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Mathematical Model of Pressure Formation Process along the Helix Channel Length of Screw Grinder

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Abstract

The improvement of the technical equipment effectiveness is currently becoming particularly important. This applies not only to large and high-energy-intensive machines, but also to household appliances, the total energy consumption of which often exceeds the energy consumption of the overall equipment. These types of devices include, in particular, grinding and cutting equipment. The mathematical description of the processes carried out on this equipment is generalized and can be extended to a wider class of machines, including waste processing and mining equipment. The technological parameters, the design of screw grinders, and the processes of movement, deformation, extrusion and cutting carried out in them are characterized by a significant number of factors affecting the energy intensity. The main ones are the geometric parameters of the screw, machine's body, cross knife, grinding plate's thickness, the number and diameter of holes in it, as well as the product's physical-mechanical characteristics and operating conditions. The most important for the mathematical description are the zones and processes where the main share of the consumed power is spent. The complexity of their analytical description is due to a simplified consideration of either individual technological zones of grinders' existing designs, or the use of unreasonable simplifications.

Keywords: screw, grinder, chopper, helix channel, friction force, mechanical resistance, deformation of collapse and shear, anti-rotation

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1. Introduction

A screw grinder's constructive feature is the presence on the inner body's surface of the anti-rotation flanges. The latter ones exclude the rotation of the product and ensure its forward movement and the pressure formation required for extrusion. Such factors complicate the modelling process. Pressure generated at the output of the grinder and the law of its distribution along the helix channel is the main power characteristics determining energy consumption of the grinding, cutting and extrusion processes. In this case the determination of the pressure dependence of pressure on the length of the helix is the aim of the study. The solution of this problem under the conditions of the inhibitory forces action on the fish product from the side of anti-rotation (anti-skid) elements fundamentally distinguishes it from existing approaches that take into account the action of only friction forces. The definition of the pressure formation law along the length of the movement trajectory under the friction forces braking action on the product from the surface of the screw was carried out in [1-8]. An analytical solution to a more complex problem, taking into account the influence of counter-rotation (anti-skid) flanges, we carry out by integrating the differential equation of product motion in the helix channel. For the mathematical description of the processes under consideration, we use the well-known physical model for moving the product plug in reverse motion along the helix channel's surface. Additionally we consider the influence of such inhibitory factors on the external contour as the friction force and mechanical resistance of the counter-rotation elements.

2. Objects, Methods and Equipment

The study objects are processes of fish raw material interaction with the screw grinder's constructive elements when the movement, deformation and extrusion of solid material in the conditions of backpressure from the anti-rotation flanges made on the inner surface of the machine's body.

Mathematical modeling of the process, solution of the defining differential equations and experimental method of evaluation of the obtained theoretical results adequacy are chosen.

For the experimental studies a unit was designed, a photograph of which is shown in Figure 1. This experimental unit is made on the basis of the analog grinder 8MMof JSC «Lenpoligrafmash». The grinder is a typical representative of the machines used by small and medium-sized enterprises processing food raw materials.

The experimental unit is equipped with a 1 kW electric motor (9) with an LGSY 004ic5 frequency converter. The productivity of a serial meat grinder 8MM for meat using a grinding plate with a diameter of 82 mm and holes of 5 mm is 300 kg/h. The unit is equipped with a computer (5), on which pressure sensors (3) are recorded during the grinding process by the measuring unit (4) and (7).

Raw materials (1) from the hopper (10) are fed by a pusher (8) into the hopper and the helix channel (11). On the body (11), along the product's movement, pressure sensors (3) and temperature are fixed. The drive is turned on and reversed by a switch (6).

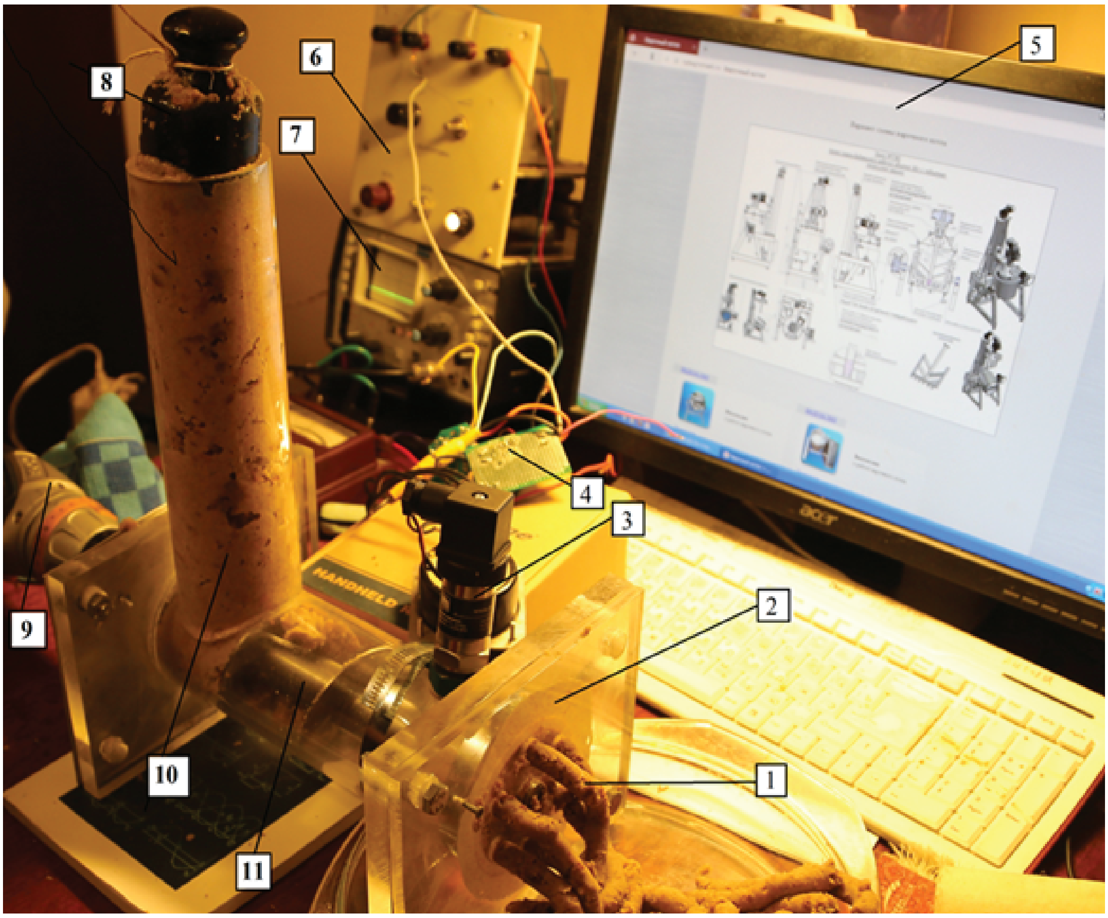

Figure 1: Photograph of an experimental unit.

The experimental setup, P401 has pressure sensors, which are used to measure the relative pressure in the screw body. The range of measured pressures is up to 2.5 MPa with a possible excess of 1.5 MPa. The sensor works complete with a measuring unit and a personal computer. Work is carried out at an ambient temperature from +10 °C to +35 °C. The main reduced error is not more than 2 %, the temperature range of the investigated raw materials varies from +1 °C to +30 °C.

When conducting experiments, both natural fish raw materials and a substitute are used. When conducting experiments that require a significant amount of raw materials, in order to reduce costs, substitutes can be used, but since the rheological properties of the substitute cannot fully correspond to real raw materials, in order to obtain reliable data, it is ultimately necessary to check on a natural product.

3. Results

As a mathematical model of the process, we take the well-known hydrodynamic equations of motion in the form of Euler for the product mass of a unit volume [5].

Considering the case of material's one-dimensional movement along a screw expanded surface of the helix channel along the natural coordinate «L» (coordinate «L» is the expanded helix line of the screw), we write:

$$\frac{\partial V_L}{\partial t}+\frac{\partial V_L}{\partial L}V_L = R_L - \frac{1}{\rho}\frac{\partial P}{\partial L}, \tag{1}$$

where t is the time of the product transfer process in the grinder channel, s; V_L - projection of the velocity vector on the coordinate axis «L», m/s; ρ is the density of the food material, kg/m^3; P is the current value of the pressure of the food raw material in the helix channel, depending on the coordinate «L», Pa; RL = R is the projection onto the «L» axis of the main vector of external forces (the friction force of the material on the screw surface of the screw and the mechanical resistance of anti-skid elements acting on the mass of a unit volume of material), m/s^2.

Given the solid nature of the product's structure in question, we neglect the processes of product circulation in directions perpendicular to the longitudinal axis of the screw channel, and consider the process itself to be stationary. Then, passing to the traditional variables, the equation of motion in the Euler's form for the product's mass M = ρQ can be written as

$$\rho Q\frac{\partial Vx}{\partial x}Vx = \rho Q R - Q\frac{\partial P}{\partial x}, \tag{2}$$

where Vx = VL; x = L; M is the mass of food material, kg; Q-volume of the transported food mass, m^3.

The volume of the product's transported mass is determined by the volume of the helix channel and can be determined by the ratio linear with respect to «L»:

$$Q = L(a+b)\frac{D_2 - D_1}{2}.$$

where a, b are the dimensions of the bases of the trapezoidal section of the screw channel, m; D_2 - diameter of the outer forming cylinder of the screw, m; D_1 - diameter of the inner forming cylinder of the screw surface of the screw, m.

Designating the left-hand side of (2) as the inertial component with the value of F_i, we write the relation

$$F_i = \rho Q\frac{\partial Vx}{\partial x}Vx = \rho Q\left(\frac{\partial^2 x}{\partial t \partial x}\right)\frac{\partial x}{\partial t}. \tag{3}$$

In its physical sense, the Euler equation, written in the form (2), is an equation of equilibrium of forces acting on a displaced material through a helix channel.

The analysis of the accepted physical model shows that the force factor ρQR of the product pressure profile formation along the screw helix line (surface) consists of two components.

The first component is associated with the presence of material sliding forces on the screw surface and the inner surface of the cylinder body.

The second determining reason for the increase in pressure in the helix channel as it moves from the loading zone in the exit direction is the presence of mechanical resistance from the side of the anti-skid flanges, which can be performed coaxially to the body or in the form of multi-screw elements. Under constrained compression, the friction force is determined by the equation:

$$F_{fr} = \frac{\nu}{1-\nu}\mu P S, \quad (4)$$

where F_{fr} is the friction force, H; μ is the coefficient of friction of the product sliding along the flow surface; ν is the Poisson's ratio of the product material; S is the area of friction of the material on the screw surface and body's cylinder, m^2.

The friction area is determined by the perimeter of the helix channel cross section and its length in accordance with the obvious ratio:

$$S = L[(a+b) + 2(D_2 - D_1)],$$

where a, b are the dimensions of the trapezoidal section bases of the screw helix channel, m; D_2, D_1 - diameters of the outer and inner cylinders forming the body of screw grinder, m.

The strength of the mechanical resistance from the side of the anti-skid elements is recorded:

$$F_{cr} = F_{fl} = \sigma_{cr} S_{cr}, \quad (5)$$

where F_{cr} = F_{fl} is the strength of mechanical resistance to the fish raw materials movement in the helix channel from the side of the protruding anti-skid flange perpendicular to it, N; σ_{cr} is the shear stress of the raw material, Pa; S_{cr} - the area of the crease surface, determined by the corresponding area of the anti-skid flange, m^2.

Wherein:

$$S_{cr} = \Delta L_{cr}, \quad (6)$$

where D is the height of the protrusion of the anti-skid ribs (shoulder), m; L_{cr} - the length of the anti-skid line (crushing line), m.

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For braking force from the side of the anti-skid flange we get:

$$F_{fl} = \sigma_{cr} \Delta L_{cr}. \tag{7}$$

Given the physical picture of the product's interaction with a helical surface and with anti-skid flanges, we determine the length of the material's shear line by the ratio:

$$L_{cr} = L_{cp} = L_s N_s, \tag{8}$$

where N_s - the number of sections of anti-skid flanges intersected by a helix along its entire length; L_{sh} - the length of the shear line; L_s - the length of one anti-skid section located between adjacent turns of the helix line (in fact, this is the end step of the screw), m.

The length of one section of the anti-skid flange is determined by the length of the entire helix line, the number of turns and the angle of inclination of the normal screw surface to its axis in accordance with the obvious ratio:

$$L_s = \frac{L sin\gamma}{N_t}, \tag{9}$$

where N_t is the number of turns of the helix line; γ is the angle of normal inclination of the screw surface to its axis.

Thus, the length of the deformation line of the collapse or shear after substituting relation (9) in equation (8) will take the form of the expression:

$$L_{cr} = L_{sh} = \frac{L N_s sin\gamma}{N_t}. \tag{10}$$

In this case, the force of crushing of the material, forming the law of pressure distribution along the helix line length can be written in the form:

$$F_{fl} = \frac{\sigma_{cr} \Delta L N_s sin\gamma}{N_t}. \tag{11}$$

Similarly, for the case of anti-skid flanges in the form of slots buried on the inner surface of the screw mechanism housing, we write down the corresponding shear mechanical resistance in the form of two possible ratios:

$$F_{fl} = \frac{\sigma_{sh} b L N_s sin\gamma}{N_t} = \frac{P_{sh} L N_s sin\gamma}{N_t}. \tag{12}$$

where σ_{sh} is the shear stress of the material of the fish product, N/m^2; b - the width of the buried slot, m; P_{sh} - specific cutting force of the chopped fish material, N/m.

Estimates show that the order of values calculated by relations (11) and (12) is the same. For technological reasons, we take equation (11) as the basis for further calculations.

The general expression for the force factor ρQR, which includes the sliding friction force and the mechanical resistance of the anti-skid flanges, can be written as the sum of relations (4) and (11): ρQR = F_{fr} + F_{fl} or

$$\rho QR = \frac{v}{1-v}\mu PS + \frac{\sigma_{cr}\Delta LN_s sin\gamma}{N_t}. \tag{13}$$

Let us consider in more detail the left side of the Euler equation (2).

With a constant pitch of the helix line of the screw in the case of a steady-state process mode, the analyzed factor does not give an external power component, since it is a kinematic, inertial component and does not form inertia forces in the stationary mode, that is, it is equal to zero.

In the case of a variable pitch of the screw helix, an inertial component appears, therefore, it is necessary to set the law of the change in speed Vx depending on the longitudinal coordinate of the helix channel.

This law can be specified in a linear form:

$$Vx = VL = V_0(1-kL), \tag{14}$$

where k is the coefficient of decrease in the speed of movement of the material, determined by the compression ratio of the pitch of the screw helix, m^{-1}.

The coefficient k is determined through the initial and final angles of inclination of the helix, as well as its full length:

$$k = \frac{\gamma_1 - \gamma_2}{L_{scr}}, \tag{15}$$

where γ_1 is the angle of inclination of the helix to the axis of the screw at its beginning; γ_2 is the angle of inclination of the helix to the axis of the screw at its end; L_{scr} - the full length of the screw line of the screw, m; V_o is the initial value of the speed of the fish material in the zone of its loading, m/s; k is the proportionality coefficient characterizing the rate of decrease of the axial velocity of the food material in the grinder (k = 0.2) m^{-1}; L = x - linear coordinate along the length of the screw surface, m.

From geometric considerations, in accordance with the scan of the helix line of the screw, the following dependence is obvious:

$$V_0 = \frac{\omega\ D_2}{2cos\gamma_1}, \tag{16}$$

where ω is the angular speed of screw rotation, s^{-1}; D_2 is the value of the outer diameter of the forming cylinder of the screw helix surface, m.

In this case, for an unsteady inertial power component due to the inconsistency of the screw pitch, we can write:

$$F_i = \rho Q\frac{\partial Vx}{\partial x}Vx = \rho QVL\frac{\partial VL}{\partial L} = -\rho Q{V_0}^2 k(1-kL) = -\rho Q(\frac{\omega\ D_2}{2cos\gamma_1})^2 k(1-kL). \tag{17}$$

Thus, the equation of motion (2) in the form of Euler for moving food material in a helix channel with a variable pitch in the general case takes the form:

$$\frac{\partial P}{\partial L}=\frac{\nu}{1-\nu}\frac{\mu PS}{Q}+\frac{\sigma_{cr}\Delta LN_{s}sin\gamma}{N_{t}Q}+\rho{V_{0}}^{2}k(1-kL). \tag{18}$$

After rearrangement of terms homogeneous in L and P, we obtain:

$$\frac{\partial P}{\partial L}=\frac{\nu}{1-\nu}\frac{\mu PS}{Q}+(\frac{\sigma_{cr}\Delta LN_{s}sin\gamma}{N_{t}Q}-\rho{V_{0}}^{2}k^{2})L+\rho{V_{0}}^{2}k. \tag{19}$$

Considering that S and Q are functionally related to the helix length L linearly, the ratio of S and Q for a particular screw mechanism is constant. Assuming μ = const, we obtain from (15) a first-order linear differential equation with constant coefficients m, n, and w in the following form:

$$\frac{\partial P}{\partial L}+mP=nL+w, \tag{20}$$

where m = $\frac{-\nu}{1-\nu}\frac{\mu S}{Q}$= const is a constant value, m^{-1}; n =$\frac{\sigma_{cr}\Delta N_{s}sin\gamma}{N_{t}Q}$- ρ$V_0^2k^2$ is a constant value, Н/м^4; w = ρV_0^2k is a free term of the equation independent of L and P, Н/м^3.

To solve this differential equation, we use the Bernoulli method of changing the variable.

As a result, the expression for the law of pressure change P of the product along the length of the helix channel L is obtained in the form:

$$P=[n(\frac{L}{m}-\frac{1}{m^{2}})+\frac{w}{m}]+(P_{0}+\frac{n}{m^{2}}-\frac{w}{m})e^{-mL}. \tag{21}$$

After substituting the obtained ratio of the coefficients values n, m and w, according to the accepted notation, we write the solution of differential equation (18) in the following form:

$$\begin{aligned} P &= [(\tfrac{\sigma_{cr}\Delta N_{s}sin\gamma}{N_{t}Q}-\rho{V_{0}}^{2}k^{2})[L(\tfrac{-\nu}{1-\nu}\tfrac{\mu S}{Q})^{-1}--(\tfrac{-\nu}{1-\nu}\tfrac{\mu S}{Q})^{-2}]+[\rho{V_{0}}^{2}k(\tfrac{\nu}{1-\nu}\tfrac{\mu S}{Q})^{-1}]+ \\ &\quad [P_{0}+(\tfrac{\sigma_{cr}\Delta N_{s}sin\gamma}{N_{t}Q}-\rho{V_{0}}^{2}k^{2})(\tfrac{\nu}{1-\nu}\tfrac{\mu S}{Q})^{-2}+\rho{V_{0}}^{2}k(\tfrac{\nu}{1-\nu}\tfrac{\mu S}{Q})^{-1}]e^{\frac{\nu}{1-\nu}\frac{\mu S}{QL}}. \end{aligned} \tag{22}$$

Thus, we found the analytical dependence (18) of the product's pressure change P along the length L of the helix channel on sixteen characteristics: σ_{cr}, D, N_s, N_t, ρ, n, m, P_0, a, b, D_2, D_1, L_{scr}, γ_2, γ_1, ω.

In accordance with the actual numerical values of these characteristics, we determine the order of food material pressure magnitude in the helix channel of the screw and plot the dependence P = P (L).

Consider fish raw materials with the following characteristics:

σ_{cr}=0,75 MPa; Δ=2·10^{-3} m; N_s=4; N_t=4; ρ=0,9·10^3 kg/m^3; ν=0,47; μ=0,015; P_0=0,1MPa; a=1,9·10^2 m; b=3,1·10^{-2} m; D_2=60·10^{-3} m; D_1=30·10^{-3} m; L_{scr}=0,650 m; γ_1 =π/4; γ_2=π/20; ω=5π c^{-1}.

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In this case, we obtain: S=6·10^{-2} m^2; Q=3·10^{-4} m^3.

Then the desired dependence will take the following form:

$$P = 183 - 801637(L + 0,3211) + (P_0 + 256973)e^{3,01L}. \tag{23}$$

For L_{scr}=0.75 m we get the maximum possible achievable pressure value of meat raw materials in this screw mechanism P=2.19 MPa. According to the results of experimental measurements, P_{max}=1.91 MPa.

4. Discussion

As a result of theoretical studies and a series of field tests, analytical equation (22) and graphs of the fish product pressure dependence along the length of the screw helix line are obtained. Based on experimental measurements, the maximum pressure at the grinder's outlet was P_{max}=1.91 MPa.

Figure 2 shows graphs of the theoretical and experimental curves of the pressure distribution in food raw materials along the length of the screw.

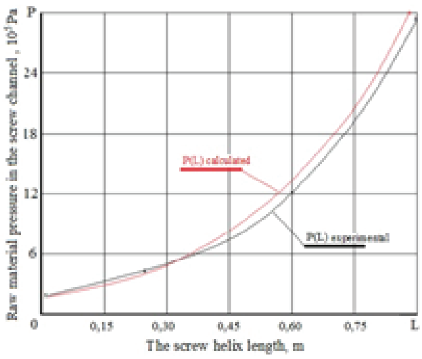

Figure 2: Theoretical and experimental dependence of pressure on the screw helix length.

Statistical processing of the experimental results showed that with a confidence probability of 95 % (quantile of the standard normal distribution of 1.645 at the level of 0.05), the deviation of the experimental data from the calculated is about 12.6 %.

5. Conclusion

The problem of increasing the technology efficiency has necessitated new research [9-12] in this direction. This research also belongs to such a number of studies, taking into account the design features of screw grinding and cutting machines and allowing optimizing their design and technological parameters in the direction of reducing energy intensity. An adequate mathematical model of the pressure distribution law of food raw materials along the length of the helix channel of the screw grinder, depending on the screw grinder's parameters, including the anti-skid elements influence, has been worked out.

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Conflict of Interest

The authors have no conflict of interest to declare.

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