Minimum Energy Expenditure of Arm and Leg Motions

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Abstract

The purpose of the study is to find the optimized arm and leg motions by computer simulation. The research method of this study was based on Lagrange-Euler equations (LEE) of motion and the seven types of homogeneous transformation matrices (CH-7T) defined by Chiu [1]. A dynamic system with 10 degrees of freedom (DoF) was established for segments of the right arm, while segments of the right leg were defined as a dynamic system with 9 DoF. A video camera was used to shoot two motions of 16 male subjects. For the first motion, subjects stood motionless in the initial state, and then they were free to adopt what they consider the most comfortable way to lift their right arm until it rested alongside the head in the final state. For the second motion, subjects were required to lift their right leg. Then applying LEE of motion in analyzing the video, this study calculated the energy expenditure for the arm motion and the leg motion. Additionally, the video was used to locate the boundary conditions for the initial state and the final state of two motions. With the data on boundary conditions, the minimum energy expenditure(MEE) was calculated. There were three major findings in the analytic results. First, the calculated energy expenditure for simulated optimal arm motion of 16 subjects was 93% of the energy expenditure for the real arm motion done in an individualized way. Second, the calculated energy expenditure for simulated optimal leg motion of 16 subjects was 81% of the energy expenditure for the real leg motion done in an individualized way. It was obvious that adopting the simulated optimal trajectory required less energy than completing the motion in individualized ways. Third, it was found that performing arm motion required less energy than performing leg motion. The result demonstrated that the former had more efficiency than the latter, evidencing that humans manipulate their arm better than their leg. Whether there are other factors involved in the difference of efficiency between arm and leg motion will be reserved for further research.

Keywords: Computer simulation, Segments, Boundary conditions

Introduction

Whether in work or sports, arms and legs are frequently-used body segments. For example, such activities as playing tennis, driving a car, running, walking, long jump and high jump depend on arm and leg motions. Therefore, the more times we are able to perform arm and leg motions, the longer duration we can get involved in these activities. To increase the number of arm and leg motions, it is necessary to reduce energy expenditure.

Arm and leg motions are characterized by nonlinear dynamics. And the mathematics models of the dynamic systems for arms and legs are very complicated. In the past, studies adopted Newton or D'Alembert equations of motion to design dynamic systems. However, these systems are not comprehensive because problems will arise in the process of transforming the dynamic system into the state-space of control system. Conversely, the above problems don't exist if LEE are used to design the human motion system. Besides, there are other advantages in adopting LEE. First, LEE are succinct. Second, with LEE, it is easier to transform a dynamic system involving multiple variables into a control system. And when the system is based on LEE, the inertial acceleration-related symmetric matrix, nonlinear Coriolis centrifugal force vector, and gravity loading force vector can be presented in matrix form [2,3]. Finally, the LEE have been widely used in the optimal control theory. In view of the above merits, the LEE were adopted in designing the dynamic system of this study.

In the past, studies on human arms focused on the mathematics model for motions of arm joints or strength in contracting muscles [4,5]. Few studies explored the energy expenditure for arm motions. As for the studies on energy expenditure for leg motion, Beckett and Chang [6] adopted LEE to establish a dynamic system for legs. They calculated the MEE for thigh and shank motions in the swing phase of walking. But the problem is that Beckett et al. didn't limit the time spent in completing the motions. Beckett et al. also failed to explore the difference between the optimized motion and real motion in energy expenditure. Since the study by Beckett et al. leaves room for improvement, the researcher of this study attempts to refine it. The researcher once designed a dynamic system with 14 DoF, applying the principle of MEE to simulate front chin-ups. The results showed that the subjects could complete the motion within the limited time and expended the minimum energy [7-8]. Since the method is proved to be useful, it is applied to the present study on arm and leg motions.

To confirm the simulated motions to be the optimal ones, this study will adopt the following three steps: (1) design an algorism to calculate the MEE for arm and leg motion, (2) locate the optimal trajectory of leg and arm motions by computer simulation, (3) explore the difference between optimized motions and real motions in energy expenditure.

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Materials and Methods

Subjects

This study took 16 males as the subjects. Their mean age is 22.8±1.6 years, mean height 168.7±5.1cm, and mean weight 62.7±6.7kg.

Shooting the arm and leg motions

A video camera was set to shoot subjects' motions at 60 Hz. For the video shooting (2D), each of the 16 male subjects was required to stand motionless in the initial state, and then move his segments in his habitual way until they reached the final state. The video was analyzed to calculate the energy expended for these real motions. On the other hand, from the video this study collected the data on boundary conditions, which are necessary for the simulation of optimal trajectory requiring the MEE.

The arm model and leg model

This study adopted LEE of motion to establish dynamic systems for the right arm and the right leg(Fig.1). The arm model comprised three segments(upper, forearm and hand) and 10 DoF ($\boldsymbol{q}_i$ =generalized coordinate).The leg model comprised three segments(thigh, shank and foot) and 9 DoF.

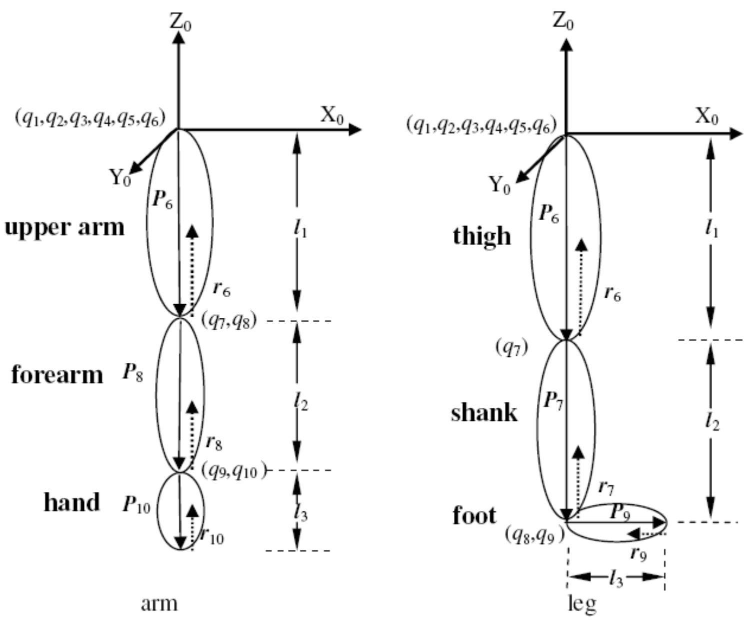


Figure 1. The arm and leg models and the controlled DoF. The arm and leg motions were shot and analyzed in 2 dimensions, so in the arm model the variables related to the study were five ($\boldsymbol{q}_1$, $\boldsymbol{q}_3$, $\boldsymbol{q}_4$, $\boldsymbol{q}_7$, $\boldsymbol{q}_9$), and in the leg model the variables related to the study were five ($\boldsymbol{q}_1$, $\boldsymbol{q}_3$, $\boldsymbol{q}_4$, $\boldsymbol{q}_7$, $\boldsymbol{q}_8$).

Table 1. Parameters of the homogeneous coordinate transformation matrices for the arm

$\boldsymbol{q}_i$	${}^{i-1}E_i$	${}^{0}E_i$	x_i	y_i	z_i
$\boldsymbol{q}_1$	${}^{0}E_1=\boldsymbol{E}_1^{(i)}$	${}^{0}E_1=\boldsymbol{E}_1^{(i)}$	-	-	-
$\boldsymbol{q}_2$	${}^{1}E_2=\boldsymbol{E}_2^{(j)}$	${}^{0}E_2=\boldsymbol{E}_1^{(i)}\ \boldsymbol{E}_2^{(j)}$	-	-	-
$\boldsymbol{q}_3$	${}^{2}E_3=\boldsymbol{E}_3^{(k)}$	${}^{0}E_3=\boldsymbol{E}_1^{(i)}\ \boldsymbol{E}_2^{(j)}\ \boldsymbol{E}_3^{(k)}$	-	-	-
$\boldsymbol{q}_4$	${}^{3}E_4=\boldsymbol{E}_4^{(y)}$	${}^{0}E_4=\boldsymbol{E}_1^{(i)}\ \boldsymbol{E}_2^{(j)}\ \boldsymbol{E}_3^{(k)}\ \boldsymbol{E}_4^{(y)}$	0	0	0
$\boldsymbol{q}_5$	${}^{4}E_5=\boldsymbol{E}_5^{(x)}$	${}^{0}E_5=\boldsymbol{E}_1^{(i)}\ \boldsymbol{E}_2^{(j)}\ \boldsymbol{E}_3^{(k)}\ \boldsymbol{E}_4^{(y)}\ \boldsymbol{E}_5^{(x)}$	0	0	0
$\boldsymbol{q}_6$	${}^{5}E_6=\boldsymbol{E}_6^{(z)}$	${}^{0}E_6=\boldsymbol{E}_1^{(i)}\ \boldsymbol{E}_2^{(j)}\ \boldsymbol{E}_3^{(k)}\ \boldsymbol{E}_4^{(y)}\ \boldsymbol{E}_5^{(x)}\ \boldsymbol{E}_6^{(z)}$	0	0	$\boldsymbol{l}_1$
$\boldsymbol{q}_7$	${}^{6}E_7=\boldsymbol{E}_7^{(y)}$	${}^{0}E_7=\boldsymbol{E}_1^{(i)}\ \boldsymbol{E}_2^{(j)}\ \boldsymbol{E}_3^{(k)}\ \boldsymbol{E}_4^{(y)}\ \boldsymbol{E}_5^{(x)}\ \boldsymbol{E}_6^{(z)}\ \boldsymbol{E}_7^{(y)}$	0	0	0
$\boldsymbol{q}_8$	${}^{7}E_8=\boldsymbol{E}_8^{(z)}$	${}^{0}E_8=\boldsymbol{E}_1^{(i)}\ \boldsymbol{E}_2^{(j)}\ \boldsymbol{E}_3^{(k)}\ \boldsymbol{E}_4^{(y)}\ \boldsymbol{E}_5^{(x)}\ \boldsymbol{E}_6^{(z)}\ \boldsymbol{E}_7^{(y)}\ \boldsymbol{E}_8^{(z)}$	0	0	$-\boldsymbol{l}_2$
$\boldsymbol{q}_9$	${}^{8}E_9=\boldsymbol{E}_9^{(y)}$	${}^{0}E_9=\boldsymbol{E}_1^{(i)}\ \boldsymbol{E}_2^{(j)}\ \boldsymbol{E}_3^{(k)}\ \boldsymbol{E}_4^{(y)}\ \boldsymbol{E}_5^{(x)}\ \boldsymbol{E}_6^{(z)}\ \boldsymbol{E}_7^{(y)}\ \boldsymbol{E}_8^{(z)}\ \boldsymbol{E}_9^{(y)}$	0	0	0
$\boldsymbol{q}_{10}$	${}^{9}E_{10}=\boldsymbol{E}_{10}^{(x)}$	${}^{0}E_{10}=\boldsymbol{E}_1^{(i)}\ \boldsymbol{E}_2^{(j)}\ \boldsymbol{E}_3^{(k)}\ \boldsymbol{E}_4^{(y)}\ \boldsymbol{E}_5^{(x)}\ \boldsymbol{E}_6^{(z)}\ \boldsymbol{E}_7^{(y)}\ \boldsymbol{E}_8^{(z)}\ \boldsymbol{E}_9^{(y)}\ \boldsymbol{E}_{10}^{(x)}$	0	0	$-\boldsymbol{l}_3$

- means that ${}^{i-1}E_i$ had no position vector p_i

Table 2. Parameters of the homogeneous coordinate transformation matrices for the leg

q_i	${}^{i-1}E_i$	${}^{0}E_i$	x_i	y_i	z_i
q_1	${}^{0}E_1=E_1^{(i)}$	${}^{0}E_1=E_1^{(i)}$	-	-	-
q_2	${}^{1}E_2=E_2^{(j)}$	${}^{0}E_2=E_1^{(i)}\ E_2^{(j)}$	-	-	-
q_3	${}^{2}E_3=E_3^{(k)}$	${}^{0}E_3=E_1^{(i)}\ E_2^{(j)}\ E_3^{(k)}$	-	-	-
q_4	${}^{3}E_4=E_4^{(y)}$	${}^{0}E_4=E_1^{(i)}\ E_2^{(j)}\ E_3^{(k)}\ E_4^{(y)}$	0	0	0
q_5	${}^{4}E_5=E_5^{(x)}$	${}^{0}E_5=E_1^{(i)}\ E_2^{(j)}\ E_3^{(k)}\ E_4^{(y)}\ E_5^{(x)}$	0	0	0
q_6	${}^{5}E_6=E_6^{(z)}$	${}^{0}E_6=E_1^{(i)}\ E_2^{(j)}\ E_3^{(k)}\ E_4^{(y)}\ E_5^{(x)}\ E_6^{(z)}$	0	0	$-l_1$
q_7	${}^{6}E_7=E_7^{(y)}$	${}^{0}E_7=E_1^{(i)}\ E_2^{(j)}\ E_3^{(k)}\ E_4^{(y)}\ E_5^{(x)}\ E_6^{(z)}\ E_7^{(y)}$	0	0	$-l_2$
q_8	${}^{7}E_8=E_8^{(y)}$	${}^{0}E_8=E_1^{(i)}\ E_2^{(j)}\ E_3^{(k)}\ E_4^{(y)}\ E_5^{(x)}\ E_6^{(z)}\ E_7^{(y)}\ E_8^{(y)}$	0	0	0
q_9	${}^{8}E_9=E_9^{(z)}$	${}^{0}E_9=E_1^{(i)}\ E_2^{(j)}\ E_3^{(k)}\ E_4^{(y)}\ E_5^{(x)}\ E_6^{(z)}\ E_7^{(y)}\ E_8^{(y)}\ E_9^{(z)}$	l_3	0	0

- means that ${}^{i-1}E_i$ had no position vector p_i

The parameters indispensable to the dynamic systems are segment lengths, center of mass position, segment weights, and mass moment of inertia (Appendix A). In Fig. 1, li represented the length of each arm and leg segment. The segment lengths of 16 subjects were obtained by measuring the subjects' arms and legs. To obtain the data on other parameters--center of mass position, segment weights, and mass moment of inertia, the regression equations by Zatsiorsky and Seluyanov were adopted with subjects' heights and weights as input values [9]. In this study, the symbol r_i was used to represent the center of mass vector of link i from the ith link coordinate frame and expressed in link coordinate frame, and p_i , the relative position vector between two ends of a certain body segment. In the arm coordinate system, the shoulder joint was set to comprise six DoF, three of which involved the translation of three axes and the remaining three degrees involved the rotation of the three axes. The elbow joint had two DoF, and the wrist joint comprised two DoF. In the leg coordinate system, the hip joint was assumed to comprise six DoF, three of which involved the translation of three axes and the remaining three degrees involved the rotation of the three axes. The knee joint had one DoF, and the ankle joint comprised two DoF.

Equations of motion

For the arm and leg models, the generalized torque τ_i , defined in accordance with LEE of motion, was written as [3,8,10]:

$$\tau_i=\sum_{j=i}^{n}\sum_{k=1}^{j} Trace(W_{jk}J_jW_{ji}^T)\ddot{q}_k+\sum_{j=i}^{n}\sum_{k=i}^{j}\sum_{m=1}^{j} Trace(W_{jkm}J_jW_{ji}^T)\dot{q}_k\dot{q}_m-\sum_{j=i}^{n}(m_jgW_{ji}r_j)\quad i=1,2,3,\ldots,n \tag{1}$$

where:

q_i =generalized coordinate

$\dot{q}_i$ =generalized velocity

$\ddot{q}_i$ =generalized acceleration

${}^{i-1}E_i$ =homogeneous transformation matrix of the ith coordinate frame relative to the i-1th coordinate frame

${}^{0}E_i$ =homogeneous transformation matrix from 0 coordinate frame to the ith coordinate frame; ${}^{0}E_i={}^{0}E_1{}^{1}E_2{}^{2}E_3\ldots{}^{i-1}E_i$

W_{ij} $=\partial^0A_i/\partial q_j$

W_{ijk} $=\partial W_{ij}/\partial q_k$ (i,j,k=1,2,3,…,n)

g $=[0,0,-|g|,0]$, $g=9.8062\ m/s^2$

m_j = the mass of the jth link (Appendix A)

r_j $=(\bar{x}_j,\bar{y}_j,\bar{z}_j,1)^T$, position of the center of mass for jth link (Appendix A)

J_j = pseudo-inertia matrix (Appendix A)

p_i $=(xi, yi, zi)^T$

The mechanical energy expenditure Ei for link i during the period of time from t_0 to t_1 was written as [11]:

$$E_i=\int_{t0}^{t1}|\tau_i\dot{q}_i|dt\qquad i=1,2,3,\ldots\ldots\ldots,n \tag{2}$$

The total mechanical energy expenditure E during the period of time from t_0 to t_1 was written as :

$$E=\sum_{i=1}^{n}E_i \tag{3}$$

Seven types of homogeneous coordinate transformation matrices

In this study, the homogeneous coordinate transformation matrices used in LEE of motion were based on CH-7T proposed by Chiu. This study adopted six types of CH-7T to design the needed equations of motion, as shown in Eq.(4) to Eq.(9). In the basic homogeneous rotation matrices, the generalized coordinate q_i was abbreviated as $cq_i(=\cos q_i)$ and $sq_i(=\sin q_i)$. In these matrices, p_i = (xi,yi,zi)^T, which represented the translation from the origin of the coordinate frame for link i to the coordinate frame for link i-1.

$$E_i^{(x)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cq_i & -sq_i & 0 \\ 0 & sq_i & cq_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & x_i \\ 0 & 1 & 0 & y_i \\ 0 & 0 & 1 & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$E_i^{(y)} = \begin{bmatrix} cq_i & 0 & sq_i & 0 \\ 0 & 1 & 0 & 0 \\ -sq_i & 0 & cq_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & x_i \\ 0 & 1 & 0 & y_i \\ 0 & 0 & 1 & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$E_i^{(z)} = \begin{bmatrix} cq_i & -sq_i & 0 & 0 \\ sq_i & cq_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & x_i \\ 0 & 1 & 0 & y_i \\ 0 & 0 & 1 & z_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$E_i^{(i)} = \begin{bmatrix} 1 & 0 & 0 & q_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$E_i^{(j)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & q_i \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

$$E_i^{(k)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (9)$$

Variables of the dynamic system

The homogeneous transformation from the base coordinate frame OXoYoZo to each arm segment of the dynamic system was presented in Table 1. Table 2 presented the homogeneous transformation from the base coordinate frame OXoYoZo to each leg segment of the dynamic systems. The values of position vector pi on X-axis, Y-axis, and Z-axis were also listed in Table 1 and Table 2.

Optimal control

LEE of motion was written in a matrix form as [12]:

$$\boldsymbol{\tau} = \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{V}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q}) \quad (10)$$

In Eg.(10), the following representations were used: $\boldsymbol{\tau}=[\tau_1, \tau_2,\dots, \tau_n]^T$, an n×1 generalized torque vector; $\boldsymbol{q}=[q_1, q_2, \dots, q_n]^T$, an n×1 vector of the joint variable; $\dot{\boldsymbol{q}}=[\dot{q}_1, \dot{q}_2, \dots, \dot{q}_n]^T$, an n×1 vector of the joint velocity; $\ddot{\boldsymbol{q}}=[\ddot{q}_1, \ddot{q}_2, \dots, \ddot{q}_n]^T$, an n×1 vector of the acceleration; $\boldsymbol{M}(\boldsymbol{q})$, an n×n inertial acceleration; $\boldsymbol{V}(\boldsymbol{q}, \dot{\boldsymbol{q}})$, an n×1 nonlinear Coriolis and centrifugal force vector; $\boldsymbol{G}(\boldsymbol{q})$, an n×1 gravity loading force vector. The state variables was written as [12-17,]:

$$\boldsymbol{x} = [\boldsymbol{q}^T \ , \dot{\boldsymbol{q}}^T]^T \quad (11)$$

In Eq.(11), $\boldsymbol{q}^T$ represented the position vector, and $\dot{\boldsymbol{q}}^T$ represented the velocity vector. Consequently, the equation for the state-space of the control system was defined as :

$$\dot{x} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} x + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \end{bmatrix} \boldsymbol{u} \quad (12)$$

In Eq.(12), I represented an n×n identity matrix, and $\boldsymbol{u}=[\ddot{q}_1, \ddot{q}_2, \dots, \ddot{q}_n]^T$ represented an n-dimensional input vector, which was written as :

$$\boldsymbol{u} = -\boldsymbol{M}^{-1}(\boldsymbol{q})[\boldsymbol{V}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{G}(\boldsymbol{q})] + \boldsymbol{M}^{-1}(\boldsymbol{q})\boldsymbol{\tau} \quad (13)$$

The dynamic systems designed in this study adopted the Open-loop Linear Quadratic Controller as the control method. The entire process of completing the arm motion and the leg motion was divided into two time points. The first time point was defined as t0 with $x(t_0)$ as the initial state. The other time point was defined as t1 with $x(t_1)$ as the final state. The state variables $x(t_0)$ and $x(t_1)$ for these two time points were defined as established boundary conditions. Then J, the cost function of the MEE during the interval of $[t_0, t_1]$, could be written as followed [12,14,15,16]:

$$\mathrm{J} = \frac{1}{2}\int_{t_0}^{t_1} \boldsymbol{u}^T \boldsymbol{R} \boldsymbol{u} \, dt \quad (14)$$

R was a symmetric matrix($\boldsymbol{R}$=[I]).

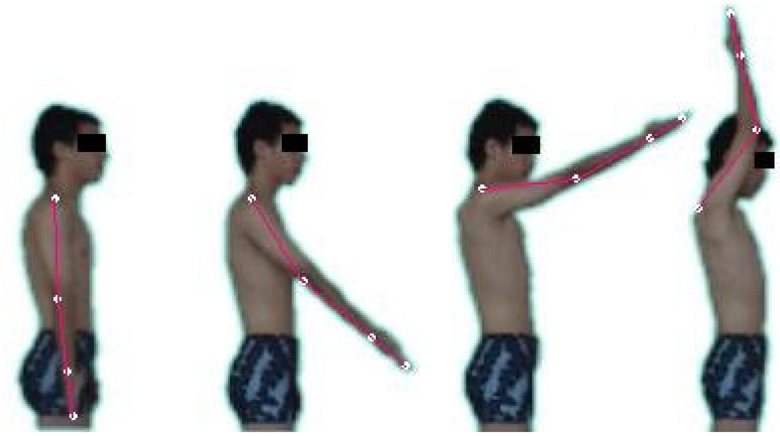
Boundary conditions

In this study, the boundary conditions acting as inputs for the equations of optimal control referred to the initial state and the final state of each controlled variable. For the arm motion, the initial state referred to the state where each subject put his right arm naturally along his side, which comprised the initial position and the initial velocity. The final state referred to the state where the right arm of each subject rested alongside his head, which comprised the final position and the final velocity. For the leg motion, this study focused on the swing leg instead of the support leg. In the initial state of the leg motion, subject A stood and had his right leg naturally relaxed, while the highest point the lifted leg could reach was called the final state.

Results

In this study, subject A was taken as an example to elaborate the arm and leg motions. In the initial state subject A put his right arm naturally along his side (Fig. 2(a)). Then subject A lifted his right arm upward. In the final state the right arm of subject A rested alongside his head (Fig. 2(d)). The time for subject A to complete the motion was 1.23sec. The completed motion was dissected into four parts, and the interval between two continuous parts was 0.41sec.

The simulated optimal trajectory and the actual trajectory for subject A's shoulder joint q_4 were presented in Fig. 3(a). The simulated optimal trajectory and the actual trajectory for subject A's elbow joint q_7 were also presented (Fig. 3(b)).



(a) (b) (c) (d)

Figure 2. Arm motion(a)initial state(d) final state.

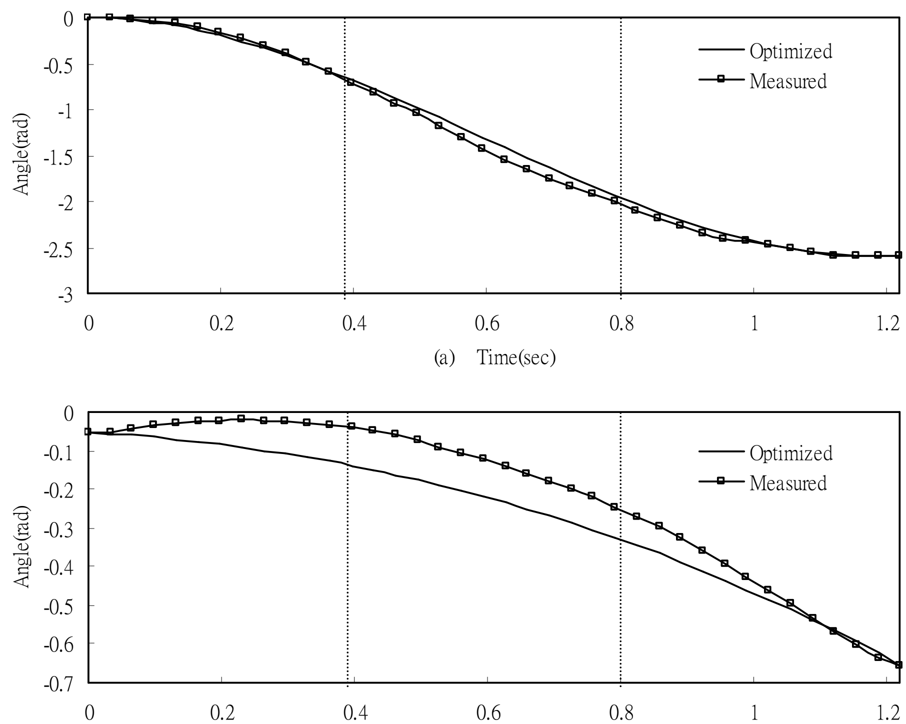


Figure 3. (a)The simulated optimal trajectory and the actual trajectory on Y-axis for the shoulder joint q_4 of subject A's arm motion (b) The simulated optimal trajectory and the actual trajectory on Y-axis for the elbow joint q_7 of subject A's arm motion[8].

For the leg motion, each of the subjects was required to swing his right leg. Fig. 4(a) showed the initial position of subject A's leg in the initial state. Then subject A performed the swing leg motion. Fig. 4(d) demonstrated the highest point that the lifted leg of subject A reached in the final state. The time for subject A to complete the leg motion was 0.99sec. The completed motion was dissected into four parts, and the interval between two continuous parts was 0.033sec.

The simulated optimal trajectory and the actual trajectory of leg motion for subject A's hip joint q_4 were presented in Fig. 5(a). As for the simulated optimal trajectory and the actual trajectory of leg motion for subject A's knee joint q_7, they were presented in Fig.5(b). Table 3 presented the mean and percentage of mechanical energy expended for optimized arm motion and leg motion, and those for actual arm motion and leg motion.

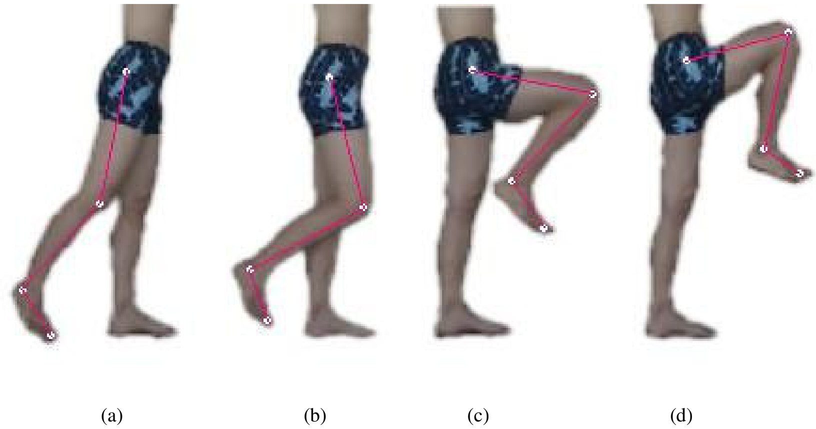


Figure 4. Swing leg motion(a) initial state(d) final state

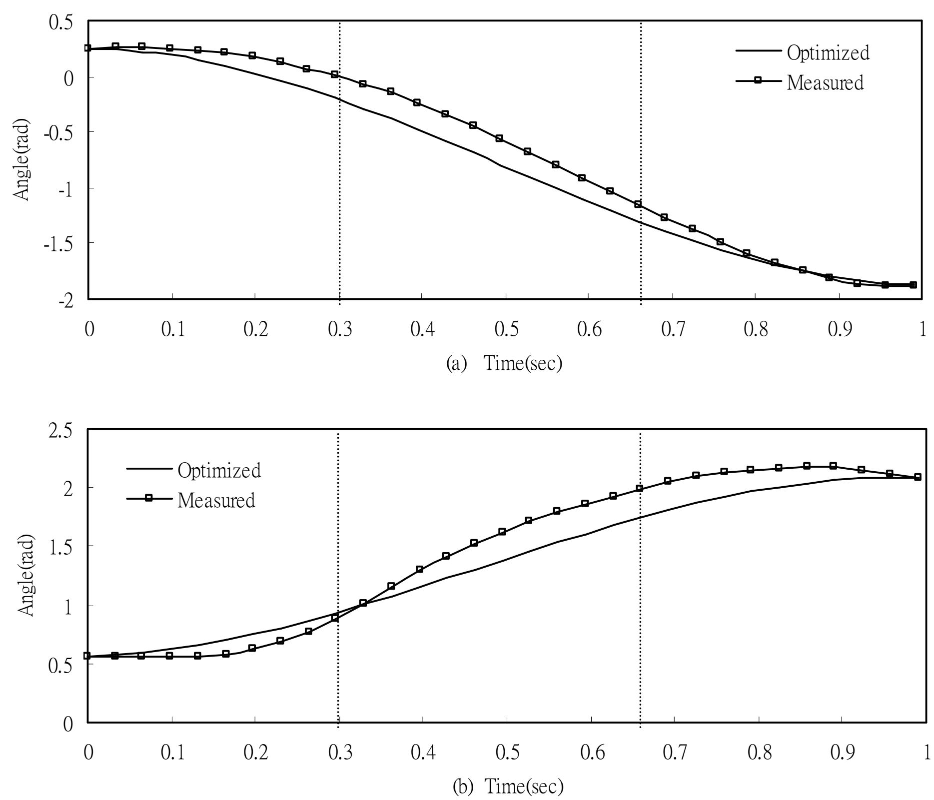


Figure 5. (a) The simulated optimal trajectory and the actual trajectory on Y-axis for the hip joint q_4 of subject A's leg motion (b) The simulated optimal trajectory and the actual trajectory on Y-axis for the knee joint q_7 of subject A's leg motion[8].

Discussion

The arm lifting motion of subject A was presented in Fig.2. As for the simulated optimal trajectory and the actual trajectory for subject A's shoulder joint and elbow joint, they were presented in Fig. 3. It was found in Fig. 3 that the optimal trajectory starting from the initial state and ending in the final state was smooth and had no peak value. On the other hand, the trajectory for actual arm motion fluctuated more widely. The mean time for the 16 subjects to complete the arm motion was 1.16±0.21sec (Table 3). Also presented in Table 3 were the mean and percentage of mechanical energy expended for optimized arm motion and those for actual arm motion. The mean of the energy expenditure for adopting the optimal trajectory was 15.55(J), while actual motion expended the energy at the mean of 16.64(J). That is, adopting the optimal trajectory required less energy, which was 93% of the mechanical energy expended in real motion. This result showed that adopting the optimal trajectory in lifting arms had more efficiency than adopting an individualized way.

As for the leg motion, the optimal trajectory for hip joint in the simulated motion is smoother, while the trajectory for actual leg motion fluctuated more widely (Fig.5(a)). The mean time for the 16 subjects to complete the leg motion was 1.09±0.07 sec (Table 3). Table 3 also demonstrated that the mean of the mechanical energy expenditure for real leg motion was 42.15±9.03(J), and that adopting the optimal trajectory required the mean energy expenditure of 34.13±4.4(J). It was found that adopting the optimal trajectory to swing the leg required less energy, which was 81% of the energy expenditure for real leg motion. This result showed that adopting the optimal trajectory in swinging the leg had more efficiency than adopting individualized ways.

Table 3. mean and percentage of mechanical energy expenditure for 16 subjects[8]

	Optimized(J) Mean±SD	Measured(J) Mean±SD	Time(sec) Mean±SD	Optimized/Measured (%)
Arm	15.55±3.07	16.64±3.79	1.16±0.21	93
Leg	34.13±4.4	42.15±9.03	1.09±0.07	81

The above results demonstrated that adopting the simulated optimal trajectory to perform the arm and leg motions required less energy than taking individualized ways. First, the energy expenditure for optimized arm motion of 16 subjects was 93 % of the energy expenditure for real arm motion done in an individualized way. Second, the energy expenditure for optimized leg motion of 16 subjects was 81% of the energy expenditure for real leg motion done in an individualized way. Therefore, what is viewed as the most comfortable way of exercising is not necessarily the most efficient one.

Another finding was that performing arm motion required less energy than performing leg motion. The result demonstrated that the former had more efficiency than the latter. This phenomenon evidenced that humans manipulate their arm better than their leg. Whether there are other factors contributing to this phenomenon is left for further research.

Appendix A. Matrix Ji

The *Ji* matrix of LEE could be written as followed[11,15]:

$$J_i=\begin{bmatrix} (-I_{xx}+I_{yy}+I_{zz})/2 & I_{xy} & I_{xz} & m\bar{x}_i \\ I_{xy} & (I_{xx}-I_{yy}+I_{zz})/2 & I_{yz} & m\bar{y}_i \\ I_{xz} & I_{yz} & (I_{xx}+I_{yy}-I_{zz})/2 & m\bar{z}_i \\ m\bar{x}_i & m\bar{y}_i & m\bar{z}_i & m_i \end{bmatrix}$$

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