissolved gases (SO_2, CO, et al.), and also interact with dissolved Cu_2O and restore it by reaction:

$$Cu_2O + H_2 = 2Cu + H_2O \quad (5)$$

$$Cu_2O + CO = 2Cu + CO_2 \quad (6)$$

$$4Cu_2O + CH_4 = 8Cu + CO_2 + 2H_2O \quad (7)$$

The duration of the reduction period of copper is determined by the degree of oxygen saturation of the copper melt during the oxidation.

2.1. Examples of modeling of metallurgical process of copper refining

Let us consider a technique of creation of mathematical model for the decision single-criterion and multicriteria problem of copper fire refining using correlation regression analysis [13–17].

To solve this such tasks, we use the condition, that the interpolation dependence between the variables is constructed on the basis of experimental data, processed by methods mathematical statistics using the application package *Microsoft Excel*.

A mathematical model for solving the of the task single-criterion.

We introduce assumptions and restrictions:

– batch consists of blister copper suppliers plants ($Ш_1$-$Ш_n$); the temperature in the oven is constant; oxygen concentration in the batch not taken into account; the relationship between the investigated quantities is linear.

Experimental data are selected from the database on each download batch.

From equations (1–4) it can be seen that the concentration of the required amount of oxygen, providing sufficiently complete oxidation of impurities of blister copper depends on their mass in the composition of the batch. Using equations (1–4) and the method of passive experiment will create a model for the dependence of the oxygen concentration (%, by mass) in the melt, on the mass Cu_2O (tn) x_1, (Figure 2):

$$C([O_2]) = f(Cu_2O) \tag{8}$$

Drafting algorithm models.

1. For one melt batch blister copper consistently calculate the total mass of impurities in the batch $\sum M_{Mej}$; mass of copper oxide (I), necessary for removal of these impurities; value of the theoretical mass of oxygen required for the oxidation of copper $M_{[O]}$. By the size of the furnace load, calculate the value of the oxygen concentration, $[O_2]$;
2. Determine the number of swimming trunks (sample) and for each of them we shall make a calculation by the method p. 1;
3. Based on the selected number of swimming trunks we identify and describe regularity of dependence (8). Proceeding from the proposed assumptions, represent this dependence by the equation of simple linear regression:

$$y = b_0 + b_1 x_1 \tag{9}$$

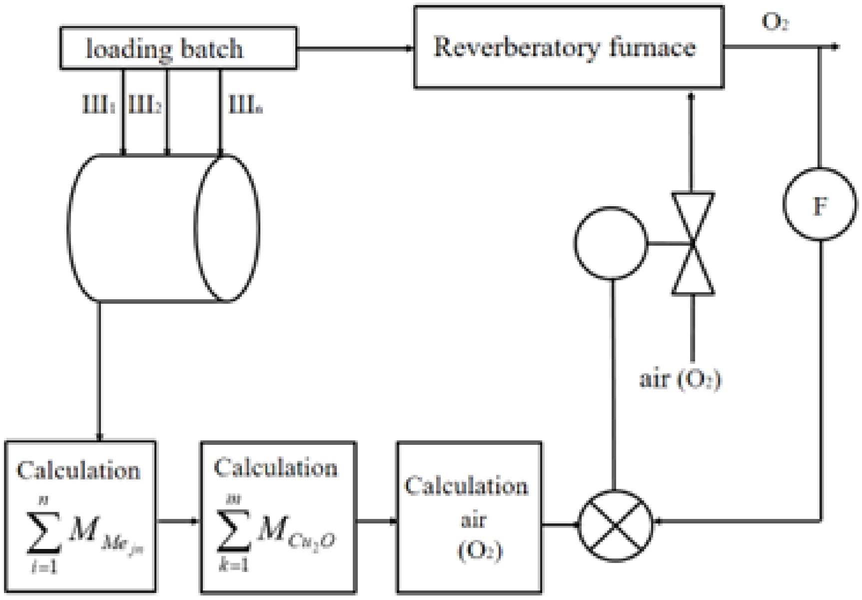

Figure 2: The scheme of application of the mathematical model in the solution of the task single-criterion *C([O_2]) = f (Cu_2O)*.

4. We formulate the null hypothesis: parameter – x_1 (mass Cu_2O) do not affect *y* (concentration of oxygen in the melt);
5. Using the method of least squares (MLS), we find the coefficients b_0 и b_1;
6. We determine the statistical significance of the regression coefficients;
7. We check the adequacy of the obtained equation (9) experimental data by one selected criterion;
8. We draw conclusions, prepare proposals and recommendations.

For this model, the coefficient b_1 shows the average change in the productive index oxygen concentration (%, by mass) fed into the melt from the change in mass Cu_2O (tn), coefficient b_0 formally shows the predicted level of oxygen concentration in the melt. By the sign of the coefficient b1, one can judge the 'direct connection' between the parameters considered.

Mathematical model for solving a multicriteria task.

Analysis of physical and chemical processes of fire refining shows, that the concentration of oxygen in the melt is mainly dependent on the concentration of impurities, from the temperature of the melt and the hydrodynamics of the purging of the liquid bath. Using equations (1–4) and the method of passive experiment We shall create a model for the dependence of the oxygen concentration in the melt y on the mass Cu_2O x_1, melt temperature x_2 and the intensity of purging the liquid bath x_3 (Figure 3):

$$C([O_2]) = f(Cu_2O, T, V) \tag{10}$$

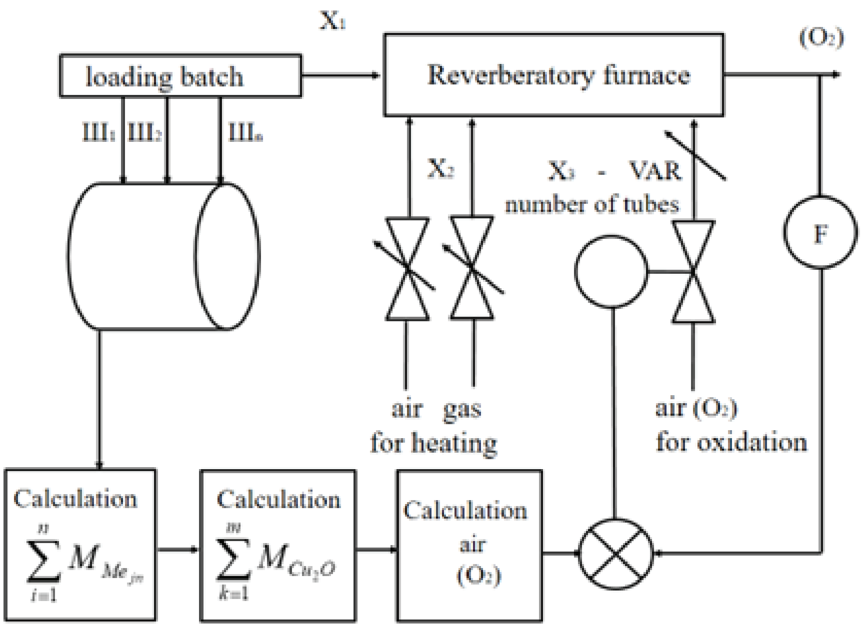

Figure 3: The scheme of application of the mathematical model in the solution of the multicriteria task $C([O_2]) = f\ (Cu_2O, T, V)$.

Drafting algorithm models.

1. Perform p.p. 1–2 algorithm for creating a model task single-criterion;
2. Define the limits of change and the step of variables x_2 and x_3;
3. Based on the selected number of swimming trunks we identify and describe regularity of dependence (10). Proceeding from the proposed assumptions, represent this dependence by the equation of multiple linear regression:

$$y = b_0 + b_1 x_1 + b_2\ x_2\ + b_3\ x_3 \tag{11}$$

4. We formulate the null hypothesis: parameters – x_1, x_2, x_3 do not affect the concentration of oxygen in the melt;
5. Using the method of least squares (MLS), we find the coefficients b_0, b_1, b_2, b_3;
6. We determine the statistical significance of the regression coefficients;

7. We check the adequacy of the obtained equation (11) experimental data by one selected criterion;

8. We draw conclusions, prepare proposals and recommendations.

For this model, the interpretation of the coefficients b_0, $b_{1,} b_2$, b_3 similar to single-criterion model.

Depending on the task of the study, there are features of the calculation when creating a multicriteria model. Let us consider their methods.

Suppose it is necessary to determine the joint influence of the parameters – x_1, x_2, x_3на productive variable y.

In this case:

– we calculate the paired correlation coefficients between the exponents by the formula:

$$r_{xy} = \frac{\overline{XY} - \overline{X} \cdot \overline{Y}}{\sigma_x \sigma_y} \tag{12}$$

– we find the coefficient of multiple correlation:

$$R_{1.2.3..n} = \sqrt{1 - \frac{D}{D_{11}}} \tag{13}$$

– we check the statistical significance of the coefficient of multiple correlation by the Fisher criterion;

– we draw a conclusion, for example, when $F_{obser} > F_{crit}$ the resulting multiple correlation coefficient is statistically significant.

Suppose, it is necessary to determine the degree of influence x_1, x_2, $x_3$on the concentration of oxygen in the melt y.

In this case:

– using the previous example by formula (12) find the paired correlation coefficients between y and x_1, y and x_2, y and x_3;

– we analyze the values obtained and determine the quality of the 'correlation connection'. If the correlation between y and one or more parameters x_1, x_2, $x_3$strong, then this parameter is left, otherwise the parameter is not taken into account to compose the regression equation;

– we find the pair correlation coefficients between the parameters x_1, x_2, x_3;

– we analyze the obtained values and determine the quality 'correlation connection'. If the correlation between two parameters of (x_1, x_2, x_3) strong, then there is no point in the regression equation include both, because one of them strongly repeats the dynamics of another. In this case, for the equation, we choose one that has a greater effect on *y*;

– write the final equation;

– draw a conclusion.

Quite often, it is necessary to evaluate the degree of connection between the oxygen concentrations in the melt *y* with one input parameter, excluding the influence of all other.

In this case:

– we find private correlation coefficients. This is the so-called 'pure' correlation, for example, between the quality of blister copper and the composition of the batch, excluding the impact on this relationship of other parameters. It is most convenient to search through the linear correlation coefficient by the formula:

$$r_{yx1.x2} = \frac{r_{yx1} - r_{yx2} \cdot r_{x1x2}}{\sqrt{(1 - r_{yx2}^2} \cdot \sqrt{1 - r_{x1x2}^2}} \tag{14}$$

– draw a conclusion.

It is obvious that mathematical models allow processing a large number of variables. In this connection, in order to calculate the values of the chosen number of coefficients of the multiple linear regression equation, it is expedient to use the matrix method:

$$\beta = (X^T \cdot X)^{-1} \cdot X^T \cdot Y \tag{15}$$

The calculated coefficients are substituted into equation (11).

3. Summary

In this article, we consider the method of creating mathematical models of various levels in the example of fire refining of blister copper.

Mathematical models have been created linking the oxygen concentration necessary for the oxidation of blister copper impurities to the main indicators of the metallurgical process: mass of impurities in the composition batch blister copper from various plants, melt temperature, rate of saturation of the bath with oxygen.

The models allow the operative calculation of the required amount of oxygen, for oxidation of impurities of blister copper with variable composition taking into account the influence of the selected parameters.

The results of mathematical modeling are consistent with the general theory of fire refining of copper, and may be of interest can be used for forecasting and control of anode melting.

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