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Evidence of the QCD tricritical endpoint existence at NICA-FAIR energies

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Abstract

We present a summary of possible signals of the chiral symmetry restoration and deconfinement phase transitions which may be, respectively, probed at the center of mass collision energies at 4.3-4.9 GeV and above 8.7-9.2 GeV. It is argued that these signals may evidence for an existence of the tricritical endpoint of QCD phase diagram at the collision energy around 8.7-9.2 GeV. The equation of state hadronic matter with the restored chiral symmetry is discussed and the number of bosonic and fermionic degrees of freedom is found.

1. Introduction

One of the most important directions of physics of heavy ion collisions is related to a location of the (tri)critical endpoint ((3)CEP) of the quantum chromodynamics (QCD) phase diagram. At present the lattice QCD cannot tell us whether at high baryonic charge densities the chiral symmetry restoration (CSR) phase transition (PT) and the deconfinement one of color degrees of freedom are two different PTs or a single one. On the other hand, a few advances approaches support an idea that at finite values of baryonic chemical potential a phase with a partial CSR occurs before the deconfinement PT [1–3]. However, one of the hardest problem of QCD phenomenology is to determine the collision energy threshold of these PTs using the existing experimental data obtained in the central nucleus-nucleus collisions.

TABLE 1: The summary of possible PT signals. The column II gives short description of the signal, while the columns III and IV indicate its location, status and references.

No and Type	Signal	C.-m. energy $\sqrt{s}$ (GeV) Status	C.-m. energy $\sqrt{s}$ (GeV) Status
1. Hydrodynamic	Highly correlated quasi-plateaus in entropy/baryon, thermal pion number/baryon and total pionnumber/baryon. Suggested in [11, 12].	Seen at **3.8-4.9 GeV** [4, 5]. Explained by the shock adiabat model [4, 5]. Require an explanation.	Seen at **7.6-9.2 GeV** [4, 5].
2. Thermodynamic	Minimum of the chemical freeze-out volume V_{CFO}.	In the one component HRGM it is seen at **4.3-4.9 GeV** [13]. In the multicomponent HRGM it is seen at **4.9 GeV** [14]. Explained by the shock adiabat model [4, 5].	Not seen.
3. Hydrodynamic	Minimum of the generalized specific volume $X = \frac{\epsilon+p}{\rho_b^2}$ at chemical freeze-out.	Seen at **4.9 GeV** [4]. Explained by the shock adiabat model [4, 5].	Seen at **9.2 GeV** [4]. Require an explanation
4. Thermodynamic	Peak of the trace anomaly $\delta = \frac{\epsilon-3p}{T^4}$.	Strong peak is seen at **4.9 GeV** [5]. Is generated by the δ peak on the shock adiabat at high density end of the mixed phase [5].	Small peak is seen at **9.2 GeV** [5]. Require an explanation
5. Thermodynamic	Peak of the baryonic density ρ_b.	Strong peak is seen at **4.9 GeV** [10]. Is explained by $\min\{V_{CFO}\}$ [14].	Strong peak is seen at **9.2 GeV** [10]. Require an explanation
6. Thermodynamic	Apparent chemical equilibrium of strange charge.	$\gamma_s = 1$ is seen at **4.9 GeV** [10]. Explained by thermostatic properties of mixed phase at $p = const$ [10].	$\gamma_s = 1$ is seen at $\sqrt{s} \geq$ **8.8 GeV** [10, 13]. Explained by thermostatic properties of QG bags with Hagedorn mass spectrum [10].
7. Fluctuational (statistical mechanics)	Enhancement of fluctuations	N/A	Seen at **8.8 GeV** [9]. Can be explained by CEP [9] or 3CEP formation [10].
8. Microscopic	Strangeness Horn (K^+/π^+ ratio)	N/A	Seen at **7.6 GeV**. Can be explained by the onset of deconfinement at [15]/above [8] **8.7 GeV**.

Fortunately, during last few years an essential progress in resolving such a problem was achieved [4–9]. In particular, two sets of remarkable hydrodynamic and thermodynamic signals of two PTs at the center of mass collision energies $\sqrt{s_{NN}} = 4.3 - 4.9$ GeV and $\sqrt{s_{NN}} = 7.6 - 9.2$ GeV were found in [4] and the hypothesis of their possible observation at these energies of collision was first formulated in [4–6]. In the works

[7, 8] a very good description of the large massive of experimental data on nuclear collisions was first achieved with the Parton-Hadron-Sring-Dynamics (PHSD) model by assuming an existence of CSR PT at about $\sqrt{s_{NN}} \simeq 4$ GeV in a hadronic phase and a deconfinement one at $\sqrt{s_{NN}} \simeq 9 - 10$ GeV. In 2017 the group of scientists analyzed the fluctuations of light nuclei and came to a conclusion that the vicinity of collision energy $\sqrt{s_{NN}} \simeq 8.8$ GeV is a nearest vicinity of the critical endpoint of the QCD phase diagram [9]. The arguments of such a statistical signal of the endpoint were essentially enhanced in [10] and the conclusion of the tricritical endpoint existence in QCD at or slightly above $\sqrt{s_{NN}} \simeq 8.7 - 9.2$ GeV was first formulated in [10]. The summary of found signals is given in Table 1.

Although the PHSD model provides us with some hints about the properties of the phase existing at the collision energy range $\sqrt{s_{NN}} \simeq 4.9 - 9.2$ GeV, the question is whether one can independently get the properties of this matter. In this work we briefly show how one can get them from the equation of state (EoS) which is obtained in [4, 5] from fitting the data.

2. Hadron Resonance Gas Model with Hard-Core Repulsion

The possible PTs signals 1-6 presented in Table 1 were obtained with the help of the multicomponent Hadron Resonance Gas Model (HRGM) [4–6, 14, 16–19], which, in contrast to the HRGM with one or two hard core radii of hadrons [13] has the following hard-core radii of pions R_π=0.15 fm, kaons R_K=0.395 fm, Λ-hyperons R_Λ=0.085 fm, other baryons R_b=0.365 fm and other mesons R_m=0.42 fm. Thus, having only 2 or 3 additional global fitting parameters compared to the usual HRGM [13], one can get extremely good description of the hadronic multiplicity ratios measured at AGS, SPS and RHIC energies with a high quality $\chi^2/dof \simeq 1.04$ [6, 19], including traditionally the most problematic ones for the usual HRGM [13], i.e. K^+/π^+, Λ/π^+ and $\bar{\Lambda}/\pi^-$ ratios.

A high quality fit of hadronic multiplicity ratios achieved by the multicomponent HRGM gives us a high confidence that the EoS of hadronic matter is now fixed with high accuracy in the wide range of chemical freeze-out (CFO) temperature T and baryonic chemical potential μ_B. This conclusion was thoroughly verified recently with the newest version [18, 19] of the multicomponent HRGM which allows one to go beyond the Van der Waals approximation traditionally used in HRGM. In the simplest case of a single hard-core radius of hadrons R the HRGM pressure in the grand canonical ensemble is

$$p = \sum_n p_n^{id}(T, \mu_n - b\,p), \quad p_n^{id}(T,\mu) = g_n \int \frac{dk}{(2\pi^3)} \frac{k^2}{3\,E_n(k)} \frac{1}{\exp\left(\frac{E_n(k)-\mu}{T}\right) + \zeta_n} \tag{1}$$

where the sum is running over all particles (and antiparticles) with the chemical potentials μ_n, $b = 4V_0 = 4\frac{4}{3}\pi R^3$ is the excluded volume of hadrons and V_0 is their proper volume. Here $p_n^{id}(T,\mu)$ denotes the partial pressure of the point-like hadrons of sort n with the degeneracy g_n and the mass m_n, while $E_n(k) = \sqrt{\vec{k}^2 + m_n^2}$ is the energy of particle with the 3-momentum $\vec{k}$ and μ is the effective chemical potential. The parameter ζ_n defines the Fermi ($\zeta_n = 1$), the Bose ($\zeta_n = -1$) or the Boltzmann ($\zeta_n = 0$) statistics. Then the thermal density of particles of sort l is defined as

$$n_l \equiv \frac{\partial p}{\partial \mu_l} = \frac{n_l^{id}}{1 + b\sum_k n_k^{id}}\,, \quad n_l^{id}(T,\nu,\zeta_l) = g_l \int \frac{dk}{(2\pi^3)} \frac{1}{\exp\left(\frac{E_l(k)-\nu}{T}\right) + \zeta_l}\,, \tag{2}$$

where $n_l^{id}(T,\nu,\zeta_l)$ denotes the particle number density of point-like hadrons of sort l.

3. Necessity of Multicomponent HRGM

There are two main reasons of why the HRGM of Eqs. (1) and (2) with the Van der Waals is used to determine the CFO parameters. The main reason is that for the hard-core repulsion the energy per particle is same as in the ideal gas and, therefore, there is no need to transform the potential energy of the system into the kinetic energy of particles. Of course, one could add the attractive term $P_{attr}(\{n_k\})$ to the pressure (1), but in this case one would face a hard mathematical problem to convert the interacting gas into free streaming hadrons [20–22] measured in the experiments.

The second reason is that only the hard-core repulsion provides the consistence with the lattice QCD results. In other words, if one takes into account all hadrons as the point-like particles with $b = 0$, then it is well-known that at high T and μ_B their pressure will dramatically exceed the pressure of quarks and gluons. However, in order to provide a high quality fit of the data such a simplified model should be modified in two respects. First of all one should remember that the quantum second virial coefficients of particle of sort k interacting with the particle of sort l is [23]

$$a_{2,kl}^{Q} = b + a_{2,k}^{(0)}\delta_{kl} - \frac{a_{kl}^{attr}}{T} = b + \frac{\zeta_k}{2}\frac{n_k^{id}(T/2,0,0)}{\left[n_k^{id}(T,0,0)\right]^2}\delta_{kl} - \frac{1}{T}\lim_{\{n_m\}\to 0}\frac{\partial^2 P_{attr}(\{n_m\})}{\partial n_k\,\partial n_l}\,, \tag{3}$$

where the term $a_{2,k}^{(0)}$ is the virial coefficient due to quantum statistics of hadron of sort k which is expressed in terms of the densities $n_k^{id}(T,0,0)$ of auxiliary Boltzmann hadrons of the same sort k, and the term a_{kl}^{attr} is due to attractive interaction. This equation shows that the gas pressure (1) with the hard-core repulsion, indeed, accounts for the quantum properties of hadrons, if $\zeta_k \neq 0$. It also shows that, if one introduces the different hard-core radii of hadrons, then one can even account for the attraction between them at the level of the second virial coefficient which is sufficient for the low particle densities at CFO. Of course, for all hadrons the second virial coefficients (3) are temperature dependent, but fortunately, at high temperatures such a dependence is not strong [23] and, hence, to a leading order one can restrict the treatment by the constant hard-core radii.

In addition, the HRGM pressure corresponds to the hadron resonances of vanishing width. This is, of course, a rough approximation because at CFO the density is sufficiently low that the inelastic reactions between hadrons can be neglected and, hence, the hadrons and their resonances should get their vacuum masses and vacuum widths before going into detector. The other reason to introduce the widths is the practical one. Thus, using the Briet-Wigner parameterization of resonance width of all hadronic resonances one can describe the hadron rations essentially better than with the Gaussian one or with the vanishing width [24]. Therefore, it seems that the most efficient way to account for the residual attractive interaction between hadrons at CFO and to achieve a high quality of the hadron multiplicity description is to generalize the one component HRGM (1), (2) to the multicomponent case, i.e. to account for different hard-core radii and then to determine these radii from the fit of experimental data. This is exactly what was done in Refs. [4–6, 14, 16–19, 24] during last five years. It is evident that the hard-core radii determined in this way are effective ones by construction.

4. EoS of Hadronic Matter with CSR

Using the multicomponent HRGM in Refs. [4, 5] it was possible from fitting the entropy per baryon s/ρ_B along the shock adiabat [11, 12] to determine the EoS of the phase existing at the collision energy range $\sqrt{s_{NN}} \simeq 4.9 - 9.2$ GeV. This EoS is similar to the MIT-Bag model

$$p_{Chiral} = A_0 T^4 + A_2 T^2 \mu^2 + A_4 \mu^4 - B\,, \tag{4}$$

but the coefficients $A_0 \simeq 2.53 \cdot 10^{-5}\ MeV^{-3} fm^{-3}$, $A_2 \simeq 1.51 \cdot 10^{-6}\ MeV^{-3} fm^{-3}$, $A_4 \simeq 1.001 \cdot 10^{-9}\ MeV^{-3} fm^{-3}$, and $B \simeq 9488\ MeV\ fm^{-3}$ are rather different from what is predicted by the perturbative QCD for massless gluons and (anti)quarks. In Ref. [10] the EoS (4) was suggested to find out the number of bosonic and fermionic degrees of freedom of this phase. Recalling that first three terms of the EoS (4) correspond to the gas of massless particles and noting that the coefficient A_4 is small and its value is comparable to its own error, we could determine the numbers of total N_{dof}^{tot}, bosonic N_b^{eff} and fermionic N_f^{eff} degrees of freedom as

$$N_{dof}^{tot} = \frac{90}{\pi^2} A_0 \hbar^3 \simeq 1770\,, \quad N_f^{eff} = 12\, A_2 \hbar^3 \simeq 141\,, \quad N_b^{eff} = N_{dof}^{tot} - \frac{7}{4} N_f^{eff} \simeq 1523\,. \tag{5}$$

Since the numbers N_b^{eff} and N_f^{eff} are much larger than the corresponding number of degrees of freedom in perturbative QCD, but at the same time they are close to the total number of spin-isospin degeneracies of all known hadrons, in Ref. [10] we, independently of the works [7, 8], concluded that the EoS (4) corresponds to the gas of massless hadrons with strong attraction given by the vacuum pressure B.

5. Conclusions

Here we present a summary of possible signals of CSR and deconfinement PTs which may be, respectively, probed at the collision energies at $\sqrt{s_{NN}} \simeq 4.3 - 4.9$ GeV and above $\sqrt{s_{NN}} \geq 8.7 - 9.2$ GeV. Also these signals may evidence for an existence of the tricritical endpoint of QCD phase diagram at the collision energy around or slightly above $\sqrt{s_{NN}} \geq 8.7 - 9.2$ GeV. The EoS of the hadronic matter with CSR is discussed and the number of bosonic and fermionic degrees of freedom is found.

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