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J Intell Robot Syst (2006) 47: 317–340
DOI 10.1007/s10846-006-9072-2

Dynamic Modeling and Tracking Control of a Nonholonomic Wheeled Mobile Manipulator with Dual Arms

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Received: 15 July 2006 / Accepted: 31 July 2006 /
Published online: 31 October 2006
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Abstract This paper presents methodologies for dynamic modeling and trajectory tracking of a nonholonomic wheeled mobile manipulator (WMM) with dual arms. The complete dynamic model of such a manipulator is easily established using the Lagrange's equation and MATHEMATICA. The structural properties of the overall system along with its subsystems are also well investigated and then exploited in further controller synthesis. The derived model is shown valid by reducing it to agree well with the mobile platform model. In order to solve the path tracking control problem of the wheeled mobile manipulator, a novel kinematic control scheme is proposed to deal with the nonholonomic constraints. With the backstepping technique and the filtered-error method, the nonlinear tracking control laws for the mobile manipulator system are constructed based on the Lyapunov stability theory. The proposed control scheme not only achieves simultaneous trajectory and velocity tracking, but also compensates for the dynamic interactions caused by the motions of the mobile platform and the two onboard manipulators. Simulation results are performed to illustrate the efficacy of the proposed control strategy.

Key words backstepping control · Lyapunov stability · mobile robot · nonholonomic system

1 Introduction

Over the past and present decades, there has been increasingly interest in the challenging field of wheeled mobile manipulators [1–6]. A wheeled mobile manipulator typically consists of a mobile platform with wheeled mobility configuration and one or two more robotic arm(s) mounted on the mobile platform. Such a manipulator has been extensively used in many applications such as manufacturing, warehouse, health-care, hazardous exploration and etc. Due to the mobility of the platform and the manipulability of the arm(s), the wheeled mobile manipulator is capable of performing dexterous manipulation tasks in a much larger workspace than a fixed-based manipulator. However, it will give rise to many new challenges

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that are not easily solvable by considering each subsystem separately. For example, it will be more realistic to consider the tracking problem using the overall dynamic model of wheeled mobile manipulators due to their significant dynamics [7], and address the path planning problem by considering the redundancy between the mobile platform and the mounted arm(s). Furthermore, the more arms mounted on the platform, the more laborious and complicated procedures required in the complete modeling and the controller synthesis of the wheeled mobile manipulator. In recent years, the dual-arm mobile manipulators have played a critical role in designing service robots and humanoid robots with dexterous arms, which are one of the fantastic applications of the dual-arm configuration. In particular, the two-arm mobile manipulators have been demonstrated the potential in many tasks such as home cleaning and cooperation [8]. Hence, this article will revisit and give new insights for the problems of dynamic modeling and control of a wheeled mobile manipulator with dual arms.

Owing to the kinematic constraints, the wheeled mobile manipulators are regarded as a category of the nonholonomic dynamic system which is richly documented from the comprehensive book [9] to more recent developments [10]. It is clear that the modeling of the mobile manipulators provides a basis on the study of stability analysis, feedback control synthesis, computer simulation, implementation, and so on. The dynamic modeling of mobile manipulators with one arm has been addressed by many researchers. Generally specking, the mobile manipulator has been derived based on four approaches: Newton-Euler method, Lagrange's equation [1], forward recursive formulation [2], and Kane's approach [3]. Yamamoto and Yun [1] utilized the recursive form of Lagrange's approach to separately derive the motion equations of the platform and arm. Yu and Chen [2] employed the forward recursive formulation for the dynamics of multi-body system to obtain the governing equation; in their approaches, the additional terms were introduced to take into account the dynamic interactions between the mobile platform and the onboard manipulator. On the other hand, Lin and Goldenberg [4] proposed the on-line neural network estimators to identify the unknown system dynamics, which possibly leads to long-time learning and computational requirement. Moreover, to the best of the author's knowledge, the problem of modeling the mobile manipulator with dual arms are seldom addressed in literature, except that Yamamoto and Yun [5] presented that the motion equations of a wheeled mobile manipulator with two arms are constructed based on the modular approach [1].

From dynamic modeling aspect, two-arm mobile manipulators have a much more complicated dynamic behavior than one-arm mobile manipulators because of larger dimensions and highly coupled dynamics. Although several approaches using the Lagrange equations of motion with multipliers have been proposed in modeling both two types of mobile manipulators, their derivation procedures of finding dynamic models are too complicated and time-consuming. To circumvent the difficulty, this paper contributes a direct system modeling approach for the two-arm mobile manipulator through the use of the commercial package, MATHEMATICA. By virtue of the advantages of the symbolic computation in MATHEMATICA, the dynamic model of the two-arm mobile manipulator can be easily developed and then represented in a state-space form. Similar to conventional one-arm mobile manipulators, the developed model even shows that there are more complicated dynamic coupling effects between the platform and the dual arms. With this model, the procedures of controller synthesis and numerical simulation will be easy and efficient.

From controller design aspect, the complicated dynamics of two-arm mobile robot indicates that the controller syntheses for two-arm mobile manipulators are nontrivial to achieve their defined tasks or mission and compensate for the dynamic coupling effects between the platform and the dual arms. Compared to the well-known wheeled mobile manipulators with one arm, two-arm mobile manipulators play a more important role in achieving object delivering and dexterous applications for mobile service robots. With the two onboard arms, this kind of mobile manipulator needs to deal with more complicated control problems, such as trajectory tracking, two-arm trajectory planning without collision, cooperation, compliance, and manipulability, etc. Furthermore, the study on control of the mobile manipulator with dual arms can provide physical insights in designing more powerful service robots and humanoid robots with dexterous arms.

For the nonholonomic constraints and highly coupling dynamics, a great deal of approaches have been presented to the trajectory control of mobile manipulators with one arm, but few methods have paid attention to the two-arm case. For instance, Yamamoto and Yun [6] applied the feedback linearization technique to propose a coordination algorithm for a two-link planar mobile manipulator that is capable of fully compensating the dynamic interactions. With the function approximation ability in neural network, Lin and Golderberg [4] developed an on-line turning neural network controller to the position control for a one-arm mobile manipulator with unknown dynamics and disturbances. On the other hand, for avoidance to deal with nonholonomic constraints directly, Dong [11] utilized the diffeomorphic state transformation to transform the kinematic equation into a well-known chained form to acquire the reduce model, and proposed an adaptive controller to attack the trajectory tracking with parameter uncertainties. In the field of dual-arm mobile manipulator, few researchers have undertaken controller synthesis for implementing their desired tasks or missions. For examples, Takayasu et al. [8] proposed a motion planning for the realization of grasping/returning a book, and Hirata et al. [9] presented a motion control algorithm for handing an object in coordination. However, the simultaneous tracking control problems for the dual-arm mobile manipulators have not been completely addressed yet.

In comparison with the well-known dynamic modeling and control problems of the mobile manipulators with dual arms, this paper is written with two principal contributions. One is that dynamic modeling procedure of the dual-arm mobile manipulators is presented in a more systematic and detailed way and the model can be automatically constructed using the commercial package, MATHEMATICA. This presented model can be simplified to match many well-known models, and it also plays an active role in facilitating further studies, such as system analysis, controller synthesis and computer simulation. The other is that simultaneous position and velocity tracking problem of the mobile manipulator with dual arms is solved using the aforementioned dynamic model and the Lyapunov control strategy. With the proposed nonlinear tracking control law, the entire state of the system to asymptotically track to the desired trajectory is definitely ensured and dynamic coupling effects between the mobile platform and the onboard dual arms are fully compensated. Furthermore, the proposed methods can be expected to be pragmatic and effective in modeling and controlling the nonholonomic mobile manipulator with dual n-link arms.

The remainder of this paper is organized as follows. Section 2 derives and validates the constraint equations and dynamic model of a wheeled mobile manipulator, and then investigates the structural properties of the model. In Section 3, a globally asymptotical nonlinear tracking controller is proposed to track any desired smooth position and velocity trajectories simultaneously. Section 4 presents numerical results to show the effectiveness of the proposed tracking control scheme. Finally, conclusions are presented in Section 5.

2 System Modeling

This section presents the complete governing equations of the mobile manipulator system, investigates the structural properties of the derived models and validates the developed model in comparison with other's well-known work.

2.1 System Description

Consider the wheeled mobile manipulator with dual arms shown in Figure 1, which is equipped with three subsystems: a wheeled mobile platform and two individual three-link rigid robotic manipulators. The mobile platform has two driving wheels driven independently by two DC servomotors, and four passive supporting wheels at the corners. All the three joints of two manipulators are torque-driven by separate DC servomotors, respectively. To simplify the modeling derivation of the mobile manipulator system, the following assumptions are made: 1) the mobile platform moves in a flat and smooth terrain; 2) no slippage occurs between the wheels and the ground; 3) the two arms are individually mounted at symmetrical location p_r and p_l with respect to the central line of the platform.

Following the work [1], the subsequent notation will be used in the formulation of the constraint equations and the system motion equations of the wheeled mobile manipulator (see Figures 2 and 3). The subscripts r and l denote the right and left manipulators, respectively.

P_o	the intersection of the axis of symmetry with the driving wheels axis;
P_c	the location of the center of mass of the platform;
P_r, P_l	the locations of the two manipulators on the platform;
e	the distance from the manipulator to the central line;
d	the distance from P_o to P_c;
b	the distance between the driving wheels and the axis of symmetry;
R_w	the radius of each driving wheel;
m_c	the mass of the platform without the driving wheels and the rotors of the DC motors;
m_w	the mass of each driving wheel with its motor;
I_c	the moment of inertia of the platform, without the driving wheels and the motors, about a vertical axis through P_c;
I_m, I_w	the moment of inertia of each wheel and the motor rotor about the wheel axis and wheel diameter, respectively;

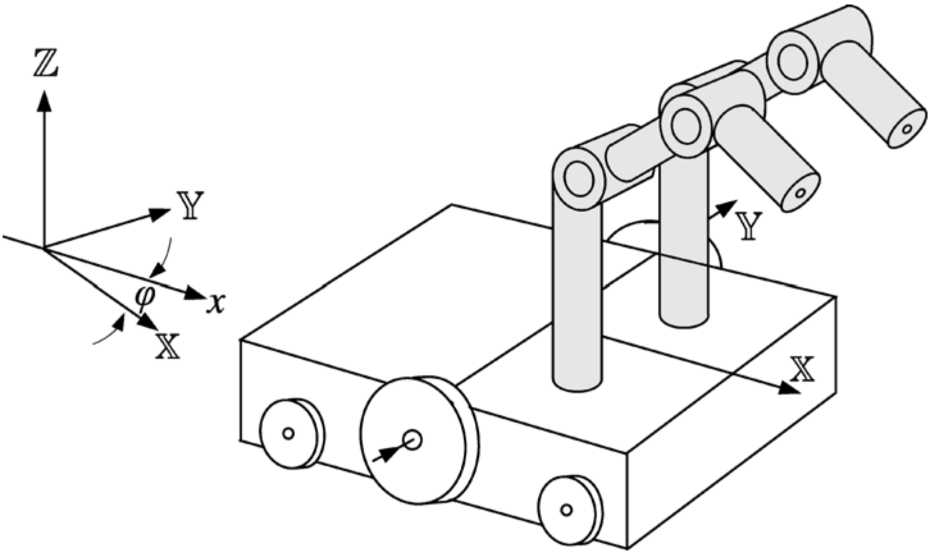

Figure 1 Schematics of the nonholonomic wheeled mobile manipulator with dual arms.

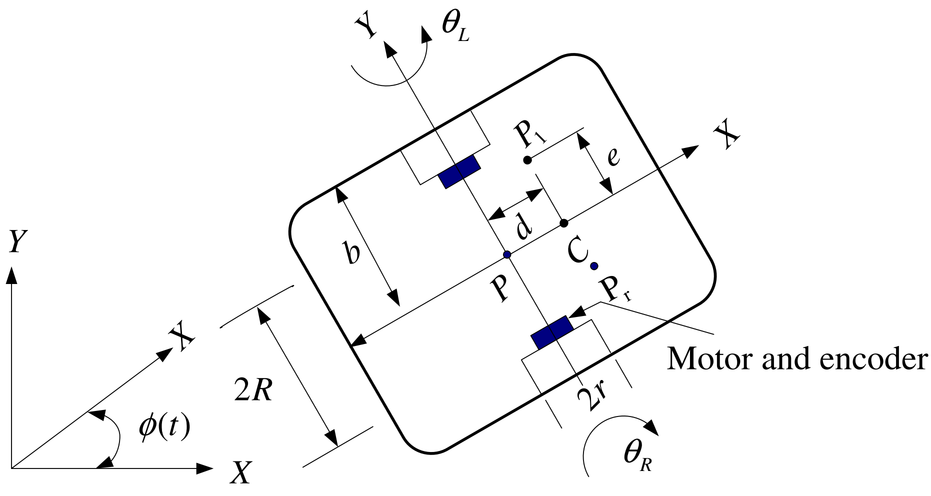

Figure 2 Top view of a mobile platform.

θ_{ri}, θ_{li}, L_{ri}, L_{li}, Z_{ri}, Z_{li}	the rotational angles, lengths and centroids of the *i*-th link of the two manipulators, respectively;
$\overline{X}_{ri}$, $\overline{X}_{li}$, $\overline{Z}_{ri}$, $\overline{Z}_{li}$	the position vectors of the mass centroid of *i*-th link of the two arms with respect to the moving frame *xyz*, respectively.
C_ϕ, S_ϕ	the cosine and sine functions of the variable ϕ.

In order to find the kinematics of the mobile manipulator system, two frames including the world frame $\langle XYZ \rangle$ and the moving frame $\langle xyz \rangle$ are adopted in the subsequent

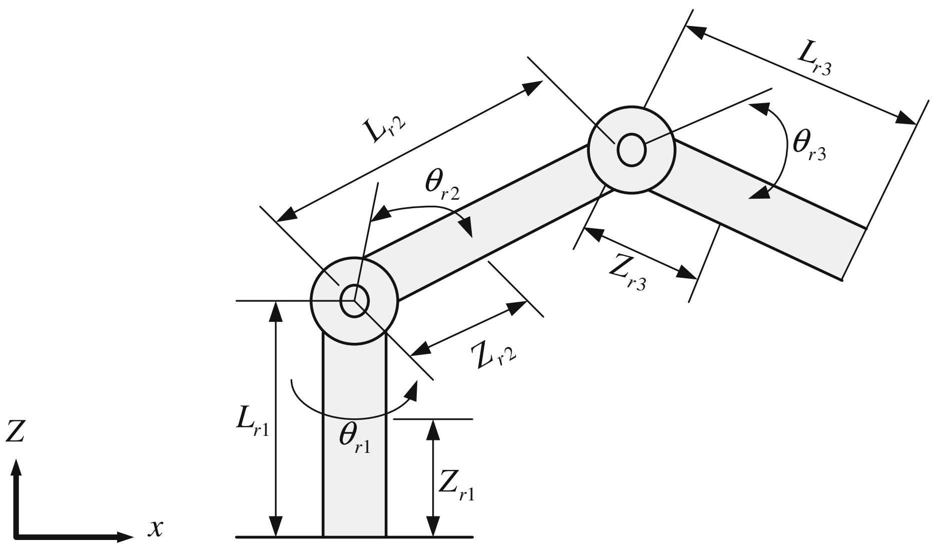

Figure 3 Side view of the right manipulator.

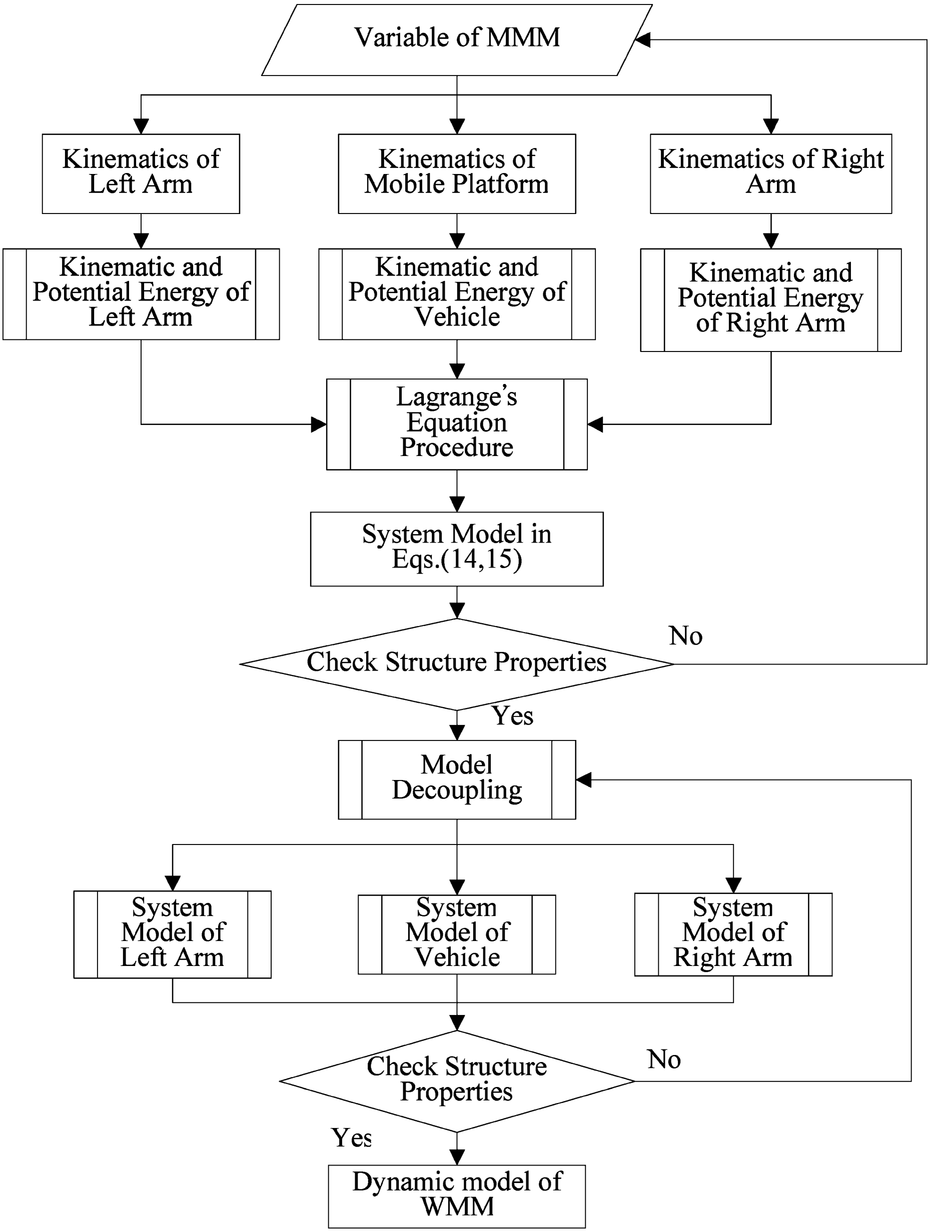

Figure 4 Flow chart of the dynamic modeling of the mobile manipulator using MATHEMATICA.

modelling process. Figure 2 illustrates the coordination transformation between the two frames.

$$\begin{bmatrix} \vec{e}_1 \\ \vec{e}_2 \\ \vec{e}_3 \end{bmatrix} = \begin{bmatrix} C_\phi & S_\phi & 0 \\ -S_\phi & C_\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{bmatrix} = T_{f/i}(\phi) \cdot \begin{bmatrix} \vec{i} \\ \vec{j} \\ \vec{k} \end{bmatrix} \tag{1}$$

where $\left[\overrightarrow{i}, \overrightarrow{j}, \overrightarrow{k}\right]^T$ and $\left[\overrightarrow{e}_1, \overrightarrow{e}_2, \overrightarrow{e}_3\right]^T$ represent the unit basis vectors of the two frames, $\langle XYZ\rangle$ and $\langle xyz\rangle$, respectively. The superscript T denotes the transpose of a matrix, and $T_{f/i}(\phi)$ is the coordinate transformation matrix from the world frame $\langle XYZ\rangle$ to the moving frame $\langle xyz\rangle$.

2.2 Kinematics of the Wheeled Mobile Manipulator

The subsection aims at developing the kinematics equations of the mobile manipulator; the equations will be exploited in the next subsection to find the kinetic energy of the system. Let $(x, y)^T$ represent the displacement of P_o along the X–Y axes, and $(\phi_r, \phi_l)^T$ denote the corresponding angular displacements of the right and left wheels.

2.2.1 Kinematics of the Wheeled Mobile Platform

The position vector of P_c with respect to the frame $\langle XYZ\rangle$ are denoted by $\vec{r}_c(t)$, satisfying

$$\vec{r}_c = \left(x + d\cdot C_\phi\right)\vec{i} + \left(y + d\cdot S_\phi\right)\vec{j} \tag{2}$$

which leads to obtain its velocity vector $\vec{V}_c(t)$ expressed by

$$\vec{V}_c = \left(\dot{x} - d\cdot\dot{\phi}\cdot S_\phi\right)\overrightarrow{i} + \left(\dot{y} + d\cdot\dot{\phi}\cdot C_\phi\right)\overrightarrow{j} \tag{3}$$

Due to the similarity, the position and velocity of the two driving wheels are described by

$$\begin{aligned} \vec{r}_{wr} &= \left(x + b\cdot S_\phi\right)\overrightarrow{i} + \left(y - b\cdot C_\phi\right)\overrightarrow{j}, \\ \vec{V}_{wr} &= \left(\dot{x}C_\phi + \dot{y}S_\phi + b\dot{\phi}\right)\overrightarrow{e}_1 + \left(\dot{y}C_\phi - \dot{x}S_\phi\right)\overrightarrow{e}_2, \end{aligned} \tag{4}$$

$$\begin{aligned} \vec{r}_{wl} &= \left(x - b\cdot S_\phi\right)\overrightarrow{i} + \left(y + b\cdot C_\phi\right)\overrightarrow{j}, \\ \vec{V}_{wl} &= \left(\dot{x}C_\phi + \dot{y}S_\phi - b\dot{\phi}\right)\overrightarrow{e}_1 + \left(\dot{y}C_\phi - \dot{x}S_\phi\right)\overrightarrow{e}_2, \end{aligned} \tag{5}$$

Let T_p be the total kinetic energy of the mobile platform, comprising the platform and two driving wheels. Hence it follows that

$$T_P = \frac{1}{2}m_p\left(\dot{x}^2 + \dot{y}^2\right) + m_c d\dot{\phi}\left(-\dot{x}S_\phi + \dot{y}C_\phi\right) + \frac{1}{2}I_P\dot{\phi}^2 + \frac{1}{2}I_w\left(\dot{\phi}_l^2 + \dot{\phi}_r^2\right) \tag{6}$$

where m_p represents the total mass of the platform, i.e., $m_p = m_c + 2m_w$, and I_P is the total inertia moment of the whole mobile platform about the vertical axis through P_o, i.e.,

$$I_P = I_c + m_c d^2 + 2I_m + 2m_w b^2$$

2.2.2 Kinematics of the Two Onboard Arms

Let the generalized coordinates of the right arm be $q_r = [\theta_{r1}, \theta_{r2}, \theta_{r3}]^T \in \mathbb{R}^{n_r}$, and assign the variables ($\overline{X}_{r1} = 0$, $\overline{X}_{r2} = z_{r2}S_{\theta_{r2}}$, $\overline{X}_{r3} = L_{r2}S_{\theta_{r2}} + z_{r3}S_{\theta_{r3}}$, $\overline{Z}_{r1} = z_{r1}$, $\overline{Z}_{r2} = L_{r1}+$

$z_{r2} \cdot C_{\theta_{r2}}$, and $\overline{Z}_{r3} = L_{r1} + L_{r2} C_{\theta_{r2}} + z_{r3} C_{\theta_{r3}}$) to represent the relative displacement of the each joint corresponding to the moving frame ⟨*xyz*⟩. Hence, the absolute position vectors of the *i*-link of the right arm in the world frame are described by

$$\overrightarrow{r}_{mri}\Big/_{o} = H_r \cdot \left\{ \overline{X}_{ri},\, 0,\, \overline{Z}_{ri},\, 1 \right\}^T,\; i = 1, \cdots, n_r \tag{7}$$

where $H_r \in R^{4\times 4}$ is the homogeneous transform matrix given by

$$H_r = \begin{bmatrix} C_{\theta_{r1}} & S_{\theta_{r1}} & 0 & d \\ -S_{\theta_{r1}} & C_{\theta_{r1}} & 0 & e \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

Deleting the last row of (7) and representing it in the world frame <*XYZ*>, one obtains

$$\begin{aligned} \overrightarrow{r}_{mri} &= \overrightarrow{r}_o + T_{f/_i}(\phi) \cdot \overrightarrow{r}_{mri/_o} \\ &= \left[x + d \cdot C_\phi - eS_\phi + \overline{X}_{ri} C_{(\phi+\theta_{r1})},\, y + d \cdot S_\phi + eC_\phi + \overline{X}_{ri} S_{(\phi+\theta_{r1})},\, \overline{Z}_{ri} \right]^T \end{aligned} \tag{9}$$

For the purpose of calculating the kinematics energy, the time derivative of (9) is found to obtain the velocity vector of each link. Similarly, the same procedure is also applicable to the left arm.

2.3 Constraint Equations

Due to the fact that the wheeled mobile platform cannot move in the lateral direction, this system can be considered as one of typical nonholonomic dynamic systems. In other words, the system satisfies the conditions of pure rolling and non-slipping so that the *y*-component of the velocities of the two driving wheels in (4) and (5) must be zero to obtain the following three kinematics constraints, which are consistent with the result in [6]

$$\dot{y} \cdot C_\phi - \dot{x} \cdot S_\phi = 0 \tag{10}$$

$$\dot{x} \cdot C_\phi + \dot{y} \cdot S_\phi + b\dot{\phi} = R_w \dot{\theta}_r \tag{11}$$

$$\dot{x} \cdot C_\phi + \dot{y} \cdot S_\phi - b\dot{\phi} = R_w \dot{\theta}_l \tag{12}$$

Let $q_v = \{x,\, y,\, \phi,\, \theta_r,\, \theta_l\}^T \in R^{n_v}$ be the generalized coordinates of the mobile platform, and the constraint equations (10–12) be represented in the following matrix form

$$A_v(q_v)\dot{q}_v = 0 \tag{13}$$

where $A_v(q_v) = \begin{bmatrix} -S_\phi & C_\phi & 0 & 0 & 0 \\ C_\phi & S_\phi & b & -R_w & 0 \\ C_\phi & S_\phi & -b & 0 & -R_w \end{bmatrix} \in R^{3\times n_v}$.

According to the results in [14], this wheeled mobile manipulator system has two nonholonomic constraints and one holonomic constraint, giving rise to the complexity in the controller synthesis and computer simulation.

2.4 Dynamic Equations of the Wheeled Mobile Manipulator

In this subsection, the Lagrange's equation with multipliers is employed to derive the overall dynamic equations of the two-arm mobile manipulator system. By virtue of the advantages of the symbolic computation in MATHEMATICA the system dynamics is easily derived based on the flow chart shown in Figure 4 and it can be expressed in the following compact form [4]:

$$M_s(q_s)\ddot{q}_s + C_{ms}\left(q_s, \dot{q}_s\right)\dot{q}_s + G_s(q_s) = E_s(q_s)\tau_s - A_s^T\lambda \tag{14}$$

where m kinematics constraints are described by

$$A_s\left(\dot{q}_s\right)\dot{q}_s = 0. \tag{15}$$

and $q_s = \left\{q_v^T(t), q_r^T(t), q_l^T(t)\right\}^T \in \mathbb{R}^{n\times 1}$ is the generalized coordinates of the overall system, which is composed of the three subsystems: the mobile platform and the two independent robotic arms. Moreover, $M_s(q_s) \in \mathbb{R}^{n\times n}$ is the symmetric and positive definite inertia matrix, $C_{ms}(q_s, \dot{q}_s) \in \mathbb{R}^{n\times n}$ the centripetal and *Coriolis* matrix, $G_s(q_s) \in \mathbb{R}^{n\times 1}$ the gravitational vector, $E_s(q_s) \in \mathbb{R}^{n\times(n-m)}$ the input transformation matrix, $A_s(q_s) \in \mathbb{R}^{m\times n}$ the constraint matrix associated with the m motion constraints (10–12), $\lambda \in \mathbb{R}^{m\times 1}$ the vector of constraint forces, and $\tau_s \in \mathbb{R}^{n-m}$ the torque input vector. All the detailed components in (14) are referred to [11].

To understand the dynamic coupling effects between the platform and the dual arms, the overall system model (14) can be decomposed into three subsystems whose mutual coupling relations are given by

$$M_v\ddot{q}_v + C_{mv}\dot{q}_v = E_v(q_v)\tau_v - A_v^T(q_v)\lambda - f_{vr} - f_{vl} \tag{16}$$

$$M_r\ddot{q}_r + C_{mr}\dot{q}_r + G_r = \tau_r - f_{rv} \tag{17}$$

$$M_l\ddot{q}_l + C_{ml}\dot{q}_l + G_l = \tau_l - f_{lv} \tag{18}$$

where $f_{ij}(f_{ij} \equiv M_{ij}\ddot{q}_j + C_{mij}\dot{q}_j,\ i \neq j,\ i, j = v,\ r,$ and $l)$ denotes the dynamic interactions of the i-th subsystem caused by the motion of j-th subsystem. From (17) and (18), it can be noted that two independent arms do not affect each other unless they interact with each other via the mobile platform. Obviously, the three decoupled equations (16–18) can be easily used to study the dynamic interactions and simulate their system responses.

2.5 State Space Realization

To facilitate the system analysis and controller synthesis, this subsection aims at rewriting Eqs. (16–18) in a state space form. Based on the matrix theory, it is always possible to find a full m rank matrix $S_v(q_v) \in \mathbb{R}^{n_v\times m}$ formed by a set of smooth and linearly independent vectors to span the null space of A_v, i.e., $S_v^T A_v^T = 0$, where

$$S_v(q_v) = \begin{bmatrix} cb\cdot C_\phi & cb\cdot S_\phi & -C & 0 & 1 \\ cb\cdot C_\phi & cb\cdot S_\phi & C & 1 & 0 \end{bmatrix}^T \tag{19}$$

and $C = R_w/2b$. The dynamic equation (16) can be transformed into two parts: kinematics equation and a more appropriate representation dynamic equation via an auxiliary vector $\eta_v = \{v_c, w_c\}^T$ such that, for all t,

$$\dot{q}_v = S_v(q_v)\eta_v. \tag{20}$$

Equation (20) is called a well-known steering system, where η_v is an input velocity vector to steer the state vector q_v in the state space. Substituting (20) into the system model (16) and then pre-multiplying both sides of (16) by S_v^T to eliminate the constraint force term λ, one obtains

$$\overline{M}_v\dot{\eta}_v + \overline{C}_v\eta_v + \overline{f}_{vr} + \overline{f}_{vl} = \overline{\tau}_v \tag{21}$$

$$M_r\ddot{q}_r + C_{mr}\dot{q}_r + G_r = \tau_r - M_{rv}\left(\dot{S}_v\eta_v + S_v\dot{\eta}\right) - C_{mrv}S_v\eta_v \tag{22}$$

$$M_l\ddot{q}_l + C_{ml}\dot{q}_l + G_l = \tau_l - M_{lv}\left(\dot{S}_v\eta_v + S_v\dot{\eta}_v\right) - C_{mlv}S_v\eta_v \tag{23}$$

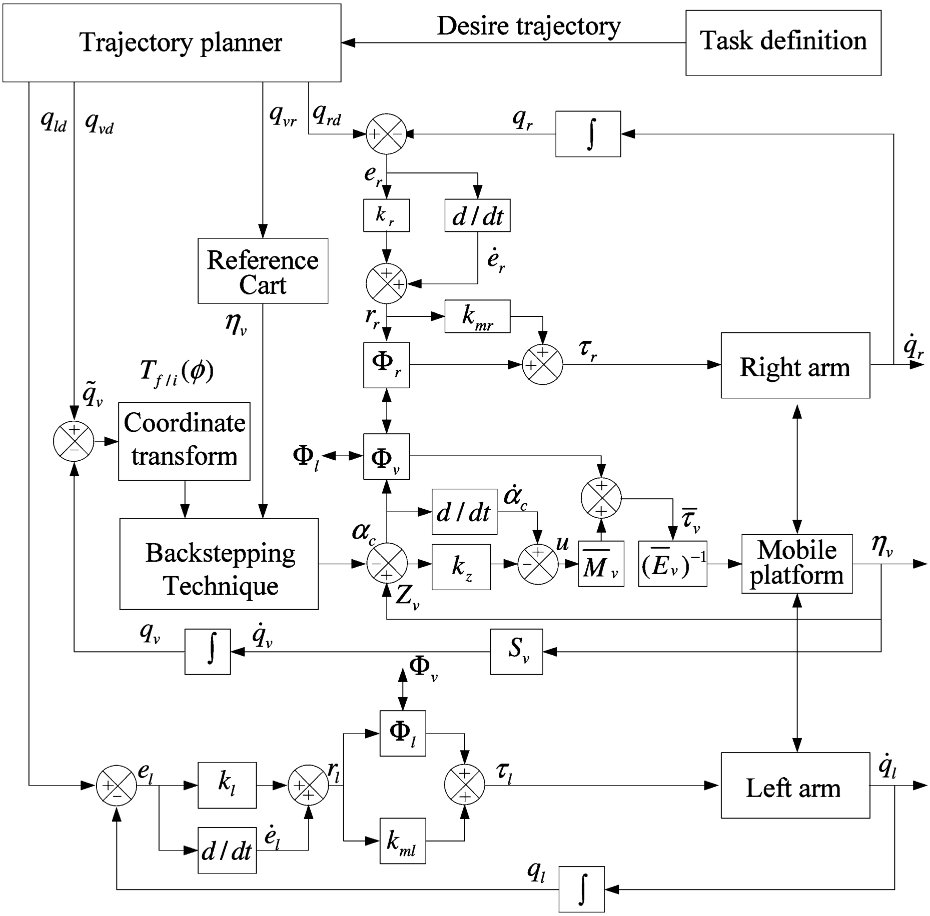

Figure 5 Block diagram of the proposed tracking control architecture.

where $\overline{M}_v = S_v^T M_v S_v$, $\overline{C}_v = S_v^T(M_v\dot{S}_v + C_{mv}S_v)$, $\overline{f}_{vr} = S_v^T f_{vr}$, $\overline{f}_{vl} = S_v^T f_{vl}$, $\overline{E}_v = S_v^T E_v$, and $\overline{\tau}_v = \overline{E}_v\tau_v$. For the convenience of numerical simulation, the decoupled system equations of the wheeled mobile manipulator are expressed in the matrix form by rearranging (21–23) in the form

$$P\dot{x} = \xi + Q\tau \tag{24}$$

where $x = \begin{bmatrix} \eta \\ \dot{q}_r \\ \dot{q}_l \end{bmatrix}$, $P = \begin{bmatrix} \overline{M}_v & S_v^T M_{vr} & S_v^T M_{vl} \\ M_{rv}S_v & M_r & \overline{0}_3 \\ M_{lv}S_v & \overline{0}_3 & M_l \end{bmatrix}$, $\xi = \begin{bmatrix} -\overline{C}_v\eta_v - S_v^T C_{mvr}\dot{q}_r - s_v^T C_{mvl}\dot{q}_l \\ -C_{mr}\dot{q}_r - G_r - M_{rv}\dot{S}_v\eta_v - C_{mrv}\dot{q}_v \\ -C_{ml}\dot{q}_l - G_l - M_{lv}\dot{S}_v\eta_v - C_{mlv}\dot{q}_v \end{bmatrix}$, $\tau = \begin{bmatrix} \tau_v \\ \tau_r \\ \tau_l \end{bmatrix}$,

and $Q = \begin{bmatrix} \overline{E}_v & \overline{0}_3 & \overline{0}_3 \\ \overline{0}_3 & \overline{I}_3 & \overline{0}_3 \\ \overline{0}_3 & \overline{0}_3 & \overline{I}_3 \end{bmatrix}$, $\overline{0}_i$ and $\overline{I}_i$ denote the i-th order null and identity matrices, respectively. Note that the previous results agree well with those proposed in [16]. Moreover, if the matrix P is invertible, then the model (24) can be transformed into an affine state-space form, i.e.,

$$\dot{x} = P^{-1}\xi + P^{-1}Q\cdot\tau = f(x) + g(x)u \tag{25}$$

2.6 Model Validation and Structural Properties

To verify the validity of the derived model, (14) can be reduced to the case without dual arms, i.e.,

$$\begin{aligned} m_P\ddot{x} - m_c d\left(\ddot{\phi}S_\phi + \dot{\phi}^2 C_\phi\right) - \lambda_1 S_\phi - (\lambda_1+\lambda_2)C_\phi &= 0 \\ m_P\ddot{y} + m_c d\left(\ddot{\phi}C_\phi - \dot{\phi}^2 S_\phi\right) + \lambda_1 C_\phi - (\lambda_1+\lambda_2)S_\phi &= 0 \\ -m_c d\left(\ddot{x}S\phi - \ddot{y}C\phi\right) + I_P\ddot{\phi} + b(\lambda_3 - \lambda_2) &= 0 \\ I_w\ddot{\theta}_r + \lambda_2 R_w &= \tau_r \\ I_w\ddot{\theta}_l + \lambda_3 R_w &= \tau_l \end{aligned} \tag{26}$$

It is clear that the simplified equations completely coincide with the dynamic equations of the wheeled mobile robot studied in [12]. In what follows briefly discuss three important structural properties, such as boundness and skew-symmetricity, of the derived model (21–23). Detailed proof procedures are referred to [11].

Property 2.1 All the mass matrices (M_s, M_v, $\overline{M}_v$, M_r, and M_l) the centrifugal matrices (C_{ms}, C_{mv}, $\overline{C}_v$, C_{mr}, and C_{ml}), and the gravitational matrices (G_s, G_r, and G_l) are bounded in the sense that there exist two positive scalars u_{i1} and u_{i2}, two positive functions $k_{ic}(q)$ and $k_{jg}(q)$ such that $u_{i1} \le \|M_i\| \le u_{i2}$, $\|C_{mi}\| \le k_{ic}(q)$ for i=s, v, r, l, and $\|G_j\| \le k_{jg}(q)$ for j=s, r, l; moreover, $\overline{u}_{v1} \le \|\overline{M}_v\| \le u_{v2}$ and $\|C_v\| \le k_v(q)$ where $\overline{u}_{v1}$ and $\overline{u}_{v2}$ are two positive real constants, and $k_v(q)$ is also a positive function.

Property 2.2 $(\dot{M}_i - 2C_{mi})$ is skew-symmetric for i=s, v, r, l, where M_i and C_{mi} are respectively the mass matrix and centrifugal matrix in the overall system and three subsystems in (16–18) and (21). Moreover, $\dot{M}_i = C_{mi} + C_{mi}^T$, or $x^T\left(\dot{M}_i - 2C_{mi}\right)x = 0,\ \forall x \neq 0$.

Property 2.3 $\dot{M}_{vj} = C_{mjv} + C_{mvj}^T$, $\dot{M}_{jv} = C_{mjv} + C_{mvj}^T$, $j=r,\ l$ where M_{vj}, M_{jv} and C_{mvj} are the matrices of the dynamic interactions in the mobile platform caused from the two three-link arms in (16–18).

Proof Since the mass and centrifugal matrices of (14) can be expressed by

$$M_s = \begin{bmatrix} M_v & M_{vr} & M_{vl} \\ M_{rv} & M_r & \overline{0}_3 \\ M_{lv} & \overline{0}_3 & M_l \end{bmatrix},\quad C_{ms} = \begin{bmatrix} C_{mv} & C_{mvr} & C_{mvl} \\ C_{mrv} & C_{mr} & \overline{0}_3 \\ C_{mlv} & \overline{0}_3 & C_{ml} \end{bmatrix}$$

Property 2.3 can be easily proven by using the result of Property 2.2 and the fact that M_{vr} and M_{vl} are 3 by 3 square matrices.

Remark 1 Property 2.3 can be shown generally true for the mobile manipulators without two three-link arms. For example, M_{vr} and M_{vl} used for computer simulation in Section 4 are 3 by 2 matrices, not square matrices, but Property 2.3 is also true for the two matrices. Because Property 2.3 is used for the derivation of the nonlinear tracking control law in Section 3, the proposed tracking controller will be shown to work well for the mobile manipulators with two two-link arms. Moreover, the proposed modeling approach and tracking control method can be applicable to mobile manipulators with two n-link arms where n can be a positive and finite integer.

3 Nonlinear Tracking Control Design

On the basis of the previous dynamic model and the essential control concepts in [8], this section develops a novel kinematics path tracking law for the mobile vehicle and a Lyapunov-based trajectory tracking control scheme for the wheeled mobile manipulator. In controller synthesis, the idea of backstepping control together with the filtered error method is used to tackle with the kinematics motion constraints and compensate for the highly nonlinear dynamics of the dual arms. It is well-known that if there exit both nonholonomic and holonomic constraints in system models, it will result in an elaborate procedure in controller design and computer simulation [15]. Thus, one substitutes the constraint equations

$$\dot{\theta}_r = \frac{1}{r}\left(\dot{x}\cdot C_\phi + \dot{y}\cdot S_\phi + b\dot{\phi}\right),\ \dot{\theta}_l = \frac{1}{r}\left(\dot{x}\cdot C_\phi + \dot{y}\cdot S_\phi - b\dot{\phi}\right), \tag{27}$$

into the dynamic models (21–23) and also ignores the mass of the driving wheels to obtain a completely nonholonomic WMM system with dual arms. For details, see [13].

Assume that there are no uncertainties and external disturbances applied to the WMM system, and all the parameters in the system models are known in advance or measured from the experimental data. Then the tracking problem is formulated as follows. For the purpose of tracking the desired twice differentiable trajectories $q_d(t) = \left[q_{vd}^T,\ q_{rd}^T,\ q_{ld}^T\right]^T$, the control objective is to find a tracking torque control law $\tau=[\tau_v,\ \tau_r,\ \tau_l]^T$ such that the position/

velocity tracking errors of the platform and the two arms approach zero as time goes to infinity, i.e.,

$$\begin{gathered}\lim_{t\to\infty}(q_{vd}-q_v)=0;\ \lim_{t\to\infty}(q_{rd}-q_r)=0;\ \lim_{t\to\infty}(q_{ld}-q_l)=0;\\ \lim_{t\to\infty}\left(\dot{q}_{vd}-\dot{q}_v\right)=0;\ \lim_{t\to\infty}\left(\dot{q}_{rd}-\dot{q}_r\right)=0;\ \lim_{t\to\infty}\left(\dot{q}_{ld}-\dot{q}_l\right)=0.\end{gathered} \tag{28}$$

3.1 A Novel Globally Kinematics Path Tracking Control Law for the Mobile Platform

Given the reference mobile platform subject to the nonholonomic kinematics constraints,

$$\dot{q}_{vd}=S(q_{vd})\eta_d=\begin{bmatrix}\dot{x}_d\\ \dot{y}_d\\ \dot{\phi}_d\end{bmatrix}=\begin{bmatrix}C_{\theta_d} & 0\\ S_{\theta_d} & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}v_d\\ w_d\end{bmatrix}, \tag{29}$$

where v_d and w_d are the reference linear and angular velocities, the control objective is to propose a smooth auxiliary velocity control law $\alpha_c(t)$ to track any smooth reference ground path.

In doing so, the tracking error vector of the mobile platform is defined by $\widetilde{q}_v=\left[\widetilde{x}_v,\ \widetilde{y}_v,\ \widetilde{\phi}_v\right]^T$, where $\widetilde{x}_v=x_d-x$, $\widetilde{y}_v=y_d-y$, and $\widetilde{\phi}_v=\phi_d-\phi$, and the tracking errors in the moving frame $\langle xyz\rangle$ is represented by

$$e_v=\left[e_x,\ e_y,\ e_z\right]^T=T_{f/_i}(\phi)\cdot\widetilde{q}_v, \tag{30}$$

where e_x, e_y, and e_z are the tangential, normal and orientation errors, respectively. Taking the time derivative of (30) gives the following error dynamics of the mobile platform [16]

$$\dot{e}_v=\begin{bmatrix}\omega_c e_y-v_c+v_d C_{e_z}\\ -\omega_c e_x+v_d S_{e_z}\\ w_d-\omega_c\end{bmatrix}. \tag{31}$$

Because the normal error e_y cannot be directly controlled using the control vector $\alpha_c(t)=(v_c,\omega_c)^T$, an auxiliary variable $\overline{e}_z$ is introduced to propose a novel globally path tracking controller for the mobile platform.

$$\overline{e}_z=e_z+\alpha\cdot e_y,\ \text{where } \alpha\neq 0 \tag{32}$$

The following theorem is provided to design the novel kinematics path tracking law for the mobile platform.

Theorem 3.1 *Assume that q_{vd}, v_d and w_d are continuous, bounded, and differentiable at least twice on the time interval* $[0,\infty)$. *If the following smooth velocity law $\alpha_c(t)$ of* (33) *is applied to the system model* (20), *then the tracking errors of a mobile base system* (31) *are globally asymptotically stable, i.e., e_x, e_y, and e_z will approach zero as $t\to\infty$.*

$$\alpha_c=\begin{bmatrix}v_c\\ \omega_c\end{bmatrix}=\begin{bmatrix}v_d C_{e_z}+k_1 e_x\\ \frac{1}{1+\alpha e_x}\left(w_d+\alpha v_d S_{e_z}+k_\delta\delta\cdot\overline{e}_z+\delta\frac{v_d S_{e_z}}{\overline{e}_z}e_y\right)\end{bmatrix} \tag{33}$$

where k_1, k_δ, δ, are positive constants, and $0\leq\alpha<\frac{1}{\|e_x(0)\|_2}$ provided that the starting and destination poses of the robot are known. Note that $\|e_x(0)\|_2$ denotes the 2-norm of the tangential error at time 0.

Proof Select a radially unbounded Lyapunov function candidate for the steering system as

$$V_p = \frac{1}{2}\left(e_x^2 + e_y^2\right) + \frac{1}{2\delta}\overline{e}_z^2. \tag{34}$$

Differentiating (34) and using (31–33) yields

$$\dot{V}_p = -k_1 e_x^2 - k_\delta \overline{e}_z^2 \le 0. \tag{35}$$

Using Barbalat's lemma, one shows that $e_x(t)$ and $\overline{e}_z(t)$ approach zero when time goes to infinity. With the proof procedure proposed by Dixon et al. [4], it is concluded what if the reference linear velocity satisfies $\lim_{t\to\infty} v_d \neq 0$, then $\lim_{t\to\infty} e_y(t) = 0$ and $\lim_{t\to\infty} e_z(t) = 0$.

3.2 Lyapunov Function for the Mobile Platform

By defining the backstepping error z_v as $z_v = \eta_v - \alpha_c$, and rearranging the dynamics of a mobile platform (21) in terms of the backstepping error z_v, it yields

$$\overline{M}_v \dot{z}_v + \overline{C}_v z_v + \overline{f}_{vr} + \overline{f}_{vl} + \overline{M}_v \dot{\alpha}_c + \overline{c}_v \alpha_c = \overline{\tau}_v. \tag{36}$$

Select the following a Lyapunov candidate function V_1 for the mobile platform

$$V_1 = \frac{1}{2} z_v^T \overline{M}_v z_v. \tag{37}$$

Based on Property 2.2, the time derivative of V_1 along the system trajectory is

$$\dot{V}_1 = \frac{1}{2} z_v^T \dot{\overline{M}}_v z_v + z_v^T \overline{M}_v \dot{z}_v = z_v^T \left(\overline{\tau}_v - \overline{f}_{vr} - \overline{f}_{vl} - \overline{M}_v \dot{\alpha}_c - \overline{c}_v \alpha_c\right) \tag{38}$$

3.3 Lyapunov Functions for the Dual Arms

This subsection aims at selecting a Lyapunov function for the dual arms in order to find the nonlinear tracking control law. In doing so, we take into account the dynamics of the dual arms, and define the arm position tracking errors e_j and the filtered tracking errors r_j of the dual arms by

$$e_j = q_{jd} - q_j,\ r_j = \dot{e}_j + k_j e_j,\ \text{with constant matrices } k_j = k_j^T > 0, j = r, l. \tag{39}$$

Table I Physical parameters of the LABMATE platform [6]

Parameter	Value	Unit
Mass of the platform m_c	94.0	kg
Mass of the driving wheel m_w	5.0	kg
Inertia of the platform I_c	6.609	kg m^2
Inertia of the driving wheel I_w	0.01	kg m^2
Radial of the wheel r	0.075	m
High width of the platform b	0.171	m

From (39), it is easy to show that $e_j \rightarrow 0$ as $r_j \rightarrow 0$ via the linear control theory. By reformulating the dynamics models of the two arms in terms of their filtered errors r_j and the time derivatives $\dot{r}_j$, one obtains

$$M_j\dot{r}_j = M_j\ddot{q}_{jd} + C_{mj}\dot{q}_j + G_j + f_{jv} - \tau_j + M_jk_j\left(r_j - k_je_j\right),\ j = r,\ l \tag{40}$$

Similarly, the two Lyapunov function candidates for the dual arms are selected by

$$V_j = \frac{1}{2}r_j^TM_jr_j,\ j = r,\ l. \tag{41}$$

Taking the time derivative of (41) and applying Property 2.2 of the two arms model give

$$\dot{V}_j = \frac{1}{2}r_j^T\dot{M}_jr_j + r_j^TM_j\dot{r}_j = r_j^T\left(-\tau_j + M_jk_jr_j - \left(C_{mj} + M_jk_j\right)k_je_j + \widehat{F}_{jd} + f_{jv} + G_j\right) \tag{42}$$

where $\widehat{F}_{jd} = M_j\ddot{q}_{jd} + C_{mj}\dot{q}_{jd}$, j=r, l.

3.4 Lyapunov Functions for the Overall System

To show the asymptotical stability for the overall manipulator system, the following Lyapunov function candidate is chosen as

$$V = V_p + \frac{1}{2}\begin{bmatrix} S_vz_v \\ -r_r \\ -r_l \end{bmatrix}^T M_s \begin{bmatrix} S_vz_v \\ -r_r \\ -r_l \end{bmatrix} = V_p + \frac{1}{2}\left[z_v^TS_v^T,\ -r_r^T,\ -r_l^T\right] \cdot \begin{bmatrix} M_v & M_{vr} & M_{vl} \\ M_{rv} & M_r & \overline{0}_3 \\ M_{lv} & \overline{0}_3 & M_l \end{bmatrix} \cdot \begin{bmatrix} S_vz_v \\ -r_r \\ -r_l \end{bmatrix}$$

$$= V_p + \frac{1}{2}z_v^T\overline{M}_vz_v + \frac{1}{2}\sum_{j=r}^{1}\left(r_j^TM_jr_j - z_v^TS_V^TM_{vj}r_j - r_j^TM_{jv}S_vz_v\right),\ j = r,\ l. \tag{43}$$

With the symmetric property of $z_v^TS_v^TM_{vj}r_j = r_j^TM_{jv}S_vz_v$, j=r, l (43) becomes

$$V = V_p + V_1 + V_r + V_l - \sum_{j=r}^{l}r_j^TM_{vj}S_vz_v,\ j = r,\ l. \tag{44}$$

Table II Physical parameters of the two two-link onboard manipulators [6]

Parameter j=r, l	Link 1, i=1	Link 2, i=2
Mass of the link m_{ji} (kg)	5.0	2.0
Length of the link L_{ji} (kg)	0.5	1.0
Center of mass z_{ji} (m)	0.25	0.5
Inertia tensor (kg m^2)		
Inertia about x-axis $I_{m\ jxi}$		0.011
Inertia about y-axis $I_{m\ jyi}$		0.167
Inertia about z-axis $I_{m\ jzi}$	0.00625	0.168

Taking the derivative of V along the system trajectories and after some algebraic manipulations give

$$\dot{V} = \dot{V}_p + z_v^T\left\{\overline{\tau_v} - \overline{M}_v\dot{\alpha}_c - \overline{C}_v\alpha_c - \overline{f}_{vr} - \overline{f}_{vl}\right\}$$
$$+\sum_{j=r}^{l} r_j^T\left\{-\tau_j + M_jk_jr_j - \left(C_{mj} + M_jk_j\right)k_je_j + \widehat{F}_{jd} + f_{jv} + G_j\right\}$$
$$-\sum_{j=r}^{l}\frac{d}{dt}\left\{r_j^T M_{vj}S_v z_v\right\}, \quad j = r,\, l \tag{45}$$

To find the negative semi-definiteness of (45), let us consider the dynamic interactions between the platform and the dual arms term by term.

(1) From the definition of $\overline{f}_{vj}$ in (21) implies that

$$\begin{aligned}\overline{f}_{vj} &= S_v^T\left\{M_{vj}\left(\ddot{q}_{jd} - \dot{r}_j + k_j\left(r_j - k_je_j\right)\right) + C_{mvy}\left(\dot{q}_{jd} - \left(r_j - k_je_j\right)\right)\right\} \\ &= \overline{F}_{vjd} - S_v^T\left\{M_{vj}\dot{r}_j + \left(C_{mvj} - M_{vj}k_j\right)\left(r_j - k_je_j\right)\right\}, \quad j = r,\, l\end{aligned} \tag{46}$$

where $\overline{F}_{vjd} = S_v^T\left(M_{vj}\ddot{q}_{jd} + C_{mvj}\dot{q}_{jd}\right), j = r,\, l$ represents the desired coupling dynamics in the mobile vehicle affected by the dual arms.

(2) From (22) and (23), the dynamic interaction force f_{jv} can be represented in terms of z_v

$$\begin{aligned}f_v &= M_{jv}\left(\dot{S}_v\eta_v + S_v\dot{\eta}_v\right) + C_{mjv}S_v\eta_v \\ &= M_{jv}S_v\left(\dot{z}_v + \dot{\alpha}_c\right) + \left(M_{jv}\dot{S}_v + C_{mjv}S_v\right)(z_v + \alpha_c),\ j = r, l.\end{aligned} \tag{47}$$

By substituting the previous results (46–47) into (45), one obtains

$$\begin{aligned}\dot{V} = \dot{V}_p + z_v^T\left\{\overline{\tau}_v - \overline{M}_v\dot{\alpha}_c - \overline{C}_v\alpha_c\right\} + \sum_{j=r}^{l} r_j^T\left\{-\tau_j + M_jk_jr_j + \left(C_{mj} - M_jk_j\right)k_je_j + G_j\right\} \\ +\sum_{j=r}^{l}\left(-z_v^T\overline{f}_{vj} + r_j^T\widehat{F}_{jd} + r_j^Tf_{jv}\right) - \sum_{j=r}^{l}\frac{d}{dt}\left\{r_j^TM_{vj}S_vz_v\right\}, \quad j = r,\, l\end{aligned} \tag{48}$$

The last bracket in (48) can be simplified by using Property 2.3, $\dot{M}_{vj} = C_{mjv} + C_{mvj}^T$, yields

$$\begin{aligned}&-z_v^T\overline{f}_{vj} + r_j^T\widehat{F}_{jd} + r_j^Tf_{jv} - \frac{d}{dt}\left\{r_j^TM_{vj}S_vz_v\right\} \\ &= -z_v^T\overline{F}_{vjd} + r_j^T\widehat{F}_{jd} + (S_vz_v)^T\left\{-C_{mvj}k_je_j + M_{vj}k_j\left(r_j - k_je_j\right)\right\} \\ &+r_j^T\left(M_{jv}S_v\dot{\alpha}_c + C_{mjv}S_v\alpha_c\right),\ j = r,\, l\end{aligned} \tag{49}$$

which leads to the following result by substituting (49) into (48).

$$\dot{V} = -k_1e_x^2 - k_\delta\overline{e}_z^2 + z_v^T\left\{\overline{\tau}_v - \overline{M}_v\dot{\alpha}_c - \overline{C}_v\alpha_c - \overline{F}_{vrd} - \overline{F}_{vld} - W_{vr} - W_{vl}\right\}$$
$$+\sum_{j=r}^{l} r_j^T\left\{-\tau_j + M_jk_jr_j - \left(C_{mj} + M_jk_j\right)k_je_j + G_j + \widehat{F}_{jd} + W_{jv}\right\} \tag{50}$$

where $W_{vj} = S_v^T\left(C_{mvj}k_jr_j + M_{jv}k_j\left(r_j - k_je_j\right)\right)$, $W_{jv} = M_{jv}S_v\dot{\alpha}_c + C_{mjv}S_v\alpha_c$, j=r, l. The following theorem summarizes the aforementioned results.

Theorem 3.2 *Assume that desired trajectories* $[q_{vd}, q_{rd}, q_{ld}, \dot{q}_{vd}, \dot{q}_{rd}, \dot{q}_{ld}]$ *are continuous, bounded, and differentiable at least twice on the time interval* $[0, \infty)$. *Under the assumption that there are no unmodeled dynamics and external disturbances exerted in the system models, if the nonlinear tracking control laws* (51–52) *illustrated in* Figure 5 *are applied to a wheeled mobile manipulator system governed by* (21–23), *then the position and velocity tracking errors of the closed-loop system approach zero as* $t\to\infty$; *accordingly, the closed-loop system is globally asymptotically stable.*

$$\overline{\tau}_v = \overline{M}_v u + \Phi_v \tag{51}$$

$$\tau_j = k_{mj}r_j + \Phi_j,\ j = r, l \tag{52}$$

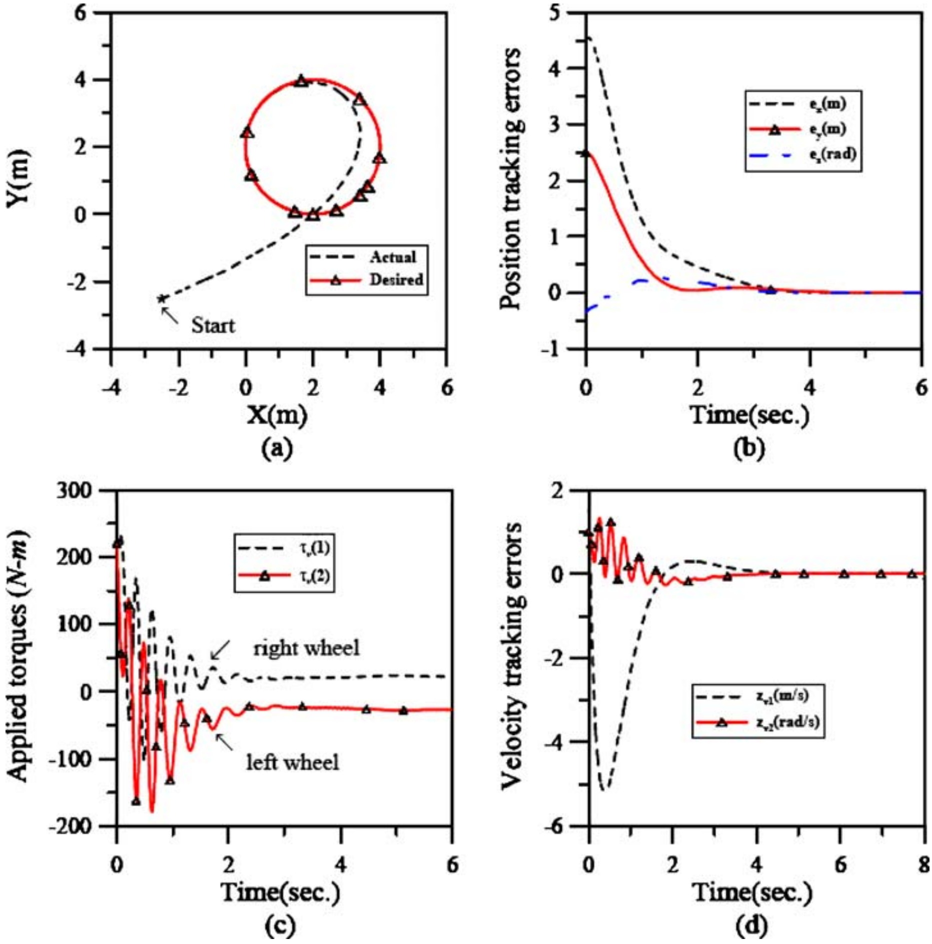

Figure 6 Control performance in the mobile platform. (a) X–Y trajectories, (b) position tracking errors, (c) applied control torques in the two wheels, (d) velocity tracking errors.

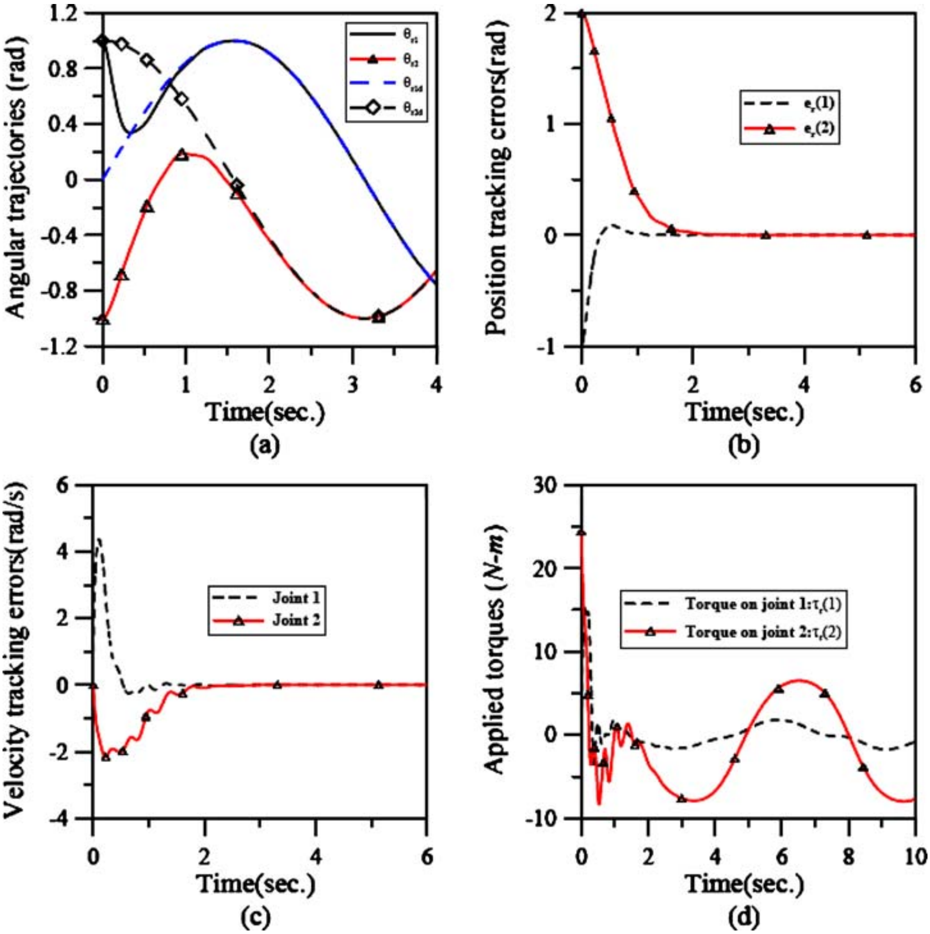

Figure 7 Control performance in the right arm. (a) Angular trajectories, (b) angular position tracking errors, (c) angular velocity tracking errors, (d) applied torques in the two joints.

where $\Phi_v = \overline{C}_v\alpha_c + \overline{F}_{vrd} + \overline{F}_{vld} + W_{vr} + W_{vl}$, $\Phi_j = M_jk_jr_j - \left(C_{mj} + M_jk_j\right)k_je_j + G_j + \widehat{F}_{jd} + f_{jv}$, $u = \dot{\alpha}_c - k_zz_v$ denotes the nonlinear feedback term, and k_z, k_{mr} and k_{ml} are the positive controller gains.

Proof Using the radially unbounded Lyapunov function candidate V for the overall system (43) and substituting the proposed control laws into (50), we obtain

$$\dot{V} \leq -k_1\|e_x\|^2 - k_\delta\|\overline{e}_z\|^2 - k_z\|z_v\|^2 - k_{mr}\|r_r\|^2 - k_{ml}\|r_l\|^2 \leq 0 \tag{53}$$

It is obvious that $\dot{V}$ is negative semi-definite such that the entire errors are globally bounded. The use of Barbalat's lemma implies that the entire errors asymptotically approach zero as time goes to infinity. Note that the parameters in the proposed control strategy are predetermined so that the control signals can be directly computed without any learning via neural networks [4]; moreover, the main differences between the proposed control laws (51) and the control approach in [16] are the novel kinematics control law α_c and the effective nonlinear feedback control u.

3.5 Asymptotical Stability of the Closed-Loop Error Dynamics

To investigate the asymptotical stability of the closed-loop system, the proposed control laws (51–52) are applied to the wheeled mobile manipulator dynamics (36) and (40) so that the closed-loop dynamic equations of the mobile platform and the dual arms are obtained as follows;

$$\overline{M}_v\dot{z}_v + \left(\overline{C}_v + k_z\cdot\overline{I}_2\right)z_v = 0 \tag{54}$$

$$M_j\dot{r}_j + \left(C_{mj} + k_{mj}\overline{I}_3\right)r_j = 0,\ j = r,\ l. \tag{55}$$

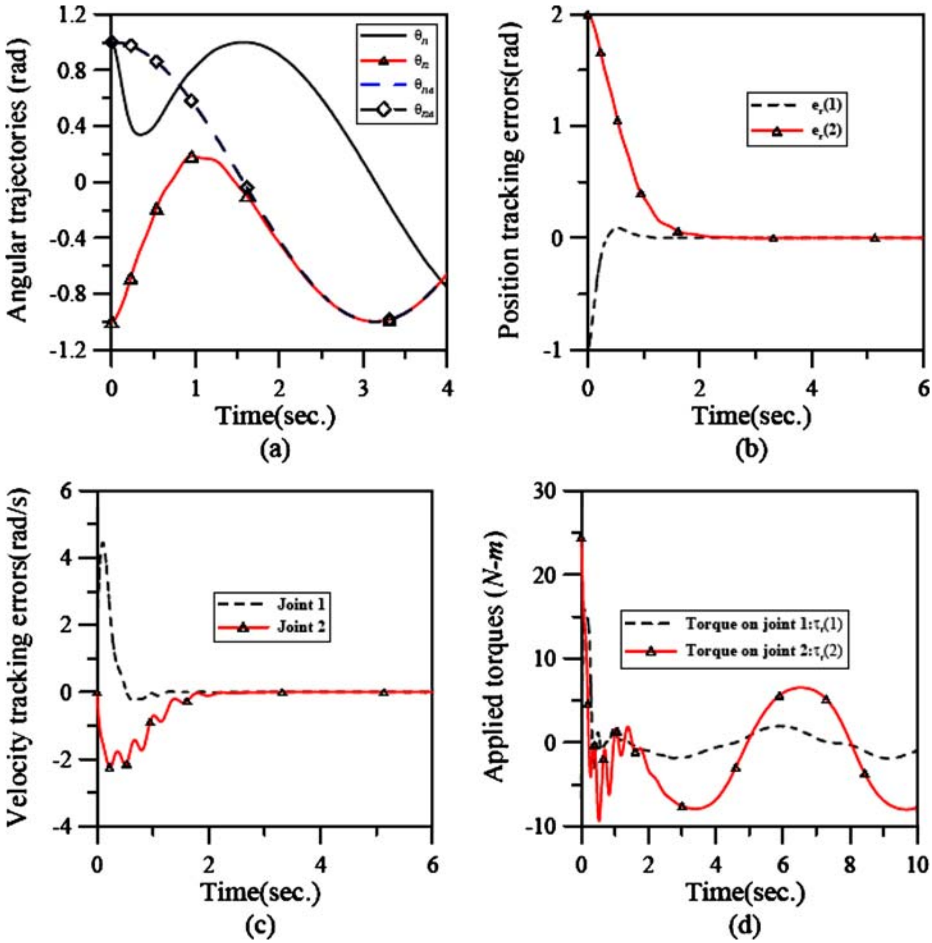

Figure 8 Control performance in the left arm. (a) Angular trajectories, (b) angular position tracking errors, (c) angular velocity tracking errors, (d) torques applied in the two joints.

Equation (54) implies that $z_v \rightarrow 0$ and $\eta_v \rightarrow \alpha_c$ as $t \rightarrow \infty$. From the result in [6] and Barbalat's lemma, it is concluded that if the velocity of the mobile vehicle satisfies $\eta_v \rightarrow \alpha_c$ as $t \rightarrow \infty$, then the equilibrium point $e_v=0$ of the mobile platform is globally asymptotically stable. From (55), it follows that, for the dual arms, their filter errors and position and velocity tracking errors approach zero as $t \rightarrow \infty$.

4 Simulation Results and Discussion

Numerical simulations using MATLAB 6.5 were conducted in this section to examine the feasibility and effectiveness of the proposed controller. Without loss of generality, the simulated wheeled mobile manipulator is equipped with one TRC LABMATE as the mobile platform and two PUMA-250 manipulators with two links as the dual arms. Tables I and II list the parameters of the mobile platform and the dual two-link arms. The detailed

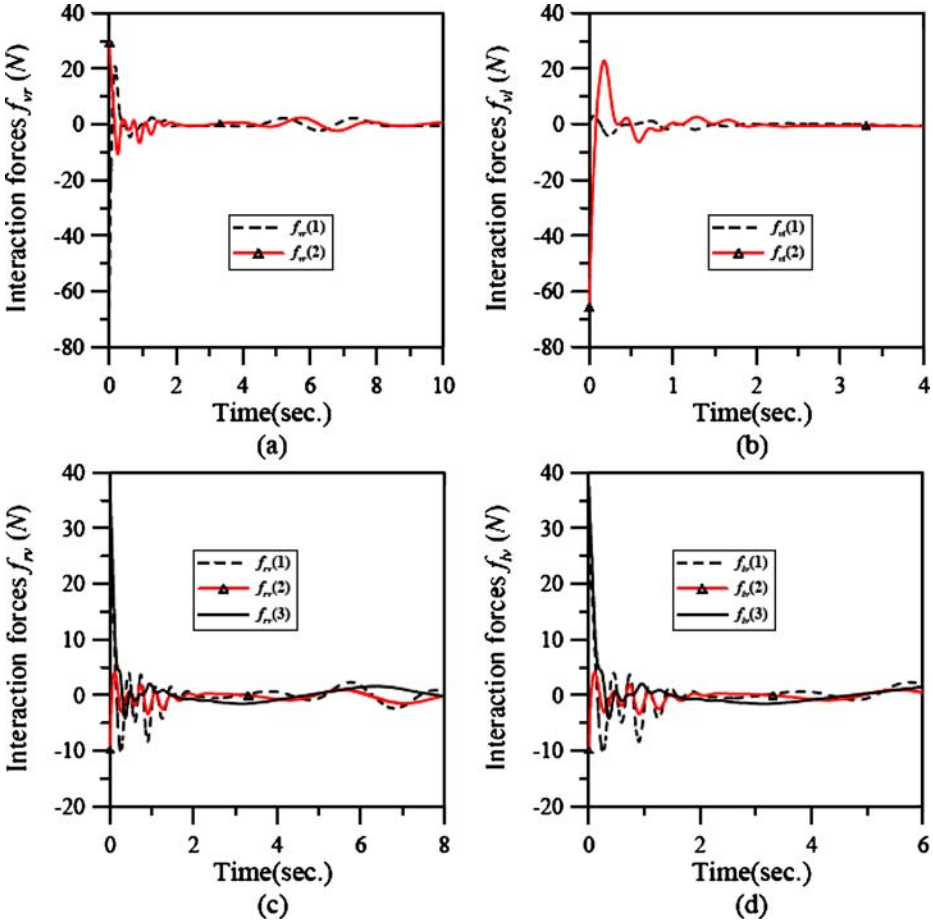

Figure 9 Dynamic interactions between the mobile platform and the dual arms. (a) Dynamic coupling forces in the mobile platform caused by the right arm; (b) dynamic coupling forces in the mobile platform caused by the left arm; (c) dynamic coupling forces in the right arm caused by the mobile platform; (d) dynamic coupling forces in the left arm caused by the mobile platform.

system parameters given in the Appendix is automatically derived using MATHEMATICA and the modeling procedure shown in Figure 4. The desired trajectories for the overall system were given as follows; the circular trajectory with a radius of 2 m was for the mobile platform and the same sinusoid trajectories were for the dual arms, i.e., $q_{vd}(t) = [x_d(t), y_d(t), \phi_d(t)]^T = \left[2 + 2S_{\phi_d}, 2 - 2C_{\phi_d}, w \cdot t\right]^T$, $\theta_{j1d}(t) = S_\theta$ and $\theta_{j2d}(t) = C_\theta$, j=r, l. Furthermore, the corresponding reference velocities of the mobile platform were selected by $\eta_d = [v_d, w_d]^T = \left[2m/s, 2rad/s\right]^T$. The simulations were conducted using the following initial conditions of the system: $q_v(0) = [-2.5, -2.5, 20^0]^T$, $\eta_v(0)$=[0, 0]^T, $q_j(0) = [1, -1]^T$, and $\dot{q}_j = [0, 0]^T$, j=r, l. The nonlinear tracking control laws (51–52) were applied to control the three subsystems by setting the following controller gains: [k_1, k_δ, k_z, k_r, k_l, k_{mr}, k_{ml}]=[2, 2, 5, 4, 4, 4, 4], α=0.0001 and δ=10. Figures 6, 7, and 8 depict the control performance of the mobile platform, the right arm and the left arm, respectively. The results in

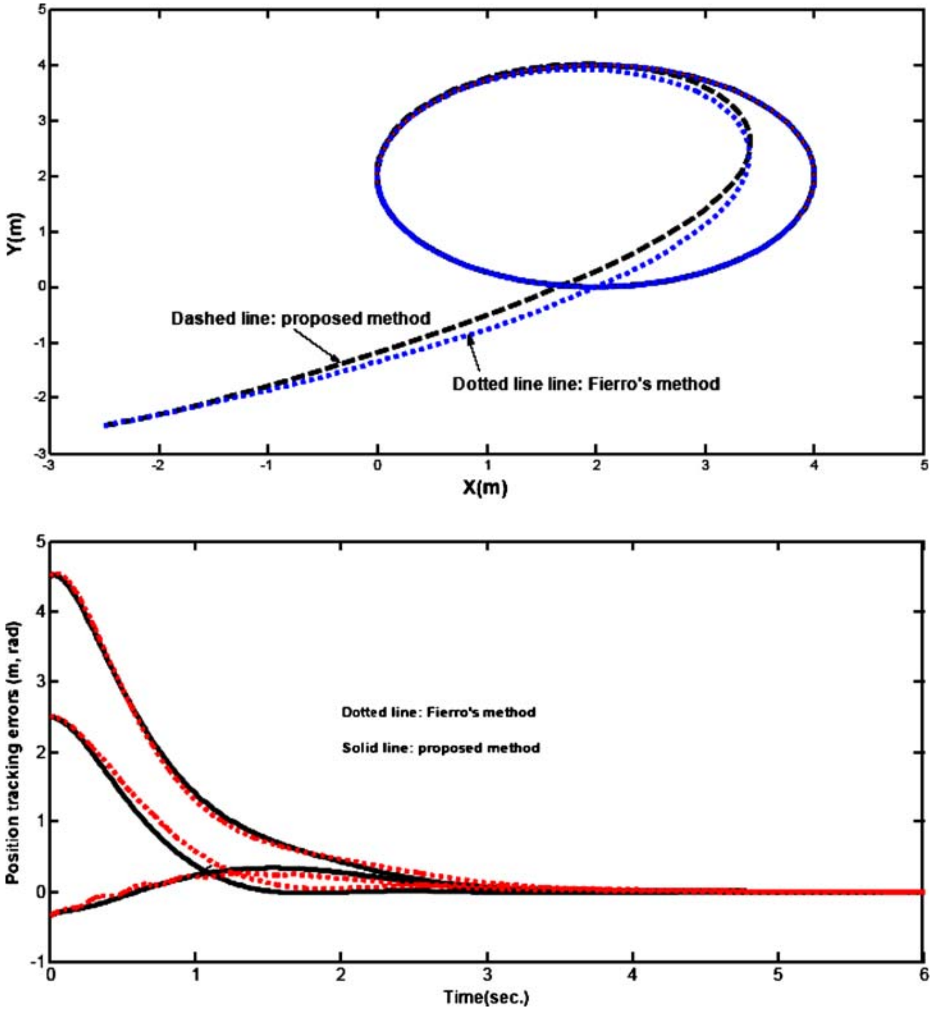

Figure 10 Performance comparison of the proposed controller with the Fierro method [6]. *Up*: Trajectories of the mobile platform; *bottom*: tracking position errors of the mobile platform.

Figures 6, 7, and 8 reveal that all position and velocity tracking errors asymptotically approach zero, as predicted by Theorem 3.2. Figure 9 shows that the dynamic interactive forces between the mobile platform and the dual arms are fully compensated. In addition, Figure 10 compares the performance of the proposed method and the Fierro method in [6] for the case of steering the mobile platform to follow a circular path. The simulation results also indicate that the proposed method in Theorem 3.1 outperforms the Fierro method slightly.

5 Conclusions

This paper has developed methodologies for dynamic modelling and tracking control of a wheeled mobile manipulator with dual arms. With the Lagrange's equations and MATHEMATICA, the accurate dynamic model and structural properties of the system have been easily established and then investigated. The derived model has been shown valid in comparison with other well-known models. Via the Lyapunov stability theory, the nonlinear trajectory tracking controller with fully dynamic compensation ability has been proposed to achieve both velocity and position tracking for the wheeled mobile manipulator system. Numerical results indicate that proposed control method not only satisfies the requirement of velocity and position tracking performance, but also compensates for the dynamic interactions between the mobile platform and the two arms. An interesting topic for future work would be to combine recurrent neural networks and nonlinear control for developing a new structure of an adaptive and robust controller for this kind of manipulator with uncertainties caused from mass change and static and dynamic frictions.

Acknowledgment The author gratefully acknowledges financial support from the National Science Council, Taiwan, R.O.C., under the project contract NSC92-2213-E-005 -009.

Appendix

For simplifying the repression of system models, the subscript {0, 1, 2, 3, 4, 5, 6} in triangular functions are adopted to represent the rotational angles of a mobile platform and the two-link dual arms, i.e., $S_0 = \mathrm{Sin}(\phi)$, $C_6 = \mathrm{Cos}(\theta_{l3})$, and $S_{04} = \mathrm{Sin}(\phi + \theta_{l1})$.

(1) Mass matrix: M_s

$$M_s(1, 1) = m_s + 2I_A C_0^2; \quad M_s(1, 2) = 2I_A S_0 C_0;$$
$$M_s(1, 3) = -m_s d S_0 - (m_r - m_l) e C_0 - \Omega_{r2} S_2 S_{02} - \Omega_{l2} S_4 S_{04}; \quad M_s(1, 4) = -\Omega_{r2} S_2 S_{01};$$
$$M_s(1, 5) = \Omega_{r2} C_2 C_{01}; \quad M_s(1, 6) = -\Omega_{l2} S_4 S_{03}; \quad M_s(1, 7) = \Omega_{l2} C_4 C_{03};\ M_s(2, 1) = 2I_A S_0 C_0;$$
$$M_s(2, 2) = m_s + 2I_A S_0^2;\ M_s(2, 3) = m_s d C_0 - (m_r - m_l) e S_0 + \Omega_{r2} S_2 C_{01} + \Omega_{l2} S_4 C_{03};$$
$$M_s(2, 4) = \Omega_{r2} S_2 C_{01}; \quad M_s(2, 5) = \Omega_{l2} C_2 S_{02}; \quad M_s(2, 6) = \Omega_{l2} S_4 C_{03}; \quad M_s(2, 7) = \Omega_{l2} C_4 S_{04};$$
$$M_s(3, 1) = -m_s d S_0 - (m_r - m_l) e C_0 - \Omega_{r2} S_2 S_{01} - \Omega_{l2} S_4 S_{03};$$
$$M_s(3, 2) = m_s d C_0 - (m_r - m_l) e S_0 + \Omega_{r2} S_2 C_{01} + \Omega_{l2} S_4 C_{03};$$
$$M_s(3, 3) = 2I_0 + I_{rs} + I_{ls} + (m_r + m_l)\left(d^2 + e^2\right) + 2\Omega_{r2} S_2 (dC_1 + eS_1) + H_{r2} S_2^2$$
$$+2\Omega_{l2} S_4 (dC_3 + eS_3) + H_{l2} S_4^2;$$
$$M_s(3, 4) = \Omega_{r2} S_2 (dC_1 + eS_1) + H_{r2} S_2^2; \quad M_s(3, 5) = \Omega_{r2} C_2 (dS_1 - eC_1)$$

$M_s(3,6) = \Omega_{l2}S_4(dC_3 - eS_3) + H_{l2}S_4^2;\quad M_s(3,7) = \Omega_{l2}C_4(dS_3 + eC_3);\ M_s(4,1) = -\Omega_{r2}S_2S_{01};$
$M_s(4,2) = \Omega_{r2}S_2C_{01};\ M_s(4,3) = \Omega_{r2}S_2(dC_1 + eS_1) + H_{r2}S_4^2;$
$M_s(4,4) = I_r + H_{r2}S_2^2;\quad M_s(4,5) = M_s(4,6) = M_s(4,7) = 0;\quad M_s(5,1) = \Omega_{r2}C_2C_{01};$
$M_s(5,2) = \Omega_{r2}C_2S_{01};\ M_s(5,3) = \Omega_{r2}C_2(dS_1 - eC_1);\ M_s(5,4) = 0;\ M_s(5,5) = I_{mY2} + H_{r2};$
$M_s(5,6) = M_s(5,7) = 0;\quad M_s(6,1) = -\Omega_{l2}S_4S_{03};\quad M_s(6,2) = \Omega_{l2}S_4C_{03};$
$M_s(6,3) = \Omega_{l2}S_4(dC_3 - eS_3) + H_{l2}S_3^2;\ M_s(6,4) = 0;\ M_s(6,5) = 0;\ M_s(6,6) = I_l + H_{l2}S_4^2;$
$M_s(6,7) = 0;\ M_s(7,1) = \Omega_{l2}C_4C_{03};\ M_s(7,2) = \Omega_{l2}C_4S_{03};\ M_s(7,3) = \Omega_{l2}C_4(dS_3 + eC_3);$
$M_s(7,4) = M_s(7,5) = M_s(7,6) = 0;\ M_s(7,7) = I_{lY2} + H_{l2};$

(2) Gravity matrix: $G_v = [0,\ 0,\ 0]^T$, $G_r = [0,\ -\Omega_{r2}S_2]^T$, $G_l = [0,\ -\Omega_{l2}S_4]^T$;
(3)Centripetal matrix:

$C_s(1,1) = -2I_A\dot{\phi}S_0C_0;\ C_s(1,2) = I_A\dot{\phi}\left(C_0^2 - S_0^2\right);$

$$C_s(1,3) = -2I_A\dot{x}S_0C_0 + I_A\dot{y}\left(C_0^2 - S_0^2\right) - m_sd\dot{\phi}C_0 + (m_r - m_l)\dot{\phi}eS_0$$
$$-\Omega_{r2}\left(\dot{\phi} + \dot{\theta}_{r1}\right)S_2C_{01} - \Omega_{l2}\left(\dot{\phi} + \dot{\theta}_{l1}\right)S_4C_{03} - \Omega_{r2}\dot{\theta}_{r2}C_2S_{01} - \Omega_{l2}\dot{\theta}_{l2}C_4S_{03};$$

$C_s(1,4) = -\Omega_{r2}S_2\left(\dot{\phi} + \dot{\theta}_{r1}\right)C_{01} - \Omega_{r2}\dot{\theta}_{r2}C_2S_{01};\ C_s(1,5) = -\Omega_{r2}C_2\left(\dot{\phi} + \dot{\theta}_{r1}\right)S_{01} - \Omega_{r2}\dot{\theta}_2S_2C_{01};$

$C_s(1,6) = -\Omega_{l2}S_4\left(\dot{\phi} + \dot{\theta}_{l1}\right)C_{03} - \Omega_{l2}\dot{\theta}_{l2}C_4S_{03};\ C_s(1,7) = -\Omega_{l2}C_4\left(\dot{\phi} + \dot{\theta}_{l1}\right)S_{03} - \Omega_{l2}\dot{\theta}_{l2}S_4C_{03};$

$C_s(2,1) = I_A\dot{\phi}C_{2\phi};\quad C_s(2,2) = 2I_A\dot{\phi}C_0S_0;$

$$C_s(2,3) = I_A\dot{x}\left(C_0^2 - S_0^2\right) + 2I_A\dot{y}S_0C_0 - m_sd\dot{\phi}S_0 - (m_r - m_l)\dot{\phi}eC_0$$
$$-\Omega_{r2}\left(\dot{\phi} + \dot{\theta}_{r1}\right)S_2S_{01} - \Omega_{l2}\left(\dot{\phi} + \dot{\theta}_{l1}\right)S_4S_{03} + \Omega_{r2}\dot{\theta}_{r2}C_2C_{01} + \Omega_{l2}\dot{\theta}_{l2}C_4C_{03};$$

$C_s(2,4) = -\Omega_{r2}\left(\dot{\phi} + \dot{\theta}_{r1}\right)S_2S_{01} + \Omega_{r2}\dot{\theta}_{r2}C_2C_{01};\ C_s(2,5) = \Omega_{r2}\left(\dot{\phi} + \dot{\theta}_{r1}\right)C_2C_{01} - \Omega_{r2}\dot{\theta}_{r2}S_2S_{01};$

$C_s(2,6) = -\Omega_{l2}\left(\dot{\phi} + \dot{\theta}_{l1}\right)S_4S_{03} + \Omega_{r2}\dot{\theta}_{l2}C_4C_{03};\ C_s(2,7) = \Omega_{l2}\left(\dot{\phi} + \dot{\theta}_{l1}\right)C_4C_{03} - \Omega_{l2}\dot{\theta}_{l2}S_4S_{03};$

$C_s(3,1) = 2I_A\dot{x}S_0C_0 - I_A\dot{y}\left(C_0^2 - S_0^2\right);\ C_s(3,2) = -I_A\dot{x}\left(C_0^2 - S_0^2\right) - 2I_A\dot{y}S_0C_0;$

$$C_s(3,3) = \Omega_{r2}\dot{\theta}_{r1}S_2(-dS_1 + eC_1) + \Omega_{r2}\dot{\theta}_2C_2(dC_1 + eS_1) + H_{r2}\dot{\theta}_2S_2C_2$$
$$-\Omega_{l2}\dot{\theta}_{l1}S_4(dS_3 + eC_3) + \Omega_{l2}\dot{\theta}_{l2}C_4(dC_3 - eS_3) + H_{l2}\dot{\theta}_{l2}S_4C_4;$$

$C_s(3,4) = -\Omega_{r2}S_2\left(\dot{\phi} + \dot{\theta}_{r1}\right)(dS_1 - eC_1) + \Omega_{r2}\dot{\theta}_{r2}C_2(dC_1 + eS_1) + H_{r2}\dot{\theta}_{r2}C_2S_2;$

$C_s(3,5) = \Omega_{r2}C_2\left(\dot{\phi} + \dot{\theta}_{r1}\right)(dC_1 + eS_1) + H_{r2}\left(\dot{\phi} + \dot{\theta}_{r1}\right)C_2S_2 + \Omega_{r2}S_2\dot{\theta}_{r2}(-dS_1 + eC_1);$

$C_s(3,6) = -\Omega_{r2}S_4\left(\dot{\phi} + \dot{\theta}_{l1}\right)(dS_3 + eC_3) + \Omega_{r2}\dot{\theta}_{l2}C_4(dC_3 - eS_3) + H_{r2}\dot{\theta}_{l2}S_4C_4;$

$C_s(3,7) = \Omega_{l2}C_4\left(\dot{\phi} + \dot{\theta}_{l1}\right)(dC_3 - eS_3) + H_{l2}\left(\dot{\phi} + \dot{\theta}_{l1}\right)S_4C_4 - \Omega_{l2}\dot{\theta}_{l2}S_4(dS_3 + eC_3);$

$C_s(4,1) = C_s(4,2) = 0;;C_s(4,3) = \Omega_{r2}\dot{\phi}S_2(dS_1 - eC_1) + H_{r2}\dot{\theta}_{r2}S_2C_2;$
$C_s(4,4) = H_{r2}\dot{\theta}_{r2}S_2C_2;\quad C_s(4,5) = H_{r2}\left(\dot{\phi} + \dot{\theta}_{r1}\right)S_2C_2;\quad C_s(4,6) = C_s(4,7) = 0;$

$C_s(5,1) = \ C_s(5,2) = 0;\ \ C_s(5,3) = -\Omega_{r2}\dot{\phi}C_2(dC_1 + eS_1) - H_{r2}\dot{\phi}S_2C_2 - H_{r2}\dot{\theta}_{r1}S_2C_2$
$C_s(5,4) = -H_{r2}\left(\dot{\phi} + \dot{\theta}_{r1}\right)S_2C_2;\ C_s(5,5) = C_s(5,6) = C_s(5,7) = 0;$

$C_s(6,1) = C_s(6,2) = 0;\ C_s(6,3) = \Omega_{l2}\dot{\phi}S_4(dS_3 + eC_3) + H_{r2}\dot{\theta}_{l2}S_3C_3$

$$C_s(6,4) = C_s(6,5) = 0;\; C_s(6,6) = H_{l2}\dot{\theta}_{l2}S_4C_4;\; C_s(6,7) = H_{l2}\left(\dot{\phi} + \dot{\theta}_{l1}\right)S_4C_4;$$

$$C_s(7,1) = C_s(7,2) = C_s(7,4) = C_s(7,5) = C_s(7,7) = 0;$$

$$C_s(7,3) = -\Omega_{l2}\dot{\phi}C_4(dC_3 - eS_3) - H_{l2}\left(\dot{\phi} + \dot{\theta}_{l1}\right)S_4C_4 = C_s(7,6) = -H_{l2}\left(\dot{\phi} + \dot{\theta}_{l1}\right)S_4C_4.$$

where $I_A = \frac{I_w}{R_w^2}$, $I_o = I_A b^2 + \frac{I_c}{2} + \frac{m_c d^2}{2}$, $m_j = \sum_{i=1}^{2} m_{ji}$, $m_s = m_c + m_r + m_l$, $I_{js} = \sum_{i=1}^{2} I_{jzi}$, $\Omega_{li} = m_{ji}\overline{X}_{ji}$, and $H_{li} = m_{ji}\overline{X}_{ji}^2$.

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