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Gravitational Search Algorithm for NURBS Curve Fitting

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Abstract

By providing great flexibility non-uniform rational B-spline (NURBS) curves and surfaces are reason of preferability on areas like computer aided design, medical imaging and computer graphics. Knots, control points and weights provide this flexibility. Computation of these parameters makes the problem as a non-linear combinational optimization problem on a process of reverse engineering. The ability of solving these problems using meta-heuristics instead of conventional methods attracts researchers. In this paper, NURBS curve estimation is carried out by a novel optimization method namely gravitational search algorithm. Both knots and knots together weights simultaneous optimization process is implemented by using research agents. The high performance of the proposed method on NURBS curve fitting is showed by obtained results.

Keywords: Non-uniform rational B-spline, gravitational search algorithm, meta-heuristic

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1. Introduction

Representation of the object curves on the computer which was obtained as 2-dimensional point cloud from areas like 2-dimensional medical imaging, geographical contour detection, noise reduction in image processing, computerized animation, computer aided design and edge screening on fabrication is expected to be appeared as smooth curves.

Inclusion of untidy, irregular and noise on these points make the process of regular curve fitting harder. In mathematics, a curve is described by split polynomials. Commonly determination of polynomial value is preferred on obtaining curve values within intermediate value determination problems. Shape of the curve is tried to fit by an equation as if possible. On computer graphics, computer aided design and subfields of computer science, curves which couldn't be represented by a simple polynomial are tried to display by joining polynomials especially obtained from splitting the parts with rich of arcs. These form of curves represented by parametric equations due to request of some demands such as ease of definition, less memory requirement for recording, ease of visualization and ease of manipulations on curves. The use of B-spline curves is a good choice in this situation. Non-uniform rational B-spline (NURBS) curves being a generalized and specialized form of B-spline curves are the most widely used parametric curve type on industry and production field having some special features as providing interactivity in design and improving flexibility.

Many researches are carried out on the data fitting topic using B-spline and NURBS curves in past 2 decades. Curve fitting process on a point cloud using NURBS curves is commonly implemented by using least square method employing knots, control points, weights and parameters corresponding to data points. Therefore, NURBS curve fitting process is implemented in five steps; parameterization, knot placement, control point estimation, weight adjustment and minimization with least squares method. There are many methods like equal distance uniform, girder length and centripetal method are available for parameterization of data points. Selection of the knots in point of placement as well as quantity is very important on determination of shape of the curve. These are appended into the unknown list, and transform the problem into a multi variable and multi-mode optimization problem. After the placement of knots, control points are obtained by solving non-linear equations. A weight value is assigned for each control point. Prediction of the weight values are also a different complexity issue. Existence of weight on both portion and denominator on common NURBS equation makes the programming complexity harder. Curve fitting process considering these whole parameters together with minimal an error reveals non-linear consistent combinational optimization problem.

Numerous studies has been carried out for the problem of curve fitting on 2 dimension data point set, and these are given in related title. The main contribution of this study is implementing an alternative method in this area using gravitational search algorithm (GSA). There are four fundamental interaction; the gravitation, electromagnetic force, weak nuclear force and strong nuclear force. GSA is based on gravitation. Also, it is a novel optimization algorithm based on the law of gravity and mass interactions, and it has been used for solution of various non-linear functions lately. This paper is focused on curve fitting using GSA as a step in NURBS unknowns' process in curve fitting problem using NURBS curves on regular point cloud.

1.1. Related Work

NURBS surfaces are generalization of the Bezier and B-spline surfaces, and they are industrial standard tools used for geometric design and display. While B-spline curves and surfaces are describing by control points and knot values uniquely, weight vector is determined for NURBS curve and surfaces additionally. Because they have control inspectors like control points, knot vectors and weights, designers have to make a decision on parameter selection on NURBS's in order to obtain desired shapes and modify them. This is one of the most important issue in computer aided graphical design (CAGD).

Prochaskova and Prochazka deal the effect of knot vectors and knot types on NURBS surface shapes. Authors were also researched the effect of the position of the control points and the weights in the same study [1].

Leal et al. used SOM and evolutional strategy together for enhancement of NURBS surfaces by improving sharp features like high arcs, edges and corners. First, the data which would be fitted was separated into subsets employing SOM, and then each subset is evaluated utilizing evolutional strategy to minimize the fitting error and smooth the sharp features in order to find NURBS suitable weights. Clustering approach decreases computational cost of memory and process due to local effect of control points of NURBS surfaces. Method could be applied to both regular and irregular surfaces [2].

When a point set given on a plane, the first task is describing the sequence of the points for interpolation and prediction methods in order to fit a curve. Kohonen network was used to sequencing the points by Hoffman and Varady [3]. B-spline curves were used to predict and implement interpolation with a specific sequence. Then, a modification of Kohonen networks algorithm was introduced by Hoffman [4]. Author suggested continuously decreasing neighborhood instead of old neighborhood structure in order to achieve better results on surface reconstruction problems. This continuously decreasing neighborhood assists to pull neurons through input points better, and eventually, output layer formalize smoother rectangular grid on distributed input data. After these preprocessing steps standard free-shape algorithm could be used for surface reconstruction. Hoffman implemented digital control of Kohonen neural network for distributed data prediction [5]. Fernand et al. studied on the topic of sequencing distributed 3D data and parameterization for B-spline surface prediction [6]. Gaussian mapping extended to the surface and a segmentation plan which provides unmatched parameterization was used in order to achieve a parameterization.

Juhász illustrated how does shape modification specified by NURBS curves by utilizing NURBS control points weights modification were performed [7]. Piegl and Tiller implemented estimation of the knots values and optimization of values of the points [8]. Then, authors illustrated how does the number of the control points degraded preserving accurate interpolation [9]. The idea was providing flexible knots so that each line could be interpolated by adding less enough knots possible.

Prahasto and Bedi implemented knot vector optimization for prediction of multi-curve B-spline [10]. Pottman et al. presented an active frame model for prediction of parametric curve and surface [11]. Lee presented a curve fitting method for trapezoidal shapes utilizing 2 dimensional B-spline curves [12]. Hongwei et al. constructed an iterative NURBS curve and surface in order to fit given data points. The process started by a starter NURBS curve or surface segment where the given point set is handled as control point set. Then, by slightly adapting the control points with an iterative formula, gradually growing non-uniform B-spline curves group were obtained [13].

Sarfraz et al. researched selection of the knots to obtain an optimized curve for shape design in curve fitting problems [14]. While Safraz and Raza introduced selection of the knots on curve fitting problems by employing B-spline, they introduced an evolutional heuristic technique which is known as Simulated Evolution on curve fitting problems by using NURBS [15]. ibrahim et al. utilized particle swarm optimization (PSO) as a swarm algorithm to reveal NURBS knots starting with largest number of control points and knots. In their study, PSO was used to identify error tolerance. Tests were performed by simple objects like FRD Oil Tanker, Firebot, Laughing Buddha and Standford Dragon instead of complex objects like ship's hull [16]. Kim and Lee handled both knot positioning and weight control on NURBS curve fitting. Authors implemented knot positioning by generating a pleasing curve, and on the parameterization process they used chord and centripetal methods. They exploit quadratic programming on prediction of weights [17].

Goldenthal and Bercovier introduced a multi objective optimization algorithm for design by curve and surface optimization, and they unify this algorithm with evolutionary algorithms. They used the introduced MOO algorithm for optimization of NURBS curves and surfaces. Optimization variables were knot vectors, parameterizations and NURBS weights [18].

Sarfraz and Riyazuddin performed simulated annealing to NURBS's knots optimization for curve data. This process was implemented by dealing sum of squared error as fitness function, fixed number of control points calculated using leased square technique and one unit weight vector [19]. Sarfraz et al. studied for optimization of NURBS's weights to data display using simulated annealing. The aim of this method was displaying the data by decreasing the error and obtaining a smooth curve [20]. Then, in addition to the studies on which NURBS's weights and knot parameters optimized, Sarfraz et al. developed a separate study in order to optimize weights and knots simultaneously. Optimized NURBS models were adjusted on planar shapes edge data for eventual and automatic output. Also, a web-sided system was developed for effective and evolutionary usage [21]. Jing et al. firstly developed tests using simulated annealing independently, and then they implemented test using only weight optimization. After the experiments, authors asserted weight optimization was a better choice by seeing the fact that knots optimization was depend on the initial values of the knot vector after their experiments. Authors were also indicated that the proposed algorithm was well functional on single segmented images but not functional on double segmented images [22]. Adi et al. implemented NURBS curve fitting process generating NURBS curves control points and weights using PSO utilizing uniform knot vectors produced by chord length parameterization method. Authors compared PSO slightly by a conventional method for a curve consist of 5 points, thus, they need to improve their experiments [23]. Sarfraz improved the study implemented on B-spline, and presented the implementation of his technique on curve and surface fitting problem employing NURBS. This study presented a method for NURBS estimation utilizing digitalized data. To solve the problem, a novel method based on simulated evolution was developed instead of classic optimization approach [24].

In this study, an alternative method is presented for both knot and weight optimization on NURBS curve fitting. Besides, being a novel heuristic method, GSA is used for both objectives. Both details of NURBS curve fitting and GSA is presented oncoming sections.

2. Materials and Methods

2.1. NURBS Curve

A NURBS curve is a curve consists of one dimensional parametric input and S points of output. It can be generalized into 2 and 3 dimension using tensor product. A planar NURBS curve identified by $S(t)$ given in (1) as follows;

$$S(t) = \frac{\sum_{i=1}^{n_c} P_i w_i N_{i,k}(t)}{\sum_{i=1}^{n_c} w_i N_{i,k}(t)} \quad a \le t \le b \tag{1}$$

where P is a vector identifies ith control points position, w_i is a positive value which determines ith precise control points weights and $N_{i,k}(t)$ is B-spline blending function given as a parametric function of t. The t parameter identifies a position in curve equivalent to a point on a curve presented by $S(t)$ vector. B-spline blending function is a recursive function identified by (2) and (3).

$$N_{i,k}(t) = \left(\frac{t - u_i}{u_{i+k-1} - u_i}\right) N_{i,k-1}(t) + \left(\frac{u_{i+k} - t}{u_{i+k} - u_{i+1}}\right) N_{i+1,k-1}(t) \tag{2}$$

$$N_{i,1}(t) = \begin{cases} 1 & u_i \le t \le u_{i+1} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

In the related equations, u correspond to knots vector as sequence of parameter values which identifies control points impact regions on a NURBS curve. Impact point in question for ith control point is determined by k. Degree of the curve represented by its order, and $k = d+1$. In B-spline base function, mathematical 0/0 ambiguity condition is used as 0 for calculation purposes, and shows exterior impact region of the control points. Blending function value is 1 in control points impact region. Curve's order must be equal or less then control points value, so $2 \le k \le nC$.

NURBS curve and surfaces, positions of control points, control points weights (homogeneous coordinate of control points), knot vectors and curve order uses k in order to produce a high order curve identification. Weighted sum of a few control points identified by curve order produces a NURBS curve. While control points weights identifies strength of the impact, each control points impact region is identified by knots vectors. More information of the details of NURBS can be reached from [8].

2.2. Gravitational Search Algorithm

GSA is a novel optimization method based on Newtonian gravity and laws of motion and it is firstly introduced by Rashedi et al. in 2009 [25]. According to GSA, the population is composed of agents which are considered as objects, and performances of these

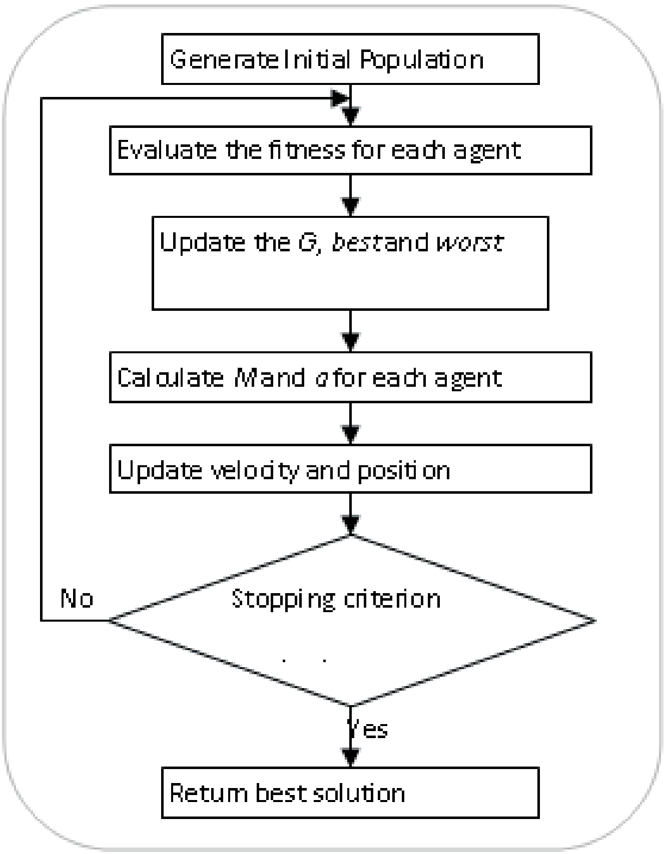


Figure 1: The diagrammatized form of GSA [25].

agents are measured by their masses. Because GSA is based on Newtonian laws, all objects namely masses attract each other by the gravity force. This force is a constant and defined at the beginning. The movement by the gravity force causes a global movement. All objects move towards the object with heavier masses. In GSA, objects with heavy masses are accepted as good solutions, thus objects with light masses are accepted as bad solutions. Objects with heavy masses move more slowly than objects with lighter masses. This situation provides exploitation step of the GSA [25].

In GSA, each agent presents a solution and has four specifications as position, inertial mass, active gravitational mass passive gravitational mass. The algorithm is navigated by properly adjusting gravitational and inertial masses which are obtained by using a fitness function. The diagrammatized form of GSA is given in Fig. 1.

2.3. NURBS Curve Fitting with GSA

Given S measured points, Q_s, there are several ways to fit NURBS curve; repositioning control points, manipulating weights and modification of knots vector. Calculating control points using least squares method can be distinguished as the first step of curve fitting. In order to reach a more fitting accuracy optimization of knot values and weights are implemented. The error value between the fitted curve and measured points in each process can be calculated using the following equation;

$$E = \left(\sum_{i=0}^{s} \left| Q_i - S(\alpha_1, \dots, \alpha_n) \right|^r / s \right)^{1/r} \quad (4)$$

where Q represents set of measured points of target curve, $S(\alpha_1, \dots, \alpha_n)$ is the geometric model of fitted curve, and $(\alpha_1, \dots, \alpha_n)$ is the parameters of the fitted curve; s is the number of the measured point and r is a power between 1 and infinite. Fitting process can be shown as optimizing curve parameters $(\alpha_1, \dots, \alpha_n)$ in order to minimize error E (or cost). If r value equals to 2 then equation above degraded into the least squares function.

NURBS curve fitting process consists of four steps. Parameterization, control points production, knot values production and weight optimization. These four steps are repeated until the error reaches a certain tolerance in order to reach a better curve. In parameterization process, t parametric value assigned for each Q data point. This value is the measure of data point through the curve. Some of the parameterization methods utilized in NURBS curves are; uniform, girder length, centripetal and hybrid. Centripetal parameterization method is used in this paper, and U knot vector is described as follows;

$$\begin{gathered} U = \{0, 0, \dots, 0, u_{k+1}, \dots, u_n, 1, 1, \dots, 1\} \\ u_{j+k} = \frac{1}{k} \sum_{i=j}^{j+k-1} \overline{u_i} \quad \text{for } j = 1, \dots, n-k \end{gathered} \quad (5)$$

After the parameterization process, control points are calculated using least squares method. Number of the control points are expected to be less than the number of the given measured data points. General NURBS equation considering in matrix form can be calculated as matrix operations and control points as follows;

$$Q_k = C\left(\overline{u_k}\right) = \sum_{i=0}^{n} P_i R_{i,k}\left(\overline{u_k}\right) \quad k = 0, \dots, n \quad (6)$$

or

$$\begin{bmatrix} R_{0,p}(\overline{u_0}) & \cdots & R_{m,p}(\overline{u_0}) \\ R_{0,p}(\overline{u_1}) & \cdots & R_{m,p}(\overline{u_1}) \\ \vdots & \ddots & \vdots \\ R_{0,p}(\overline{u_n}) & \cdots & R_{m,p}(\overline{u_n}) \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ \vdots \\ P_n \end{bmatrix} = \begin{bmatrix} Q_0 \\ Q_1 \\ \vdots \\ Q_n \end{bmatrix} \quad (7)$$

or

$$[C]\,[P] = [Q] \tag{8}$$

by matrix processes

$$[P] = [C]^T\,[C]^{-1}\,[C]^T\,[Q] \tag{9}$$

where $\overline{u_k}$ is the parameter value assigned to each data point. Production of knots vector can be implemented uniform and non-uniform. In this paper non-uniform knots vector is adopted. In order to obtain better curves, selection of the knots can be implemented by fixed weights as optimizing weights by fixed knots. Selection of knots is implemented using GSA in this paper.

In knots selection using GSA, each agent is described as a vector of potential knots positions. After masses are calculated and evaluated at each iteration, repetition of the potential knots positions are checked, and if there is a repetition, one of the repeating positions is randomly adopted with an unused position. After this check, the potential knots positions are ready to be qualified using the fitness function. In this study, fitness function is the RMS value obtained from the curve fitting algorithm.

In weight optimization process using GSA, each agent is described as a vector of potential weights within the range [0, 1]. During the iteration process, potential weights are evaluated using the same fitness function as in knots selection process given above.

All iterations are implemented with a minimization aim of RMS value obtained from the fitness function for both knots selection and weights optimization processes.

The curve obtained by knots selection comparatively more suitable to measured points. But, in order to minimize the fitting error weight optimization can be also implemented. In this paper weight optimization is implemented using GSA. Weights represent wide independent variables the number of which is equal to the number control points. This situation cause extend on search space. Because of this, global optimization techniques must be used. Preliminary solution of the weight vector is achieved producing random values in the range [0, 1] in the weight optimization process. In later process, weights must be adjusted with GSA in order to minimize the error in each iteration. Optimization is implemented with an approach similar to knots optimization. NURBS curve fitting algorithm utilized with GSA can be illustrated as following diagram given as Fig. 2.

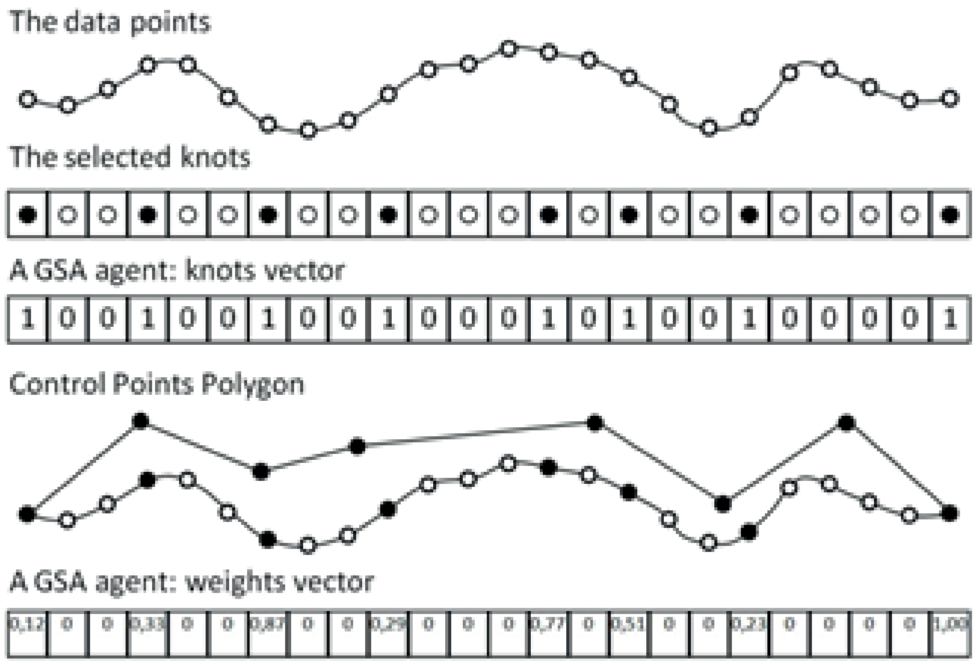
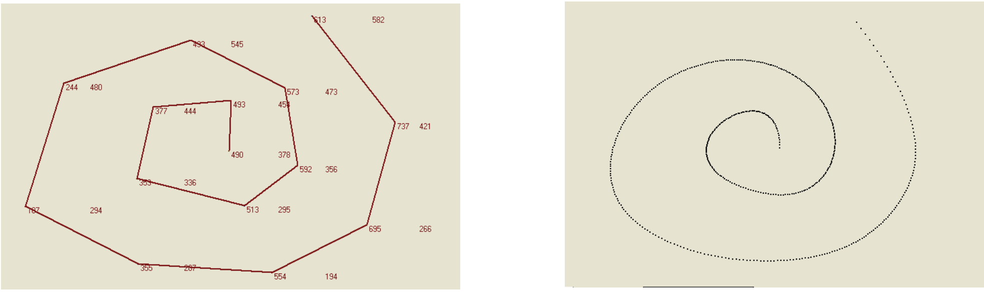


Figure 2: The NURBS curve fitting algorithm: Knots and weights optimization using GSA.

3. Results

The comparison is performed on the results of previous knot positioning algorithm implemented by genetic algorithm (GA) with the results of the weight optimization. In order to evaluate the developed NURBS curve fitting algorithm utilized with GSA, 15 control points are generated, and curve sample having a noise ratio of 16% which is given in Fig. 3(a) is utilized. The efficient value (RMS) error between modeled curve based on masses on search agents and the point cloud is given in Table 1. The initial population was iterated through 100 iterations. At the first population, the error between the real curve and supplementary antibody observed quite high. Fig. 3(b) presents converged pattern after 100 iterations. The curve obeys better to data points after 100 iterations. During the iteration process suitability increases and error decreases. The slope of the curve shows the probability of convergence still exists during the later iterations. Statistics of NURBS curve fitting algorithm utilized with GSA is given in Table 2.

In order to see the convergence speed of the developed NURBS curve fitting algorithm, outputs are recorded on some iterations during the training process. According to this outputs, average fitness values of the masses, average RMS, maximum fitness values and maximum RMS values are given in Table 3.



(a) (b)

Figure 3: Test 1; NURBS weight optimization with GSA approach of curve in dataset (a) a curve having 15 control points and noise ratio of 16%, (b) parameterized fitted curve after 100 iterations.

Table 1: RMS values obtained from NURBS weight optimization of sample curve given in Fig. 3.

Population	Best RMS (GSA)
Beginning	0.5467
5	0.5401
10	0.5135
20	0.4923
30	0.2864
50	0.2109
70	0.1204
90	0.1081
100	0.0998

Table 2: Parameter Setup.

Parameter	GSA	GA
Population size	20	20
String Size	200 (Mass)	200 (chromosome)
Mutation rate	N/A	0.001
Cross-over rate	N/A	0.7
Number of the variety	6 (30%)	6 (30%)
Alpha	20	N/A
Gravitational Constant	100	N/A
Final percentage (for elitisim)	2	N/A

Experiments are repeated by adding a noise ratio of %5 and %10 to the two curve given in Fig. 4 for extending the NURBS curve fitting tests. During the tests, only knots

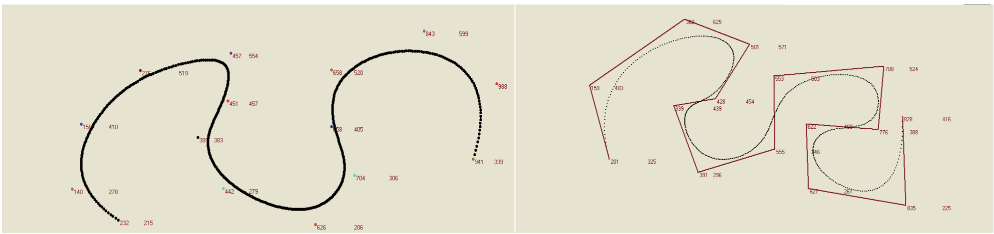


Figure 4: Two different test curves used to extend the experiments.

optimization and only weight optimization is implemented. After then, both knot and weight optimizations are implemented simultaneously. It is demonstrated that the process of simultaneous knot and weight optimization has produce better predicted curves than both only knots optimization and only weights optimization.

Table 3: AIC and RMS statistics obtained by NURBS curve optimization based on AIS given in Fig. 3.

	GSA			
	Maximum AIC (Fitness)	Maximum RMS	Average AIC (Fitness)	Average RMS
Beginning	1422	0.5726	1622	0.5672
5	799	0.5501	814	0.5431
10	768	0.5231	792	0.5217
20	673	0.5141	702	0.5083
30	592	0.3101	602	0.3074
50	508	0.2461	535	0.2371
70	499	0.1379	501	0.1363
90	490	0.1220	492	0.1191
100	478	0.1091	489	0.1042

4. Conclusion

In this paper, a framework is developed to predict a section of a geometric object or a curve within the data point set using GSA. NURBS curves are considered as prediction curves in curve fitting process in the proposed algorithm. When a reverse engineering process is implemented, calculating NURBS parameters transform the problem to non-linear combinational optimization problem. Two staged method is assimilated in the proposed study. In the first stage, search space is designed considering the knots positions as agents of GSA, and best knots are obtained. Control points are calculated using obtained knots via reverse engineering. In the second stage, weight values of each control point can take are selected as search agents and weight values which fits the curve best are obtained searching the search space with GSA. It is shown by the experiments that not only knots optimization could be performed but also both knots and weights optimization could be performed by the proposed method.

NURBS surfaces could be predicted by extending NURBS curves into the surfaces in the future works. After the development of the algorithm, optimization of the source code didn't performed, and this effects the performance negatively. Not only source code optimization could be performed in the future works but also source code could be re-designed including parallel computing architecture. GSA is used on the proposed algorithm. Performance and integrity comparisons could be implemented for the works achieved by using other heuristic and meta-heuristic methods.

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