

Triad Instability of Planetary Rossby Waves

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ABSTRACT

The triad instability of the large-scale, first-mode, baroclinic Rossby waves is studied in the context of the planetary scale when the Coriolis parameter is to its lowest order varying with latitude. Accordingly, rather than remain constant as in quasigeostrophic theory, the deformation radius also changes with latitude, yielding new and interesting features to the propagation and triad instability processes. On the planetary scale, baroclinic waves vary their meridional wavenumbers along group velocity rays while they conserve both frequencies and zonal wavenumbers. The amplitudes of both barotropic and baroclinic waves would change with latitude along a ray path in the same way that the Coriolis parameter does if effects of the nonlinear interaction are ignored. The triad interaction for a specific triad is localized within a small latitudinal band where the resonance conditions are satisfied and quasigeostrophic theory is applicable locally. Using the growth rate from that theory as a measure, at each latitude along the ray path of the basic wave, a barotropic wave and a secondary baroclinic wave are picked up to form the most unstable triad and the distribution of this maximum growth rate is examined. It is found to increase southward under the assumption that triad interactions do not cause a noticeable decrease in the quantity of the basic wave's amplitude divided by the Coriolis parameter. Different barotropic waves that maximize the growth rate at different latitudes have almost the same meridional length scale, on the order of the deformation radius. With many rays starting from different latitudes on the eastern boundary and with wavenumbers on each of them satisfying the no-normal-flow condition, the resulting two-dimensional distribution of the growth rate is a complicated function of the relative relations of zonal wavenumbers or frequencies on different rays and the orientation of the eastern boundary. In general, the growth rate is largest on rays originating to the north.

1. Introduction

As revealed by analysis of the Ocean Topography Experiment (TOPEX)/Poseidon data of sea surface height (Chelton and Schlax 1996), the first-mode baroclinic Rossby waves emanating from the eastern boundary of the Pacific Ocean can propagate westward across the basin only in low-latitude regions (see their Fig. 4). In middle-latitude regions, the wave patterns of the sea level signals fade soon after they leave the coast and, at

the same time, an eddy field emerges. To explain this latitudinal structure, Qiu et al. (1997) studied the decay of the first-mode baroclinic Rossby waves due to a prescribed horizontal eddy viscosity and found that the e -folding distance of the decaying waves is smaller toward higher latitudes, indicating that those signals tend to be more trapped to the eastern boundary. This behavior is due entirely to the much slower Rossby wave speed in high-latitude regions where the Rossby deformation radius, L_d , is smaller.

In their recent study, LaCasce and Pedlosky (2004), using a two-layer quasigeostrophic model, investigated the instability of the free, first-mode baroclinic Rossby waves. They described the confinement of the wave patterns to the eastern boundary as the process by

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which the basic wave loses energy to an eddy field growing out of instability. Whether the basic wave can successfully cross the basin is determined by the critical parameter Z , the ratio of the time for the long basic wave to traverse the basin to the e -folding growth time of parasitic instabilities. Here Z is proportional to the characteristic fluid velocity V of the basic wave divided by L_d^3 , where L_d is the deformation radius. More specifically,

$$Z = \frac{VL}{\beta L_d^3},$$

where L is the characteristic length scale of the basin and the zonal length scale of the wave. Within the quasigeostrophic framework, the Coriolis parameter to the lowest order is constant despite its meridional gradient β , so the deformation radius, the gravity wave speed divided by the Coriolis parameter, is constant with latitude and the key factor for the size of Z is the shear associated with the basic wave. For small Z or small characteristic fluid velocity, the instability is manifested as the triad interaction between the basic wave and two “daughter” waves that feed on its energy, but the decay of the basic wave by the instability is so slow that it can successfully cross the basin without losing its wave character. For large Z , conventional baroclinic instability is the main mechanism of decay, and the basic wave breaks down rapidly into an eddy field before it reaches the western boundary. By considering the parametric dependence of the ratio on latitude, possible only on the planetary scale, one could draw the conclusion that Z increases toward the high-latitude regions as the Coriolis parameter increases northward, which, as suggested by LaCasce and Pedlosky, is the reason for the confinement of wave patterns found in satellite measurements. This argument is promising; however, by only examining the ratio Z parametrically, it failed to consider any other effects that the latitudinal variation of the deformation radius has on the instability process. One obvious shortcoming of the application of the quasigeostrophic theory in this case is that the latitudinal variation of the Coriolis parameter, as it alters the deformation radius on the planetary scale, is not consistently treated. For example, the characteristics of the waves themselves will change with latitude as they propagate, which can in turn affect the instability process, but we do not know in what way the waves change and how the instability is influenced as long as we are still within the quasigeostrophic framework. Thus, an important further step is to reconsider the issue on the planetary scale, which is the primary motivation of the present study.

In this paper, we study the propagation and instability of the planetary baroclinic Rossby waves, and only examine the triad instability corresponding to the regime of small Z . Our aim is to find out how the planetary-scale propagation of waves as well as the parasitic triad instability varies with latitude when the deformation radius is varying with latitude.

2. The model

To simplify the analysis, we use Cartesian coordinates and approximate the Coriolis parameter as a linear function of meridional distance from a reference latitude θ_0 , that is, $f_{\text{dim}} = f_0 + \beta y_{\text{dim}}$, where $\beta = 2\Omega \cos\theta_0/R$ and $f_0 = 2\Omega \sin\theta_0$; f_{dim} is allowed to vary by an order-1 amount.

We use a two-layer model with flat surface and bottom. For simplicity, the two layers are set to have the same depth, H . Motion in the model is hydrostatic and, to the lowest order, geostrophic. The horizontal momentum equations and the continuity equations for the two layers are as follows:

$$\frac{\partial \mathbf{u}_1}{\partial t} + \mathbf{u}_1 \cdot \nabla \mathbf{u}_1 + f \mathbf{k} \times \mathbf{u}_1 = -\nabla p_1, \quad (1)$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0, \quad (2)$$

$$\frac{\partial \mathbf{u}_2}{\partial t} + \mathbf{u}_2 \cdot \nabla \mathbf{u}_2 + f \mathbf{k} \times \mathbf{u}_2 = -\nabla p_2, \quad \text{and} \quad (3)$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial w_2}{\partial z} = 0. \quad (4)$$

In the above equations p_n is the pressure divided by the mean density ρ_0 .

Adding and subtracting momentum equations lead to

$$\frac{\partial \mathbf{u}_B}{\partial t} + \mathbf{u}_B \cdot \nabla \mathbf{u}_B + \mathbf{u}_T \cdot \nabla \mathbf{u}_T + f \mathbf{k} \times \mathbf{u}_B = -\nabla p_B \quad (5)$$

and

$$\frac{\partial \mathbf{u}_T}{\partial t} + \mathbf{u}_B \cdot \nabla \mathbf{u}_T + \mathbf{u}_T \cdot \nabla \mathbf{u}_B + f \mathbf{k} \times \mathbf{u}_T = -\nabla p_T, \quad (6)$$

where the subscript B denotes barotropic motion, T denotes baroclinic motion,

$$\mathbf{u}_B = \frac{\mathbf{u}_1 + \mathbf{u}_2}{2}, \quad \mathbf{u}_T = \frac{\mathbf{u}_1 - \mathbf{u}_2}{2}, \quad p_B = \frac{p_1 + p_2}{2},$$

and

$$p_T = \frac{p_1 - p_2}{2}.$$

As to the continuity equations, we can first integrate them through the depth of each layer and then manipulate them in the same way as we did to the momentum equations:

$$\frac{\partial u_B}{\partial x} + \frac{\partial v_B}{\partial y} - \frac{\partial}{\partial x} \left(u_T \frac{\eta}{H} \right) - \frac{\partial}{\partial y} \left(v_T \frac{\eta}{H} \right) = 0 \quad (7)$$

and

$$\frac{\partial \eta}{\partial t} + \mathbf{u}_B \cdot \nabla \eta + \eta \left(\frac{\partial u_B}{\partial x} + \frac{\partial v_B}{\partial y} \right) - H \left(\frac{\partial u_T}{\partial x} + \frac{\partial v_T}{\partial y} \right) = 0, \quad (8)$$

where η is the interface elevation. Equations (5)–(8) are a complete set because using hydrostatic relation η can be linearly related to the dynamical pressure difference between the two layers as $g'\eta = p_2 - p_1 = -2p_T$.

As shown by LaCasce and Pedlosky (2004), the ratio of two time scales determines the fate of the long baroclinic Rossby waves. The first one is the time taken to traverse the basin by the basic wave, which is

$$T_R = \frac{L}{\beta L_d^2}$$

with the characteristic scale of the basin L , and is also the characteristic period of the Rossby wave with wavelength L . The second time scale is the e -folding growth time of instability, which is $T_g = L_d/V$ for both the triad interaction and baroclinic instability (LaCasce and Pedlosky 2004), where V is the characteristic fluid velocity of the basic wave.

The dispersion relation for linear, freely propagating baroclinic Rossby waves,

$$\omega = - \frac{\beta k}{k^2 + l^2 + 1/L_d^2},$$

explicitly depends on latitude. For the first mode, L_d is smaller than 80 km north of 20°N and more than 200 km in the Tropics (Chelton et al. 1998). According to the wave theory in the Wentzel–Kramers–Brillouin limit (Schopf et al. 1981), if the dispersion relation only explicitly depends on the meridional coordinate, the frequency and zonal wavenumber remain constant following the wave propagating with the group velocity, while the meridional wavenumber varies with the deformation radius. To guarantee the validity of the linear dispersion relation and yet make the variation of the meridional wavenumber discernible, we assume the meridional wavenumbers are small but not much smaller than $1/L_d$. Besides, perturbations growing out

of the baroclinic or triad instability have scales of the deformation radius, so we pick L_d as the first important length scale. On the other hand, the largest length scale that we are dealing with is the scale of the earth's radius, much larger than that of the deformation radius. Therefore, we have two distinct length scales in the meridional direction: one is L_d , over which the wave oscillates with a locally constant meridional wavelength comparable to L_d ; the other is L , over which the Coriolis parameter varies and the wave responds to that variation by changing its meridional wavelength as it propagates along a group velocity ray. In the zonal direction, there is only one scale L , the scale of the basin and zonal wavelength of the basic wave.

It is natural to define multiple time and length variables using those scales:

$$t_r = \frac{t_{\text{dim}}}{T_R}, \quad t_g = \frac{t_{\text{dim}}}{T_g}, \quad s = \frac{y_{\text{dim}}}{L_d},$$

$$Y = \frac{y_{\text{dim}}}{L}, \quad \text{and} \quad x = \frac{x_{\text{dim}}}{L}.$$

The nondimensional form of the Coriolis parameter is

$$f = \frac{f_{\text{dim}}}{f_0} = 1 + b(Y - Y_0),$$

where $b = \beta L/f_0$, and is small in quasigeostrophy when $O(L) \leq O(L_d)$ but is order 1 on the planetary scale. Because the meridional length scale is much less than the zonal one, the characteristic fluid velocity has different scales in the two directions as a result of the geostrophic balance. In addition, the scalings for pressure and interface elevation can also be obtained from that balance:

$$(u, v) \rightarrow V \left(u \frac{L}{L_d}, v \right) \quad \frac{\partial}{\partial t} \rightarrow \frac{\beta L_d^2}{L} \left(\frac{\partial}{\partial t_r} + Z \frac{\partial}{\partial t_g} \right)$$

$$p \rightarrow f_0 V L p \quad \frac{\partial}{\partial y} \rightarrow \frac{1}{L_d} \left(\frac{\partial}{\partial s} + \varepsilon \frac{\partial}{\partial Y} \right),$$

$$\eta \rightarrow \frac{f_0 V L}{g'} \eta \quad \frac{\partial}{\partial x} \rightarrow \frac{1}{L} \frac{\partial}{\partial x}$$

where

$$Z = \frac{T_r}{T_g} = \frac{V L}{\beta L_d^3} \quad \text{and} \quad \varepsilon = \frac{L_d}{L}.$$

Now, the nondimensional forms of the original equations are

$$\varepsilon b \left(\frac{\partial}{\partial t_r} + Z \frac{\partial}{\partial t_g} \right) u_B + \varepsilon b Z [u_B u_{B_x} + v_B u_{B_s} + u_T u_{T_x} + v_T u_{T_s} + \varepsilon (v_B u_{B_Y} + v_T u_{T_Y})] - f v_B = - \frac{\partial p_B}{\partial x}, \quad (9)$$

$$\varepsilon b \left(\frac{\partial}{\partial t_r} + Z \frac{\partial}{\partial t_g} \right) u_T + \varepsilon b Z [u_B u_{T_x} + v_B u_{T_s} + u_T u_{B_x} + v_T u_{B_s} + \varepsilon (v_B u_{T_Y} + v_T u_{B_Y})] - f v_T = - \frac{\partial p_T}{\partial x}, \quad (10)$$

$$\varepsilon^3 b \left(\frac{\partial}{\partial t_r} + Z \frac{\partial}{\partial t_g} \right) v_B + \varepsilon^3 b Z [u_B v_{B_x} + v_B v_{B_s} + u_T v_{T_x} + v_T v_{T_s} + \varepsilon (v_B v_{B_Y} + v_T v_{T_Y})] + f u_B = - \frac{\partial p_B}{\partial s} - \varepsilon \frac{\partial p_B}{\partial Y}, \quad (11)$$

$$\varepsilon^3 b \left(\frac{\partial}{\partial t_r} + Z \frac{\partial}{\partial t_g} \right) v_T + \varepsilon^3 b Z [u_B v_{T_x} + v_B v_{T_s} + u_T v_{B_x} + v_T v_{B_s} + \varepsilon (v_B v_{T_Y} + v_T v_{B_Y})] + f u_T = - \frac{\partial p_T}{\partial s} - \varepsilon \frac{\partial p_T}{\partial Y}, \quad (12)$$

$$(u_{B_x} + v_{B_s} + \varepsilon v_{B_Y}) - \varepsilon b Z [(u_T \eta)_x + (v_T \eta)_s + \varepsilon (v_T \eta)_Y] = 0, \quad \text{and} \quad (13)$$

$$\varepsilon b \left(\frac{\partial}{\partial t_r} + Z \frac{\partial}{\partial t_g} \right) \eta - (u_{T_x} + v_{T_s} + \varepsilon v_{T_Y}) + \varepsilon b Z [(u_B \eta)_x + (v_B \eta)_s + \varepsilon (v_B \eta)_Y] = 0. \quad (14)$$

There are three nondimensional parameters in these equations: ε , b , and Z . For motion on the planetary scale, $\varepsilon = L_d/L$ is much less than 1 while $b = \beta L/f_0$ is of order 1. Only the parameter Z could be large or small corresponding to different dynamical regimes. Since the regime where the basic wave may successfully cross the basin despite triad interactions is the focus of our study, we assume that $Z = \varepsilon \ll 1$. This allows us to examine the effects of β dispersion on the growth (Schopf et al. 1981). The opposing limit $Z \gg 1$ has been investigated by Isachsen et al. (2007). In the limit $Z \ll 1$ the energy-releasing process is the triad instability of the basic wave, which can be analyzed by expanding variables in a series of ε or Z using the ordering relation $Z = O(\varepsilon)$.

3. Triad instability

We expand the pressure as

$$\begin{aligned} p_B &= p_B^{(0)} + \varepsilon p_B^{(1)} + \varepsilon^2 p_B^{(2)} + \dots \quad \text{and} \\ p_T &= p_T^{(0)} + \varepsilon p_T^{(1)} + \varepsilon^2 p_T^{(2)} + \dots \end{aligned} \quad (15)$$

and then substitute into the Eqs. (9)–(14) to find solutions at each order.

To the lowest order, motion is steady and geostrophic without the variation on slow length scales entering explicitly, that is,

$$\begin{aligned} f v_B^{(0)} &= \frac{\partial p_B^{(0)}}{\partial x} & f v_T^{(0)} &= \frac{\partial p_T^{(0)}}{\partial x} \\ f u_B^{(0)} &= - \frac{\partial p_B^{(0)}}{\partial y} & f u_T^{(0)} &= - \frac{\partial p_T^{(0)}}{\partial y} \\ \frac{\partial u_B^{(0)}}{\partial x} + \frac{\partial v_B^{(0)}}{\partial s} &= 0 & \frac{\partial u_T^{(0)}}{\partial x} + \frac{\partial v_T^{(0)}}{\partial s} &= 0 \end{aligned} \quad (16)$$

To order $O(\varepsilon)$, time dependence on the fast time scale is determined by two linear equations:

$$\begin{aligned} \frac{\partial}{\partial t_r} \left[\frac{\partial^2 p_B^{(0)}}{\partial s^2} \right] + \frac{\partial p_B^{(0)}}{\partial x} &= 0 \quad \text{and} \\ \frac{\partial}{\partial t_r} \left[\frac{\partial^2 p_T^{(0)}}{\partial s^2} - 2f^2 p_T^{(0)} \right] + \frac{\partial p_T^{(0)}}{\partial x} &= 0. \end{aligned} \quad (17)$$

Both free barotropic and baroclinic Rossby waves and their superpositions can satisfy the above equations. To study triad interactions, we add two other wave solutions besides the basic one: a secondary baroclinic Rossby wave and a barotropic Rossby wave. The total solutions are, thus

$$\begin{aligned} p_B^{(0)} &= A_B e^{i\Theta_B} + * \quad \text{and} \\ p_T^{(0)} &= A_T e^{i\Theta_T} + A_{To} e^{i\Theta_{To}} + *, \end{aligned} \quad (18)$$

where the asterisks represent the complex conjugates of the preceding functions and the wave amplitudes have no dependence on t_r , x , or y . The dispersion relations for the three waves are

$$\omega_T = - \frac{k_T}{l_T^2 + 2f(Y)^2}, \quad \omega_{To} = - \frac{k_{To}}{l_{To}^2 + 2f(Y)^2},$$

and

$$\omega_B = - \frac{k_B}{l_B^2}.$$

Zonal wavenumbers are absent from denominators because they are much less than the meridional ones. As expected, the dispersion relations of the baroclinic waves are functions of Y , so their meridional wavenumbers, while constant locally, vary with latitude. The phases of baroclinic waves are therefore functions of Y too:

$$\begin{aligned}\Theta_T &= -\omega_T t_r + k_T x + \theta_T(Y)/\varepsilon \quad \text{and} \\ \Theta_{T_o} &= -\omega_{T_o} t_r + k_{T_o} x + \theta_{T_o}(Y)/\varepsilon,\end{aligned}\quad (19)$$

whose derivatives with respect to s yield meridional wavenumbers,

$$\begin{aligned}l_T &= \frac{\partial \Theta_T}{\partial s} = \frac{\partial \theta_T}{\partial Y} \quad \text{and} \\ l_{T_o} &= \frac{\partial \Theta_{T_o}}{\partial s} = \frac{\partial \theta_{T_o}}{\partial Y}.\end{aligned}\quad (20)$$

The barotropic wave has constant frequency and wavenumber because its dispersion relation is independent of Y , so its phase linearly depends on the fast time and spatial variables, $\Theta_B = -\omega_B t_r + k_B x + l_B s$.

In the limit of small Z , the variation on the growth time t_g is of order $O(Z)$ compared with that in the traverse time t_r , implying that effects of instability are not apparent until we go to the next order.

The equations at order $O(\varepsilon^2)$ become nonlinear,

$$\frac{\partial}{\partial t_r} \left[\frac{\partial^2 p_B^{(1)}}{\partial x} \right] + \frac{\partial p_B^{(1)}}{\partial x} = -2 \frac{\partial}{\partial t_r} \left[\frac{\partial^2 p_B^{(0)}}{\partial Y \partial s} \right] + 2 \frac{b}{f} \frac{\partial}{\partial t_r} \left[\frac{\partial p_B^{(0)}}{\partial s} \right] - \left\{ \frac{\partial}{\partial t_g} \left[\frac{\partial^2 p_B^{(0)}}{\partial s^2} \right] + \frac{1}{f} J \left[p_B^{(0)}, \frac{\partial^2 p_B^{(0)}}{\partial s^2} \right] + \frac{1}{f} J \left[p_T^{(0)}, \frac{\partial^2 p_T^{(0)}}{\partial s^2} \right] \right\}$$

and

$$\begin{aligned}\frac{\partial}{\partial t_r} \left[\frac{\partial^2 p_T^{(1)}}{\partial s^2} - 2f^2 p_T^{(1)} \right] + \frac{\partial p_T^{(1)}}{\partial x} &= -2 \frac{\partial}{\partial t_r} \left[\frac{\partial^2 p_T^{(0)}}{\partial Y \partial s} \right] + 2 \frac{b}{f} \frac{\partial}{\partial t_r} \left[\frac{\partial p_T^{(0)}}{\partial s} \right] - \left\{ \frac{\partial}{\partial t_g} \left[\frac{\partial^2 p_T^{(0)}}{\partial s^2} - 2f^2 p_T^{(0)} \right] + \frac{1}{f} J \left[p_T^{(0)}, \frac{\partial^2 p_B^{(0)}}{\partial s^2} \right] \right. \\ &\quad \left. + \frac{1}{f} J \left[p_B^{(0)}, \frac{\partial^2 p_T^{(0)}}{\partial s^2} - 2f^2 p_T^{(0)} \right] \right\}.\end{aligned}\quad (21)$$

The motion at order $O(\varepsilon)$ is not free but *forced* by variations of the zero-order fields on slow spatial and time scales as well as the nonlinear interactions among them. Any two of the three waves interact with each other at zero order, producing forcing with frequencies equal to the sum and the difference of the two. If the following conditions are satisfied by the three waves,

$$\begin{aligned}k_T + k_{T_o} + k_B &= 0, \\ l_T + l_{T_o} + l_B &= 0, \quad \text{and} \\ \omega_T + \omega_{T_o} + \omega_B &= 0,\end{aligned}\quad (22)$$

a triad forms and resonant interaction occurs. The motions at first order will grow linearly with time,

rendering the expansion shown in Eq. (15) invalid after a time $t_r = O(1/\varepsilon)$. The present problem is peculiar compared with triad problem that is usually encountered because the meridional wavenumbers of the two baroclinic waves vary when propagating along rays. Consequently, the three waves that form a triad at some time and position along a group velocity ray may fail to satisfy Eq. (22) further along the ray path.

To ensure the expansion remains valid in the presence of the triad interaction and for at least a period of time as long as $t_r = O(1/\varepsilon)$, we eliminate the possible resonant terms of the “forcing” on the right-hand sides of the equations:

$$\begin{aligned}\frac{\partial}{\partial t_g} \left(\frac{A_B}{f} \right) + c_{gBy} \frac{\partial}{\partial Y} \left(\frac{A_B}{f} \right) + \frac{(k_T l_{T_o} - k_{T_o} l_T)(l_T^2 - l_{T_o}^2)}{l_B^2} \frac{A_{T_o}^* A_T^*}{f^2} e^{-ix} &= 0, \\ \frac{\partial}{\partial t_g} \left(\frac{A_T}{f} \right) + c_{gTy} \frac{\partial}{\partial Y} \left(\frac{A_T}{f} \right) + \frac{(k_B l_{T_o} - k_{T_o} l_B)(l_B^2 - l_{T_o}^2 - 2f^2)}{l_B^2} \frac{A_B^* A_{T_o}^*}{f^2} e^{-ix} &= 0, \quad \text{and} \\ \frac{\partial}{\partial t_g} \left(\frac{A_{T_o}}{f} \right) + c_{gT_o y} \frac{\partial}{\partial Y} \left(\frac{A_{T_o}}{f} \right) + \frac{(k_B l_T - k_T l_B)(l_B^2 - l_T^2 - 2f^2)}{l_B^2} \frac{A_B^* A_T^*}{f^2} e^{-ix} &= 0,\end{aligned}\quad (23)$$

where c_g represents the group velocity and $\chi = [l_B Y + \theta_T(Y) + \theta_{T_o}(Y)]/\varepsilon$ is the part of the sum of phases of the three waves that depends on Y . The first three terms in each equation have a clear physical meaning: they describe the evolution of the pressure amplitude divided by the Coriolis parameter (the geostrophic streamfunction amplitude) following the waves along the group velocity ray. If the nonlinear interaction were absent, those three terms would sum to zero and streamfunction amplitudes would be conserved, implying decrease of wave amplitude as wave energy propagates southward. This invariance is related to the conservation of energy. The last term in each equation is what can cause growth or decay of the streamfunction amplitude, and is what normally appears in the triad interaction problem except for the additional term $e^{-i\chi}$, a fast-oscillating term in Y due to ε in the denominator of χ . From the theory of stationary phase, we know that the main contribution to the integration of this term in Y comes from a narrow range with width of order $O(\sqrt{\varepsilon})$ and centered at the point where $\partial\chi/\partial Y = l_B + l_T + l_{T_o} = 0$, that is, where the resonance conditions are exactly satisfied. Integration over the remaining meridional intervals would be close to zero because of cancellation from fast oscillations of this final factor. This behavior is the mathematic manifestation of the important physical feature described before: because baroclinic waves change meridional wavenumbers as they propagate, the effects of resonant interaction on amplitudes are localized within a narrow latitudinal band where resonance conditions are well satisfied.

For the moment, consider waves whose streamfunction amplitudes do not vary with Y . If we linearize the equations around the basic wave's initial amplitude,

$$\begin{aligned} A_T &= A_0 + a, \\ A_{T_o} &= a_{T_o}, \quad \text{and} \\ A_B &= a_B, \end{aligned} \quad (24)$$

where a , a_{T_o} , and a_T are small when compared with A_0 , we can find exponential growth for the two small waves with growth rate σ :

$$\sigma^2 = \frac{(k_B l_T - k_T l_B)^2 (l_T^2 - l_{T_o}^2)(l_B^2 - l_T^2 - 2f^2) |A_0|^2}{l_B^2 (l_{T_o}^2 + 2f^2) f^2}. \quad (25)$$

This gives us a general idea of how fast the initially small amplitude can grow, though it is not the actual growth rate of the original problem that also has latitudinal dependence. Still, we can regard it as an upper limit of actual growth rate and find the wavenumbers of the two parasitic waves that can maximize σ as they

interact with a known basic wave. The underlying assumption is that the available parasitic perturbations have all possible wavenumbers so that the basic wave can select any two waves to form the most unstable triad. For Eq. (23), we believe the most important new feature is the meridional variation of the deformation radius, and this is the soul of the problem. Therefore, as the first step, we assume that the streamfunction amplitude, A/f , is independent of time along the ray, which is possible when all transient motions are ignored. Among waves with energy propagating in all possible directions, which are determined by group velocities associated with the wave vectors, we only consider, by choosing sign relations between zonal and meridional wavenumbers of each wave, the basic and the secondary baroclinic waves with energy propagating southward from the northern end of a region in Y , while the barotropic wave propagates northward. In both Figs. 1 and 2, the three waves start with zonally oriented wave vectors as they leave the northern and southern limits of the domain. After that, the two baroclinic waves change their meridional wavenumbers with Y to keep frequencies constant. Because of that variation, we can only have triad conditions strictly satisfied at one point, which is set to be in the middle of the range in these two figures. The wavenumbers of the two parasitic waves are selected in such a way that the triad interaction is the most unstable one according to Eq. (25) among all possible triads associated with the basic wave at that latitude. Near this latitude, the streamfunction amplitude of the basic wave decreases as its energy is drained, while those of the other two waves increase. Through the wave interaction, A/f of each of the two small waves increases to several times their initial values; the decrease of A_T/f is of order of the square of A_{T_o}/f or A_B/f and so is very small. When the value of ε is one-tenth of that in Fig. 1, variations of A/f in Fig. 2 are smaller and more localized. Outside the interaction zone, this quantity for all waves is nearly constant in Y indicating conservation mentioned before. In the interaction zone A_{T_o} and A_B vary in a similar way with similar magnitudes to A_{T_o}/f and A_B/f , but A_T decreases all the way southward with a variation a thousand times bigger than that of variation in A_T/f because again, the variation of the latter is of order square of A_{T_o}/f or A_B/f , which is only 0.01 in our calculations.

Triad interaction among three specific waves can only occur at a specific latitude. The basic wave, however, is subject to triad instability everywhere along its path by forming different triads with different parasitic waves, given that the ocean is always populated by perturbations with a wide range of spatial and time scales. An interesting question is then whether there are any

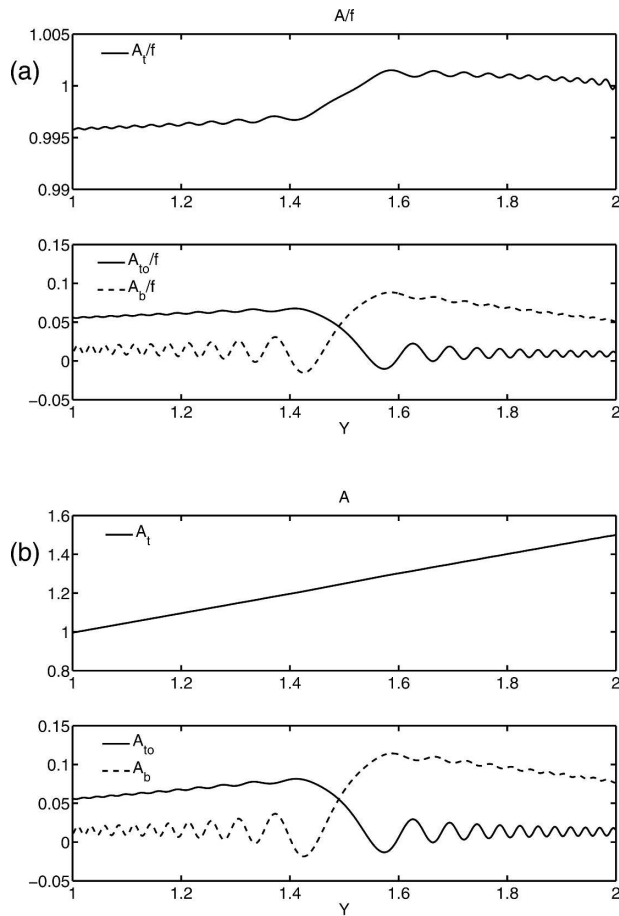


FIG. 1. The one-dimensional steady problem: (a) variations of amplitude divided by the Coriolis parameter in Y ; (b) the amplitudes themselves. The upper panel in (a) and (b) is for the basic wave, the solid line in the lower panel is for the secondary baroclinic wave, and the dotted line is for the barotropic wave. In both (a) and (b), $\varepsilon = 0.01$, $f = 1 + 0.5(Y - Y_s)$, $k_T = 1$, $l_T(Y_N) = 0$, $A_T/f(Y_N) = 1$, and $A_{To}/f(Y_N) = A_B/f(Y_s) = 0.01$.

latitudinal bands where a single basic wave is more vulnerable to the triad interaction or, in other words, where the instability is especially strong. To compare the effects of different triads on the same basic wave, we need to track many triads including many parasitic waves besides the basic one using Eq. (23), as we did for a single triad in Figs. 1 and 2. Meanwhile, outside the Tropics, ε is smaller than $O(0.01)$ so the interaction for a specific triad is limited to a very narrow region with a scale roughly of the deformation radius. Within such a small region, the variation of the Coriolis parameter is small compared with the Coriolis parameter itself. We can use quasi geostrophy locally to study the interaction and at the same time regard this local interaction as one of many events that happen to the basic wave, which, besides suffering from triad instability, varies its me-

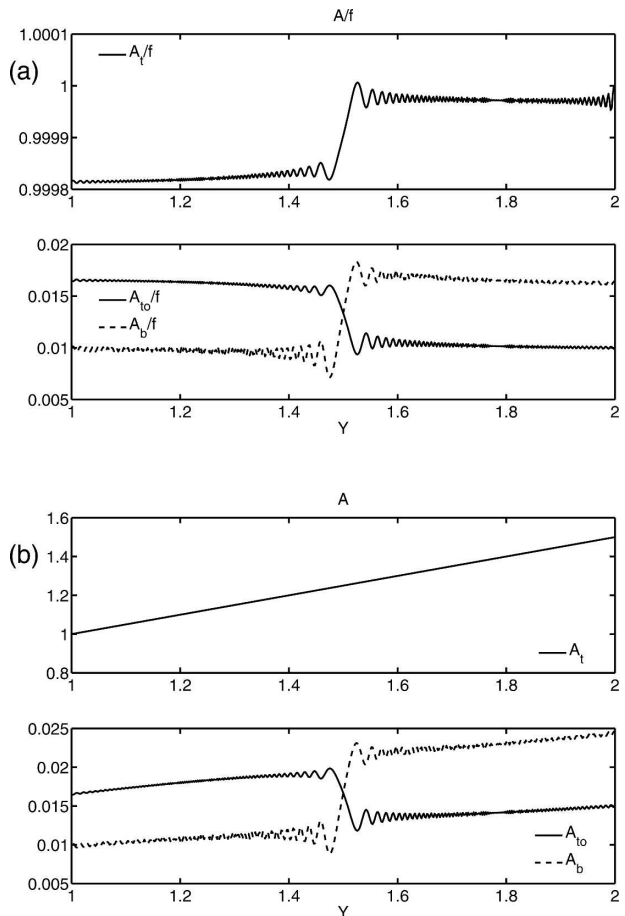


FIG. 2. As in Fig. 1 except that $\varepsilon = 0.001$.

ridional wavenumber and its amplitude as it propagates. From Figs. 1 and 2, we know that, when the barotropic and baroclinic Rossby waves have very small amplitudes when compared with that of the basic one, the decrease of the basic wave's amplitude due to one single triad interaction is negligible compared with that due to variation of the Coriolis parameter. We assume that, although the basic wave interacts with parasitic waves everywhere along its path, the variation of its amplitude is still dominated by that of the Coriolis parameter, so its streamfunction amplitude remains constant along the path. Therefore, as long as the wavenumbers and amplitude of the basic wave are known at one location on the path, we will know their values on other locations.

The linear growth rate of the triad interaction in quasigeostrophic theory is

$$\sigma^2 = \frac{(K_{To}^2 - K_T^2)(K_T^2 + F - K_B^2)(k_T l_{To} - k_{To} l_T)^2}{K_B^2(K_{To}^2 + F)} (A_T/f)^2, \quad (26)$$

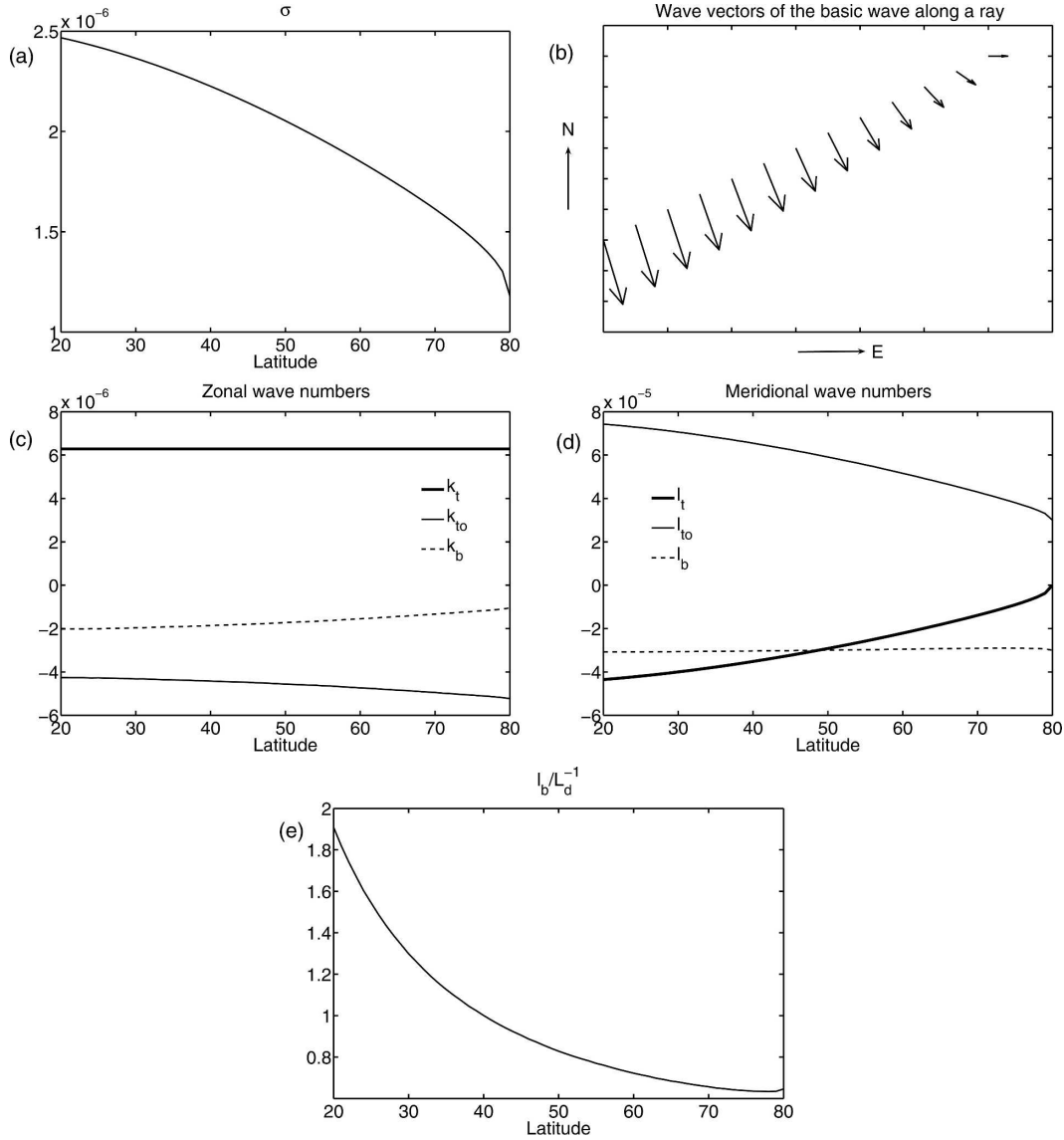


FIG. 3. (a) Latitudinal variation of the maximum growth rate (thick solid line) for the basic wave starting from the eastern boundary with $k_T = 2\pi/1000$ km and $l_T = 0$; the corresponding period is about 4 yr. First-mode baroclinic deformation radius is set to be 30 km at 45°N. (b) Wave vectors of the basic wave along the ray. (c), (d) Wavenumbers of the basic wave and the other two parasitic waves that maximize the growth rate. (e) The ratio of the meridional wavenumber of the barotropic wave to L_d^{-1} .

where $K^2 = k^2 + l^2$,

$$F = \frac{2f^2}{g'H} = \frac{1}{L_d^2},$$

A_T denotes the magnitude of pressure associated with the basic wave, and A_T/f represents the geostrophic streamfunction amplitude. It is important to remember that this formula is only applicable locally. Using this growth rate as a measure of the strength of the triad instability, we can find out the maximum growth rate

among all the triads that the basic wave can have at a specific latitude. Applying it to different latitudes with different l_T , A_T , and f , we can then examine the latitudinal distribution of the triad growth associated with this specific planetary wave. What is shown in Fig. 3 is the result of triad interactions occurring along the path of a basic wave starting from 80°N with $l_T = 0$ and $k_T = 2\pi/1000$ km, propagating with southward group velocity. The basic wave's frequency is assumed big enough to allow the ray to bend all the way down to 20°N. Along the path, the meridional wavenumber of the ba-

sic wave varies with latitude; so does the wave vector as displayed by Fig. 3b. Because the quantity A_T/f is constant along the path, we believe its specific value does not affect the latitudinal distribution of growth rate and set the value of A_T in a way that the geostrophic shear at 45°N is about a few centimeters per second. At each latitude along the path, the most unstable triad among all triads containing the basic wave is found, and the growth rate is calculated using Eq. (26), which is of order $O(10^{-6}) \text{ s}^{-1}$, as shown in Fig. 3a. Regarding the triad instability as a classic baroclinic instability (LaCasce and Pedlosky 2004), we would be tempted to expect decreasing growth rate with latitude because the basic wave's vector is turning more and more meridionally as the magnitude of the basic wave's meridional wavenumber increases. To extract potential energy from the basic wave by crossing the shear, perturbation motions need to be increasingly in the meridional direction, experiencing more and more the stabilizing effect of β . However, the result is opposite to our expectations: the locally maximum growth rate increases southward along the ray rather than decreases. From Fig. 3b, the plot of the basic wave's wavenumber vector, we find that the orientation of the basic wave's wave vector does not change much with latitude except at the far northern end of the interval where the meridional wavenumber is small compared with the zonal one, so the wave vector orients more zonally. The increase of the magnitude of the total wavenumber with latitude, however, is clear along the ray. Rewriting the growth rate formula leads to

$$\sigma^2 = \frac{(K_{T_0}^2 - K_T^2)(K_T^2 + F - K_B^2)}{K_B^2(K_{T_0}^2 + F)} (\mathbf{u}_T \cdot \mathbf{K}_B)^2, \quad (27)$$

where resonance conditions and geostrophic relations $u_T = -f^{-1}l_T A_T$ and $v_T = f^{-1}k_T A_T$ have been used. The dot product describes the projection of the basic shear onto the wave vector of the barotropic wave (or the secondary baroclinic wave). From the continuity equation, we know that the wave motion is perpendicular to its wave vector, so the dot product indeed represents the part of the basic shear that is perpendicular to the barotropic perturbation, or the basic shear felt by perturbations. For planetary Rossby waves that are examined, the zonal wavenumber is at least one order smaller than the meridional one, the basic shear is therefore primarily in zonal direction. Since the streamfunction amplitude is constant along a ray, the southward increase of magnitude of the meridional wavenumber causes an increase of the basic shear and contributes to a southward increase of the locally maximum growth rate. At any latitude, the wavenumbers of the basic wave are

determined by their initial conditions at the starting latitude, but the wavenumbers of the locally excited barotropic wave and the secondary baroclinic wave could have any values as long as they can produce the maximum growth rate. Surprisingly, when we look at the wavenumbers of the parasitic waves, the meridional wavenumber of the barotropic wave is almost constant with latitude, which means the barotropic perturbations growing out of triad instabilities at different latitudes have almost the same meridional length scale. The ratio of this wavenumber to L_d^{-1} is about 0.64 at the starting latitude when the basic wave has zero meridional wavenumber, consistent with the result of LaCasce and Pedlosky (2004), and the ratio increases southward afterward. If we decrease the zonal wavenumber of the basic wave, which still remains constant along a ray path while we let the meridional wavenumber of the basic wave experience the same variation process, starting as zero from 80°N and increasing southward subsequently according to the relation $l_T^2 + F = F(80^\circ\text{N})$ to 20°N, the overall magnitude of growth rate is smaller, but the shape of the growth rate curve hardly changes at all, as shown by Fig. 4a. In this figure, three basic waves propagating along the rays that bend from 80° to 20°N are considered. All three waves have the same meridional wavenumber, $l_i = 0$, at starting points, but different zonal wavenumbers along the three rays, which are $k_i = 2\pi/1000 \text{ km}$, $k_i = 2\pi/3000 \text{ km}$, and $k_i = 2\pi/6000 \text{ km}$, respectively. By calculating the ratio of growth rate between two different curves, that is, the ratio of growth rate on the curve of the basic wave with $k_i = 2\pi/3000 \text{ km}$ to that on the curve of the basic wave with $k_i = 2\pi/1000 \text{ km}$, as shown by the solid line in Fig. 4a, we find that changing the value of the basic wave's zonal wavenumber does change the magnitude of the growth rate, but does little to the shape of the growth rate curve. Another result, not shown by the figure, is that, when the zonal wavenumber of the basic wave is different, zonal wavenumbers of the two parasitic waves at each latitude along the ray are of course different, but the magnitudes of meridional wavenumbers are little different from those displayed in Fig. 3. If we change the initial orientation of the basic wave's wave vector by changing its meridional wavenumber at the starting latitude, the meridional wavenumber will evolve differently from that in Fig. 3. There are three basic waves studied in Fig. 4b; each of them has the same zonal wavenumber $k_i = 2\pi/1000 \text{ km}$ along their rays but different meridional wavenumbers at the starting latitude. When the magnitude of initial meridional wavenumber is greater than zero, the overall magnitude of growth rate is larger and, apparently, the shape of the growth rate curve is different, although the

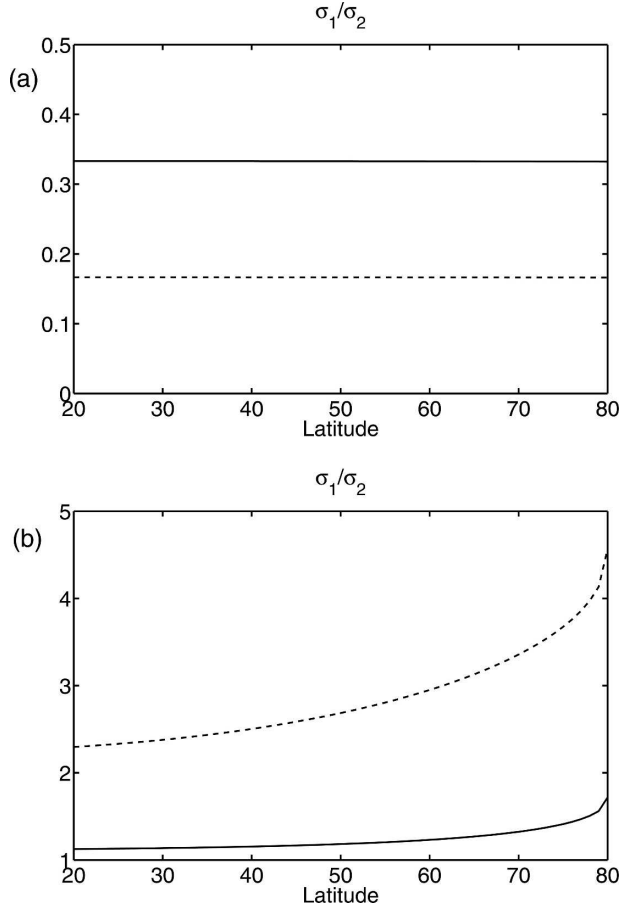


FIG. 4. The ratio of growth rate values of different basic waves. There are three different basic waves and growth rate curves involved in each figure. (a) The three basic waves have the same meridional wavenumber $l_i(80^\circ\text{N}) = 0$ at starting points of their rays, but different zonal wavenumbers: the first one has $k_i = 2\pi/1000$ km; the second one has $k_i = 2\pi/3000$ km; the third one has $k_i = 2\pi/6000$ km. There are three growth rate curves corresponding to these three basic waves. The solid line represents the ratio of growth rate on the second curve to that on the first curve at each latitude; the dash-dot line represents the ratio of growth rate on the third curve to that on the first curve. (b) The three waves have the same zonal wavenumber along their rays, $k_i = 2\pi/1000$ km but different meridional wavenumbers at their starting latitude 80°N : the first one has $l_i(80^\circ\text{N}) = 0$; the second one has $l_i(80^\circ\text{N}) = -0.5/L_d(80^\circ\text{N})$; the third one has $l_i(80^\circ\text{N}) = -2/L_d(80^\circ\text{N})$. The solid line represents the ratio of growth rate on the second curve to that on the first curve at each latitude; the dash-dot line represents the ratio of growth rate on the third curve to that on the first curve.

growth rate is still increasing southward along each ray. One thing that should be mentioned is that the locally maximum growth rate calculated according to Eq. (26) does show a somewhat surprising increase toward the south. However, this does not take into account the effects of nonlinear interactions on the basic wave's streamfunction amplitude. With energy continuously

transferred to parasitic waves in triad interactions, the streamfunction amplitude of the basic wave would decrease as it propagates southward along the ray, so the increase of the growth rate seen in Figs. 3 and 4 is certainly an overestimate. It is reasonable to expect a slower increase, or even an opposite trend, of the growth rate southward along the ray, especially when amplitudes of those perturbing waves are not so small and the basic wave is drained of energy as it propagates southward.

In the above calculations, the locally maximum growth rate is compared along a single group velocity path of a basic wave, but at any time in the real ocean, we could have many different large-scale Rossby waves propagating along their respective rays. The growth of parasitic waves arising out of triad interaction at a specific latitude is related to an ensemble of basic waves propagating, not along one single path, but along an ensemble of paths. Another consideration is that, although rays bend southward, the meridional range of bending will generally not be as large as what we have assumed in previous cases when we consider basic waves with very long periods. Along a ray, the meridional and zonal coordinates satisfy the differential relation,

$$\frac{dY}{dx} = \frac{c_{gTy}}{c_{gTx}} = \frac{2\omega_T l_T / \beta}{(1 + 2\omega_T k_T / \beta)}. \quad (28)$$

In Eq. (28), $l_T = -\sqrt{l_{TN}^2 + 2(f_N^2 - f^2)/c^2}$ and $df/dY = \beta$, where the subscript N represents the value at the northern starting point of the ray. The above equation yields the variation of the Coriolis parameter with x along the ray:

$$\frac{df}{dx} = \frac{-2\sqrt{2}\omega_T}{c(1 + 2\omega_T k_T / \beta)} \sqrt{\frac{c^2 l_{TN}^2}{2} + (f_N^2 - f^2)}, \quad (29)$$

which is readily integrated to obtain

$$f = A \cos \left[\theta_N + \frac{2\sqrt{2}\omega_T}{c\sqrt{1 + 2\omega_T k_T / \beta}} (x - x_e) \right],$$

where

$$\cos \theta_N = \frac{f_N}{\sqrt{c^2 l_{TN}^2 / 2 + f_N^2}},$$

$A = \sqrt{l_{TN}^2 c^2 / 2 + f_N^2}$, and x_e is the zonal coordinate of the starting point on the eastern boundary. Since low-frequency waves tend to bend very little, there is not much latitudinal variation of the deformation radius along a ray and the growth rate would not increase as much as in previous cases when a large bending is assumed a priori. Taking that into account, we examine

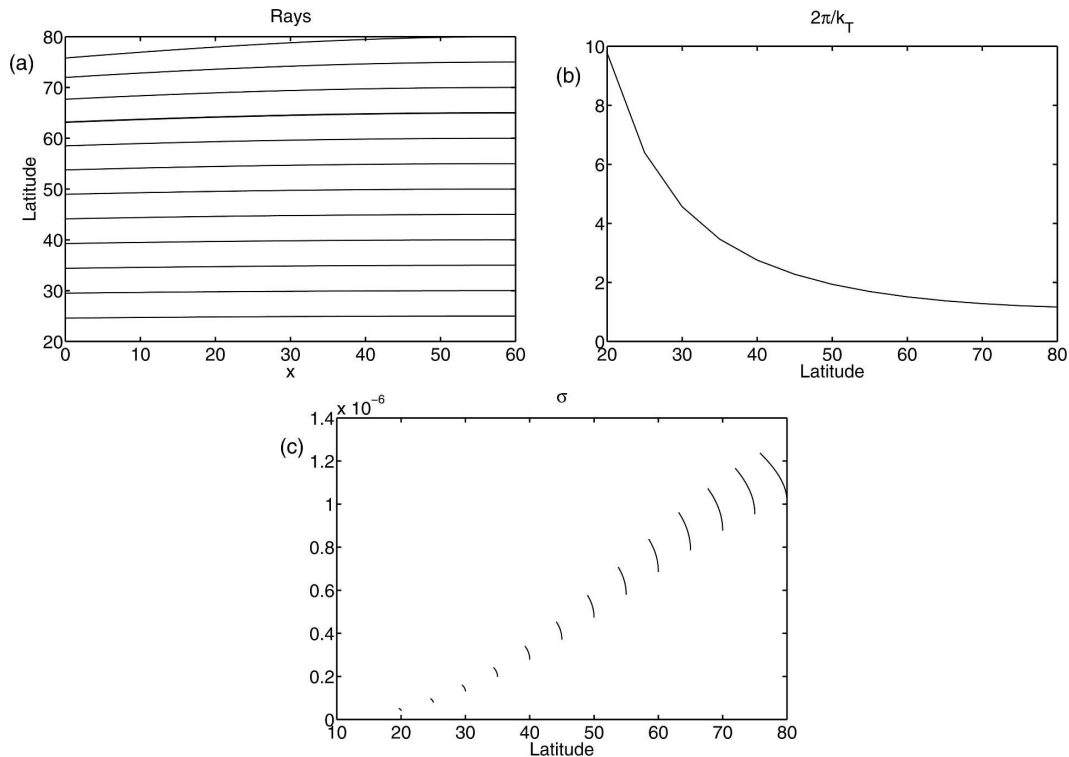


FIG. 5. (a) Rays of basic waves starting from the eastern boundary with zero meridional wavenumbers and the same period $T = 5$ yr. (b) Variation of zonal wavelength (unit: 1000 km) with starting latitudes of rays (x axes). (c) Each curve represents variation of growth rate along a ray that starts from the latitude corresponding to the northern end of the curve.

the case of many rays starting from different latitudes along the eastern boundary and study the two-dimensional distribution of growth rate associated with the basic waves propagating along those rays.

The first case that we examine is a basin with the eastern boundary lying parallel to meridians, so the initial meridional wavenumbers on the eastern boundary are zero for all rays in order to satisfy the boundary condition of no zonal velocity. For all rays shown in Fig. 5a, the basic wave's period is fixed as 5 yr, so its zonal wavenumber decreases from ray to ray as the ray's starting point on the eastern boundary is more to the south. For simplicity, the streamfunction amplitude is constant for all rays. As the starting point of the ray moves southward, the bending of the ray is less and less. For a ray starting from high latitudes, the growth rate curve, as illustrated by Fig. 5c, is relatively long since the ray transverses a great latitudinal range and shows a clear increase trend with decreasing latitude. Rays from very low latitudes, on the contrary, traverse a rather small latitudinal range, so the growth rate curves are short and appear nearly as dots. A comparison of growth rate among different rays is not easy in this case because, among different rays, the growth rate de-

creases with decreasing zonal wavenumber, while along each ray the growth rate increases as shown in the single ray case. But generally, the important qualitative result is that high growth rates appear in high-latitude regions. When the period of the basic wave is longer, that is, 10 yr as shown in Fig. 6, zonal wavenumbers along rays are even smaller. The rays are more flat and the magnitude of variation of growth rate along a ray is smaller, but the weaker growth still appears in the south.

In the last two figures, stronger instability appears in high-latitude regions. It is interesting to know what will happen if the eastern boundary is not meridional but extends from southwest to northeast, as for the Atlantic Ocean, or from southeast to northwest for the Pacific Ocean. For basic waves with energy propagating southward, the product of zonal wavenumber and meridional wavenumber is negative, so the no-normal-flow condition can only be satisfied on an eastern boundary that lies southwest–northeast. In Fig. 7, wavenumbers of the basic waves satisfy the relation $k_T/l_T = -\tan 60^\circ$ on the eastern boundary and so satisfy the no-normal-flow condition for the boundary that lies from southwest to northeast with an angle of 60° to the east. With $k_T =$

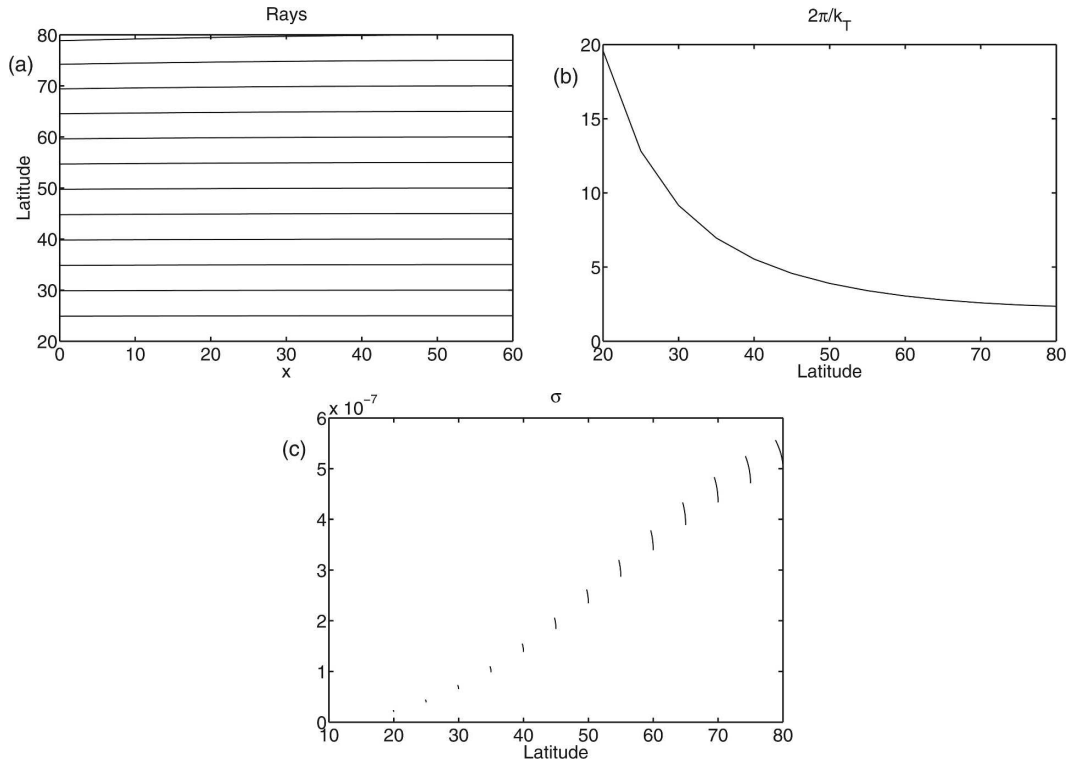


FIG. 6. (a) Rays of basic waves starting from the eastern boundary with zero meridional wavenumbers and the same period $T = 10$ yr. (b) Variation of zonal wavelength (unit: 1000 km) with starting latitudes of rays (x axes). (c) Each curve represents variation of growth rate along a ray that starts from the latitude corresponding to the northern end of the curve.

$2\pi/500$ km on all rays, l_T also has the same value for all rays on the eastern boundary. Again, the streamfunction amplitude is set to be the same for all rays. Owing to the orientation of the boundary, rays in this case occupy much larger latitudinal ranges (as large as 40° of latitude) than in previous ones, and the growth rate along all the rays shows a much larger variation with latitude. More importantly, the growth rate curve for any ray is capped by that for a ray starting north of it. As the starting latitude of a ray moves southward, the growth rate everywhere along it becomes smaller. Although all basic waves starting from north are unstable, the one that emanates from the northernmost latitude will suffer the strongest instability. For specific longitude, that is, at $x = 20$ in Fig. 7, when we are at 25°N , the basic wave that can be seen at this position is the one that propagates along a ray starting from about 40°N ; when we are at 60°N , however, the starting latitude of the ray is as high as 80°N . From the growth rate distribution in Fig. 7c, we easily recognize that the growth rate along the ray starting from 80°N is totally above the ray starting from 40°N , so along this longitude the triad growth is getting much stronger as we move northward. This is a picture reminiscent of global

satellite observations. If we assign different zonal wavenumbers to each ray, that is, smaller zonal wavenumbers to rays starting from south and bigger ones to rays starting from north, initial meridional wavenumbers will also decrease from ray to ray as the starting latitude decreases to satisfy the boundary condition. On the basis of our analysis of growth rate along a single ray, we would expect both of these two changes to cause an increase of the overall growth rate along rays starting from higher latitudes; therefore, there will be even greater increase in growth rate as we move northward along a longitude.

4. Summary

The triad interaction among a basic, first-mode baroclinic Rossby wave and two other parasitic baroclinic and barotropic waves has been studied on the planetary scale where, to the first order, the Coriolis parameter is not constant. The zonal wavelengths of waves examined are on the planetary scale, while their meridional wavelengths are comparable to the deformation radius. Baroclinic waves conserve their frequencies and zonal wavenumbers as they propagate along rays, but adjust

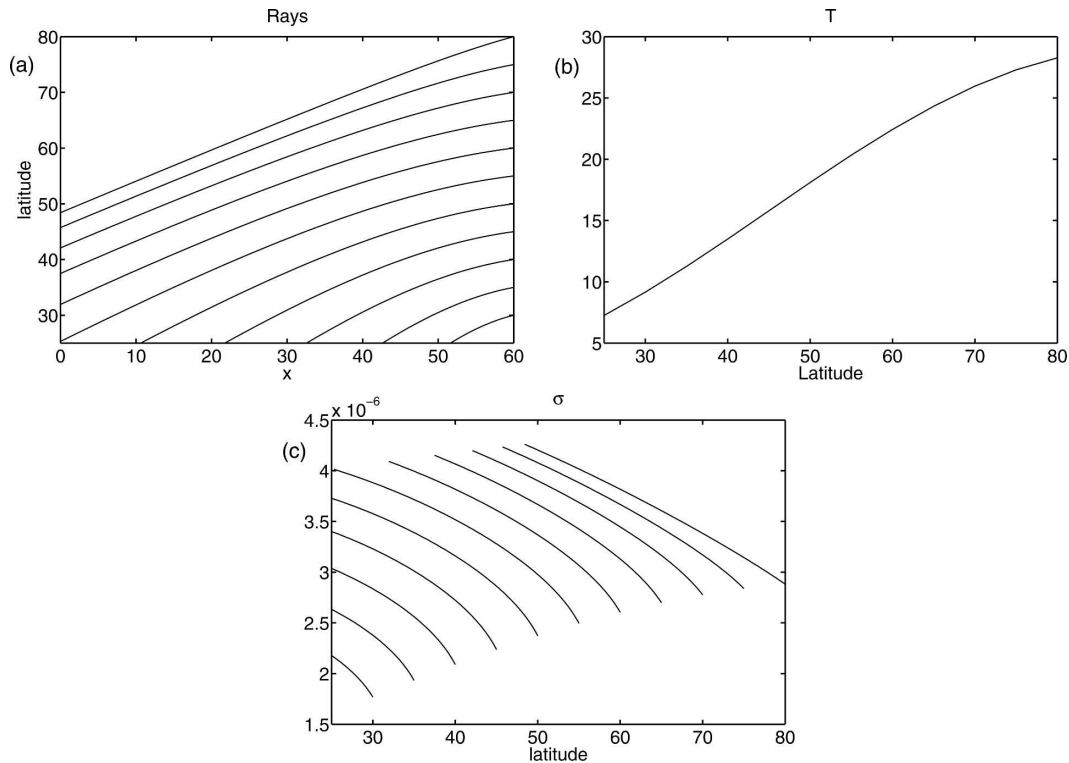


FIG. 7. (a) Rays of basic waves starting from the eastern boundary that lies from southwest to northeast with 60° to the zonal direction. Basic waves along different rays have the same zonal wavenumber $k_z = 2\pi/500$ km and $k_T/l_T = -\tan 60^\circ$ on the eastern boundary. (b) Variation of basic wave periods (unit: yr) with starting latitudes of rays (x axes). (c) Each curve represents variation of growth rate along a ray that starts from the latitude corresponding to the northern end of the curve.

their meridional wavenumbers in order to keep the quantity $l^2 + 1/L_d^2$ constant; barotropic waves, on the other hand, have their dispersion relation independent of latitude, so their wavenumbers are constant along rays. For all waves involved, their amplitudes would vary like the Coriolis parameter if there were no nonlinear interactions.

Because of the variation of meridional wavenumbers of baroclinic waves, resonance conditions can only be strictly satisfied at a specific latitude for a specific triad. As a result, the nonlinear interaction occurs within a narrow latitudinal band with a width comparable to the deformation radius. This striking feature allows the use of quasi-geostrophy theory in studying the process by which the basic wave continuously loses energy to parasitic waves as it propagates along its ray. We consider the process as a series of separate interactions of the basic wave with different triads at each latitude. At one location along the ray, the interaction among the basic wave and the other two small waves has been examined under quasi-geostrophy and the most unstable triad has been found. At a different location, the basic wave's meridional wavenumber and amplitude change from

their previous values, and the two small waves that can maximize the growth rate change accordingly. Applying this strategy everywhere along a ray, we found the latitudinal structure of the growth rate associated with a specific basic wave. With the assumption that the effect of nonlinear interactions on the basic wave's streamfunction amplitude is negligible, the growth rate increases southward. Smaller zonal wavenumber decreases the overall magnitude of growth rate but does not change the shape of the curve at all. Different initial meridional wavenumbers at the starting latitude cannot only change the magnitude of the growth rate but also change the shape of the growth rate curve. An important result is that the meridional scale of the barotropic perturbations growing out of instability associated with a basic wave is nearly constant with latitude. The initial meridional wavenumber and the starting latitude of the basic wave determine that constant.

Analyzing the growth rate along a single ray that is assumed to bend over a large latitudinal range is helpful in discovering the latitudinal dependence of triad interaction of a specific basic wave, but not so helpful in getting a general idea of how the growth rate may be

distributed in the real ocean. Therefore, an analysis of an ensemble of rays starting from different latitudes on the eastern boundary is indispensable in offering us a two-dimensional picture that is closer to reality than what can be done with quasigeostrophic theory, yet is greatly complicated by factors like the orientation of the eastern boundary and the zonal wavenumber or period for each ray. With a meridional eastern boundary, the basic waves start with zonally oriented wave vectors and the bending of rays is small, especially for waves with long periods. The resulting two-dimensional distribution of growth rate clearly indicates stronger growth in high-latitude regions. When the eastern boundary extends from southwest to northeast and the zonal wavenumber as well as the initial meridional wavenumber is the same for all rays, rays occupy a bigger latitudinal range. The overall magnitude of growth rate along a ray decreases as the starting latitude of the ray decreases. The energy source for the strongest growth seen at a specific latitude is the basic wave propagating from the northernmost latitude, and at the same longitude, the higher the latitude is, the stronger the growth is.

The dynamics of the triad interaction and conventional baroclinic instability are quite similar in the sense that both are processes by which the basic wave loses potential energy to growing perturbations; therefore, the growth rate largely depends on the magnitude of the available potential energy determined by the basic wave's amplitude and wavenumbers. Compared with conventional baroclinic instability, that is, the process that is pertinent with larger shear in the basic wave and hence greater growth rate, the triad interaction is slow and relatively weak in releasing basic wave energy. However, the small Z limit yields important insights into the evolution of the instability along the propagation paths of the basic baroclinic waves and emphasizes the strong latitudinal dependence of the instability. The southward increase trend of the growth rate along a single ray path is unexpected from the conventional baroclinic instability theory and so demonstrates one important difference in dynamics of the two processes despite the similarity that deformation-scale perturbations naturally develop in both mechanisms.

Relative to the work of LaCasce and Pedlosky (2004), the present work extends the study of the large-

scale Rossby wave triad instability problem to the planetary scale. In their work, the basic wave's instability under quasi geostrophy does have a preference for stronger growth at higher values of Z , but not in higher-latitude regions. Because in quasigeostrophic theory, the deformation radius is constant, Z does not vary with latitude. However, if the latitudinal variation of the deformation radius is parametrically allowed, the value of Z can increase northward. Based on this argument, LaCasce and Pedlosky concluded that stronger instability may appear in high-latitude regions. This is a conclusion that is made under quasi geostrophy while it is applied on the planetary scale. Some important phenomena caused by latitudinal variation of the deformation radius on the planetary scale cannot be found under quasi geostrophy, but have been exposed in our study, such as the latitudinal variation of a baroclinic wave meridional wavenumber, baroclinic and barotropic wave amplitudes, and, more importantly, the latitudinal dependence of the triad growth rate. The triad interaction of the basic wave on the planetary scale, though only one of two regimes of the instability problem, has turned out to yield illuminating insight into the connection between propagation and instability of baroclinic Rossby waves.

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