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ORIGINAL PAPER

Brazil nut effect in annular containers

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Abstract This paper investigates the motion of particles between two co-axial cylinders which are subjected to a sinusoidal vertical vibration. We measure the rising time of a large intruder from the bottom of the container to the free surface of the bed particles and find that the rising time as a function of intruder density decreases to a minimum and then increases monotonically. The result is qualitatively opposite to the previous findings in experiments using cylindrical containers where a maximal instead of minimal rising time in the small-density regime was found. The experimental results suggest that the topology of the container plays an important role in the Brazil nut effect.

Keywords Granular particles · Brazil nut effect · Annular container · Vertical vibration · Convection

When a bed of small particles is subjected to vertical vibration, a large intruder tends to rise to the free surface. This phenomenon, often called the "Brazil nut effect (BNE)", has been studied extensively [1–15]. There are three plausible mechanisms leading to the BNE. The first mechanism [12] says the intruder rides on convection currents [16,17] within the host bed. When it reaches the upper surface it is likely to remain there, being unable to enter the flow. In the second [1], the intruder is ratcheted, since during vibration voids appear below the intruder, into which bed particles move. The third is the air effect [7]. It is generally believed that the combination of them or the competition between them is able to explain the various features of the BNE. A few refined models, using ideas such as convection [12,13], drag [14], buoyancy [9,15,18], and air pressure [7,8,19,20] have been proposed. Schröter et al. [11] have systematically checked which are the important segregation mechanisms for diverse conditions. Kudrolli [21] has given a comprehensive review on the development of the BNE up to 2004.

Experimentally, the rising time of the intruder, from when it starts at a distance below the surface of the medium to when it emerges at its surface, has been measured to show its dependence on the driving frequency [4], vibration amplitude [4,5], particle size [4–9], and intruder density [7–9], etc. Möbius et al [7] measured the rising time of the intruder and found that it is a non-monotonic function of the intruder's density, indicating an intricate interplay between intruder motion, convection, and drag by interstitial air. Huerta and Ruiz-Suárez [9] also obtained similar results. Specifically, they showed that, in the case of cylindrical container, for a fixed intruder diameter, the rising time of the intruder to the free surface exhibits a pronounced peak as a function of ρ/ρ_b, where ρ and ρ_b are the density of the intruder and the bed particles, respectively. The present paper reports the rising time of the intruder in an annular container. It is found that the rising time displays a valley as a function of ρ/ρ_b (Fig. 1). An intruder with a density near that of the bed particles rises to the free surface faster than one of any of other density. The results are qualitatively the same for various driven frequencies, amplitudes, chamber pressures, and container gaps. It is the topology of the container that affects the property of the rising time as a function of ρ/ρ_b. Presumably, the convection patterns of the background particles, which acquire their forms according to the topology of the container, play a crucial role in the intruder's rising time.

The experimental system for this study consists of an acrylic annular container with two co-axial cylindrical walls of height 15 cm and radii $R_1 = 4$ cm and $R_2 < 4$ cm, respectively, mounted on a vibrator (model: Bruel & Kjaer 4808). Five different sizes of $R_2 = 0.5$, 1.0, 1.5, 2.5, and 3 cm were

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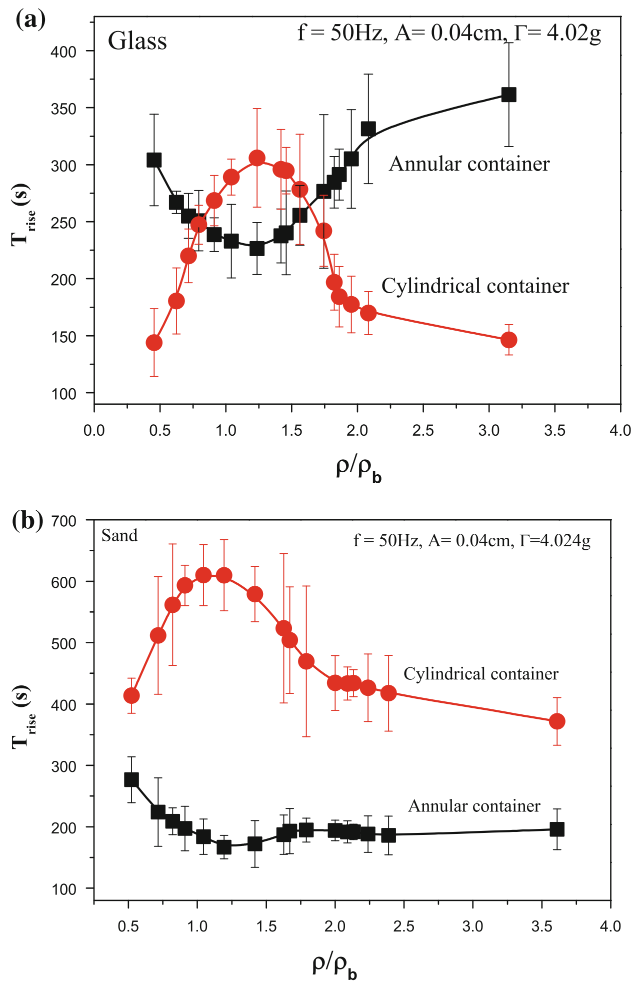

Fig. 1 Rising time of the intruder as a function of density has a minimal value near the bed-particle density for the case of the annular container, while for the cylindrical container it has a maximum. Similar results are obtained for two different types of bed particles: **a** glass and **b** sand. ($H = 6$ cm, $f = 50$ Hz, $A = 0.04$cm, $\Gamma = 4.02$g)

used in the experiment. For comparison, the experiments were also performed with a cylindrical container, which is equivalent to the case of $R_2 = 0$. In a typical experiment, the container was filled with identical bed particles up to a height H of 6 or 7 cm. A large intruder was initially put at the bottom of the container, close to the wall of the outer cylinder. For all cases except Fig. 1(b), the bed particles were glass beads of diameter $d = 0.2$ cm with a density $\rho_b = 2.77$ g/cm^3. In the experiment of Fig. 1(b), the bed particles are sands of size 0.5–0.8 mm. The intruder was made of clay and has a fixed diameter $D = 0.8$ cm. By inserting different amounts of tiny beads of various kinds of materials inside the spherical clay, the intruders were made to have various densities ρ ranging from $0.20\rho_b$ to $3.2\rho_b$. A sinusoidal signal was used to drive the system to vibrate vertically at frequency $13 \leq f \leq 50$ Hz, amplitude $0.02 \leq A \leq 0.49$ cm, and relative acceleration $2.24 \leq \Gamma/g \leq 7.18$, where g is the acceleration of gravity. A stopwatch was used to record the rising time $T_{\rm rise}$ when the intruder emerged through the surface of the bed particles. The cylinder walls and all particles were treated with an antistatic agent to avoid the buildup of static charges. The experiment was repeated 10 times for each set of parameters [22]. Relative humidity was controlled at $40 \pm 5\%$ [23].

It is well known that the friction between the particles and the walls normally induces a downward motion for the particles along the walls. Particles in a cylindrical container subjected to vibration normally form a heap near the center of the surface where particles emerge. The largest convection loops go down along the wall and merge together to go up along the axis of the cylinder. For annular containers, because particles tend to go down on the wall of either the inner or the outer cylinder, they rise to the surface at a position near the middle of the annular gap. In addition, the vibration-induced asymmetry, which is less conspicuous in cylindrical container, can be clearly seen in annular containers, especially at high vibration frequencies. The carefully-arranged initial symmetry was always broken by the vibration–the free surface of the particles tilted to one side–when experiments were conducted at 1 atm. We have checked our system for lateral motion during vibration. The maximal lateral displacement of the central axis during vibration is less than 0.02 cm for all frequencies and amplitudes we used. We suspect the azimuthal asymmetry is caused not by the small mechanical vibration unbalance but mainly by the air trapped in the bed particles [24]. Indeed, the asymmetry can be reduced to be indiscernible by lowering the air pressure inside the container below 0.2 atm [25]. Because of this asymmetry, particles on the surface had an additional tendency to move along the slope. Similarly, when particles move down or up below the surface, they have a velocity component in the azimuthal direction. The convection for particles in the vibrated annular containers thus took on a spiral-like pattern. The convection spirals could go all the way from the position where the surface had the smallest height in both φ and $-\varphi$ directions to the point at the greatest height where they merged. Figure 2 shows a schematic plot of the convection pattern that we observed using tracers.

The intruder was placed at the bottom of the container against the wall that was near the highest point of the free surface. For annular containers with any ratio value R_2/R_1 in the experiments, light and heavy intruders rose more slowly than those with intermediate densities (Fig. 3). The intruder rose faster/slower when the vibration amplitude/frequency increased (Fig. 4). However, qualitatively, the non-monotonic curve of the rising time $T_{\rm rise}$ as a function of relative density was independent of the amplitude or frequency used in the experiments. The concave down property of the $T_{\rm rise}$ curve is not due to the azimuthal asymmetry either. We obtained similar results by reducing the air pressure to below 0.2 atm so that the bed surface was flat during

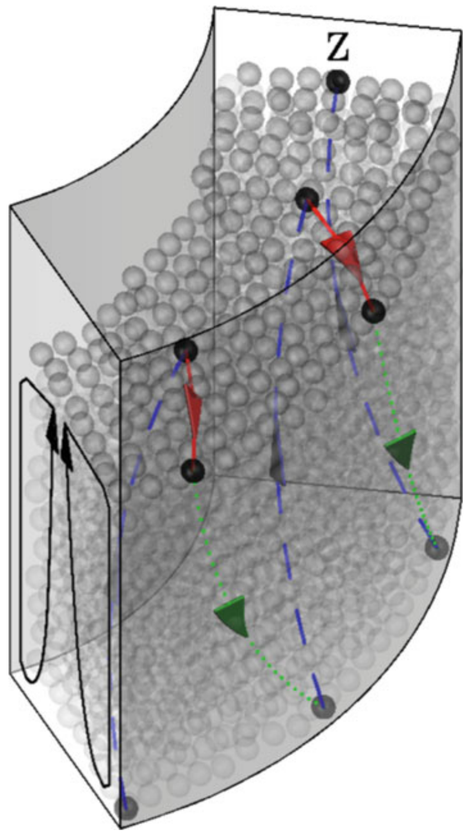

Fig. 2 Convection patterns in the annular container. At 1 atm, vibration always induces an asymmetry so that the surface of the bed particles is tilted. On the surface, a bed particle moves to either the inner or outer wall and, at the same time, slides down along the slope of the surface (*solid paths*). During the sinking (*dotted paths*) and rising (*dashed paths*), it has an azimuthal velocity toward the highest point Z of the surface. The asymmetry becomes indiscernible when the air pressure in the container is kept below 0.2 atm. The bed surface is flat and the convection loops approximately lie on the azimuthal crosssection (*black curves*)

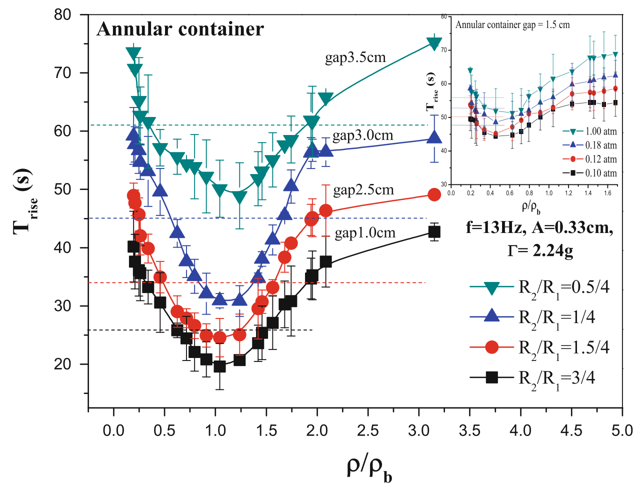

Fig. 3 Rising time as a function of density for annular containers. Curves with various gap distances collapse approximately into a single curve when the rising times are normalized by the corresponding values of the rising time of the bed particles (*dashed lines*). $H = 6$ cm, $f = 13$ Hz, $A = 0.33$ cm, $\Gamma = 2.24$g. The *inset* shows the rising time of the intruder when the air pressure inside the annular container with gap = 1.5 cm is kept at 1.0, 0.18, 0.12, and 0.10 atm respectively

vibration (inset of Fig. 3). When the rising time was normalized by convection, namely, when the rising time of the intruder was divided by the rising time of the bed particles (which was measured with no intruder inside the bed), the curves in Fig. 3 for annular containers with different gap distances approximately collapsed into a single curve. This implies that the gap distance of the annular container changes the strength of the convection but does not affect the general behavior of T_{rise} as a function of density.

It is known that the intruder can affect the strength of convection [26]. A few tracers were placed beside the intruder before vibration began. The rising time of the tracer T_b that emerged first from the surface was recorded, and it was found that T_b, as a function of intruder density, is a concave up curve for either the cylindrical or annular container, but the peak values of T_b appear at different densities for the two kinds of containers (Fig. 5). The intruder always rose faster than the bed particles for the case of $R_2/R_1 = 1/8$. The rising times of the intruder T_{rise} for the two kinds of containers approached each other in the large density region, $\rho > \rho_b$. In small density region, they behaved in the opposite way: T_{rise} had a pronounced peak for the cylindrical container but a deep trough for the annular container.

It was observed that even given a radius of the inner cylinder as small as 0.5 cm ($R_2/R_1 = 1/8$), the curve of the rising time as a function of relative density was still concave down in the small-density region, qualitatively different from the results for the case of cylindrical container. To see how the curve changed from having a minimum to a maximum, the experiments were conducted using containers whose inner cylinder had a radius of 0.5 cm but at a distance h_b above the bottom. (The lower end of the inner cylinder had been made into a shape of an inverse cone to smooth the flow of the particles.) The depth of the particles in the container was set to $H = 7$ cm prior to vibration. The measured rising-time of the intruder T_{rise} for cases of $h_b = 0, 1, 2, 4, 5, 6, 7$ cm as a function of density were plotted in Fig. 6. For the annular container ($h_b = 0$), the curve had a minimum at $\rho \approx 0.4\rho_b$. It was seen that T_{rise} decreased for all densities as h_b increased from 0 to 6 cm. However, the variation of T_{rise} caused by h_b was density-dependent so that the curve of T_{rise} as a function of density changed its form when h_b increased. Increasing the value of h_b further from 6 cm to 7 cm, the inner cylinder was raised above the surface of the bed particles and so the container became a single cylinder. In the cylinder, T_{rise} dropped substantially at small densities so that T_{rise} as a function of density became a concave up curve having a maximum at $\rho \approx 0.3\rho_b$.

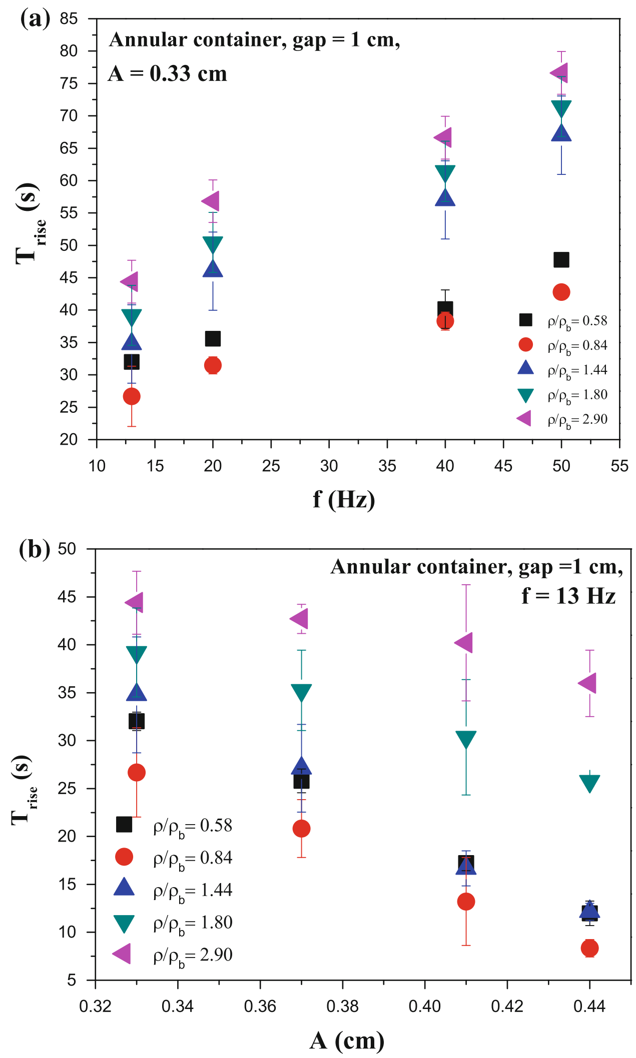

Fig. 4 Rising time of the intruder **a** increases with the frequency and **b** decreases with the amplitude

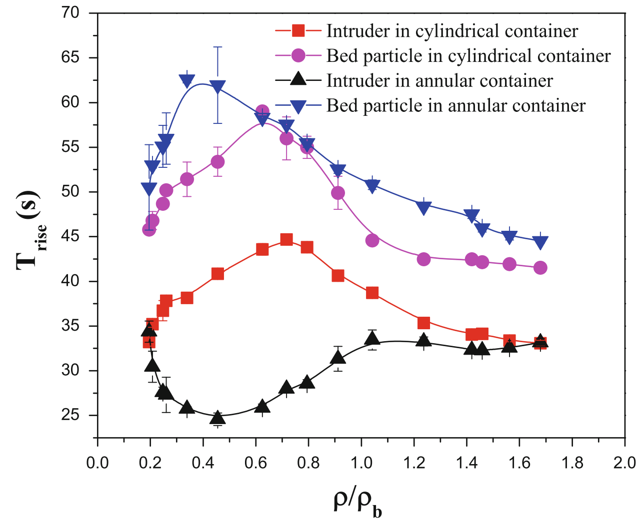

Fig. 5 Rising time of the bed particles (measured with an intruder inside the bed) and the intruder in an annular container. $H = 7$ cm, $f = 13$ Hz, $A = 0.33$ cm, $\Gamma = 2.24g$, $R_1 = 4$ cm, $R_2 = 0.5$ cm

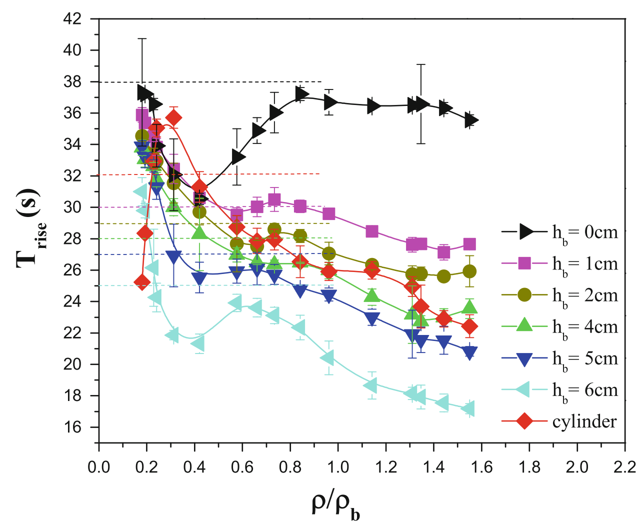

Fig. 6 Rising time of the intruder in containers with inner cylinders raised above the bottom by a height h_b. Dashed lines indicate the values of the rising time of the bed particles. $H = 7$ cm, $f = 13$ Hz, $A = 0.33$ cm, $\Gamma = 2.24g$, $R_1 = 4$ cm, $R_2 = 0.5$ cm

The variation of the rising time in containers with different h_b values was strongly related to the convection patterns of the bed particles. As h_b increased, the two loops in the azimuthal projection of the convection patterns (Fig. 2) became more asymmetric: the inner loop that was close to the inner cylinder shrank and the outer loop grew larger (Fig. 7). At the same time, outer loops began to merge in the area under the inner cylinder. Eventually, when the inner cylinder was raised above the surface so that the container became a single cylinder, the inner loop disappeared and all outer loops merged at the axis of the cylinder. It is surmised that the merging of the convection loops had an important effect on the rising time of the intruder. An experiment was also carried out using a parallelepiped container of 15 cm long, 8 cm wide and 1 cm thick. The rising time of the intruder in the rectangle container was found to have a pronounced peak at $\rho \approx 0.4\rho_b$, similar to the results for the case of the cylindrical container. Topologically speaking, a rectangular container is like an annular container except it has two walls at lateral ends while the annular one has none. Because of these two lateral walls, the convection pattern of a rectangular system is different from that of an annular system. Again, it is surmised that the pattern of convection is crucial to the rising time of the intruder.

Möbius et al have reported that the value of relative density ρ/ρ_b for the rising time of the intruder to reach a maximum in the case of cylindrical container is dependent on air pressure [7]. In the experiments conducted for this study, when the air pressure was kept at 1 atm, it was found that

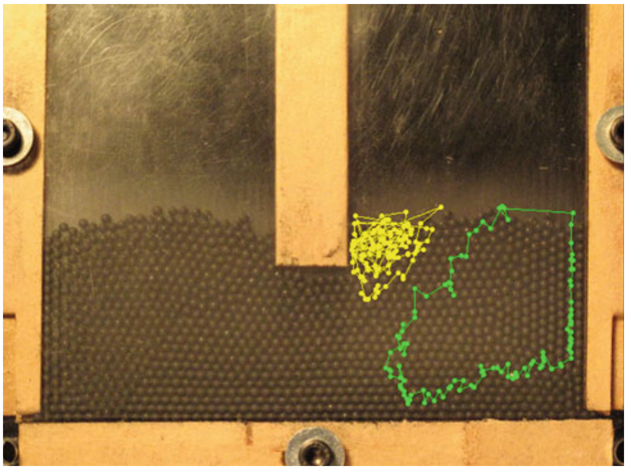

Fig. 7 The trajectories shown here are for particles in a pseudo two-dimensional rectangular system ($f = 20$ Hz, $A = 0.33$ cm, $\Gamma = 5.31g$). They are similar to the φ-projection of the particle trajectories we observed in an annular container. The tracers are migrating slowly inside the crystal-like structure of our monodisperse system. Time interval between two connected points is 2 s. The total time for both marked trajectories is 240 s. For the present, we are unable to trace quantitatively the 3-dimensional paths of the particles inside an annular container

the value of ρ/ρ_b where T_{rise} has a maximum (cylindrical container) or a minimum (annular container) is dependent on the surface roughness of the intruder. With the use of a newly-made, roughly-surfaced clay intruder, whose coefficient of restitution is measured to be values between 0.1 and 0.3 depending on its density, T_{rise} has its maximum or minimum at lower density $\rho/\rho_b \leq 0.5$ (Fig. 6, inset of Fig. 3). After the intruder's rough surface had been smoothed by long (>20 hours) collisions with the bed particles, the coefficient of restitution is found to be about 0.05 greater than that for the rough-surfaced intruder. The measured T_{rise} of the smooth-surfaced intruder had a maximum or minimum at a larger density $\rho/\rho_b \approx 1.0$ (Figs. 1, 3).

In conclusion, because of the friction between the particles and the walls, the geometry of the container influences the convection pattern of the bed particles (Fig. 2). The appearance of the intruder affects the strength of the convection in a non-linear way (Fig. 5). The resultant convection patterns play a crucial role in determining the rising time of the intruder (Fig. 6). This report has shown that the rising time of the intruder in annular containers as a function of density, in particular, has a deep trough in the small density region (Figs. 1, 3), opposite to the previous results found in experiments using cylindrical containers. The interstitial air apparently plays insignificant role for this result (inset of Fig. 3). Our findings suggest that a good understanding of how the convection is changed by the topology of the container and how it is affected by the appearance of an intruder is a crucial precondition to a full understanding of Brazil nut effect.

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