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Energy and phase transition in a horizontally vibrating granular system

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Abstract The study focuses on the average energy of a monolayer of granular particles confined in a rectangular container. The container is shaken sinusoidally in a horizontal plane. The motion of every particle is recorded by a CCD camera so that the kinetic energy of the system can be analyzed by tracking the trajectory of each particle. It is found that the average energy changes abruptly at a certain critical filling fraction while the configuration of the particles makes a transition from a disordered to a solid-like state. We determine the critical value of the filling fraction and the energy of the solid-like state using a resonant condition.

Keywords Phase transition · Horizontal vibration · Filling fraction · Resonant condition

1 Introduction

A granular system is inevitably dissipative due to interparticle frictional contacts and inelastic collisions. These inherent properties have made granular systems a challenge for physicists in their attempts to understand the link between microscopic interactions and macroscopic properties. Although granular systems have behaviors resembling those of solids, liquids, or gases [1], they do not obey conventional statistical mechanics [2–4]. In order to sustain the fluidization of a granular system, energy must be supplied. This is usually done by shaking the system mechanically. When the energy dissipated has been completely compensated for by energy input, the system reaches a steady state. There is usually more than one steady state in a vibrating granular system [5–7]. Studying the transition between those states is an interesting issue in granular physics as state transitions can be induced by changing the driving parameters, such as shaking amplitude, frequency or shear [5–8]. Recently, some experiments have also observed crystallization in a vertically vibrating granular system when the number of identical particles in a finite region has been increased [9–11]. Reports on a horizontal shaking system have since found that the transition that signals the onset of crystallization is a precondition for segregation [12,13]. Phase transition is also the focus of recent studies of colloid systems [14–17]. Zhang and Liu [14] have shown that a two-dimensional colloidal system exhibits a phase transition from liquid to crystal under external excitation.

In this report, we present an experiment of a simple granular system which shows state transitions both in structure and dynamics. The system we studied was a monolayer of spherical particles inside a rectangular container driven by a simple harmonic oscillation. By tracking the position of particles with a CCD camera, the average kinetic energy and the radial distribution function were computed. The average kinetic energy of the system as a function of filling fraction ϕ showed a dramatic leap as the system reached a resonant state in motion at a critical value of ϕ_c. Simultaneously, the radial distribution function showed a peak that indicated the formation of a hexagonal structure in the system. We found that ϕ_c decreased linearly with respect to vibration amplitude A. We have provided a possible explanation for this phenomenon based on a resonant condition.

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2 Experiment

2.1 Experimental setup

Our experimental setup, as sketched in Fig. 1, consisted of a rectangular container with length L and width W on a moving track. The container was made of transparent acrylic and coated with a layer of anti-static material. A stepping motor drove the container so that every point (x_p, y_p) on the container moved according to $x_p = A\sin(2\pi ft)$, $y_p =$ constant., where A was the amplitude adjustable from 1.0 to 2.5 cm, and driving frequency f was fixed at 1.6 Hz. The motions of the particles in the container were recorded with a CCD camera above the container. The speed of the camera was set to 250 frames per second. The system is illuminated with the light from top. A bright spot is produced near the highest point of each sphere. We can easily locate all particles using the positions of these brightest spots in the image we took with the CCD. We obtained the trajectories of particles by tracking their positions from frame to frame.

In our experiments, the system normally reached a steady state within a few oscillating periods; we, therefore, recorded the motion of the system for 20 s after 10 s of shaking for each experiment. Having obtained the positions and velocities of all particles, we then calculated the average kinetic energy and the radial distribution function of the system to see how they changed as a function of the filling fraction in different sets of parameters. We used five different sizes of containers to see the effects of different container lengths and widths. Three kinds of particles: aluminum, plastic and steel, were used in the experiment to check the dependency

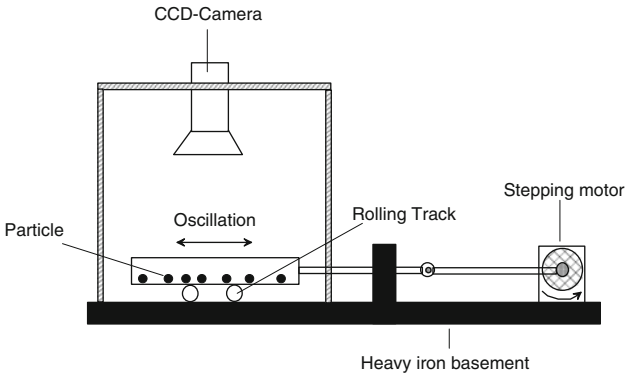

Fig. 1 Sketch of experiment setup

of the material. Aluminum particles of diameter $d = 0.8, 0.6,$ and 0.45 cm, were used to study the particle size effect. The parameters used in the experiments have been summarized in Table 1.

2.2 Alignment

The incline of the container bottom broke the symmetry of the system which was essential to the phenomenon of phase transition. Thus, a level adjustment was necessary before the data was collected. We put 40 aluminum particles of diameter $d = 0.8$ cm into the container. The horizontal level of the container was adjusted gradually during the vibration to make sure the center of the mass of particles had a bias from the center of the container of no more than $2d$ in x direction and $1d$ in y direction (Fig. 2).

Table 1 Parameters used in experiments

Diameter d	Mass/material	Amplitude/frequency	Container size $L \times W$
0.8 cm	0.7 g/Aluminum	1.45 cm /1.6 Hz	12 cm × 6 cm
		1.65 cm/1.6 Hz	
		1.9 cm/1.6 Hz	
		2.2 cm/1.6 Hz	
		2.4 cm/1.6 Hz	
0.8 cm	0.6 g/plastic	1.9 cm/1.6 Hz	12 cm × 6 cm
	2.0 g/ steel		
0.8 cm	0.7 g/Aluminum	1.9 cm/1.6 Hz	12 cm × 2.1 cm
			12 cm × 0.8 cm
0.8 cm	0.7 g/Aluminum	1.9 cm/1.6 Hz	8.5 cm × 0.8 cm
			20 cm × 0.8 cm
0.6 cm	0.7 g/Aluminum	1.9 cm/1.6 Hz	12 cm × 6 cm
0.45 cm			
0.8 cm	0.7 g/Aluminum	1.9 cm/1.2 Hz	12 cm × 6 cm
		1.9 cm / 0.8 Hz	

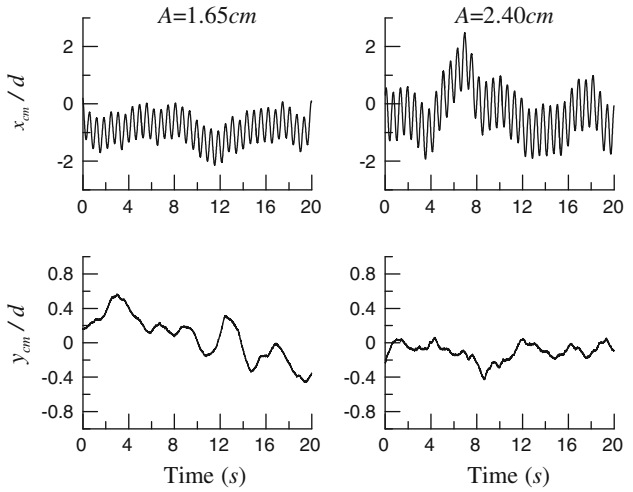

Fig. 2 Two typical trajectories of the center of mass of 40 aluminum particles. The diameter of the particles was $d = 0.8$ cm and the dimensions of container were 12 cm × 6 cm. The horizontal level of the container was adjusted so that the center of the mass had a bias from the center of the container of no more than $2d$ in x direction and $1d$ in y direction

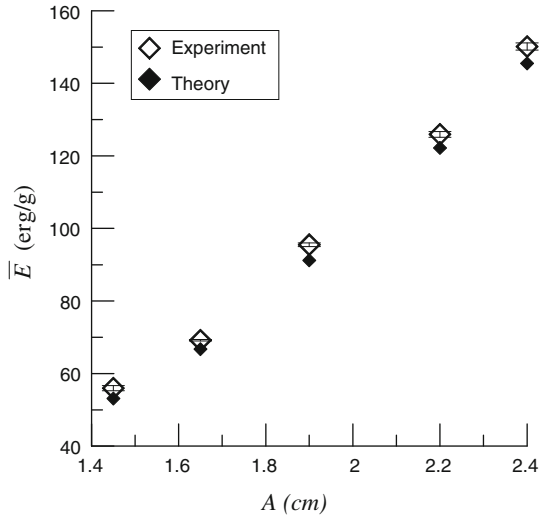

Fig. 3 Average kinetic energy of a fixed point on the container, which is doing simple harmonic motion, as calculated from experimental observation (*open diamond*). The horizontal axis was the oscillation amplitude. The frequency was fixed at 1.6 Hz. The *closed diamond* is the theoretical results given by Eq. (1)

2.3 Calibration

We next calibrated the tracking system by the following steps. First, we tracked a fixed point on the bottom of the container. From the trajectory of the point, we calculated its vibration amplitude in units of pixels and compared it with the actual amplitude of the external drive to get the proportion coefficient between cm and pixel. Since the camera speed was set to 250 frames per second, the time interval between two successive frames was 0.004 s. With this information, we were able to calculate the velocity and the average kinetic energy of the point. Theoretically, a point doing a simple harmonic motion with amplitude A and frequency f would have average kinetic energy per unit mass:

$$\overline{\mathrm{E}} = \frac{1}{4}A^2(2\pi f)^2 \tag{1}$$

This provided us with a calibration reference for our tracking system. Figure 3 shows there was on average a shift of 3.6 erg/g between the experimental and the theoretical results. This deviation was most likely caused by a small fluctuation in the tracking system. We deducted this amount of value from all our data.

3 Experimental results and discussions

The filling fraction ϕ of the system was defined as $\phi = L_C/L$, with L_C as the effective length of the compact cluster formed by particles. When a compact cluster formed in our vibrating system, it always had a row of spheres lined up

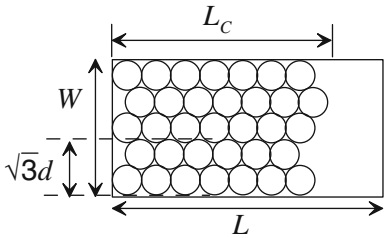

Fig. 4 Effective length of the compact cluster as shown given by $L_C = \frac{Nd}{2W/\sqrt{3}d} + \frac{1}{2}d$, where $2W/\sqrt{3}d$ is the average number of particles per row perpendicular to the vibration walls

along each wall parallel to the vibration direction (Fig. 4). For these configurations, L_C was estimated by:

$$L_C = \frac{Nd}{2W/\sqrt{3}d} + \frac{1}{2}d, \tag{2}$$

where N was the total number of particles When the filling fraction was low, each particle rolled back and forth due to friction with the bottom of the container and had little chance of colliding either with other particles or the container walls. The velocity of each particle was then given by 2/7 of the container velocity [18]. Accordingly, the average kinetic energy per unit mass was;

$$\overline{<E>} = \frac{4}{49}\pi^2 f^2 A^2 \tag{3}$$

where $\overline{X} \equiv \frac{1}{\Delta t}\int_0^{\Delta t} X(t)\mathrm{d}t$ and $<X> \equiv \frac{1}{N}\sum_{i=1}^{N} X_i$ denoted, respectively, the time and number average of the physical quantity $X_i(t)$ of i-th particle.

When the filling fraction reached a critical value of ϕ_c, the solitary oscillatory motion of each particle was upset by

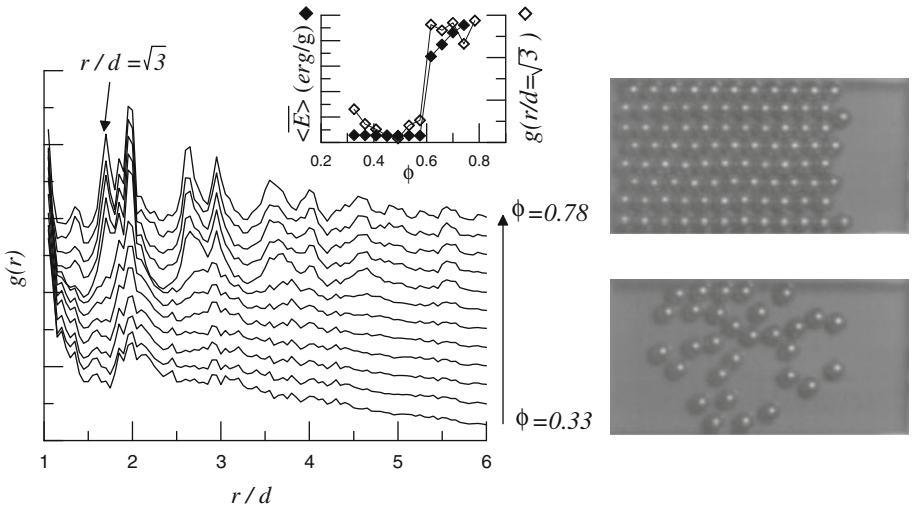

Fig. 5 Distribution function $g(r)$ in relative scale. The amplitude was $A = 2.4$ cm and the frequency 1.6 Hz. Each curve corresponded to filling fraction ϕ and was shifted by a value proportional to ϕ on a vertical axis for easy comparison. A peak started to appear at $r/d = \sqrt{3}$ when $\phi = \phi_c = 0.54$, indicating a structure transition. The *inset* shows that structure and dynamics transitions occurred at the same value of the filling fraction. The two photos on the right are the instant configurations of the particles when $\phi = 0.78$ and $\phi = 0.28$. The resolution of the photos is 138 pixel × 276 pixels

collisions between particles. All of a sudden, instead of moving chaotically, particles collapsed into a cluster and moved in resonance to the container. The hexagonal structure was clearly seen in the cluster. Painter and Behringer [19] have reported that the sliding motion after collision plays an important role in particle-particle interaction. The occurrence of sliding would possibly lead to effectively long range interactions and caused the sudden collapse of structure. To further characterize the structure of the cluster, we calculated the radial distribution function $g(r)$ defined by [20]:

$$g(r) = \frac{1}{K}\sum_{k=1}^{K}\frac{2LW}{N(N-1)a_r}\sum_{i=1}^{N}\sum_{j=1}^{i-1}\theta\left(|\vec{r}_i - \vec{r}_j|\right) \times \theta\left(r + \Delta r - |\vec{r}_i - \vec{r}_j|\right) \quad (4)$$

where $\theta(r)$ was the step function, and $K = 5000$ the number of snapshots used on average. The term $a_r = \pi(2r + \Delta r)\Delta r$ was the area of the ring with inner radius r and width Δr chosen to be $\Delta r = d/10$. With this definition for $g(r)$, $g(d)$ stood for the probability of particles that were in direct contact with other particles, while a peak of $g(r)$ at $\sqrt{3}d$ showed that the configuration had a hexagonal structure. Figure 5 shows the radial distribution function $g(r)$ for the results of the experiments at amplitude $A = 2.4$ cm. Each curve in Fig. 5 corresponds to filling fraction ϕ and it is shifted by a value proportional to ϕ in the vertical axis for easy comparison. The signal of a hexagonal structure—a peak at $r/d = \sqrt{3}$ in $g(r)$—was clearly seen for $\phi \geq \phi_c = 0.54$. In the meantime, at critical filling fraction ϕ_c, the average

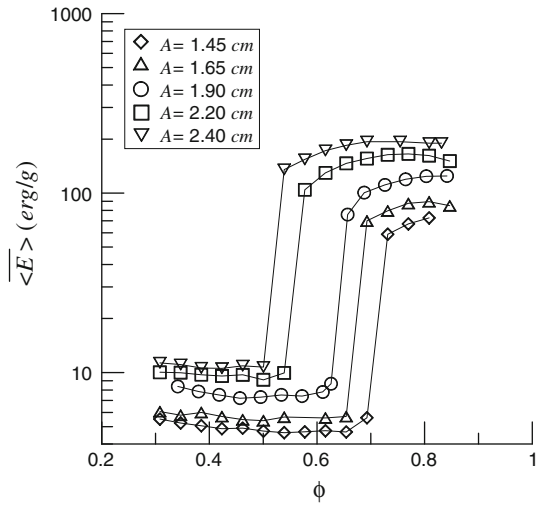

Fig. 6 Average kinetic energy as a function of the filling fraction at different shaking amplitudes. The frequency was fixed at 1.6 Hz. The particles were aluminum, 0.8 cm in diameter and the dimensions of the container were 12 cm × 6 cm

kinetic energy per unit mass changed abruptly to a higher value (inset of Fig. 5).

Value ϕ_c depended on shaking amplitude A (Fig. 6): the larger the amplitude, the less ϕ_c was needed for the onset of the collective motion. The plot of ϕ_c, as a function of relative amplitude A/L, could be well fitted by a linear relation of $\phi_c = 1.06 - 2.45A/L$ for $d = 0.8$ cm (Fig. 7). We checked that this linear relation remained good for various shaking frequencies (0.8 to 2.0 Hz with step 0.2 Hz) and container widths ($1d$ to $7d$). The value of ϕ_c was not sensitive to the materi-

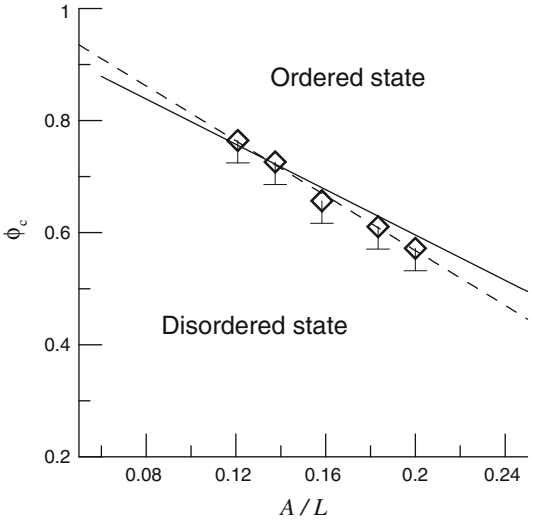

Fig. 7 Critical filling fraction ϕ_c versus A/L. *Dotted line* is the least squares fit for the experimental data (*diamond*). *Solid line* is given by Eq. (6). Particles used in this experiment are Aluminum with $d = 0.8\,\text{cm}$

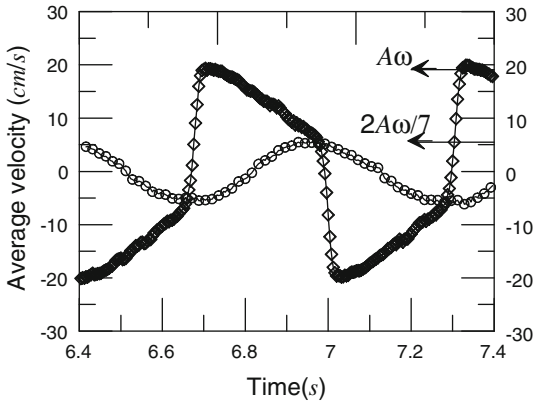

Fig. 8 Variation of average velocity in time ($A = 1.9\,\text{cm}$, $f = 1.6\,\text{Hz}$) for systems with the filling fraction at the critical value 0.65 (*diamond*) and a value (0.61) smaller than critical value (*circle*). The particles were aluminum of 0.8 cm in diameter. The dimensions of the container were 12 cm × 6 cm

als (steel, aluminum, or plastic particles) used. The relation implied that the system was easier to transit into an ordered state (smaller ϕ_c) when a larger amount of energy was supplied (larger A). This was somewhat surprising since, presumably, more input energy to a system would tend to drive the system to a disordered state. The key to this seemingly counterintuitive phenomenon was that much energy was dissipated through the friction generated between the particles and the bottom of the container so that the transition to the hexagonal state was determined, not by the amount of input energy, but by the resonant condition with an appropriate free space for the compact cluster of particles.

Figure 8 shows two typical time evolutions of the average velocity for the filling fraction at and below the critical value. For the case when the filling fraction was smaller than the critical value, average velocity of particles was about 2/7 of the container velocity. This implies that the particles rolled without sliding on the moving surface. Each particle oscillated on its own without colliding with others. Even if they were arranged in the beginning as a compact cluster, the cluster gradually broke into many solitary vibrations of particles. The situation was completely different when the filling fraction reached the critical value; particles hit the oscillating walls like a completely inelastic cluster. The cluster lost all its velocity relative to the walls after impact. Then it accelerated until its velocity reached the maximum value $A\omega$ and left the walls by inertia. After the cluster left the walls, it decelerated due to frictional force between the particles and the bottom of the container. Since the average velocity during this period of time is larger than $\frac{2}{7}A\omega$, the dominant motion should be sliding. The rolling motion was possible to be established again only when the velocity of particles decreased below $\frac{2}{7}A\omega$. However, if the particles were hit by the opposite wall and regained energy before being trapped into rolling mode, the sliding motion would continue to be dominant. Accordingly, the condition for the cluster moving resonantly to the container with a largest mean free path would be

$$\lambda_m f_c = \bar{v} \tag{5}$$

where the largest mean free path λ_m and collision frequency f_c of the cluster with the walls are $L - L_c = L - L\phi_c$ and $2f$, respectively. The mean velocity of the cluster can be approximately given by $\bar{v} = \left(A\omega + \frac{2}{7}A\omega\right)/2$. Therefore, Eq. (5) can be written as a relation between ϕ_c and A:[1]

$$\phi_C = 1 - \frac{9\pi}{14}A/L \tag{6}$$

The relation Eq. (6) says that the critical filling fraction for the emergence of phase transition decreases with the amplitude of the external drive. This was consistent with what we observed in the experiments (Fig. 7). Alternatively, from Eq. (6) we could see that, for a given number of particles in our system (equivalently, given ϕ), the system could be driven to an ordered, well-structured state by increasing external energy. This was an interesting example of "freezing by heating" [21].

The critical filling fraction did not depend on d explicitly since the effect of d had already been absorbed in the definition of ϕ through L_C (Eq. 2). Our experiments with aluminum particles of diameters 0.8 and 0.45 cm, showed that the dependence of ϕ_c on d was reasonably included implicitly

[1] To be precise, one has to take the phase of the external drive whence the velocity of the CM begins to change signs into consideration. However, this modification to Eq. (6) would not change the linear relation between ϕ_c and A. We have determined in our experiments the value of phase which is approximately equal to $\pi/2$ for the cluster of particles in resonance with the external drive. Thus, Eq. (6) is actually a very good approximation even quantitatively.

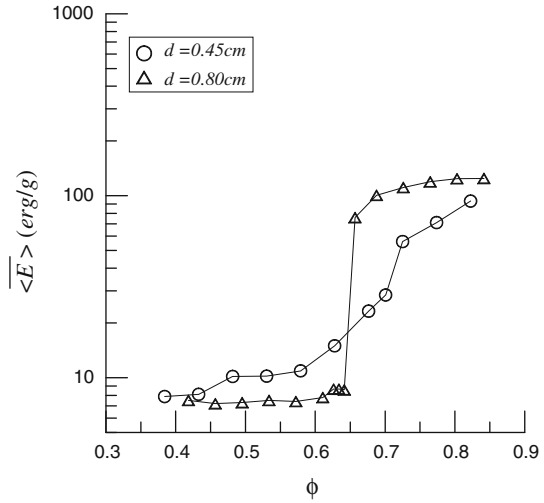

Fig. 9 Average kinetic energy per unit mass as a function of the filling fraction for three different particle sizes. The dimensions of the container were 12 cm × 6 cm. The amplitude and frequency were 1.9 cm and 1.6 Hz, respectively

in Eq. (2) (Fig. 9). In Fig. 9, however, we noticed that the transition to the ordered state was rather smooth for small $d = 0.45$ cm, which was different from the cases of larger particles where the transitions were abrupt. Presumably, the friction between the particles of small d and the bottom plate was not large enough (because small particles have small mass and therefore small friction) to make them roll without slips. The kinetic energy of small particles then consisted of contributions from rolling and translation. When ϕ increased, there were more particles gaining translational energy from collisions with the vibrating walls so that the average energy increased gradually with ϕ before the critical value was reached, as in Fig. 9 for the case of small $d = 0.45$ cm.

4 Conclusion

We studied the transition phenomenon in a horizontally vibrating granular system experimentally. The transition was identified by the discontinuous change of average kinetic energy when the filling fraction reached a critical value. At the critical filling fraction, the configuration of the particles became a solid-like hexagonal structure, while the whole group of particles moved like a giant particle in resonance with external vibrations. Experimental results showed that the critical value of the filling fraction depended linearly on relative amplitude A/L. Driving frequency and container width had no effect on the critical value of the filling fraction. We successfully derived the relation between the critical filling fraction and the relative driving amplitude based on a resonant condition.

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