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Hierarchical Fuzzy Control for C-Axis of CNC Turning Centers Using Genetic Algorithms

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Abstract. A combined PD and hierarchical fuzzy control is proposed for the low-speed control of the C-axis of CNC turning centers considering the effects of transmission flexibility and complex nonlinear friction. Learning of the hierarchical structure and parameters of the suggested control strategy is carried out by using the genetic algorithms. The proposed algorithm consists of two phases: the first one is to search the best hierarchy, and the second to tune the consequent center values of the constituent fuzzy logic systems into the hierarchy. For the least total control rule number, the hierarchical fuzzy controller is chosen to include only the simple two-input/one-output fuzzy systems, and both binary and decimal genes are used for the selection, crossover and mutation of the genetic algorithm. The proposed approach is validated by the computer simulation. Each generation consists of 30 individuals: ten reproduced from its parent generation, ten generated by crossover, and the other ten by mutation. In the simulations, the C-axis is assumed to be driven by a vector-controlled AC induction motor, and the dynamic friction model suggested by Canudas de Wit et al. in 1995 is used.

Key words: hierarchical fuzzy control, genetic algorithms, flexible C-axis, dynamic friction, low-speed control.

1. Introduction

Consideration of the effects of transmission flexibility and complex nonlinear friction often plays a key role in precision control systems design. Usually flexural couplings are introduced to accommodate undesirable error motions while being relatively rigid (with finite stiffness) along the desired motion direction. Modeling of the flexible motion and advanced control design are thus indispensable for a mechatronics device to achieve faster response and higher accuracy [1]. Nonlinear friction presents great impact on the servo dynamics especially for the fine motions and corresponding low speeds [2–4]. Stick-slip would occur when the commanded motion speed is below system's critical speed. To avoid stick-slip for much lower speed applications, such as the Czochralski silicon crystal growth systems [5] and the C-axes of CNC (computer numerical control) turning centers [6], is, thus, a challenging task considering both the effects of flexibility and friction.

Many model-based researches on friction control are known (we mention adaptive pulse width control [7], stiff PD feedback control [2], and model-based feedforward control [3]). Kato et al. [4] proposed to express the static and kinetic friction coefficients through simple formulas. Dahl [8] proposed a dynamic model for describing the spring like behavior during stiction (the so-called Dahl effect). Bo and Pavelescu [9], Hess and Soon [10] suggested traditional static friction models. Armstrong-Helouvry [3] proposed a seven-parameter friction model. Recently, Canudas de Wit et al. [11] proposed an elegant dynamic model useful for friction control and simulation. This model can capture most of the friction behaviors observed experimentally, including the Stribeck effect (the destabilizing effect at a very low velocity), hysteresis, Dahl effect, and varying break-away force.

There exist few model-free approaches for the control design of precision motion systems. Tung et al. [12] proposed a repetitive control for friction compensation to reduce the contouring error of an X-Y table. Lin and Uen [13] suggested an error back propagation-based fuzzy learning control for friction compensation. Lin and Lin [14] proposed a genetic algorithm-based fuzzy control for enhancing the transient performance and robustness of a stable adaptive control system. Genetic algorithms are the global search/optimization methods developed from the theory of biological evolution [15–18]. The structure and parameters of a fixed-parameter or adaptive fuzzy system can be systematically learned using genetic algorithms [e.g., 19, 20].

Even a single-axis system which takes into consideration the flexibility and dynamic friction behavior, is a high-order uncertain dynamical system. Introduction of a suitable fuzzy control system for enhancing its performance would need not merely two input variables but the multiple ones. As the number of the input variables increases, the required number of fuzzy rules to construct a complete fuzzy system will grow exponentially, which is identified as "the curse of dimensionality" [21–23]. The hierarchical fuzzy system is one approach to deal with the difficulty of the design and implementation of a multi-input fuzzy system [22-24]. In this paper, we will propose a combined PD and hierarchical fuzzy control with five input variables for low-speed control of the single-axis drive systems using the C-axis (servo control of the spindle) of a CNC turning center as the simulation example. The effects of the flexibility and dynamic friction are included in the learning using genetic algorithms. The suggested algorithm consists of two phases: the first one is for searching the best hierarchical structure, and the second for tuning the consequent values of the two-input/one-output constituent fuzzy systems into the hierarchy. In the simulations, vector-controlled AC induction motor and dynamic friction model suggested by Canudas de Wit et al. [11] are adopted.

2. Modeling of a Flexible C-Axis with Nonlinear Dynamic Friction

In this study, we consider the servo control of the spindle, the C-axis, of a CNC turning center as shown in Figure 1. Precision and the low-speed controls are the main problems of a C-axis design. For performance improvement, a more complete model of the flexible drive system is needed for computer simulation

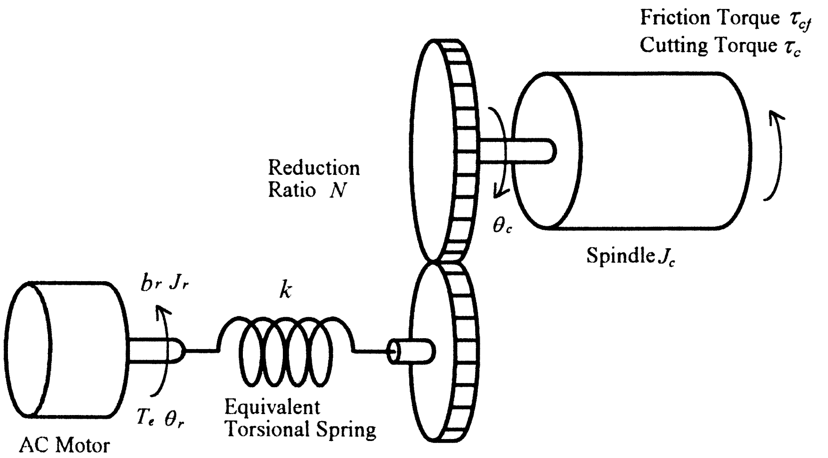


Figure 1. A flexible spindle system.

during the genetic algorithm-based design process. Mathematical model considering transmission flexibility, actuator dynamics, and friction, can be derived in a straightforward way and is listed below:

$$J_r\ddot{\theta}_r + b_r\dot{\theta}_r + k(\theta_r - N\theta_c) = T_e, \tag{1a}$$

$$J_c\ddot{\theta}_c + (\tau_{cf} + \tau_c) = Nk(\theta_r - N\theta_c), \tag{1b}$$

$$L_\sigma \dot{i}_{qs} + R_\sigma i_{qs} + \frac{L_s\lambda_r}{L_m}\dot{\theta}_r = v_{qs}, \tag{1c}$$

$$T_e = \frac{P}{2}\left(\frac{L_m\lambda_r}{L_r}\right)i_{qs}, \tag{1d}$$

$$\tau_{cf} = \sigma_0 z + \sigma_1\frac{dz}{dt} + \sigma_2\dot{\theta}_c, \tag{1e}$$

$$\frac{dz}{dt} = \dot{\theta}_c - \frac{|\dot{\theta}_c|}{g(\dot{\theta}_c)}z, \tag{1f}$$

$$\sigma_0 g(\dot{\theta}_c) = T_c + (T_s - T_c)\,e^{-(\dot{\theta}_c/\omega_s)^2}. \tag{1g}$$

Here θ_r and θ_c are, respectively, the rotor angle of the vector-controlled AC induction motor and the spindle angle; J_r and J_c are, respectively, the moments of inertia of the motor rotor and spindle; b_r is the viscous damping coefficient on the motor rotor side; k is the equivalent spring stiffness representing the transmission flexibility; N is the speed reduction ratio; T_e is the motor torque modeled by (1c) and (1d); τ_c is the load torque resulting from the cutting force; τ_{cf} is the friction torque modeled by (1e)–(1g), i.e., the new dynamic friction model suggested by Canudas de Wit et al. [11].

In the dynamics model (Equations (1c) and (1d)) of a vector-controlled AC induction motor [25], v_{qs} is the stator voltage component in the quadrature axis (q-

axis) of the rotating rotor field frame; $\dot{\theta}_r$ is the rotor velocity; i_{qs} is the stator current component in the q-axis of the rotating field frame; L_s is the stator self-inductance; L_m is the mutual inductance; L_r is the rotor self-inductance; λ_r = constant, is the rotor flux linkage;

$$L_\sigma = L_s - \frac{L_m^2}{L_r}; \qquad R_\sigma = R_s + R_r \frac{L_s}{L_r},$$

R_s and R_r are, respectively, the stator and rotor resistance; P is the number of poles.

In the dynamic friction model (1e)–(1g), $\dot{\theta}_c$ is the spindle velocity; z is the average deflection of the bristles (which can deflect like springs) between the two contact surfaces and modeled by Equation (1f); σ_0 is the stiffness, and σ_1 is the damping coefficient of the bending bristles; σ_2 is the viscous coefficient; parameterization of $\sigma_0 g(\dot{\theta}_c)$ using (1g) is used to describe the Stribeck effect; T_c is the Coulomb friction torque; T_s is the stiction torque, and ω_s is the Stribeck velocity.

Defining the state variables as $x_1 = i_{qs}$, $x_2 = \theta_r$, $x_3 = \dot{\theta}_r$, $x_4 = \theta_c$, $x_5 = \dot{\theta}_c$, and $x_6 = z$, the state equation of the dynamical C-axis system can be expressed as follows:

$$\begin{aligned}
\dot{x}_1 &= -\frac{R_\sigma}{L_\sigma}x_1 - \frac{L_s\lambda_r}{L_\sigma L_m}x_3 + \frac{1}{L_\sigma}v_{qs}, \\
\dot{x}_2 &= x_3, \\
\dot{x}_3 &= -\frac{k}{J_r}x_2 - \frac{b_r}{J_r}x_3 + \frac{kN}{J_r}x_4 + \frac{P}{2J_r}\left(\frac{L_m\lambda_r}{L_r}\right)x_1, \\
\dot{x}_4 &= x_5, \\
\dot{x}_5 &= \frac{kN}{J_c}x_2 - \frac{kN^2}{J_c}x_4 - \frac{1}{J_c}\tau_c - \frac{1}{J_c}\tau_{cf}, \\
\dot{x}_6 &= x_5 - \frac{|x_5|}{g(x_5)}x_6.
\end{aligned} \tag{2}$$

3. Hierarchical Fuzzy Control Using Genetic Algorithms

The idea of hierarchical fuzzy control systems proposed by Raju et al. [22] is to put the multi-input variables into a collection of lower-dimensional fuzzy systems, instead of only a single high-dimensional fuzzy system. In this section, design methodology of an integrated PD and hierarchical fuzzy control with five input variables using genetic algorithms [19, 20] will be proposed for the low-speed control of C-axis of a CNC turning center.

The PD control is included in the control strategy due to its well-known damping improvement capability. The hierarchical fuzzy control part is for the potential enhancement of the transient and steady state performances due to its nonlinear universal approximation characteristics [24, 26]. The possible best gains of the PD control, and the optimal structure and consequent center values of the hierarchical

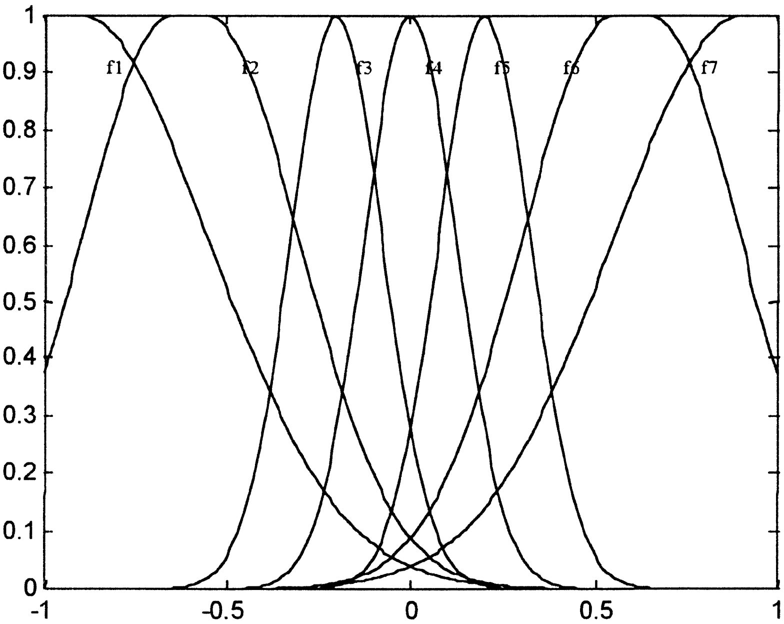


Figure 2. Membership functions of the linguistic terms for each input variable of the hierarchical fuzzy system.

fuzzy rule bases will be learned using genetic algorithms. The hierarchical fuzzy controller is set to be composed of only usual simple two-input/one-output fuzzy systems. The linguistic terms (fuzzy subsets) defined in the normalized universe, say $[-1, 1]$, of each input of the hierarchical fuzzy system are chosen as shown in Figure 2. The membership functions of used seven linguistic terms are listed as follows:

$$f_1(u^*) = \begin{cases} \exp\left[-4\left(\left|u^* - (-1)\right| - 0.1\right)^2\right] & \text{for } |u^* - (-1)| \geq 0.1, \\ 1 & \text{for } |u^* - (-1)| \leq 0.1, \end{cases} \tag{3a}$$

$$f_2(u^*) = \begin{cases} \exp\left[-8\left(\left|u^* - (-0.6)\right| - 0.05\right)^2\right] & \text{for } \left|u^* - (-0.6)\right| \geq 0.05, \\ 1 & \text{for } \left|u^* - (-0.6)\right| \leq 0.05, \end{cases} \tag{3b}$$

$$f_3(u^*) = \exp\left[-32\left(\left|u^* - (-0.2)\right|\right)^2\right], \tag{3c}$$

$$f_4(u^*) = \exp\left[-32\left(\left|u^*\right|\right)^2\right], \tag{3d}$$

$$f_5(u^*) = \exp\left[-32\left(|u^* - 0.2|\right)^2\right], \tag{3e}$$

$$f_6(u^*) = \begin{cases} \exp\left[-8\left(|u^* - 0.6| - 0.05\right)^2\right] & \text{for } |u^* - 0.6| \geq 0.05, \\ 1 & \text{for } |u^* - 0.6| \leq 0.05, \end{cases} \tag{3f}$$

$$f_7(u^*) = \begin{cases} \exp\left[-4\left(|u^* - 1| - 0.1\right)^2\right] & \text{for } |u^* - 1| \geq 0.1, \\ 1 & \text{for } |u^* - 1| \leq 0.1. \end{cases} \tag{3g}$$

Therefore, each constituent fuzzy system has only 49 rules, and their consequent values (called parameters of the hierarchical fuzzy controller) can be systematically learned using genetic algorithms.

Simplified fuzzy reasoning method using singleton fuzzifier, product inference engine, and center average defuzzifier [24, 26] is chosen for the nonlinear input/output mapping evaluation of each constituent fuzzy system:

$$\begin{aligned} y^* &= \frac{\sum_{j=1}^{n_\mathrm{r}} \mu_j w_j}{\sum_{j=1}^{n_\mathrm{r}} \mu_j}, \\ \mu_j &= f_{1j}\bigl(u_1^*\bigr) f_{2j}\bigl(u_2^*\bigr), \end{aligned} \tag{4}$$

where μ_j is the degree of firing (DOF) of the jth rule of some fuzzy system unit for the crisp input pair (u_1^*, u_2^*); $f_{1j}(u_1^*)$ is the membership degree of the first crisp input u_1^* in jth rule's first antecedent linguistic term f_{1j}; $f_{2j}(u_2^*)$ is the membership of the second crisp input u_2^* in jth rule's second antecedent linguistic term f_{2j}; w_j is jth rule's consequent center value; and $n_\mathrm{r} = 7 \times 7 = 49$ is the number of control rules of the fuzzy unit; y^* is the crisp output value for the crisp input (u_1^*, u_2^*).

3.1. BASICS OF GENETIC ALGORITHMS

Most genetic algorithms (GAs) have at least three types of operators: selection, crossover, and mutation [18].

The individuals (or called chromosomes) in a GA population (or called generation) typically take the form of binary (0 and 1) bit strings. However, decimal chromosomes could also be used for ease of computation while the number of parameters is large [27]. Each individual represents one candidate solution in the defined search space. The GA successively processes generations of individuals by replacing one parent generation with one new generation through the operators. A suitable fitness function is required for assigning a score (fitness) to each individual in the current generation. The fitness of an individual stands for how well it solves the problem. In this study, we consider the following fitness function for servo control requirement:

$$f = a\left(\frac{1000}{\sum_{t=1}^{T} |e(t)|}\right) + b\left(\frac{1000}{\sum_{t=1}^{T} |\dot{e}(t)|}\right), \tag{5}$$

where t is the discrete time index, T is the final time step, and the weights are set as $a = 3$ and $b = 0.1$ in the simulations.

SELECTION. The selection operator selects individuals in the population for reproduction. The fitter the individual, the more probability of selection it is likely to be selected to reproduce.

CROSSOVER. The crossover operator randomly chooses a locus and exchanges the subsequences before and after that locus between two selected individuals to create two offspring. This is the so-called “single-point crossover.” There are also “multi-point crossover” versions of crossover. For example, two parent strings 11000001 and 00111110 could be crossed over by randomly choosing two points $m = 2$ and $n = 7$, and exchanging the subsequences between these two points to produce the two offspring 11111111 and 00000000. Two decimal individuals could be crossed over using weighted average operator [27]. For example, by randomly choosing a number $\gamma \in (0, 1)$, two individuals A and B might be crossed over to produce two offspring: $A' = \gamma A + (1 - \gamma)B$ and $B' = \gamma B + (1 - \gamma)A$.

MUTATION. The mutation operator randomly flips some of the bits in an individual. For example, the string 11111111 might be mutated in its third and seventh bits to yield the new string 11011101. Mutation can occur at each bit position in an individual with some mutation rate. A decimal individual A could be mutated by randomly selecting a mutation rate $\beta \in (0, 1)$ to produce the new individual $A' = A + (2\beta - 1)A_{\mathrm{max}}$, where A_{max} is the selected maximum mutation.

3.2. A GENETIC ALGORITHM FOR LEARNING THE COMBINED PD AND HIERARCHICAL FUZZY CONTROLLER

Multi-input fuzzy systems are rather difficult to design via heuristic approach due to the curse of dimensionality. Hierarchical structure is a very effective way for reducing the required total rule number of a complete multivariable fuzzy system with selected fixed fuzzy partitioning of the domain. If the basic fuzzy systems constructing the hierarchical system are all two-input/one-output fuzzy systems, then the required total number of rules is minimal [24]. Usually a mechatronic system is governed both by flexibility and nonlinear friction, either for fast or low-speed operations, these effects should be considered in the servo control design for enhancing its transient and steady state performances. In this study, we consider the low-speed control for C-axis of a CNC turning center, and select a hierarchical fuzzy controller with five input variables: e (spindle tracking error), δ (deflection of the equivalent spring), $\dot{\delta}$ (time derivative of the deflection), $\dot{\theta}_{\mathrm{c}}(t)$ (current spindle velocity), and $\dot{\theta}_{\mathrm{c}}(t-1)$ (past spindle velocity at time $t-1$).

The learning flow chart of a combined PD and hierarchical fuzzy control using genetic algorithms with two phases can be shown in Figure 3, where the PD control part is

$$v_{\mathrm{c}1} = k_{\mathrm{p}}e + k_{\mathrm{d}}\dot{e}. \tag{6}$$

Here, e is the spindle tracking error, k_{p} and k_{d} are the proportional and derivative (PD) control gains. One phase is for the search of the most suitable hierarchy with all constituent two-input/one-output fuzzy systems, and the other for the search of PD gains and all the consequent centers of the constituent simple fuzzy systems.

The off-line learning algorithm is proposed as follows:

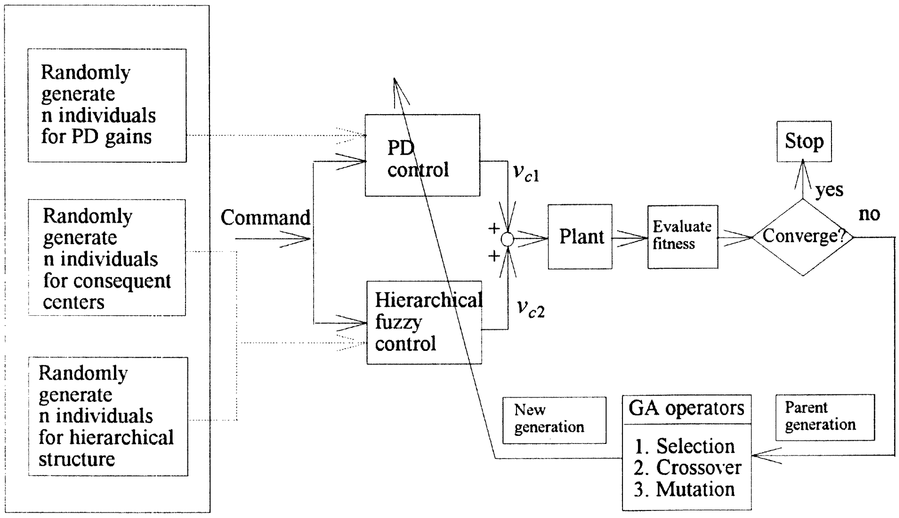


Figure 3. Learning flow chart for the combined PD and hierarchical fuzzy control system.

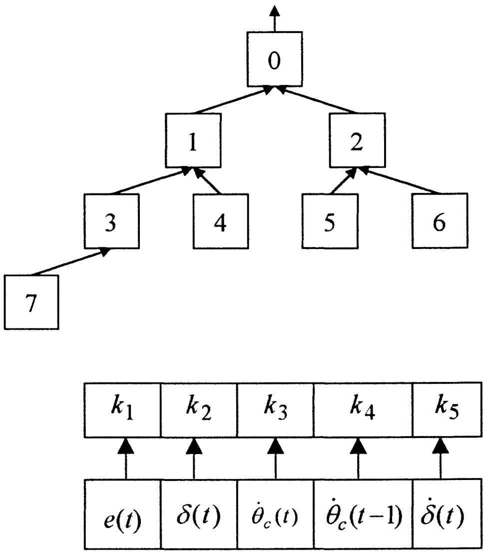


Figure 4. Initial hierarchical structure for the case with five input variables.

1. First determine the required number U of the constituent fuzzy systems: $U = 2^{i-2}$, where i is the number of input variables of the hierarchical fuzzy controller [19], and construct the hierarchical fuzzy system with binary tree structure in a top-down and left-right manner. For example, when $i = 5$ for the C-axis system, we could construct the hierarchy with at most 8 fuzzy units as shown in Figure 4; k_i, $i = 1, 2, \ldots, 5$, are the scaling factors of the input variables. Each input variable is allocated a gene in the individual for representing which fuzzy unit it will be connected to. For example, as shown in Figure 5, a

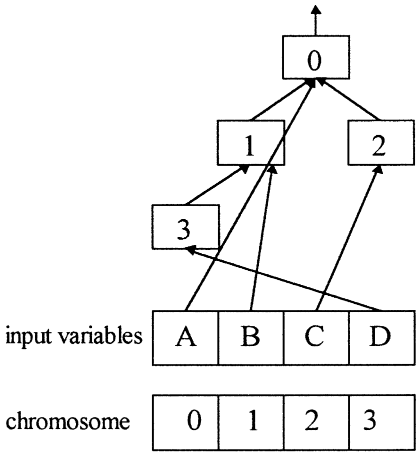


Figure 5. Connection of the input variables for the gene code 0123 when $i = 4$.

set of gene codes 0123 means that the input variable A is to be connected to the fuzzy unit 0, B connected to the fuzzy unit 1, C connected to the fuzzy unit 2, and D connected to the fuzzy unit 3.

2. **Phase (a):**
   (i) Start with a randomly generated population of n (= 30, in the simulation study) binary individuals. Each individual contains i $(i - 2)$-bit genes representing the connection between the input variables and the fuzzy units.
   (ii) Check the connection between the input variables and the fuzzy units for each individual. Delete those fuzzy units with no input variables connected to them. If a fuzzy unit has only one input (including the intermediate output from a lower-level fuzzy unit) connected to it, then it should be deleted from the structure and the very input is directly re-connected to its next higher-level fuzzy unit.
   (iii) Check whether the reduced hierarchy is composed of all two-input fuzzy units. If not, the individual must be replaced by a new randomly generated individual, until n individuals with desired simplest hierarchy are generated.
3. **Phase (b):**
   (i) Start with a randomly generated population of n (= 30) decimal individuals with each composed of k_p and k_d gains.
   (ii) Start with a randomly generated population of n (= 30) decimal individuals with each composed of all the consequent center values of the rule bases of the constituent fuzzy units. Each individual thus has $49(i-1)$ (i is the total number of input variables) real numbers belong to the normalized universes of the outputs of the fuzzy units.

Notice that the three sets of respective n individuals constitute the n composite individuals in a generation of the genetic algorithm. Each individual represents a candidate structure and parameters design of the combined PD and hierarchical fuzzy controller.

4. Conduct closed-loop system simulation and calculate the fitness f (Equation (5)) for each composite individual.
5. Repeat the following steps until n offspring have been created:
   (a) Select the top $n/3$ individuals with best fitness to reproduce the first $n/3$ composite offspring.
   (b) With crossover probability p_{c1}, two individuals (only the part describing the hierarchical structure) are crossed over using "two-point crossover" operation to form two new genes. Check whether the two genes are with simplified hierarchy, and take only the suitable genes as offspring. Repeat this crossover process until $n/3$ offspring are created. From these $n/3$ offspring, find two offspring with identical hierarchical genes. Find two parent individuals with this same hierarchical gene. With crossover probability p_{c2}, their gene codes representing consequent center values and PD gains are respectively crossed over using weighted average crossover, and then integrated with the hierarchical gene part to create two composite offspring. Find the other pairs with their same hierarchical gene and do their respective crossovers. For the remaining single new hierarchy individuals, randomly generate their respective center values and PD gains to form the second $n/3$ composite offspring.
   (c) Randomly select $n/3$ parent individuals and perform their respective mutations to produce the third $n/3$ composite offspring.
6. Replace the parent population with the new population.
7. Go to step 4 until the fitness converges.
8. Select the composite chromosome with largest fitness as the desired combined PD and hierarchical fuzzy controller.

4. Simulation Examples

In this section, the suggested genetic algorithm for the search of the best hierarchical structure, rule bases, and PD gains for C-axis of a turning center will be tested. Extensive simulations are conducted and only the representative examples are to be illustrated.

The parameters of the C-axis are selected as follows:

$$\begin{aligned}
&\lambda_r = 0.6\ \mathrm{H\,A}, \quad R_s = 1.12\ \Omega, \quad R_r = 1.05\ \Omega,\\
&L_s = 0.341\ \mathrm{H}, \quad L_r = 0.306\ \mathrm{H}, \quad L_m = 0.225\ \mathrm{H},\\
&J_r = 0.11\ \mathrm{kg\,m^2}, \quad b_r = 2.05\ \mathrm{N\,m\,s/rad}, \quad J_c = 5.102\ \mathrm{kg\,m^2},\\
&N = 1.0, \quad k = 200\ \mathrm{N\,m/rad}, \quad \sigma_0 = 17000.0, \quad \sigma_1 = 126.5, \quad \sigma_2 = 0.4,\\
&T_c = 0.7\ \mathrm{N\,m}, \quad T_s = 1.2\ \mathrm{N\,m}, \quad \text{and} \quad \omega_s = 0.003\ \mathrm{rad/s}.
\end{aligned}$$

The scaling factors for the input variables of the hierarchical fuzzy controller are selected as: $k_1 = 50$, $k_2 = 80$, $k_3 = 20$, $k_4 = 20$, $k_5 = 10$. The scaling factor for the output variable is $k_6 = 10$. The number of chromosomes in each population is $n = 30$; the crossover probability and mutation rate for the structure genes are: $p_{c1} = 0.3$, $\alpha = 0.5$; the crossover probability and mutation rate for the consequent center values and PD gains are: $p_{c2} = 0.7$, $\beta = 0.8$; and the maximum mutation value is $A_{max} = 10$

4.1. NO-DISTURBANCE TORQUE CASE

The fitness convergence is shown in Figure 6(a) for the flexible C-axis system with nonlinear friction when ramp command $\theta_{c,d}(t) = 0.02t$ (rad) is used and no-disturbance torque occurs. The learned best structure of the hierarchical fuzzy controller is shown in Figure 6(b) where the fuzzy system units are all two-input/one-output fuzzy systems with their rule bases listed in Tables I(a)–I(d). These tables show the learned consequent center values of the fuzzy rules. Notice that $\dot{\theta}_c(t)$ and $\dot{\theta}_c(t-1)$ are fed to the lowest level's fuzzy system unit 7. Output of fuzzy system unit 7 and $\dot{\delta}$ are fed to fuzzy system unit 3. Output of fuzzy system unit 3 and e are fed to fuzzy system unit 1. Output of fuzzy system unit 1 and δ are fed to the highest level's fuzzy system unit 0. The output of fuzzy unit 0 is the normalized control signal of the hierarchical fuzzy controller, and the scaling factor for it is selected as 10. Fuzzy units 2, 4, 5 and 6 are not necessary and thus excluded in the hierarchy. The learned PD gains are $k_p = 758.405$, $k_d = 743.010$. The simulation results are shown in Figures 6(c)–6(f). Figures 6(c) and 6(d) illustrate the ramp tracking response and the tracking error $(\theta_c(t) - \theta_{c,d}(t))$. Figure 6(e) shows the deflection of the equivalent spring, and Figure 6(f) presents the required control

Table I. Consequent center values of the learned hierarchical fuzzy controller: (a) rule table of fuzzy unit 7; (b) rule table of fuzzy unit 3; (c) rule table of fuzzy unit 1; (d) rule table of fuzzy unit 0

(a)

Unit 7		Input 1 ($\dot{\theta}_c(t)$)						
		f1	f2	f3	f4	f5	f6	f7
Input 2 ($\dot{\theta}_c(t-1)$)	f1	0.4479	0.4644	0.5193	0.4429	0.4474	0.5092	0.4609
	f2	0.4410	0.4987	0.4532	0.4395	0.4500	0.4598	0.4580
	f3	0.4668	0.4753	0.4449	0.5077	0.4519	0.4792	0.4425
	f4	0.4250	0.4246	0.4523	0.4853	0.4826	0.4942	0.4673
	f5	0.4944	0.3906	0.4204	0.4767	0.4868	0.4666	0.4747
	f6	0.4420	0.4209	0.5052	0.4113	0.4799	0.5038	0.5241
	f7	0.4585	0.4143	0.4921	0.3954	0.4478	0.5427	0.4560

Table I. (Continued.)

(b)

Unit 3		Input 1 (Output of unit 7)						
		f1	f2	f3	f4	f5	f6	f7
	f1	0.4532	0.4958	0.4149	0.4290	0.4036	0.4592	0.4785
	f2	0.4434	0.4608	0.4974	0.5115	0.4442	0.4814	0.4595
Input 2	f3	0.4346	0.4562	0.4834	0.4323	0.4926	0.4106	0.4683
($\dot{\delta}$)	f4	0.4544	0.4411	0.4944	0.4539	0.5006	0.5174	0.4226
	f5	0.4803	0.5058	0.4063	0.4360	0.4516	0.4327	0.5117
	f6	0.4652	0.4699	0.4651	0.4706	0.4882	0.4699	0.5079
	f7	0.5049	0.4577	0.4851	0.5124	0.4849	0.4643	0.4265

(c)

Unit 1		Input 1 (Output of unit 3)						
		f1	f2	f3	f4	f5	f6	f7
	f1	0.5577	0.4663	0.5001	0.4778	0.4531	0.4599	0.4630
	f2	0.4112	0.4472	0.4321	0.5126	0.4383	0.5112	0.4697
Input 2	f3	0.4869	0.4392	0.3917	0.4488	0.4576	0.5630	0.4529
(e)	f4	0.4915	0.5307	0.5026	0.4494	0.4507	0.4332	0.4437
	f5	0.4631	0.4287	0.4371	0.4660	0.4821	0.3741	0.4612
	f6	0.4608	0.4472	0.4977	0.4970	0.4787	0.4856	0.4357
	f7	0.4318	0.4751	0.4934	0.5007	0.4625	0.4824	0.4604

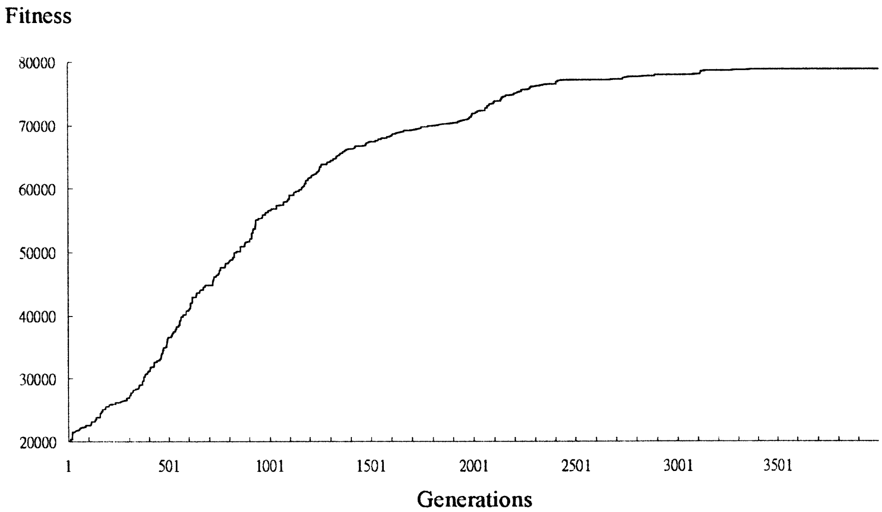
(d)

Unit 0		Input 1 (Output of unit 1)						
		f1	f2	f3	f4	f5	f6	f7
	f1	0.4862	0.5139	0.4347	0.4917	0.5337	0.3605	0.4005
	f2	0.4510	0.4805	0.4721	0.4811	0.4628	0.4058	0.3632
Input 2	f3	0.4478	0.4526	0.4254	0.4876	0.4564	0.4322	0.4714
(δ)	f4	0.4656	0.4698	0.4782	0.5103	0.4640	0.5429	0.5347
	f5	0.4422	0.4434	0.5106	0.4357	0.4315	0.5088	0.5191
	f6	0.4970	0.4342	0.4846	0.4121	0.4244	0.5335	0.5114
	f7	0.5285	0.4817	0.4522	0.4152	0.4627	0.4970	0.4745

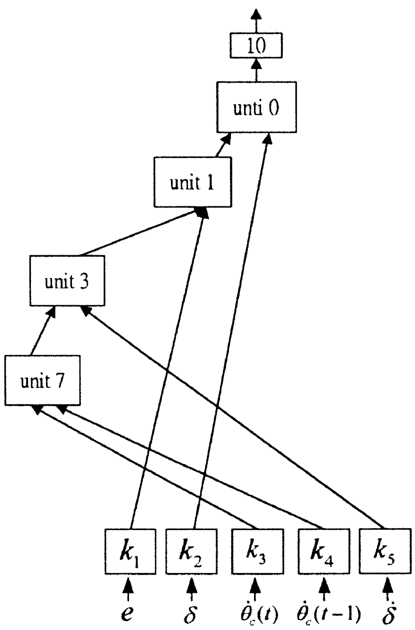
voltage. From these results we know that there exists no stick-slip and the control performance is excellent even in this very low speed (0.02 rad/s).

4.2. WITH-DISTURBANCE TORQUE CASE

Consider that the C-axis has a disturbance torque due to the metal cutting process

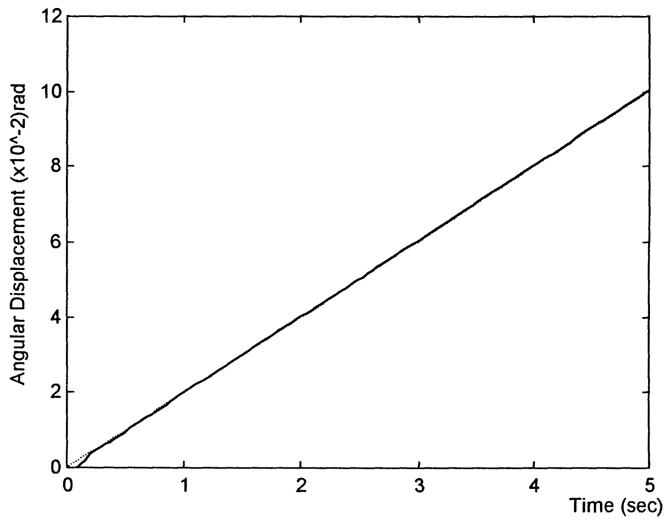


(a)

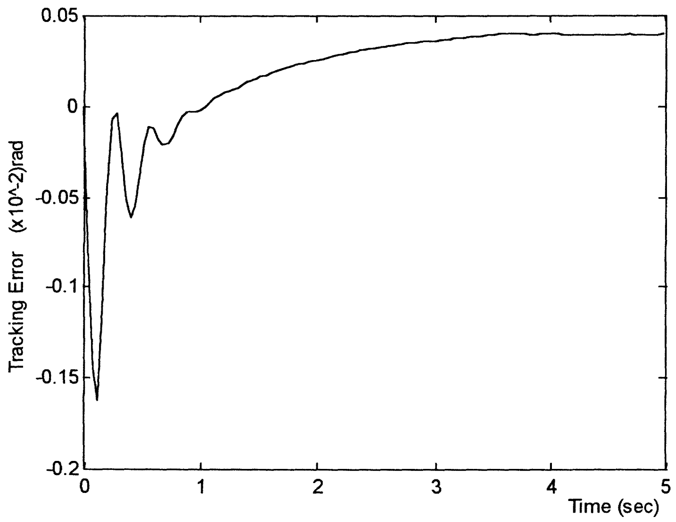


(b)

Figure 6. Learned combined PD and hierarchical fuzzy controller for low-speed control: (a) fitness convergence; (b) best hierarchical structure; (c) control response; (d) tracking error; (e) spring deflection; (f) control voltage.



(c)

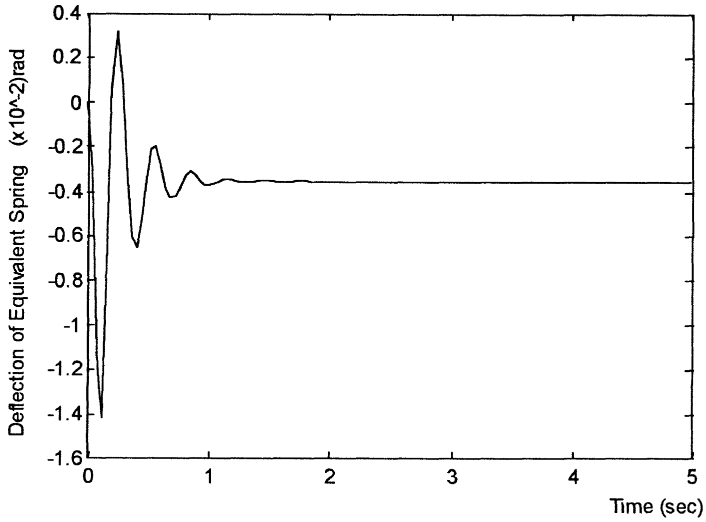


(d)

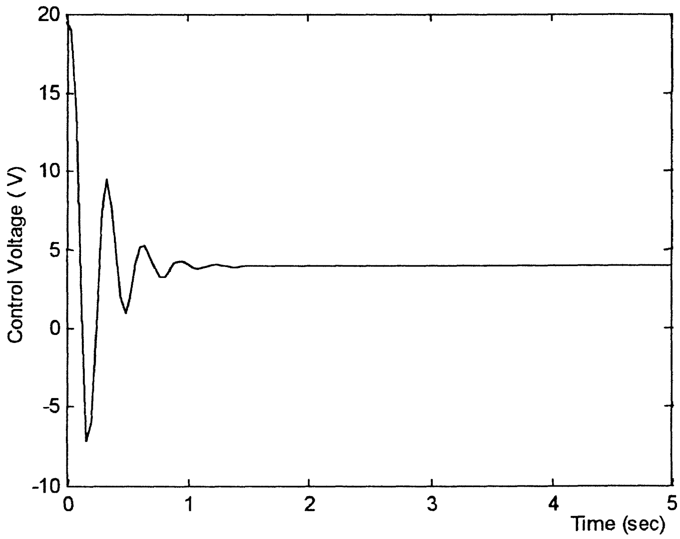
Figure 6. (Continued.)

$$\tau_c = 5 + 2\sin t + \sin\left(50t + \frac{\pi}{4}\right) \quad (\mathrm{N\,m}) \tag{7}$$

The fitness convergence of a GA learning is shown in Figure 7(a). The structure of the best hierarchical fuzzy controller is shown in Figure 7(b). Notice that e and $\dot{\theta}_c(t-1)$ are fed to the lowest level's fuzzy unit 3; output of fuzzy unit 3 and δ are fed to fuzzy unit 1; $\dot{\theta}_c(t)$ and $\dot{\delta}$ are fed to fuzzy unit 2; outputs of the fuzzy unit 1 and 2 are fed to fuzzy unit 0. The output of fuzzy unit 0 is the normalized control signal



(e)



(f)

Figure 6. (Continued.)

of the hierarchical fuzzy controller, and the scaling factor for it is also selected as 10. Fuzzy units 4–7 are not necessary and thus excluded in the hierarchy. The learned four rule bases of the fuzzy units 0–3 are shown in Table II. The learned PD control gains are $k_\mathrm{p} = 1002.076$, $k_\mathrm{d} = 135.007$. The simulation results are shown in Figures 7(c)–7(f). From these we know that the low-speed tracking purpose can still be obtained while the system has severe disturbance torque. And the effect of disturbance can be greatly attenuated by the learned PD and hierarchical fuzzy

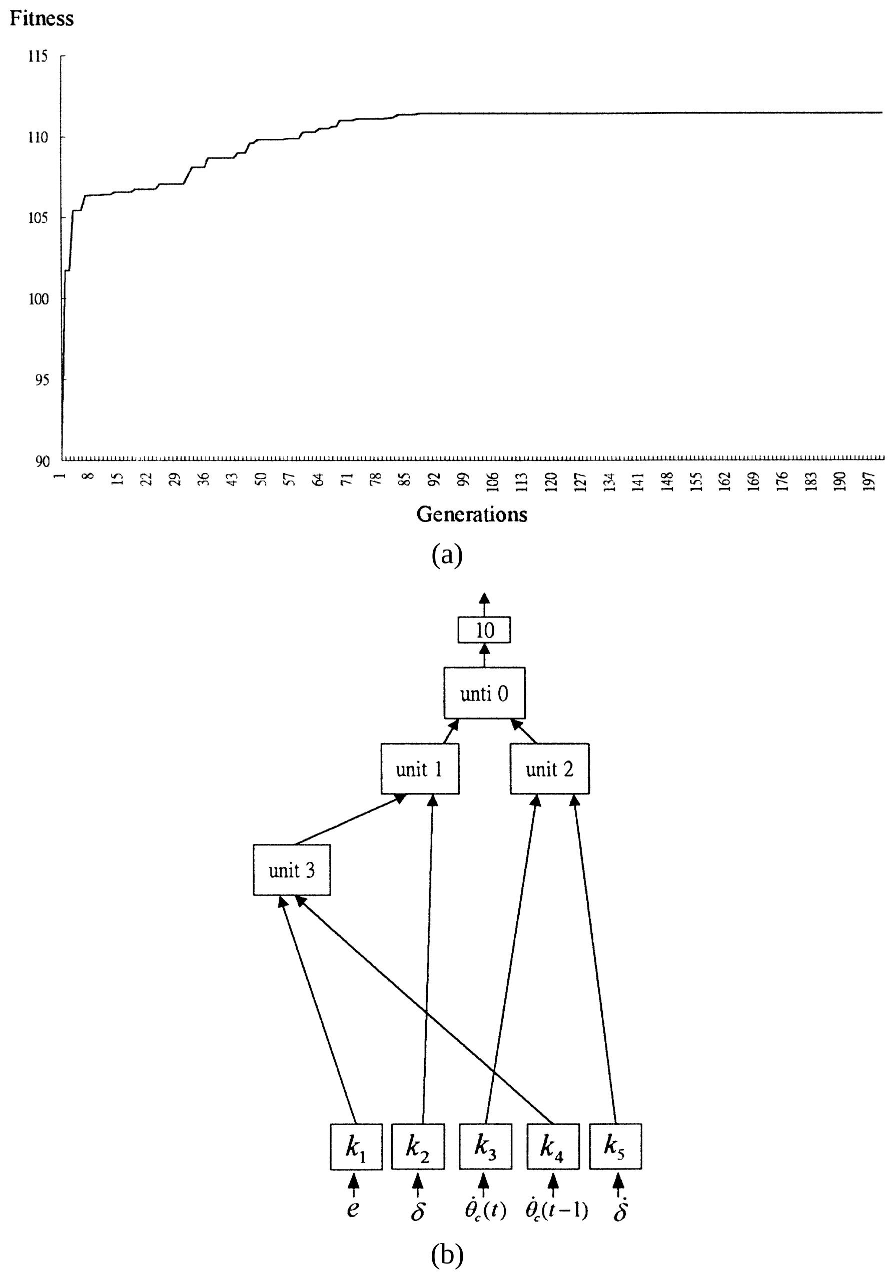
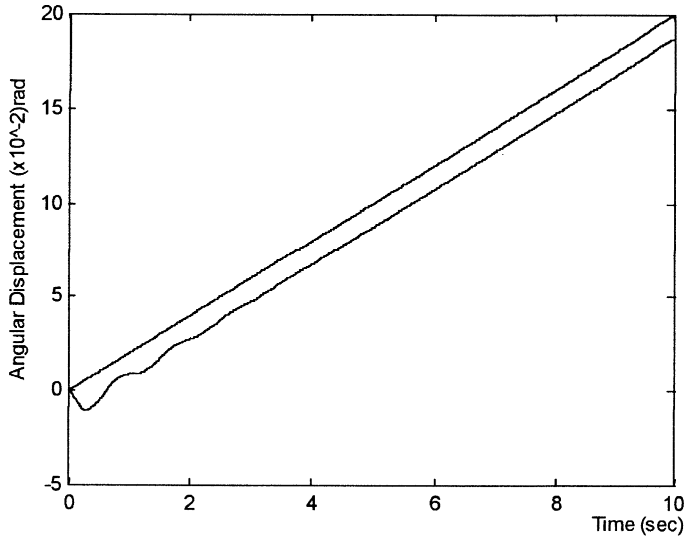
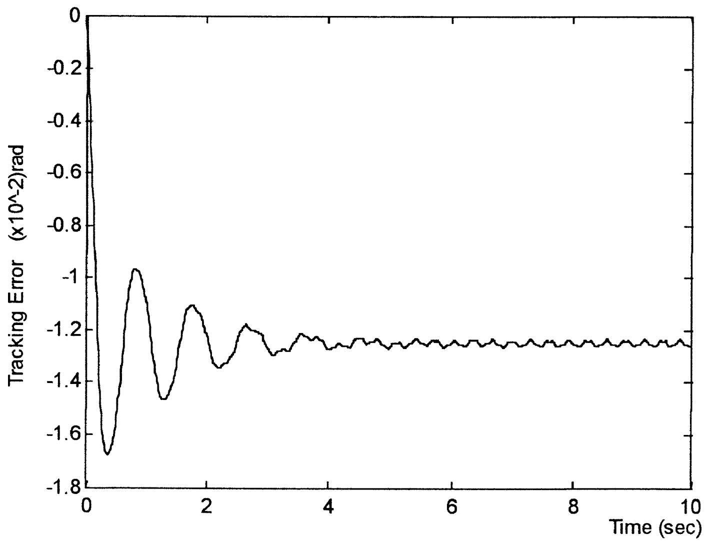


Figure 7. Learned combined PD and hierarchical fuzzy controller for low-speed control with cutting torque disturbance: (a) fitness convergence; (b) best hierarchical structure; (c) control response; (d) tracking error; (e) spring deflection; (f) control voltage.



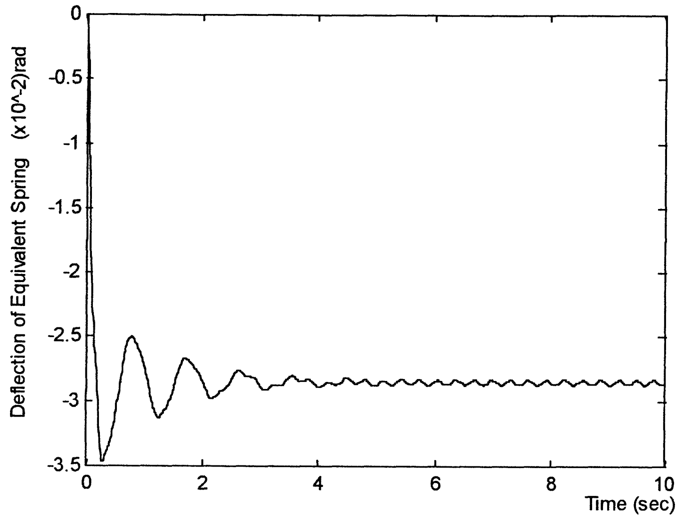
(c)



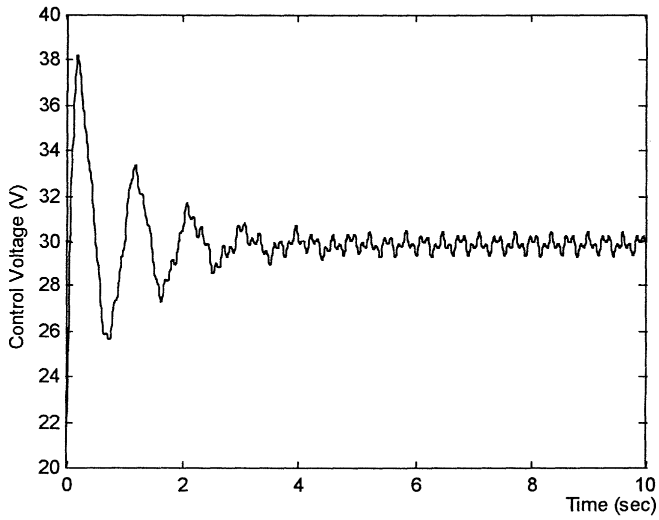
(d)

Figure 7. (Continued.)

controller. Notice that the suggested learning algorithm can automatically search the most suitable input variables pairing of the required constituent fuzzy systems. In order to compensate for the severe disturbance torque, the learned hierarchy is different from that for the nominal case. In the above two cases for low-speed smooth control, the controlled systems would find serious stick-slip phenomena while only using simple PD control.



(e)



(f)

Figure 7. (Continued.)

5. Conclusions

Since the effects of transmission flexibility and friction govern the dynamic behavior of a precision motion control system, to systematically take them into account for advanced servo system design is a very important problem in mechatronics. A combined PD and hierarchical fuzzy control for the C-axis (the spindle with servo control) of CNC turning centers is considered in this paper. A genetic algorithm-based evolutionary learning approach to the search for the best hierar-

Table II. Consequent center values of the learned hierarchical fuzzy controller with cutting torque disturbance: (a) rule table of fuzzy unit 3; (b) rule table of fuzzy unit 1; (c) rule table of fuzzy unit 2; (d) rule table of fuzzy unit 0

(a)

Unit 3		Input 1 (e)						
		f1	f2	f3	f4	f5	f6	f7
	f1	1.6274	1.5921	1.6097	1.6199	1.6045	1.5911	1.6524
	f2	1.5891	1.7120	1.6551	1.6073	1.6550	1.6539	1.6666
Input 2	f3	1.7289	1.6791	1.6704	1.5821	1.5578	1.5965	1.6874
($\dot{\theta}_C(t-1)$)	f4	1.5751	1.6794	1.7115	1.5599	1.6412	1.7350	1.5893
	f5	1.6101	1.6358	1.7218	1.6973	1.7003	1.6141	1.5716
	f6	1.6441	1.6205	1.7027	1.5643	1.6284	1.5693	1.7080
	f7	1.6110	1.6884	1.5447	1.6461	1.7235	1.5700	1.7241

(b)

Unit 1		Input 1 (Output of unit 3)						
		f1	f2	f3	f4	f5	f6	f7
	f1	1.6790	1.5474	1.5589	1.6914	1.5413	1.6040	1.7148
	f2	1.6924	1.5467	1.6181	1.6121	1.6728	1.5617	1.6870
Input 2	f3	1.6796	1.6509	1.6670	1.7081	1.6772	1.6103	1.7385
(δ)	f4	1.5738	1.7142	1.6031	1.6291	1.6026	1.7385	1.7287
	f5	1.6004	1.6147	1.6563	1.7202	1.6357	1.5469	1.7013
	f6	1.7246	1.6249	1.7282	1.6421	1.7210	1.5457	1.7171
	f7	1.5920	1.5647	1.6161	1.6574	1.6331	1.5496	1.6425

(c)

Unit 2		Input 1 $\dot{\theta}_C(t)$)						
		f1	f2	f3	f4	f5	f6	f7
	f1	1.6777	1.7269	1.6951	1.6542	1.5526	1.6286	1.5916
	f2	1.7209	1.6318	1.7346	1.6994	1.6461	1.7005	1.6017
Input 2	f3	1.6260	1.5830	1.5456	1.6801	1.6394	1.5843	1.5478
($\dot{\delta}$)	f4	1.6630	1.6502	1.6043	1.7377	1.7236	1.6872	1.5940
	f5	1.5909	1.6124	1.6317	1.6590	1.7134	1.5647	1.6970
	f6	1.5778	1.7378	1.6295	1.6442	1.7166	1.5922	1.5602
	f7	1.7161	1.5997	1.5529	1.7294	1.5466	1.6912	1.5605

chical structure of the five input-variable fuzzy controller and the parameters of the combined controller is proposed for the low-speed control. The hierarchical fuzzy controller is chosen to be composed of all two-input/one-output fuzzy logic systems for minimal total number of control rules. The fuzzy control part is integrated with the PD control to enhance the low-speed tracking capability and the

Table II. (Continued.)

(d)

Unit 0		Input 1 (Output of unit 1)						
		f1	f2	f3	f4	f5	f6	f7
	f1	1.6153	1.5485	1.7101	1.6215	1.6595	1.5460	1.7333
	f2	1.6592	1.5394	1.7326	1.6754	1.6099	1.7286	1.5729
Input 2	f3	1.6610	1.7154	1.5563	1.6631	1.7280	1.5571	1.6598
(Output	f4	1.5977	1.6623	1.6958	1.5498	1.6162	1.6549	1.6549
of unit 2)	f5	1.6526	1.6956	1.7355	1.6140	1.6696	1.5714	1.6333
	f6	1.6516	1.6522	1.6434	1.6397	1.6955	1.7299	1.6140
	f7	1.5670	1.7337	1.5858	1.6401	1.6442	1.7360	1.7340

steady-state performance. From simulations, we show that the controlled C-axis can obtain very low speed tracking without stick-slip using the learned combined PD and hierarchical fuzzy controller. The applications of genetic algorithms to construct optimal hierarchical fuzzy control for complex drive systems are challenging in precision machine design, and the efficiency improvement of the evolutionary learning algorithms deserves further studies.

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