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Brazil nut effect in a rectangular plate under horizontal vibration

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Abstract An intruder to a group of identical small beads enclosed in a rectangular plate will gradually migrate to either the center or one side of the plate when the plate is subjected to a horizontal vibration. By considering probabilities for a bead to move into and off the space between the intruder and the near side of the plate, we predict that the size ratio and the mass ratio of the intruder to small bead have equal but opposite effects in determining the direction of migration. The prediction is confirmed by a molecular dynamics simulation.

Keywords Segregation · Vibration · Horizontal · Brazil nut effect

1 Introduction

For the last 20 years, the phenomenon of the so-called Brazil nut effect (BNE) has attracted a lot of attention [1–5]. The BNE can be easily observed in a simple experiment by immersing a large granular particle in a cluster of smaller particles and shaking them vertically. One can see the large particle migrate all the way to the top, if it is not too much heavier than the small ones. There have been several explanations for the phenomenon [1,4–7]. Nevertheless, detailed experimental data [8–10] showed that a comprehensive mechanism is still lacking.

In 2005, Schnautz et al. found that beads in the circular plate under horizontal swirling motion behave similarly [11]. Namely, a large bead will migrate to either the center or the border of the plate. In this horizontal version of BNE, gravity and interstitial air obviously play no role. The system seems easier than the original BNE to study and might offer a good pathway to understand the later.

In this paper, we give an explanation for the underlying mechanism of horizontal BNE. We found that the large bead migrates only when there are enough beads present so that the energy loss due to collisions forces all the beads into a collective periodic motion, and not too many so that beads still have room to change their relative positions. We demonstrate the migration mechanism by means of a rectangular plate under one dimensional simple harmonic vibration—a simplified version of the circular horizontal BNE. In our laboratory we confined one layer of beads in a rectangular acrylic box of length 12 cm and width 6 cm and drove the box sinusoidally with a frequency of order 1 Hz in a horizontal direction parallel to one side of the box. For sufficient large amplitude but not so large that hopping is avoided, we observed that a heavy intruder tends to migrate to the center of the group of identical beads (radius $d = 0.225$ cm and $m = 0.13$ g) as shown in Animation 1 of supplementary material, while a large intruder will stay away from the center, which is qualitatively exactly the same as what have been observed in the horizontal BNE of circular vibration [11]. In this paper, we investigate the behavior of the group of identical beads theoretically using molecular dynamics (MD) simulation.

We adopt our previous simulation model [12] which has been described in details by Luding et al. [13,14]. We had shown in our previous work for the case of circular vibration [12] that it is the collisions between beads that plays the crucial role in the migration phenomenon. Though it has been shown that friction plays an essential role in some phenomena

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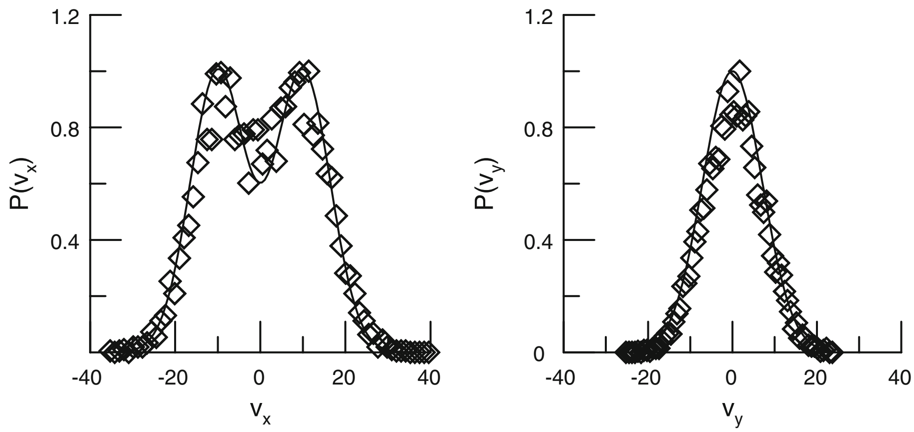

Fig. 1 Velocity distributions in the x and y directions of the beads in the box which is driven sinusoidally in the x direction with amplitude $A = 5d$, $d = 0.3$ cm and frequency $f = 1$ Hz (the unit of the velocity is cm/s)

[15,16], it appears that as far as the migration phenomenon is concerned, our simulation results produce the same results either with or without friction. In particular, since migration along the vibration direction appears in a region only when the density of the beads is small so that the Boltzmann equation is applicable [17]. We have checked that the speed distribution in the vibration direction can indeed be well approximated by the Maxwell–Boltzmann distribution both for our MD results (no friction) and for experimental data (with friction). In this theoretical work we neglect friction and focus on the movement of the intruder relative to other beads. For simplicity, the coefficient of restitution ε—the ratio of relative speeds before and after collision—is assumed to be the same for all collisions. The value of ε is taken to be 0.96 in our simulation.

We use the radius d and the mass m of the identical beads as the units of length and mass respectively in all our simulations. The size of the rectangular plate is chosen to be $l \times w = 40d \times 20d$. The relevant parameters are therefore: the vibration frequency f and amplitude A of the plate, the total number N of identical beads (or equivalently, the filling fraction μ), as well as the radius D and the mass M of the intruder. The typical values used in our simulation are $f = 1$ Hz, $N = 120(\mu = 0.47)$, $A = 5d, D = 1.0 \sim 2.4d$, $M = 1.0 \sim 2.4m$. Note that the amplitude A has been chosen large enough to generate collective motion for the beads, but not too large in avoiding the occurrence of hopping of the beads over one another. For $M/m > 0.5$ and $D/d < 3.0$, a simple estimation shows that when a small bead hits the large intruder at rest it needs at least 40 cm/s to make the large intruder jump a height of $0.5d = 1.5$ cm. Thus we set 40 cm/s as the upper bound for the speed of each bead and choose A accordingly. The velocity distributions of beads along both x and y directions are plotted in Fig. 1 for the case of $A = 5d$. We see that as long as A is $5d$ or less, the speeds of the small beads are well confined within 40 cm/s.

With appropriate frequency f and amplitude A, we found that, when N is large enough, the center of mass (CM) of the whole group of beads moves back and forth periodically with the same frequency as that of the external drive. Fig. 2 shows a typical distribution of positions and velocities of all the beads in one period at every 1/8 period time interval. The top frame of Fig. 2 shows the distribution when the CM is closest to the left wall. After a half period, the CM is closest to the right wall as shown in the 5th frame. Another half period brings the distribution back to the first frame again. We calculated the average distance between beads and average velocities as a function of position along the vibration direction, and plotted the curves on the right column of Fig. 2. We see that the average separation distance is only a little more than the diameter of the beads from one end of the plate up to a certain point and then rises to a maximal value at the other end. In the region where the separation distance is small, the beads are densely packed and are unable to change their relative positions. For the example shown in Fig. 2, a bead in the central 1/3 region has little chance to change its position at any time. Similarly, if an intruder of whatever size and mass is introduced initially into the central region, it will stay there for a long time because there is no room for it to change its relative position and no migration can be observed. If, on the other hand, the intruder is initially in the left or right 1/3 regions of the plate, it has a chance to move inward or outward. As we will explain below, it can have a definite migration direction on average if the value of $p = \left(1 + \frac{D}{d}\right) / \left(1 + \frac{M}{m}\right)$ is not close to 1.

Thus the curve of separation distances against positions plays a crucial role in the migration phenomenon. This curve is dependent on the parameters μ and A. It has a general feature that on one side there is a roughly flat region where its value is slightly larger than $2d$, and on the other side it rises to a maximal value at the wall. Figure 3 is a schematic diagram of the average distance between the beads as a function of position. Migration is not possible in the flat region where the average distance is small. The optimal choice of A and μ for observing migration is such that the flat region of the curve has a length equal to half the plate size so that the intruder can move all the way either from one wall to center or from center to one wall (Fig. 3a). For a fixed A, an increase

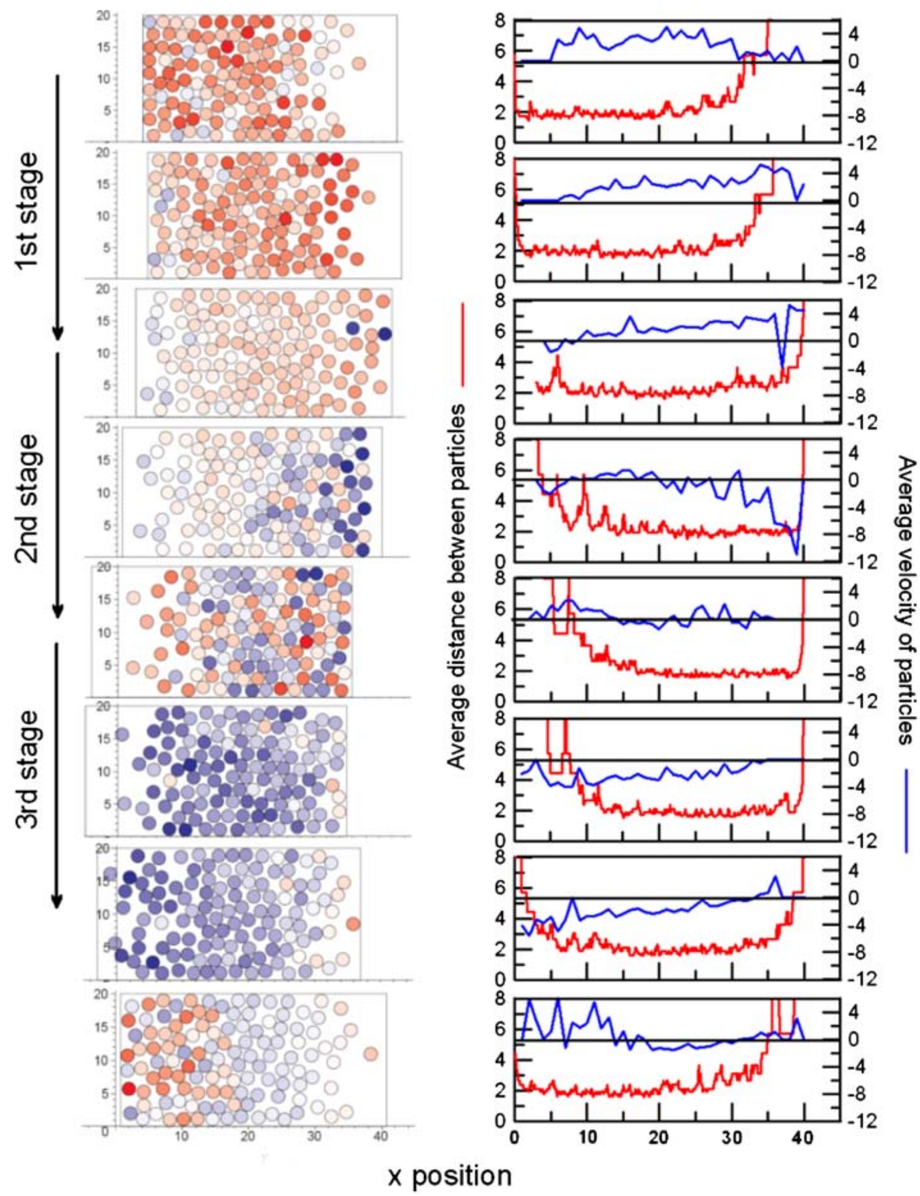

Fig. 2 Position and velocity distribution of beads at different stages of motion. *Left* column shows beads distribution in the rectangular plate in one period of vibration. Each bead is colored according to its x-component velocity, *red* for positive value and *blue* for negative. The average separation distance and average velocity as functions of position are plotted in *red* and *blue*, respectively, in *right* column. Beads can change their relative positions only when both separation distance and speed are large. These conditions are satisfied at the *right* end of the whole group of beads in the first stage (*2nd frame down*) and at the *left* end in the third stage (*6th frame down*)

of μ will decrease the migration range so that an intruder near the CM will not have directed migration (Fig. 3b). On the other hand, when μ decreases, the migration range could cover more than half of the plate size (Fig. 3c). In this case, an intruder near the CM is able to migrate in both directions and will move back and forth around the center so that it has little chance to escape to the border. Similar to the effect of μ, we found that increasing the amplitude A can decrease the migration range too. This explains why an increase of A can help to observe the migration phenomenon when μ is small, but make it disappear when μ is large in circular horizontal BNE (Fig. 5 of Ref. [11]). However, a more systematic quantitative investigation on the effect of the filling fraction is needed. In this article we only chose near optimal filling fractions ($\mu \approx 0.5$) to focus on the migration mechanism. Note that there have been a few previous works on the study of segregation phenomena under horizontal vibration [18,19]. However they focused on the patterns produced by small vibration amplitudes. The horizontal BNE could not occur in their systems mainly because the vibration amplitudes they used are an order smaller than required.

The motion of the beads can be separated into four stages in each period (Fig. 2). In the first stage, the CM begins to move to the right from its position closest to the left wall. It ends when the front beads have just reached the right wall. In the second stage, the plate begins to move to the left while the CM continues to move to the right so that at the end of this stage beads are densely packed against the right wall and every bead has gained a velocity to the left. At the end of the second stage, the CM is closest to the right wall. The third and fourth stages are similar to the first and second stages respectively except the CM moves in the opposite direction.

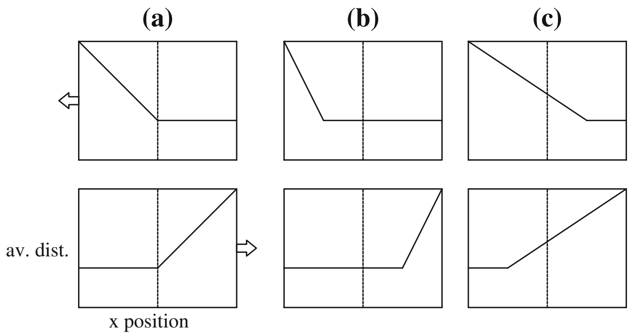

Fig. 3 Schematic diagram showing the effects of A and N on the average distance between beads as a function of position. **a** Choosing A and N appropriately, the function curve can have a flat region of half plate size. The *upper plate* is moving to the *left* and the *lower* to the *right*; **b** increasing the value of either A or N from that in (**a**) will increase the length of flat region; **c** decreasing the value of either A or N from that in (**a**) will decrease the length of flat region

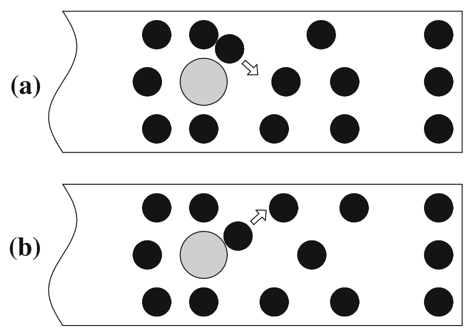

Fig. 4 Schematic diagram for the mechanism of migration of the intruder. **a** A small bead moves into the region between the intruder and its nearest wall so that the intruder is observed to migrate away from the wall. **b** A small bead is kicked off the track of the intruder so that the intruder migrates closer to the wall

Notice that in the first stage, beads in the left region where separation distance is small can hardly change their relative positions. On the other hand, beads in the right region where separation distance is large are moving to the right and making forward collisions with one another so that it is possible for them to change their relative positions. In the second stage, beads in the right region are dense so that they have no chance to change positions. Beads in the left region can hardly change relative positions either because they have too little speed to do so albeit their separation distance is large. To sum up, we find in each period of vibration, beads in the right region have good chance to change their relative positions only in the first stage, while beads in the left region can do so only in the third stage.

Consider now an intruder among the right part of the group of identical beads. According to the above discussion, it can change its relative position in the group only at the first stage of the back and forth motion. Schematically, let us consider for simplicity all beads lined up in columns. Suppose the intruder and its neighbors in the adjacent columns are at the same distance to the right border (Fig. 4) and they move to the right with the same speed (at the first stage, all beads move rightward). The intruder will migrate closer to the right wall if, before its neighbors do the same, it is hit and (1) moves to an adjacent column or (2) kicks the bead in front off its column. On the other hand, if its neighbors kick a bead into its column in front (i.e., to the right) of it or it moves to a position behind (i.e., to the left of) its neighbors, the intruder migrates away from the right wall. Thus it is the relative probability of the intruder and its neighbors to be hit and the relative speed increment of them after being hit that determines the migration direction of the intruder. The probability of a bead being hit depends on its size. For the intruder it is proportional to $D + d$. The speed increment of a bead after being hit depends on its mass. For the intruder this is proportional to $\frac{(1+\varepsilon)m}{M+m}$, where ε is the coefficient of restitution. Therefore, the relative probability and speed increment are simply $\frac{D+d}{d+d} = \frac{(1+D/d)}{2}$ and $\frac{m+m}{M+m} = \frac{2}{(1+M/m)}$, respectively. That is, the intruder tends to migrate to the border when its size D is larger than d, and to the center when its mass M is larger than m. The size and mass effects play equal but opposite roles in the migration of the intruder.

We use MD simulation to test the above explanation of the migration behavior of the intruder. We vary the values of M/m and D/d and determine the final position of the intruder. We put the intruder initially on the border of the whole group of beads and test whether it will migrate into the group or not. Specifically, we define the average inward migration distance of the intruder as

$$\Delta x(t) = \langle r - |x(t) - x_{\mathrm{CM}}(t)| \rangle \quad (1)$$

where $x(t)$ and $x_{\mathrm{CM}}(t)$ are the positions of the intruder and CM of the cluster, respectively, and r is the half size of the cluster in its compact form.[1] The notation $\langle\ \rangle$ means average over one period of vibration. The initial value for Δx is 0. We plot some typical value $\Delta x(t)$ as a function of time for $p = 0.417, 0.625, 1.25, 1.670$ in Fig. 5. One can see that it may be hard to predict the precise position of the intruder at a particular time, but beside some fluctuations, it seems for most cases the position of the intruder after 25 periods of vibrations can be identified as inside or outside the cluster without question. We therefore for each set of M/m and D/d values simulate the system 55 periods and average the intruder position of the last 30 periods. If this value is larger than r, i.e., $\Delta x > 0$, we plot a blue disk in the phase diagram of M/m versus D/d as shown in Fig. 6, otherwise a red disk. Thus a blue intruder in the phase space migrates to the center,

[1] When the cluster of beads enclosed by a rectangular boundary is in its compact hexagonal form, its width is the same with the plate, and its length can be calculated as a function of N. An explicit formula for its length is given in Ref. [13, Eq. (14)].

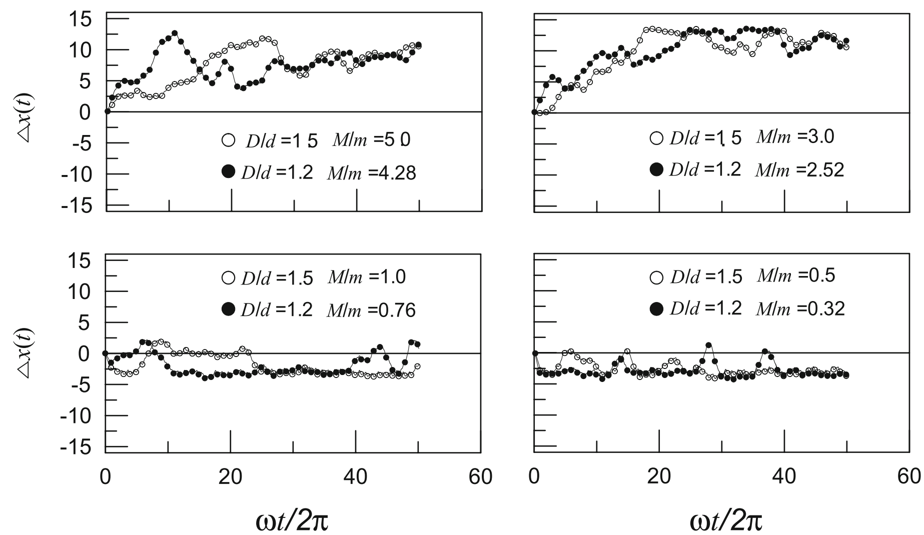

Fig. 5 Inward migration distance $\Delta x(t)$ (in units of d) as a function of time. We choose four values of $p = \frac{1+D/d}{1+M/m}$ and plot two possible combinations of size and mass ratios for each p. The intruder migrates inward for the upper two curves with p smaller than $1 (p = 0.417, 0.625)$, and stays most of time outside the cluster of the background beads for the two lower curves with p larger than 1 $(p = 1.25, 1.670)$

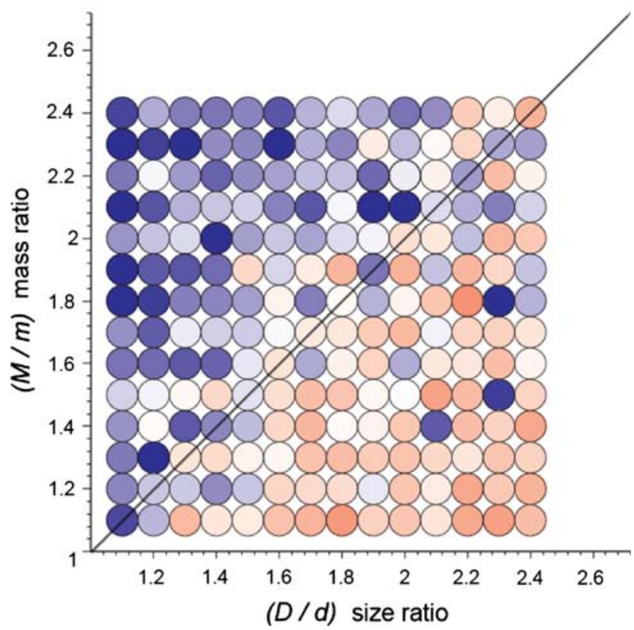

Fig. 6 Final position of the intruder shown in the M/m versus D/d phase plot. The size of the rectangular plate is $40d \times 20d$ and $f = 1$ Hz, $A = 5d$, $N = 120(\mu = 0.47)$. We calculate the average distance to the center of mass of the whole group of beads after 25 periods of vibrations and subtract it from the half of the longitudinal size of the group in its compact form. When this value is positive, we color it *blue* with hue proportional to its value. Negative values are plotted in *red*. The boundary (*solid line*) between the *blue* and *red* regions is consistent with the line $1 + M/m = 1 + D/d$ as predicted by the theory

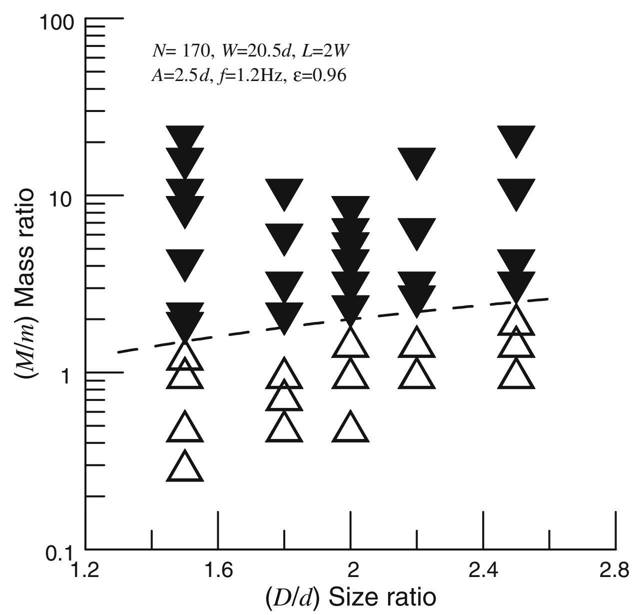

Fig. 7 Phase diagram as Fig. 6. $N = 170(\mu = 0.64)$ in this simulation. Some system parameters have changed as shown in the figure. We vary the mass ratio over a wide range from 0.1 up to 50 in this system and the resultant boundary between inward migration (*filled inverted triangle*) and outward migration (*open uptriangle*) is consistent with our theory (*dashed line*)

while a red one to the border. The simulation results show that the boundary between the blue and red regions is close to the line prescribed by the equation $1 + M/m = 1 + D/d$ as predicted by the argument given in the last paragraph.

We have also simulated the system with a larger filling fraction $\mu = 0.64(N = 170)$ than the one shown in Fig. 6 while kept the container size fixed. At four different choices of the size ratios $D/d = 1.5, 1.8, 2.0, 2.2, 2.5$, we vary the mass ratio over a wide range from 0.1 up to 50. The phase diagram shown in Fig. 7 (a semi-log plot) is also consistent with our prediction in the migration of the intruder. Based on the mechanism we proposed above, we did not expect any difference on the phase diagram of the migration direction when the system has a larger container size. A check on the system with the container size $l \times w = 53.4d \times 26.7d$ has been carried out to support this point.

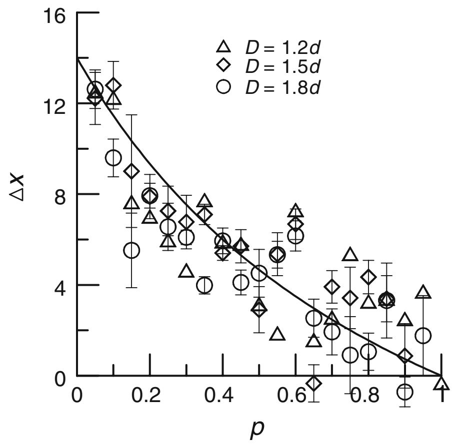

Fig. 8 Inward migration distance Δx (in units of d) of the intruder after 25 periods of vibrations as a function of $p = \frac{1+D/d}{1+M/m}$. The *solid curve* is $y(p) = 14 \cdot \frac{1-p}{p+1}$ having the function form predicted by the theory

We can further try to estimate the speed of the migration. Let p be the relative probability of the intruder and its neighbors to hit the bead in front of them. From the discussion above, $p = \frac{(1+D/d)}{2} \cdot \frac{2}{(1+M/m)} = \frac{1+D/d}{1+M/m}$. Assuming a bead being hit can have a certain constant possibility c to stay in the same column when the average distance between beads is large, and probability $(1-c)/2$ to move to each of its two neighbor columns. In each period T, the intruder has probability $(1-c) \cdot \frac{p}{p+1}$ to migrate a distance $2d$ (for most cases) closer to the border and probability $(1-c) \cdot \frac{1}{p+1}$ to migrate the same distance to the center. Putting all these together, the outward migration speed of the intruder is then given by

$$v = (1-c) \cdot \frac{p-1}{p+1} \cdot \frac{2d}{T} \cdot g(x) \tag{2}$$

where $g(x)$ is the spacing factor, depending on the distance between beads at the position x. It is equal to 1 when the distance between beads is infinite and 0 when the distance is $2d$. Assuming $g(x)$ is a slow varying function for our system, the migration distance Δx of the intruder in a given time interval is, according to Eq. 2, proportional to $|\frac{p-1}{p+1}|$. We choose $f = 1\text{Hz}$, $A = 5d$, $N = 140(\mu = 0.55)$ such that the separation curve has the form close to Fig. 2a. We test the dependence of Δx on p by putting the intruder of various p values at the border of the whole group of beads and vibrate the beads. The average inward migration distance Δx after 25 periods of vibrations is plotted against p in Fig. 8. Qualitatively, the simulation data fits the curve $y(p) = \frac{2d^2N}{w} \cdot \frac{1-p}{p+1}$, where w is the width of the plate, for $p < 1$ quite well.

In Fig. 9, we present a preliminary comparison of our simulation results with experiment using the average migration distance defined in Eq. 1. In experiment, we have tried two different background beads, aluminum and copper, for comparison. We recorded the migration distance of the intruder of various sizes and masses averaging over 10 arbitrary initial configurations. The average migration distance Δx as a function of p roughly follows Eq. 2 except for very small value of p in both aluminum and copper cases (Fig. 9a). In simulation we used the same parameters as the experimental setup: the size of the box, vibration frequency, vibration amplitude. When the same number of background beads as in experiment, i.e., $N = 300(\mu = 0.66)$, was used, the simulation results (Fig. 9b) are qualitatively closer to the results for copper than that of aluminum in experiment. Changing the value of the coefficient of restitution from 0.96 to a smaller value made little difference to our simulation results. On the other hand, using a different number of background beads, say $N = 250(\mu = 0.55)$, we could bring the simulation results to be similar to experimental results for aluminum(Fig. 9b). Presumably, the difference between the experiments and the simulations is due to the neglect of the friction between the beads and the bottom plate in our simulation. Further investigation is under way to clear this point.

In summary, we have studied the motion of an intruder among a group of mono-layer identical background beads inside a box subjected to a horizontal harmonic vibration. We observed that when the vibration frequency, amplitude, and the number of the background beads are chosen appropriately, the whole group of beads moves in a quasi-periodic motion. Under this condition, we argued that the migration direction of the intruder is completely determined (on statistical average) by the parameter $p = \left(1 + \frac{D}{d}\right) / \left(1 + \frac{M}{m}\right)$, where D/d and M/m are size and mass ratio of the intruder to the background bead, respectively. For an intruder with $p < 1$, it migrates toward the center of the cluster of the background beads. For $p > 1$, it migrates to the rim of the cluster. The prediction was supported by our molecular simulations of the system. We have further checked the inward migration speed for the cases of $p < 1$ and compared with some preliminary experiments using aluminum and copper beads as background, respectively. The simulation results are qualitatively consistent with the experiments but not in good agreement quantitatively. We believe the discrepancy between experiment and simulation is mainly due to our neglect of the friction between the beads and the bottom plate of the box in the simulations.

Finally we would like to mention two straightforward extensions of our model. One is the 2D system under 2D vibrations, which is nothing but the circular BNE system studied in Refs. [11,13]. Our analysis can be immediately applied to this case. The radial motion of beads in circular vibration is very similar to the motion of beads in rectangular plate we studied here. The motion of beads in the circular case can also separated into four stages and an intruder can migrate inward or outward only in the stage when forward collisions occur in the region where the average distance

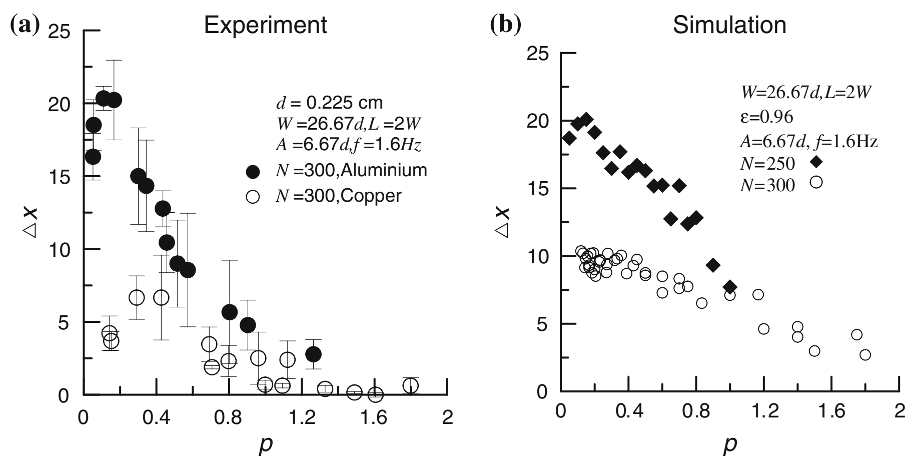

Fig. 9 Comparison between experiment (**a**) and simulation (**b**) for the inward migration distance Δx as a function of p. The simulation did not produce satisfactory result quantitatively since the friction between the beads and the bottom plate has not been taken into consideration

between beads is large. Consequently, the mechanism for an intruder to migrate to either the center or the border of the circular plate is the same and the boundary of these two cases in the phase space is determined by the same equation [12]: $p = \frac{1+D/d}{1+M/m} = 1$. Of course, there is a unique feature for the circular case: the spinning of the whole group of beads [20]. An investigation along this line has been reported in Ref. [21]. The second extension is a 3D system under 1D vibrations, which corresponds to a rectangular box containing many layers of beads under vibrations in one direction in the absence of gravity. We expect an intruder to migrate along the vibration direction exactly as in the 2D case. There is one difference, however. The probability for an intruder to be hit in the 3D case is proportional to $(D+d)^2$ instead of $D+d$. Thus the border of the two parameter regions, one for intruder to migrate outward and one inward, in phase space would be a parabola instead of a straight line. We notice that simulations performed by Hong et al. [5] for the vertical BNE system with gravity show that the borders are also a straight line and a parabola in the 2D and 3D cases respectively. It is interesting to investigate further what exactly the role of gravity is in BNE.

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