Conference Paper

Analysing Mathematical Abilities of High School Graduates

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Abstract

In this paper we are trying to differentiate the level of mathematical abilities of high school graduates in Bahrain schools. The mathematical abilities that we are trying to analyse are conceptual understanding, content knowledge and problem-solving skills. content understanding focusses on performing algorithms, while conceptual understanding focus on comprehending the concepts and relations. Problem solving needs both. We will prepare test items to measure graduates on different mathematics content domains. This research will try to identify students' weaknesses in mathematical and suggest ways to improve the understanding in mathematics.

Keywords: Conceptual understanding; content knowledge; problem solving.

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1. Introduction

Even after passing out from high school, it is being noted that students do not possess conceptual understanding in all math content domains, which might affect their fluency in problem solving. Problem Solving is one of the major processes defined in the National Council of Teachers of Mathematics (NCTM) Standards for School Mathematics (NCTM 2000). Problem solving can provide opportunities for students to apply content knowledge in all the mathematics domains. Children must learn both fundamental concepts and proper procedural knowledge for solving problem solving in all the domain s of Mathematics content. The National Research Council (2001) set forth in its document Adding It Up: Helping Children Learn Mathematics a list of five strands, which includes conceptual understanding. Conceptual understanding helps students avoid many critical errors in solving problems, particularly errors of magnitude. *Procedural fluency* refers to knowledge of procedures, knowledge of when and how to use them appropriately, and skill in performing them flexibly, accurately, and efficiently.

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The purpose of this study was to explore the relationship between conceptual understanding, content knowledge and problem-solving skills in the high school graduates in mathematics content domains.

A good starting point for us to understand conceptual understanding is to review The Learning Principle from the NCTM Principles and Standards for School Mathematics (2000). As one of the six principles put forward, this principle states:

2. Misconceptions

When students systematically use incorrect rules or correct rule in an inappropriate domain we can realize that there are misconceptions. The knowledge about understanding of mathematical concepts has been enriched by the combination of experimental, survey research and observational studies and these have challenged the theories about how children think and learn in various mathematical domains (David Wood, 10998).

The ideas about how students develop 'misconceptions' are emphasized by most of the empirical studies on learning mathematics during the last many decades. Piaget's repeated demonstration in the late 1970s that children think about the world in very different ways than adults resulted in educational researches, and people began to listen carefully what students were saying and doing on a variety of subject matter tasks (Smith J. P., 1993).

3. Mathematical Abilities

According to NCES (National Centre for Education Statistics) the following are considered as Mathematical abilities.

3.1. Conceptual understanding

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can recognize, label, and generate examples of concepts; use and interrelate models, diagrams, manipulatives, and varied representations of concepts; identify and apply principles; know and apply facts and definitions; compare, contrast, and integrate related concepts and principles; recognize, interpret, and apply the signs, symbols, and terms used to represent concepts. Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

3.2. Procedural knowledge

Students demonstrate procedural knowledge in mathematics when they select and apply appropriate procedures correctly; verify or justify the correctness of a procedure using concrete models or symbolic methods; or extend or modify procedures to deal with factors inherent in problem settings. Procedural knowledge encompasses the abilities to read and produce graphs and tables, execute geometric constructions, and perform non-computational skills such as rounding and ordering. Procedural knowledge is often reflected in a student's ability to connect an algorithmic process with a given problem situation, to employ that algorithm correctly, and to communicate the results of the algorithm in the context of the problem setting.

3.3. Problem solving

Students demonstrate problem solving in mathematics when they recognize and formulate problems; determine the consistency of data; use strategies, data, models; generate, extend, and modify procedures; use reasoning in new settings; and judge the reasonableness and correctness of solutions. Problem-solving situations require students to connect all of their mathematical knowledge of concepts, procedures, reasoning, and communication skills to solve problems.

This research follows the strands are intertwined and include the notions suggested by NCTM in its Learning Principle. To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- Strategic competence: ability to formulate, represent, and solve mathematical problems
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification

- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

And The National Assessment of Educational Progress definition for mathematical abilities is conceptual understanding, procedural knowledge, and problem solving. So; in this research participant will write a test to determine the mathematical abilities they have to answer the following questions:

1. What percentage of Mathematical abilities' does the recent high school graduates show in different mathematics domains?
2. What are students' misconceptions while solving conceptually orientated tasks involving different mathematical domains?
3. Is there a correlation between the conceptual understanding and problem solving in Mathematics?
4. Is there any differences between the Mathematical abilities test score and high school students' GPA, and specializations?

4. Literature Review

For decades, the major emphasis in school mathematics was on procedural knowledge. Rote learning was the norm, with little attention paid to understanding of mathematical concepts. Rote learning is not the answer in mathematics, especially when students do not understand the mathematics. In recent years, major efforts have been made to focus on what is necessary for students to learn mathematics, what it means for a student to be mathematically proficient (Hull & Miles). The debate over conceptual understanding versus procedural knowledge has caught the eye of many teachers in school systems all around the world. Conceptual understanding is the comprehension of not only what to do, but also why you do it. Procedural knowledge, also known as imperative knowledge, is the knowledge exercised in the performance of some task. In both cases, students understand how to complete an assignment, but the way they think about it differs. One thing that many teachers agree on is that students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge (Cummings, 2015).

The national assessment of educational progress NAEP identify the mathematical abilities as procedure knowledge, conceptual understanding and problem solving. While the National Council of Teachers of Mathematics (NCTM) identifies these three types of understanding as three of the main strands to be mathematically proficient (NCTM, 2000). NCTM defines problem solving as a strand involves students in applying four other processes: Reasoning, Communication, Connections, and Representation which provide opportunities for them to apply content knowledge in all the mathematics domains. While conceptual understanding helps students avoid many errors in solving problems, and procedural knowledge helps them to use the knowledge of procedures, the when and how, appropriately and develop skill in performing them accurately and efficiently.

Students must learn both fundamental concepts and proper procedural knowledge for solving problems in all mathematics content domains. The knowledge about understanding of mathematical concepts has been enriched by the combination of experimental, survey research and observational studies and these have challenged the theories about how children think and learn in various mathematical domains (Wood, David J. 1998). The ideas about how students develop 'misconceptions' are emphasized by most of the empirical studies on learning mathematics during the last many decades. Piaget's repeated demonstration in the late 1970s that children think about the world in very different ways than adults resulted in educational researches, and people began to listen carefully what students were saying and doing on a variety of subject matter task (Smith J. P., 1993). This help in understanding their misconceptions and why they do them. A misconception is the result of lack of understanding or in many cases a misapplication of a rule or mathematical generalization (Spooner, 2002).

Learning with understanding is essential to empower students to solve the new kinds of problems they will inevitably face in the future, but even after passing out from high school, it is being noted that some students do not possess conceptual understanding or problem solving skills in the five content domains; which are: Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability.

According to Hasnida, C., & Zakaria, E. (1991) the goal in mathematics teaching has shifted towards an emphasis on both procedural and conceptual understanding. The importance of gaining procedural and conceptual understanding is aligned with the objective of mathematics education. Using a survey method, they carried out a study in the secondary schools and the data were analyzed descriptively to determine students' procedural and conceptual understanding of mathematics. Pearson's correlation was used to determine the relationship between procedural and conceptual understanding. The findings revealed that the students' level of procedural understanding is high whereas the level of conceptual understanding is low. They suggested that a reformation in teaching is needed to boost conceptual understanding among students to minimize the use of procedures and memorization.

Jazuli and other (2017) mentioned that most students find it difficult to understand and to apply the concept of mathematics in a real-world context. They argue that the difficulty is due to the conventional learning strategy used, which is unable to improve the students' ability. They done am experimental study aimed to discover the implementation of a contextual learning strategy to improve mathematics conceptual understanding and problem-solving. The two above-mentioned issues have been examined by using a pre-test and post-test, and compared by using a control group with conventional learning. The results showed that the contextual learning strategy significantly affects the conceptual understanding and the ability to solve problems in mathematics subjects.

Johnson & Alibali, (1999) suggested that procedural knowledge may influence gains in conceptual knowledge by helping children to identify and eliminate misconceptions. Conceptual knowledge may influence improvements in procedural knowledge by improving problem representation and facilitating adaptation of known procedures in problem solving.

Children must learn both basic concepts and correct procedure to solve problems. Mathematical competencies depend on their ability to connect the knowledge of fundamental mathematical concepts and procedure to real life situations. Observations show that students who possess procedural knowledge alone couldn't solve real life problems as they lack in conceptual understanding. They were unable to connect the concepts to the problem-solving situations.

This research measures the level of mathematical abilities of high school graduates in Bahrain schools. Mathematical abilities are conceptual understanding, procedural knowledge and problem-solving skills. While procedural understanding focusses on performing facts and algorithms, conceptual understanding reflects a student's ability to reason and comprehend mathematical concepts, operations, and relations which will be helpful in solving nonroutine problems. Test items were prepared, validated and administered to recent high school graduates on five mathematics content domains: Number and Operations, Algebra, Geometry, Measurement and Data Analysis & Probability, where students demonstrate their conceptual understanding and procedural knowledge and connect them to solve problems in various real-life contexts. Correlating their performance in the test and their high school GPA this research is trying to identify the cause of the weak conceptual understanding and the difficulties in problem solving and suggesting ways to improve different type of understanding to be proficient in mathematics.

Conceptual understanding is a phrase used widely in educational literature. Even though conceptual understanding and procedural fluency are two different terms, they are inseparable. Children's conceptual understanding affect the procedures they adopt in solving problems. Children with greater conceptual understanding tend to have greater procedural skills and they are better problem solvers. The National Assessment of Educational Progress (NAEP, 2003) delineates specifically what mathematical abilities are measured by the nationwide testing program. Those abilities include conceptual understanding, procedural knowledge, and problem solving.

4.1. Conceptual understanding & procedural knowledge

Concepts are the building blocks of knowledge (Charlesworth, 2012). Conceptual understanding and procedural knowledge are essential to develop skills in problem solving (Geary, 2004). These skills contribute towards the processing of information effectively in solving problems. The five strands of mathematical proficiency, conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition, by Kilpatrick, Swafford and Findell (2001) presents the interdependence of the five components of learner's mathematics proficiency in problem solving. It starts with a clear grasp and understanding of the concept, to acquisition of mathematical concepts, strategic knowledge, which is required to help children devise and monitor a solution, is vital for solving problems successfully (Mayer, 2008).

Conceptual knowledge is in general an abstract knowledge addressing the essence of mathematical principles and relations among them, while procedural knowledge consists of symbols, conditions, and processes that can be applied to complete a given mathematical task (Hiebert & Lefevre, 1986). Procedural knowledge is meaningful only if it is connected to a theoretical fact. Faulkenberry (2003) suggests that conceptual knowledge is rich with relations, and refers to the basic mathematics constructs and relations between the ideas that illustrate mathematical procedures, and gives it a meaning. On the other hand, procedural knowledge addresses the mastery of mathematical skills, acquaintance of the procedures to determine the mathematical components, algorithms, and definitions. Many researchers suggest that both conceptual knowledge and procedural knowledge are important components in understanding mathematics (Desimone et al., 2005; Hiebert et al., 2005).

For instance, in understanding of area measurement, procedural fluency, and reflections on the accuracy of solutions for measuring areas, represents higher-order thinking skills (Lehrer, 2003). While investigating children's conceptions of Area Measurement and their strategies for solving Area Measurement problems, Huang, Witz (2012) found that children who had a good understanding of the concept of area and the area formula (by using the property of multiplication) exhibited competency in identifying geometric shapes, using formulas for determining areas, and self-correcting mistakes. The children who had a good understanding of multiplication underlying the area formula, but misunderstood the concept of area, showed some ability to use area formulas. Conversely, the children who were unable to interpret the property of multiplication underlying the area formula irrespective of their conceptions of area exhibited the common weaknesses in identifying geometric shapes and differentiating between area and perimeter. The general concept of area refers to the amount of a 2-D region within a boundary, while area measurement concerns measuring the quantity of a surface enclosed within a 2-D region (Lehrer, 2003). This incorporates the prior concept of area and measurement skills. The strategic knowledge of area measurement contains a conceptual understanding of basic facts and the knowledge of efficient strategies in solving problems with justified reasoning. Though there are various ways to solve area measurement problems, appropriate use of formulas based on conceptual understanding can be considered an efficient strategy (Lehrer, Jaslow, & Curtis, 2003).

Moreover, it is noted by Siegler, & Alibali (2005) that when comparing fractions with physical models, students could easily see the largest fraction. When the physical model was not being used, some students still had to draw the model to compare size of fractions. In order for fraction and decimal number sense to be acquired, there are three foundational concepts agreed upon by researchers (Barnett-Clarke, Fisher, Marks & Ross, 2010). These concepts along with conceptual understanding and procedural knowledge, cognitive theories, and instructional theories will create a suggested path of tasks to develop fraction and decimal number understanding and gain understanding for long term application (Van de Walle, 2007; Watanabe, 2006).

According to Hasnida, C., & Zakaria, E. (1991) the goal in mathematics teaching has shifted towards an emphasis on both procedural and conceptual understanding. Their research revealed that the students' level of procedural understanding is high whereas the level of conceptual understanding is low and hence they suggested that a reformation in teaching is needed to boost conceptual understanding among students to minimize the use of algorithms and memorization.

Conceptual understanding can be measured in various ways, mainly involving providing definitions, explanations and reasons. conceptual knowledge in a domain usually requires knowledge of many concepts. Procedural fluency can be measured by checking the accuracy or the procedure of solving problems. When interested in how flexible procedural knowledge is, researchers assess students' knowledge of multiple procedures and their ability to flexibly choose among them to solve problems efficiently (Star & Rittle-Johnson, 2008; Verschaffel, Luwel, Torbeyns, & Van Dooren, 2009). This flexibility of procedural knowledge will be a result of conceptual knowledge. The positive correlations between the two types of knowledge have been found in almost all domains. For example, in Number and Operations (Canobi & Bethune, 2008; Jordan et al., 2009), fractions and decimals (Hallett, Nunes, & Bryant, 2010; Reimer & Moyer, 2005), estimation (Dowker, 1998; Star & Rittle-Johnson, 2009), and equation solving (Durkin, Rittle-Johnson, & Star, 2011).

4.2. Problem solving

Problem Solving is one of the major processes defined in the National Council of Teachers of Mathematics (NCTM) Standards for School Mathematics (NCTM, 2000). Problem solving involves students in applying four other processes: Reasoning, Communication, Connection and Representation. Jonassen (2003) defines problem solving as an individual thought process because the previously learned law can be applied in solving problems in any situations. It is also deemed to be a new type of learning and is the result of application of knowledge and procedures of the problems (Mc Gregor, 2007).

Problem solving can also provide opportunities for students to apply content knowledge in the areas of Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability. Problem solving provides a window into children's mathematical thinking and thus is a major technique of assessment.

According to Jawhara (1995), problem solving activities can open opportunities for students to learn freely. In their own ways, students will be encouraged to investigate, seek for the truth, develop ideas, and explore the problem. These features are necessary in order to face the challenges of future (Lim et al., 1999). Kilpatrick et al. (2001), Pugale (2004, 2005), Suh and Moyer-Packenham (2007) and Ginsburg (2012) concluded that the increase on the levels of cognitive demand, mathematical intricacy and levels of abstraction balances the procedural fluency of students in problem solving tasks is based on their ability to use intellectual knowledge and skills in interpreting the problem.

Many mathematics skills are involved in problem-solving. However, large numbers of students have not acquired the basic skills they need in mathematics. As a result, many students were reported to face difficulties in mathematics particularly in mathematics problem solving (Tay Lay Heong 2005). The ability to use cognitive abilities in learning is crucial for a meaningful learning to take place (Stendall 2009). There are two major steps in problem-solving: transforming the problem into mathematical statements or equations and calculation of the required statements. Difficulties faced among students were more noticeable during the first procedural step in solving problem compared to the other. Polya (1981) stated that problem-solving is a process starting from the minute students is faced with the problem until the end when the problem is solved. Garderen (2006) stated deficiency in visual-spatial skill might cause difficulty in differentiating, relating and organizing information meaningfully.

Lack of many mathematics skills cause difficulties in problem solving. Difficulties in mathematics skills were classified into number fact, arithmetic, information, language and visual-spatial skills (Garnett 1998; Nathan et al 2002). Students are required to apply and integrate many mathematical concepts and skills during the process of making decision and problem-solving. Conceptual understanding and procedural knowledge are essential to develop skills in problem solving (Geary 2004). Language and spatial skills are also important to interpret and to tackle information effectively.

5. Research Aims

The research aims to:

- Analyze students' work according to 5 mathematical domains and three types of mathematical abilities.
- Find out the root cause of students' misconceptions and errors.
- Suggest ways to improve the conceptual understanding and problem solving skills to reach to mathematical proficiency.

6. Research Method

- Quantitative approach: By using a test comprised of questions from five domain in mathematics (Numbers & operations; Algebra; Geometry; and Measurements; Data Analysis & probability). Test consisted of 60 questions; 20 to test students' conceptual understanding, 20 to test students' procedure knowledge and 20 for students' problem-solving skills.

- Qualitative approach Interviews were conducted to collect data about students understanding. Interviews were semi structured. We have interviewed the participants who are lacking any one of the mathematical abilities and most recurring misconceptions.

After students takes the test, it will be corrected, and the results will be analysed to find out the different type of abilities they have.

Interviews will be conducted with sample of students. Depending on the students' response and type of errors committed in the test items, we will interview them to identify the root of the misconception. Then we might suggest one or two ways of teaching those concepts of mathematics to avoid misconceptions in future.

Test results will be analysed using different SPSS tests as the following:

Classified students' test score based on the Mathematical ability level.

Classified students' test score based on the accuracy level (getting 0-3).

Classified students' test score based on the mathematical domain.

Classified students' high school GPA (Below average, Average, & Above average).

High school GPA – 3 Mathematical proficiency level scores – Correlations, Pearson, Spearman, or Scheffe's test.

High school GPA – High school specialization (Science, Commercial, & Others) – ANCOVA.

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