

## Baroclinic Flow around Planetary Islands in a Double Gyre: A Mechanism for Cross-Gyre Flow

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### ABSTRACT

A quasigeostrophic, two-layer model is used to study the baroclinic circulation around a thin, meridionally elongated island. The flow is driven by either buoyancy forcing or wind stress, each of whose structure would produce an antisymmetric double-gyre flow. The ocean bottom is flat. When the island partially straddles the intergyre boundary, fluid from one gyre is forced to flow into the other. The amount of the intergyre flow depends on the island constant, that is, the value of the geostrophic streamfunction on the island in each layer. That constant is calculated in a manner similar to earlier studies and is determined by the average, along the meridional length of the island, of the interior Sverdrup solution just to the east of the island.

Explicit solutions are given for both buoyancy and wind-driven flows. The presence of an island of nonzero width requires the determination of the baroclinic streamfunction on the basin's eastern boundary. The value of the boundary term is proportional to the island's area. This adds a generally small additional baroclinic intergyre flow. In all cases, the intergyre flow produced by the island is not related to topographic steering of the flow but rather the pressure anomaly on the island as manifested by the barotropic and baroclinic island constants. The vertical structure of the flow around the island is a function of the parameterization of the vertical mixing in the problem and, in particular, the degree to which long baroclinic Rossby waves can traverse the basin before becoming thermally damped.

### 1. Introduction

The study of the role played by planetary-scale islands in the dynamics of the ocean circulation was given fresh impetus by the classic paper of Godfrey (1989). Since that study, there have been numerous extensions and elaborations of that work (for a bibliographic summary see Pedlosky et al. 2009). The papers cited in Godfrey (1989) are almost entirely barotropic in their dynamics. In this paper, we examine an important further elaboration of the original conceptual model by including the effects of baroclinicity.

Considering the role of baroclinicity in the island circulation problem allows us to discuss the question of the vertical structure of the flow, in particular the vertical structure of the transport from one subbasin to the next when the island imposes a barrier to the free circulation between the subbasins. In a previous study (Pedlosky et al. 1997, hereafter PPSH), one important consequence

of the presence of a meridionally extended island was the appearance of a recirculation region to the east of the island in the linear dynamics regime. The recirculation in quasigeostrophic theory contains fluid that is endlessly recirculating, and the meridional extent of the circulation is determined by an application of Kelvin's circulation theorem around the island. The theorem also determines the island constant itself. The vertical structure of the recirculation zone and its subsurface extent are questions that can only be addressed in a baroclinic model.

The model described in this study is driven by both buoyancy forcing (in a simple parameterization of heating and cooling) and wind stress (Ekman pumping). The buoyancy forcing naturally suggests examining a double-gyre circulation consisting of simple representations of subtropical and subpolar gyres. It will be shown in the following that, when the island extends, even slightly, from one gyre into the other, that is, across the zero line of the forcing, the presence of the island induces a substantial cross-gyre transport. This is a new mechanism for the interaction between gyres.

In section 2, we describe the basic model that is steady, quasigeostrophic, and—in this first study—entirely linear.

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In section 3 the problem of purely buoyancy forcing is taken up first. In some ways, it is algebraically simpler than the wind-driven problem since, in the present model, the response to that forcing is strictly baroclinic. Furthermore, the purely buoyancy-driven problem can be thought of as a model of the flow around and along an isolated ridge segment in the abyss, and the extended topography of the ridge across gyre boundaries suggests an additional pathway for deep flows between gyres. Section 4 discusses the wind-driven problem and the barotropic and baroclinic responses. Section 5 summarizes and discusses the principal results.

**2. The model**

We consider the circulation in a two-layer, quasigeostrophic model of the ocean circulation occurring in a rectangular basin of north–south dimension  $L$  and east–west dimension  $Lx_e$ . The upper layer has a rest thickness  $H_1$ , and the lower layer’s rest thickness is  $H_2$ . The flow is driven by a wind stress that produces an Ekman pumping  $w_e$ , assumed for simplicity to be a function only of the latitude coordinate  $y$  and by a buoyancy forcing represented as a target position,  $h(y)$ , to which the interface  $\eta$  relaxes with a time constant  $\gamma^{-1}$ . The form of each is shown in Fig. 1 and is chosen so that the integral of the forcing over the  $y$  interval  $(0, 1)$  vanishes. Both the eastward coordinate  $x$  and  $y$  are scaled with  $L$ . A scale for the geostrophic streamfunction is taken to be  $\Phi$ , which will be determined later. The Coriolis parameter is scaled by its value at the southern latitude of the basin,  $f_o$ . If  $g'$  is the reduced gravity, the deviation of the interface is scaled by  $f_o\Phi/g'$  and the pressure anomaly in each layer is scaled with  $\rho_o f_o \Phi$ . We assume that the friction can be modeled as horizontal diffusion of momentum in each layer with a mixing coefficient  $A$ . Then the linear momentum equations can be written in nondimensional form

$$\begin{aligned}
 -fv_n &= -\frac{\partial p_n}{\partial x} + \delta_m^3 \nabla^2 v_n; \\
 fu_n &= -\frac{\partial p_n}{\partial y} + \delta_m^3 \nabla^2 u_n,
 \end{aligned}
 \tag{2.1a,b}$$

where  $f = (1 + \beta y)$  and the nondimensional parameter  $\beta$  is defined,  $\beta \equiv \beta_* L/f_o$ , where the asterisk subscript denotes the dimensional beta. In quasigeostrophic theory  $\beta$  is considered a small parameter. The continuity equations in each layer, in nondimensional form, are

$$\begin{aligned}
 \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} &= -\frac{H}{H_1} \beta \left( \frac{W_e f_o L}{\beta_* H \Phi} w_e(y) - w_i \right); \\
 \frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} &= -\frac{H}{H_2} \beta w_i.
 \end{aligned}
 \tag{2.2a,b}$$

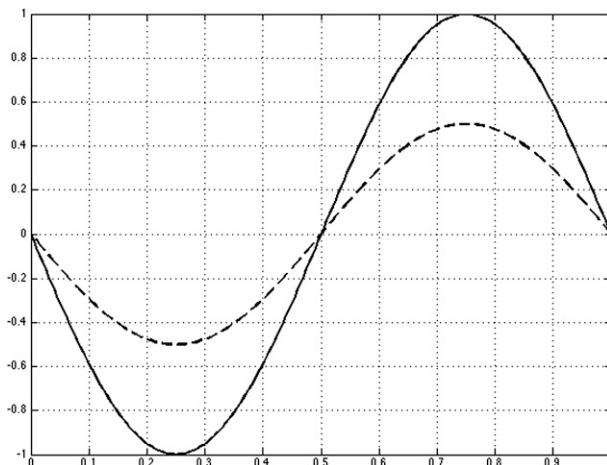


FIG. 1. The meridional profile of Ekman pumping (solid) and buoyancy forcing function  $h$  (dashed). The amplitudes of each are arbitrary.

Here the scale of the applied Ekman pumping vertical velocity is  $W_e$  and its form is given by the  $O(1)$  function  $w_e(y)$ . The function  $w_i$  is the fluid velocity at the interface. It is represented as in Pedlosky and Spall (2005): that is, in our nondimensional units,

$$w_i = \frac{H_1 H_2}{H^2} \left[ \frac{1}{\delta_T} \left( \eta - \frac{g' h_o}{f_o} h(y) \right) - \delta_K \nabla^2 \eta \right].
 \tag{2.3}$$

The nondimensional interface displacement  $\eta$  is related to the geostrophic streamfunctions in each layer by the hydrostatic relation, which in nondimensional units is simply

$$\eta = \phi_2 - \phi_1,
 \tag{2.4}$$

whereas in quasigeostrophic theory

$$u_n = -\frac{\partial \phi_n}{\partial y}; \quad v_n = \frac{\partial \phi_n}{\partial x}.
 \tag{2.5a,b}$$

The first two terms within the brackets of (2.3) represent the cross-isopycnal velocity due to the vertical displacement of the interface from the target interface  $h$ , which, aside from the scaling parameter  $g' h_o/f_o \Phi$ , is the target forcing interface height to which the interface is assumed to relax. The dimensional target displacement is  $h_* = h_o h(y)$ . Where it is positive the interface tends to rise, increasing the thickness of the denser lower layer, and hence represents cooling. A target  $h$  that is negative represents heating (a deepening of the less dense upper layer). The parameter  $\delta_T$ ,

$$\delta_T = \frac{\beta_* L_D^2}{\gamma L}; \quad L_D = \left( \frac{g' H_1 H_2}{f_o^2 H} \right)^{1/2},
 \tag{2.6a,b}$$

is the nondimensional distance a baroclinic Rossby wave can propagate westward before it decays due to the cross-isopycnal flux represented by the first two terms in (2.3), which is our representation of vertical mixing with a time scale  $\gamma^{-1}$ . The remaining term in (2.3) is our attempt to represent horizontal mixing by unresolved eddies and can be thought of as either mixing along the isopycnal surface or, especially, in boundary regions, true cross-isopycnal flux owing to lateral diffusion. It gives rise to a boundary layer scale (nondimensional) on meridional boundaries:

$$\delta_K = \frac{\kappa}{\beta_* L_D^2 L}, \tag{2.6c}$$

where  $\kappa$  is the lateral mixing coefficient. None of the results discussed in this paper depend sensitively on this parameterization of lateral mixing.

Depending on whether we consider the wind or buoyancy forcing of primary importance, we can choose the scale for the geostrophic streamfunction  $\Phi$  to make the parameter measuring the forcing in either (2.2a) or (2.3) equal to unity. Then the other parameter yields the ratio of the two forcings.

We have neglected the possible effects of bottom friction to simplify the physics and this allows us to rewrite the system of equations for the vorticity in terms of the barotropic and baroclinic streamfunctions,

$$\phi_b = \frac{H_1 \phi_1 + H_2 \phi_2}{H}; \quad \phi_t = \phi_1 - \phi_2 = -\eta, \tag{2.7a,b}$$

and

$$\begin{aligned} \frac{\partial}{\partial x} \phi_b &= \frac{f_o W_e L}{\beta_* H \Phi} w_e + \delta_m^3 \nabla^4 \phi_b; \\ \frac{\partial}{\partial x} \phi_t &= \frac{H}{H_1} \frac{f_o W_e}{\beta_* H \Phi} w_e + \frac{1}{\delta_T} \left( \phi_t + \frac{g' h_o h}{f_o \Phi} \right) \\ &\quad - \delta_K \nabla^2 \phi_t + \delta_m^3 \nabla^4 \phi_t. \end{aligned} \tag{2.8a,b}$$

Boundary conditions for the geostrophic streamfunctions are that they must be constant on the basin perimeter and on the island, shown in Fig. 2. On the basin perimeter  $C_b$ , we can take

$$\phi_b = 0 \quad \text{on } C_b. \tag{2.9}$$

On the eastern boundary of the basin (and hence on its perimeter) the baroclinic streamfunction must be a constant,  $\Phi_E$ , but that constant is not arbitrary. It represents the deviation of the interface height on the basin perimeter and must be found from the integral condition on mass in each layer:

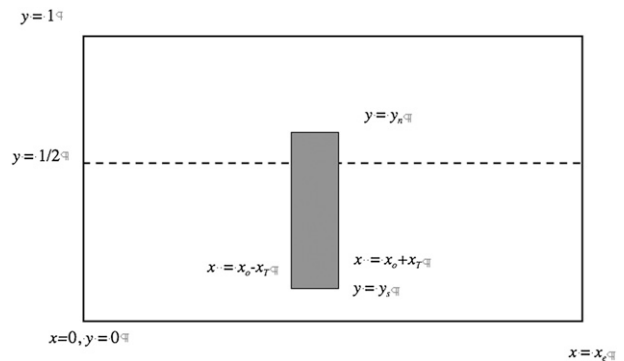


FIG. 2. The ocean basin: the island has a width  $2x_T$  and its center line is at  $x = x_o$ ; its meridional extent is from the latitude  $y_s$  to  $y_n$ . As shown, it can extend across the intergyre boundary, defined as the zero line (the dashed line) for the wind and/or buoyancy forcing.

$$\iint_A w_i dx dy = 0, \tag{2.10}$$

where the integral in (2.10) is calculated over the basin area excluding the island. If we assume that the basin boundaries are insulated so that  $\nabla \eta \cdot \mathbf{n} = 0$ , where  $\mathbf{n}$  is the normal to the solid boundary, then condition (2.10) reduces to

$$\iint_A \left( \phi_t + \frac{g' h_o}{f_o \Phi} h(y) \right) dx dy = 0, \tag{2.11}$$

where again the area of integration is over the whole basin excluding the area of the island.

On the island the geostrophic streamfunctions,  $\phi_b$  and  $\phi_t$ , are also constants,  $\Phi_{BI}$  and  $\Phi_{TI}$ , respectively. We will determine these constants in the following sections, but it is clear that they are of major importance since, along with  $\Phi_E$ , they determine the total barotropic and baroclinic mass flow around the island. We will also apply no-slip conditions on the solid boundaries.

It is possible to simultaneously consider the wind- and buoyancy-forced problems, but for clarity it is useful to consider them sequentially. Since our treatment here is linear, the results can be added together to obtain the response to the combined forcing. We start by considering the response to purely baroclinic forcing: that is, when there is no Ekman pumping—a glance at (2.8a,b) shows that the response will be purely baroclinic with no barotropic component.

### 3. The buoyancy-forced problem

We consider the forcing function  $h(y)$ , which is the nondimensional form of the target interface profile,

$$h = -\sin(2\pi y), \tag{3.1}$$

so that the region from 0 to 1/2 in  $y$  is heated and the region north of that is cooled. In particular, the integral of (3.1) over the range of  $y$ , (0, 1), is zero, so there would be no net heating or cooling in the absence of the island. However, for an island of nonzero width, as shown in Fig. 2, there will generally be a net heating or cooling (depending on the location of the island) and there will be, in response, a net rise or fall of the interface in the steady state to compensate, producing a nonzero  $\Phi_E$  on the basin perimeter.

First, we choose the scaling parameter for the geostrophic streamfunction

$$\Phi = \frac{g'h_o}{f_o} \tag{3.2}$$

so that the forcing term in (2.8b) is just unity. Then the solution of (2.8b), excluding boundary layers on the basin's western boundary and on the eastern side of the island, is

$$\phi_t = -h(1 - e^{-(x_e-x)/\delta_T}) + \Phi_E e^{-(x_e-x)/\delta_T}. \tag{3.3}$$

This solution is not valid west of the island in the latitude interval  $(y_s, y_n)$ . There instead,

$$\begin{aligned} \phi_t = & -h\{1 - e^{-(x_o-x_T-x)/\delta_T}\} \\ & + \Phi_{TI} e^{-(x_o-x_T-x)/\delta_T}, \quad x \leq x_o - x_T, \quad y_s \leq y \leq y_n. \end{aligned} \tag{3.4}$$

Note that the unknown constants  $\Phi_E$  and  $\Phi_{TI}$  produce purely meridional flows and are independent of  $y$ : that is, those flows are capable of crossing the intergyre boundary at  $y = 1/2$  where  $h = 0$ . In what follows we will talk about the direction of the flow, meaning the direction of the flow's vertical shear or, equivalently, the flow in the upper layer of this purely baroclinic solution.

To obtain the unknown constants,  $\Phi_E$  and  $\Phi_{TI}$ , we first apply the integral constraint (2.11). Now, the solution in the region east of the island needs to be supplemented by a boundary layer correction so as to satisfy the no-normal flow condition as well as the no-slip condition. These corrections to  $\phi_t$  will be of the same order as the "interior" geostrophic streamfunctions and, when integrated over the region affected by the boundary layer correction, will contribute a term on the order of the boundary layer thickness and hence, to lowest order, a small and negligible amount. Thus, for the application of (2.11) we may use the solutions in (3.3) and (3.4). The resulting calculation is straightforward and we obtain, using the fact that the integral of  $h$  over the interval (0, 1) is zero,

$$\begin{aligned} \Phi_E [1 - e^{-x_e/\delta_T} + (y_n - y_s)(e^{-x_e/\delta_T} - e^{-(x_e-x_o-x_T)/\delta_T})] \\ + \int_{y_s}^{y_n} h dy (1 - e^{-(x_o-x_T)/\delta_T} + e^{-x_e/\delta_T} - e^{-(x_e-x_o-x_T)/\delta_T}) \\ + \Phi_{TI} (1 - e^{-(x_o-x_T)/\delta_T}) = 0. \end{aligned} \tag{3.5}$$

It is important to note that the integral constraint couples the island constant  $\Phi_{TI}$  and the eastern boundary value of the baroclinic streamfunction  $\Phi_E$ . To complete the calculation of  $\Phi_E$  we therefore have to determine the island constant. To do so, we consider (2.8b) in the region east of the island between its northern and southern limits. The total baroclinic streamfunction can be written

$$\phi_t = \phi_{t\text{int}} + \phi_{t\text{bl}},$$

where the first term on the right-hand side is the streamfunction of (3.3) and the remaining term is the boundary layer correction in the narrow region just east of the island required to satisfy the no-normal flow and no-slip condition. The boundary layer term is of the same amplitude as the interior term since the two must combine to yield the island constant. Hence, as in the case for the determination of  $\Phi_E$ , the boundary layer term produces a negligible contribution to the area integral. We can therefore rewrite (2.8b) as

$$\frac{\partial}{\partial x} \phi_t = \frac{1}{\delta_T} \frac{\partial}{\partial x} \phi_{t\text{int}} - \delta_K \nabla^2 \phi_t + \delta_m^3 \nabla^4 \phi_t \tag{3.6}$$

for the purposes of the integral over the area east of the island. Carrying out that integral the diffusion term yields a contribution,  $\oint_C \delta_K \nabla \phi_t \cdot \mathbf{n} ds$ , where the contour stretches from the eastern boundary to the island, along its eastern edge, and back to the eastern boundary along which the contour closes. Along that contour the diffusion term is negligible except along the solid eastern and island boundaries where, because of the insulating (and no-slip) condition, it is zero. Hence the diffusion term makes no  $O(1)$  contribution to the area integral. The friction term in (3.6) is also negligible everywhere except along the eastern edge of the island where we anticipate a strong boundary layer. Thus, integrating (3.6) over the area east of the island yields

$$\begin{aligned} \int_{y_s}^{y_n} (\Phi_E - \Phi_{TI}) dy = \int_{y_s}^{y_n} [\Phi_E - \phi_{t\text{int}}(x_o + x_T y)] dy \\ + \delta_m^3 \int_{y_s}^{y_n} \nabla^2 v_t dy. \end{aligned} \tag{3.7}$$

Furthermore, integrating the momentum equations on a circuit encompassing the islands shows that, in the absence of an external stress, the integral of the fluid’s frictional force component tangent to the island must integrate to zero. Since that frictional force is dominated by the force in the boundary current on the eastern side of the island, it follows that the final term in (3.7) must integrate to zero (PPSH). It therefore follows that

$$\begin{aligned} \Phi_{\text{II}} &= \frac{1}{(y_n - y_s)} \int_{y_s}^{y_n} \phi_{\text{int}}(x_o + x_T y) dy \\ &= - \int_{y_s}^{y_n} \frac{h dy}{(y_n - y_s)} (1 - e^{-(x_e - x_o - x_T)/\delta_T}) \\ &\quad + \Phi_E e^{-(x_e - x_o - x_T)/\delta_T}. \end{aligned} \tag{3.8}$$

Combining (3.1), (3.5), and (3.8), a little algebra allows us to write

$$\begin{aligned} \Phi_E &= -\frac{1}{2\pi} (\cos(2\pi y_s) - \cos(2\pi y_n)) \\ &\quad \times \frac{[e^{-x_e/\delta_T} (e^{2x_T/\delta_T} - 1)]}{[1 - e^{-x_e/\delta_T} - (y_n - y_s) e^{-x_e/\delta_T} (e^{2x_T/\delta_T} - 1)]}. \end{aligned} \tag{3.9}$$

Note that, as expected, when  $x_T$ , the half width of the island, goes to zero, the eastern boundary displacement of the interface vanishes because there is then no net heating or cooling and, hence, no net cross-isopycnal flux forced by the target forcing term  $h$ . On the other hand, consider the case where the island lies in the southern subtropical gyre and where  $y_s < y_n$ . There will then be slightly more cooling than heating applied to the basin, implying more forced downwelling. To balance that there must be more upwelling provided by a positive boundary value of the interface height or a negative value of  $\Phi_E$ , as predicted by (3.9). With the eastern boundary value of the baroclinic streamfunction determined, the island constant can be written

$$\begin{aligned} \Phi_{\text{II}} &= \frac{1}{2\pi} \frac{(\cos(2\pi y_s) - \cos(2\pi y_n))}{(y_n - y_s)} (1 - e^{-(x_e - x_o - x_T)/\delta_T}) \\ &\quad + \Phi_E e^{-(x_e - x_o - x_T)/\delta_T}. \end{aligned} \tag{3.10}$$

Typically, the island will have a zonal width much smaller than the basin width, so the nondimensional parameter  $x_T$  will be small and, in turn,  $\Phi_E$  will be small. Nevertheless, both  $\Phi_E$  and  $\Phi_{\text{II}}$  will produce flows that depend on integral, rather than local, properties of the forcing. Hence, the eastern boundary constant will produce a large-scale overturning circulation, strictly meridional, and the sense of the overturning depends on the placement of the island. It is important to note that, in the region west of the island

and within its latitude range, the island constant produces a meridional flow whose sign is given by the sign of the island constant.

If the island were in the subpolar (tropical) gyre, it would yield a circulation that would be northward (southward) in the upper layer with a southward (northward) flow in the lower layer. Note that, if the island straddles the intergyre boundary at  $y = 1/2$  where  $h = 0$ , the meridional flow in the interior would be zero (aside from the contribution from the eastern boundary term), but the transport in the boundary layer on the island would be  $-\Phi_{\text{II}}$ . Normally this will be the dominant cross-gyre transport as long as the island is relatively slender.

In the region east of the island, the interior solution must be supplemented by a boundary layer correction. For the case where  $\delta_m \ll \delta_K$ , the boundary layer structure is as determined in Pedlosky and Spall (2005). There is an outer layer of width  $\delta_K$  in which the streamfunction is made to match the value on the island and an inner layer of (nondimensional) thickness  $\delta_h = \delta_m^{3/2}/\delta_K^{1/2}$  to satisfy the no-slip condition. Thus, in the region  $x_o + x_T \leq x \leq x_e$ ,  $y_s \leq y \leq y_n$ , the full solution for the streamfunction is

$$\begin{aligned} \phi_t &= [-h(1 - e^{-(x_e - x)/\delta_T}) + \Phi_E e^{-(x_e - x)/\delta_T}] \\ &\quad \times \left( 1 - e^{-(x_o + x_T - x)/\delta_K} + \frac{\delta_h}{\delta_K} e^{-(x_o + x_T - x)/\delta_h} \right) \\ &\quad + \Phi_{\text{II}} \left( e^{-(x_o + x_T - x)/\delta_K} - \frac{\delta_h}{\delta_K} e^{-(x_o + x_T - x)/\delta_h} \right) \end{aligned} \tag{3.11}$$

with a small error of order

$$\frac{\delta_h}{\delta_K} = \left( \frac{\delta_m}{\delta_K} \right)^{3/2} \ll 1.$$

Although the meridional flow in the region east of the island, including its boundary layer, is just the  $x$  derivative of (3.11) or

$$\begin{aligned} v_t &= \frac{1}{\delta_K} [-h(1 - e^{-(x_e - x)/\delta_T}) + \Phi_E e^{-(x_e - x)/\delta_T} - \Phi_{\text{II}}] \\ &\quad \times (e^{-(x - x_o - x_T)/\delta_K} - e^{-(x - x_o - x_T)/\delta_h}) \\ &\quad + \frac{1}{\delta_T} (h + \Phi_E) e^{-(x_e - x)/\delta_T} (1 - e^{-(x - x_o - x_T)/\delta_T}). \end{aligned} \tag{3.12}$$

Within the boundary layer the first term in (3.12) dominates, while in the interior it is the second term that is dominant.

Figure 3 shows the calculated, purely baroclinic streamfunction for an example in which the island lies entirely in the subtropical gyre. For this calculation  $\delta_T = 0.5$  and  $\delta_m$



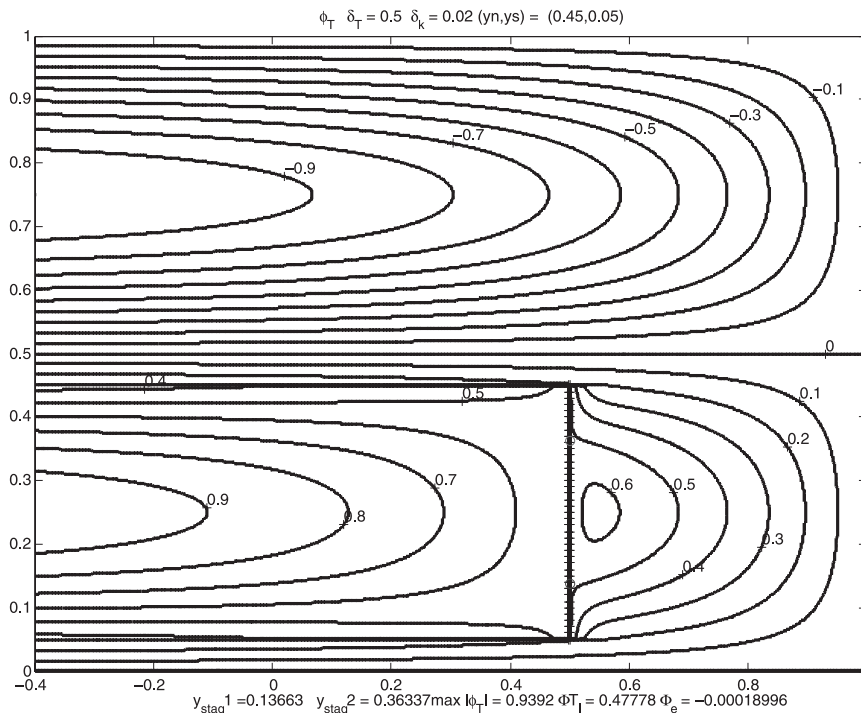


FIG. 3. The purely buoyancy-forced circulation. The island, indicated by the plus signs, is entirely in the subtropical gyre. Stagnation points are labeled by small circles. The discontinuities trailing westward from the island tips are unresolved zonal jets. Contour intervals are in units of one-tenth the absolute maximum value of the streamfunction. See text for details.

and  $\delta_K$  are 0.01 and 0.02, respectively. The northern tip of the island is at  $y_n = 0.45$  and the southern tip is at  $y_s = 0.05$ . Note the recirculation region east of the island whose meridional extent is defined by the two points where the interior streamfunction matches the island constant as in the barotropic study of PPSH. Because the island constant is positive in this case, (3.4) implies that near the intergyre boundary where  $h$  is small the meridional velocity in the region west of the island and slightly south of  $y = 1/2$  will be positive (i.e., northward rather than southward). That fluid will, however, remain in the subtropical gyre after flowing around the island's northern tip.

On the line  $y = y_n$ , the discontinuity in the streamfunction [i.e., the difference between (3.4) and (3.3) there] represents the mass flux carried eastward in a zonal jet, which we have not bothered to describe in detail as it is not the focus of this study. That transport however is easily obtained and is

$$T_{\text{east}} = e^{-(x_o - x_T - x)/\delta_T} \left[ h(y_n) - \frac{1}{(y_n - y_s)} \int_{y_s}^{y_n} h \, dy \right] \times (1 - e^{-(x_e - x_o - x_T)/\delta_T}) + \Phi_E e^{-(x_e - x)/\delta_T} (e^{2x_T} - 1). \tag{3.13}$$

The second term in (3.13) vanishes as  $x_T$  goes to zero and is normally very small. In the example shown in Fig. 3, the transport is eastward. Trailing from the southern tip of the island is a similar zonal jet flowing westward. If the detailed structure of the boundary layers along a zonal line separating the two interior regions is considered, it is easy to show that it leads to a parabolic boundary layer of the type discussed in Pedlosky (2001). The principal effect on the interior would be a contribution to the integral statement of mass conservation of order  $\delta_K^{1/2}$  and hence small.

Figure 4 shows the circulation pattern for the same parameters as in Fig. 3 except that the island extends northward into the subpolar gyre where  $h > 0$  ( $y_n = 0.65$ ,  $y_s = 0.05$ ). Fluid to the west of the island in the region  $y > 1/2$  flows northward. This is also true for some fluid in the subtropical gyre: that is, for the streamlines with values greater than zero but less than the island constant,  $\sim 0.32$ . The flow on these streamlines is into the cooling region of the subpolar gyre and then eastward in the northern tip jet of the island; it then circulates southward in the region east of the island and flows westward into the subtropical gyre west of the island to close their circuit in the western boundary layer (not shown). Fluid on streamlines with values less than zero

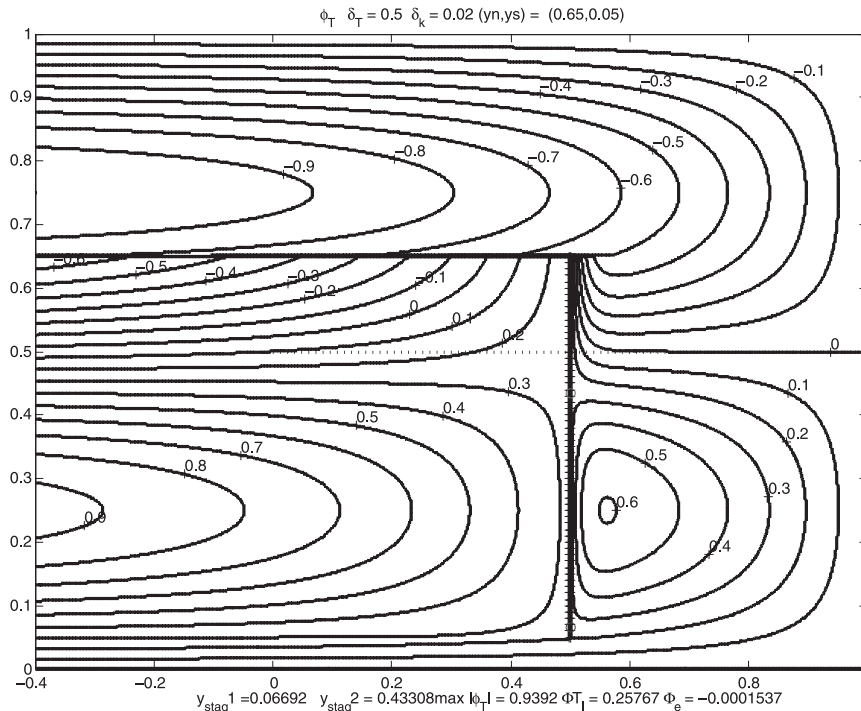


FIG. 4. As in Fig. 3, but now the northern tip of the island lies in the subpolar gyre while the major part of the island lies in the subtropical gyre. Fluid from the subpolar gyre is now drawn into the subtropical gyre.

flow around the island's north tip into the subpolar gyre and complete their circuit in a western boundary current flowing southward at the western boundary. Of course, fluid in the lower layer flows in the opposite direction. The flow across the intergyre boundary to the east of the island occurs in the island's boundary current. Figure 5a shows the meridional flow in the boundary current on the eastern side of the island at the intergyre boundary at  $y = \frac{1}{2}$ . It is flowing southward and loses flux to the subtropical gyre as it progresses. At  $y = 0.415$  a stagnation point occurs on the boundary (one of two), and the flow reverses direction and flows northward. This is in the region of the recirculation, indicated by the streamline with the value 0.4. The meridional flow across the basin to the east of the island is shown in Fig. 5b, where the northward boundary layer flow gradually merges into the interior southward velocity. Note the intensification of that southward velocity to the east as the interior, baroclinic solution decays westward due to the vertical mixing term. The decay occurs on the scale  $\delta_T$ , that is, the decay scale of the baroclinic Rossby wave.

The direction of the flow across the gyre boundary owing to the presence of the island depends only on the sign of the island constant. From (3.10) and the fact that the eastern boundary value of the streamfunction is small for slender islands, it is clear that the sign of the

island constant is determined largely by the sign of the interior streamfunction to the east of the island averaged over the length of the island. Hence, if the island lies mainly in the subtropical gyre, the island constant will be positive and the flow will be from the subpolar to subtropical gyre (in the upper layer). The intergyre flow is in the direction of the gyre containing the greater meridional extent of the island. If we think of the island as a model of a ridge system extending into the upper layer and if that ridge lies mainly in the subpolar gyre, this would produce a northward flow in the upper layer from the subtropical to subpolar gyre and a reverse flow at depth. If the island exactly straddles the gyre boundary, the island constant will be zero and there will be no cross-gyre flow. Of course, more complex distributions of forcing can alter this simple rule, but the principal remains the same.

#### 4. The wind-driven circulation

We consider now the circulation driven by a wind stress that produces the Ekman pumping in (2.8a). We thus set  $h = 0$  and choose

$$\Phi = \frac{f_o W_e L}{\beta_* H}.$$

The Ekman pumping forces both a barotropic and baroclinic response. The solutions, in all regions excluding the boundary layers and the region to the west of the island within its latitude extent, are

$$\begin{aligned} \phi_b &= -w_e(x_e - x); \\ \phi_t &= -\delta_T \frac{H}{H_1} w_e (1 - e^{-(x_e - x)/\delta_T}) + \Phi_E e^{-(x_e - x)/\delta_T}, \end{aligned} \tag{4.1a,b}$$

where, once again,  $\Phi_E$  is the value of the baroclinic streamfunction on the eastern boundary. The barotropic streamfunction, with no loss of generality, can be taken to be zero there. As before, the interface displacement from rest is, in our nondimensional units, given by  $\eta - \phi_t$ . In the region west of the island and excluding the western boundary layer,

$$\begin{aligned} \phi_b &= -w_e(x_o - x_T - x) + \Phi_{BI}; \\ \phi_t &= -w_e \delta_T \frac{H}{H_1} (1 - e^{-(x_o - x_T - x)/\delta_T}) + \Phi_{TI} e^{-(x_o - x_T - x)/\delta_T}, \end{aligned} \tag{4.2a,b}$$

where  $\Phi_{BI}$  and  $\Phi_{TI}$  are the baroclinic and barotropic components of the island constant; that is, the vertical structure of the island constant will determine the vertical structure of the flow transport around the island. So, we now have three constants that need to be determined,  $\Phi_E$ ,  $\Phi_{BI}$ , and  $\Phi_{TI}$ . In our example,  $w_e = -\sin 2\pi y$ .

To determine  $\Phi_E$  we once again appeal to mass conservation. In the absence of buoyancy forcing, the constraint (2.11) is just that the area integral of the baroclinic streamfunction over the area of the basin, excluding the island, must vanish. Essentially repeating the analysis that leads to (3.9), we now have

$$\begin{aligned} \Phi_E &= -\frac{H}{H_1} \int_{y_s}^{y_n} w_e dy \frac{[2x_T e^{-(x_o - x_T)/\delta_T} - \delta_T e^{-x_e/\delta_T} (e^{2x_T/\delta_T} - 1)]}{1 - e^{-x_e/\delta_T} - (y_n - y_s) e^{-x_e/\delta_T} (e^{2x_T/\delta_T} - 1)} \\ &= \frac{H}{H_1} \{ \cos(2\pi y_s) - \cos(2\pi y_n) \} \frac{[2x_T e^{-(x_o - x_T)/\delta_T} - \delta_T e^{-x_e/\delta_T} (e^{2x_T/\delta_T} - 1)]}{[1 - e^{-x_e/\delta_T} - (y_n - y_s) e^{-x_e/\delta_T} (e^{2x_T/\delta_T} - 1)]}, \end{aligned} \tag{4.3}$$

recalling that  $w_e = -\sin(2\pi y)$  in our scaled units. The derivation of (4.3) requires that we have first determined a relation between  $\Phi_E$  and  $\Phi_{TI}$ . That is obtained as in the derivation leading to (3.10) with the principal difference that we can no longer argue that the integral of the friction force on the island's eastern boundary is zero. Instead, an integral of the tangent

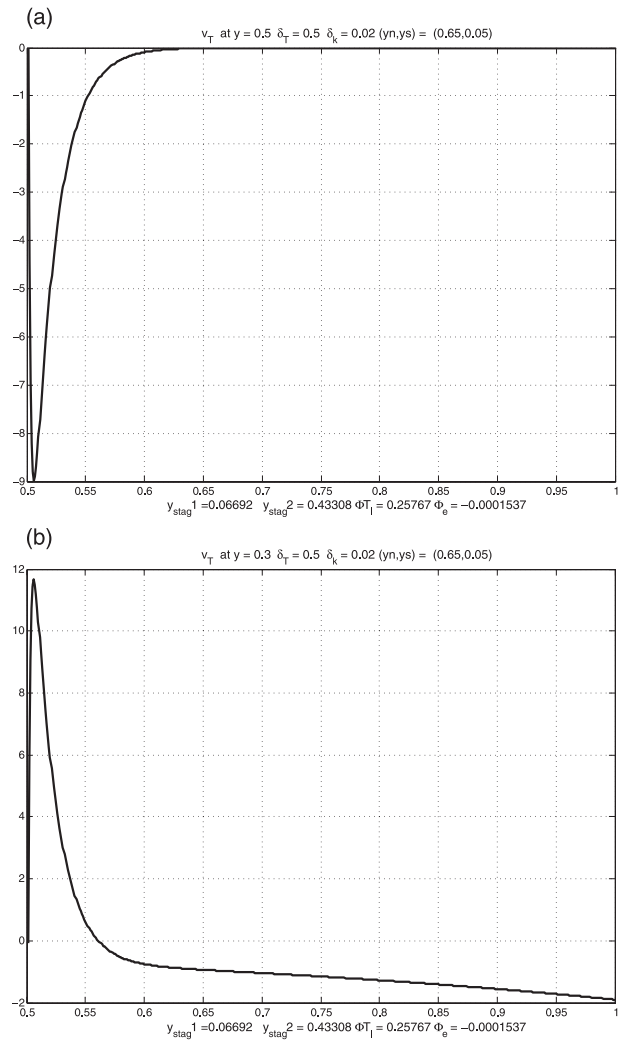


FIG. 5. (a) The meridional flow in the boundary layer on the eastern side of the island as a function of longitude at the latitude,  $y = 0.5$ , of the intergyre boundary. (b) As in (a) but at  $y = 0.3$  where we note the flow in the boundary layer is northward.

component of the momentum equation around the island shows that the frictional force when integrated around the island must equal the circulation of the wind stress around the island, and that can be directly related to the area integral of  $w_e$  over the area of the island. These manipulations can be shown to lead to the relation



$$\begin{aligned} \Phi_{\text{TI}} &= \frac{\delta_T}{2\pi(y_n - y_s)} \frac{H}{H_1} (\cos 2\pi y_s - \cos 2\pi y_n) \\ &\quad \times (1 - e^{-(x_e - x_o - x_T)/\delta_T} + 2x_T) + \Phi_E e^{-(x_e - x_o - x_T)/\delta_T}, \end{aligned} \tag{4.4}$$

which was used to derive (4.3). The barotropic island constant is derived as in PPSH and can be shown to be

$$\Phi_{\text{BI}} = \frac{(\cos 2\pi y_s - \cos 2\pi y_n)}{2\pi(y_n - y_s)} [x_e - (x_o - x_T)]. \tag{4.5}$$

We need to consider the boundary layer structure in the region to the east of the island. For the barotropic component of the flow, the only possible (linear) balance is a Munk boundary layer with thickness  $\delta_m$ ; in the region  $x_o + x_T \leq x \leq x_e, y_s \leq y \leq y_n$ , the solution for the barotropic streamfunction is

$$\begin{aligned} \phi_b &= -w_e(x_e - x)(1 - M(\xi)) + \Phi_{\text{BI}}M(\xi), \\ \xi &= (x - x_o - x_T)/\delta_m, \end{aligned}$$

and

$$M(\xi) = e^{-\xi/2} \left( \cos\left(\frac{\sqrt{3}}{2}\xi\right) + \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}\xi\right) \right) \tag{4.6a,b,c}$$

On the other hand, the nature of the baroclinic boundary layer solution depends, as in the previous section, on the relative size of  $\delta_m$  and  $\delta_K$ . As before, we choose the parameter regime where  $\delta_K \gg \delta_m$  so that, in this same region east of the island, the baroclinic streamfunction is

$$\begin{aligned} \phi_t &= \left[ -\delta_T \frac{H}{H_1} w_e (1 - e^{-(x_e - x)/\delta_T}) + \Phi_E e^{-(x_e - x)/\delta_T} \right] \\ &\quad \times (1 - K(x)) + \Phi_{\text{TI}}K(x); \\ K(x) &= e^{-(x - x_o - x_T)/\delta_K} - \frac{\delta_h}{\delta_K} e^{-(x - x_o - x_T)/\delta_h}, \end{aligned} \tag{4.7a,b}$$

where the boundary layer scales are defined as in (3.11). It is important to note that stagnation points on the eastern side of the island occur where the interior streamfunctions are equal to the respective island constants. As long as the nondimensional island thickness  $x_T$  is small, these points coincide for both the barotropic and baroclinic streamfunctions and occur at the value of  $y$  where

$$w_e(y) = \frac{1}{(y_n - y_s)} \int_{y_s}^{y_n} w_e(y) dy. \tag{4.8}$$

The ratio  $R$  of the baroclinic to the barotropic streamfunction is of the order

$$R = \frac{H \delta_T (1 - e^{-(x_e - x)/\delta_T})}{H_1 (x_e - x)}. \tag{4.9}$$

For large values of  $\delta_T$ , the ratio tends to  $H/H_1$ . This implies that the geostrophic streamfunction in the lower layer,  $\phi_2 = \phi_b - H^{-1}H_1\phi_t$ , goes to zero in that limit. That is the limit in which a baroclinic Rossby wave succeeds in crossing the basin undamped by vertical mixing. As is well known, in the absence of that vertical coupling the quasigeostrophic wind-driven circulation enters an equilibrium in which the wind-driven flow is contained entirely in the upper layer. On the other hand, if  $\delta_T$  is very small,  $R$  goes to zero and the circulation is largely barotropic except near the eastern boundary over distances  $x_e - x \ll \delta_T$ . Since the island constants are averages of the interior streamfunctions over the length of the island, their ratio will follow the same pattern as a function of  $\delta_T$ .

Figure 6 shows the result of the calculation of the wind-driven circulation around the same island as in section 3: that is, an island stretching from the subtropical gyre in to the subpolar gyre for the same parameter values. As in the buoyancy-driven circulation, a recirculation exists to the east of the island and now, external to the recirculation region, there are streamlines that have originated in the subpolar gyre, north of  $y = 1/2$ , that flow around the northern tip of the island and flow into the subtropical gyre and exit the eastern subbasin in a jet to the south of the island. The contours are labeled in units of 0.1 of the maximum value of the streamfunction. In the upper layer shown in Fig. 6a, the maximum is 1.432. Figure 6b shows the flow in the lower layer. The tight recirculation in near the northern tip of the island is an artifact of the Munk layer's oscillatory behavior. The maximum value of the lower-layer streamfunction is 0.568 so that the flow is much stronger in the upper layer. On the other hand, if  $\delta_T$  were much smaller (i.e., either because of weaker stratification or a greater magnitude of the decay rate  $\gamma$ ), the response would more closely resemble the barotropic streamfunction, which is, recall, independent of  $\delta_T$  and is shown in Fig. 6c. The baroclinic component is shown in Fig. 6d and resembles the response to purely baroclinic forcing.

Again, it is important to emphasize that the intergyre exchange is entirely due to the presence of the island, and the direction of that exchange is determined by the sign of the island constant, which in turn depends on whether the major part of the island is in the subpolar gyre or subtropical gyre. In the former case, the island constant would be negative and the cross-gyre flow would be from the subtropical to subpolar gyre. If instead, as shown in the example of Fig. 6, the island is

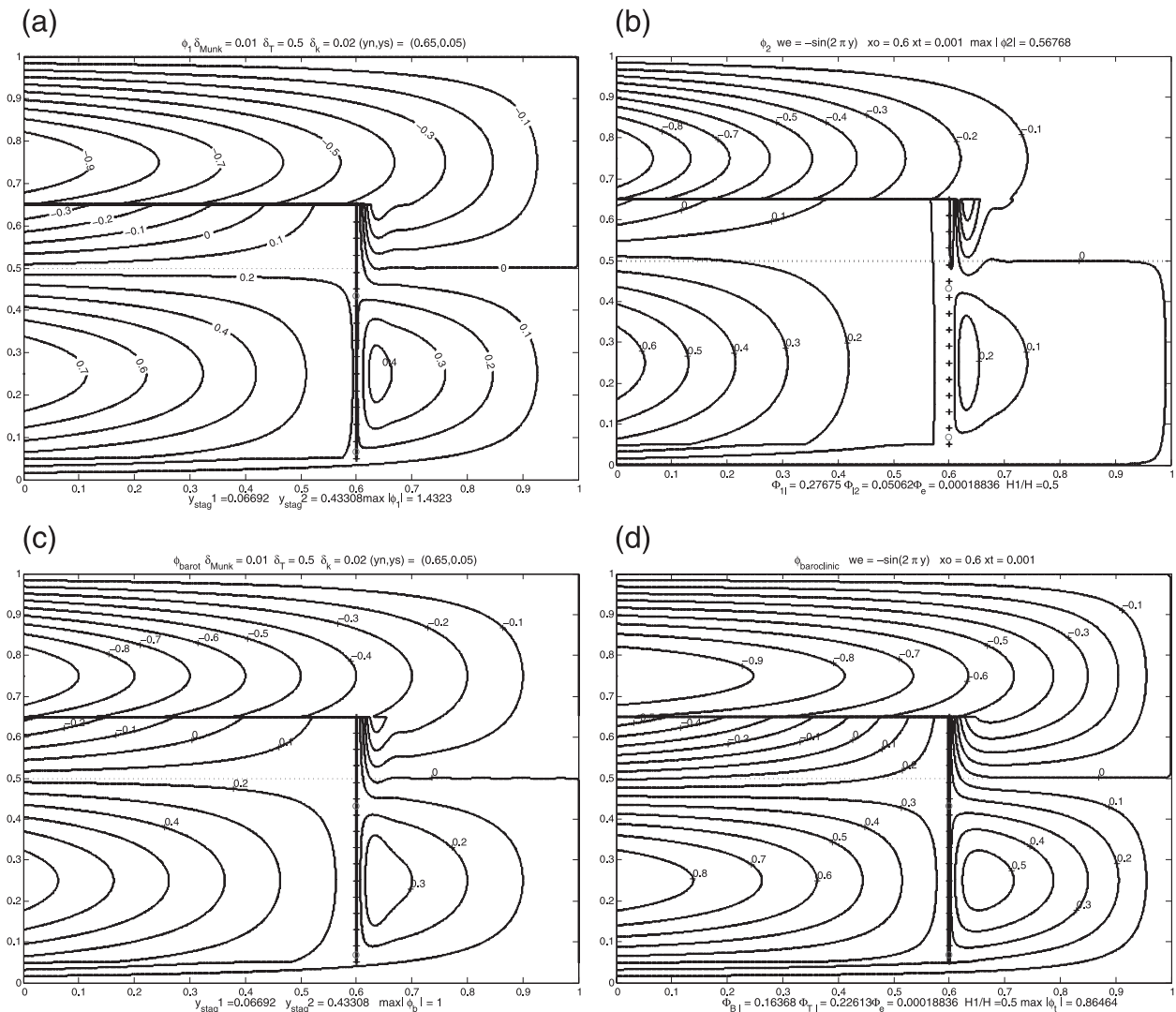


FIG. 6. The wind-driven circulation in the (a) upper and (b) lower layer for the nondimensional Ekman pumping,  $-\sin 2\pi y$ , for the same parameters as in Fig. 3 and the (c) barotropic and (d) baroclinic components of the circulation. See text for details.

predominantly in the subtropical gyre, the sign and the direction of the cross-gyre flow is reversed. Figure 7 shows the flow in the boundary layer on the eastern side of the island at the intergyre boundary for the example of Fig. 6. The left panel shows the barotropic (solid line) and baroclinic (dashed) meridional velocity. The right panel shows the velocity in the upper (solid) and lower (dashed) layers. Note the damped oscillation of the velocity in the barotropic component of the velocity.

## 5. Discussion

The addition of stratification, in the form of a two-layer model, introduces new features to the dynamics of

flow around a planetary-scale island. The model described in the present study, although quasigeostrophic and linear, demonstrates these important new elements.

In considering the central issue of the flow circulating from one subbasin to another, for example, from the western to the eastern, the island constants (for now there are two, one for each layer) determine the vertical structure of that interbasin exchange. For a purely buoyancy-driven circulation, linear dynamics necessitates an equal and opposite transport in the two layers so that the interbasin flow driven by buoyancy forcing is a two-way flow analogous to an estuarine exchange flow. In the model studied here, the cooled subpolar gyre circulates cyclonically in the upper layer and anticyclonically in the lower layer. The directions are reversed in the

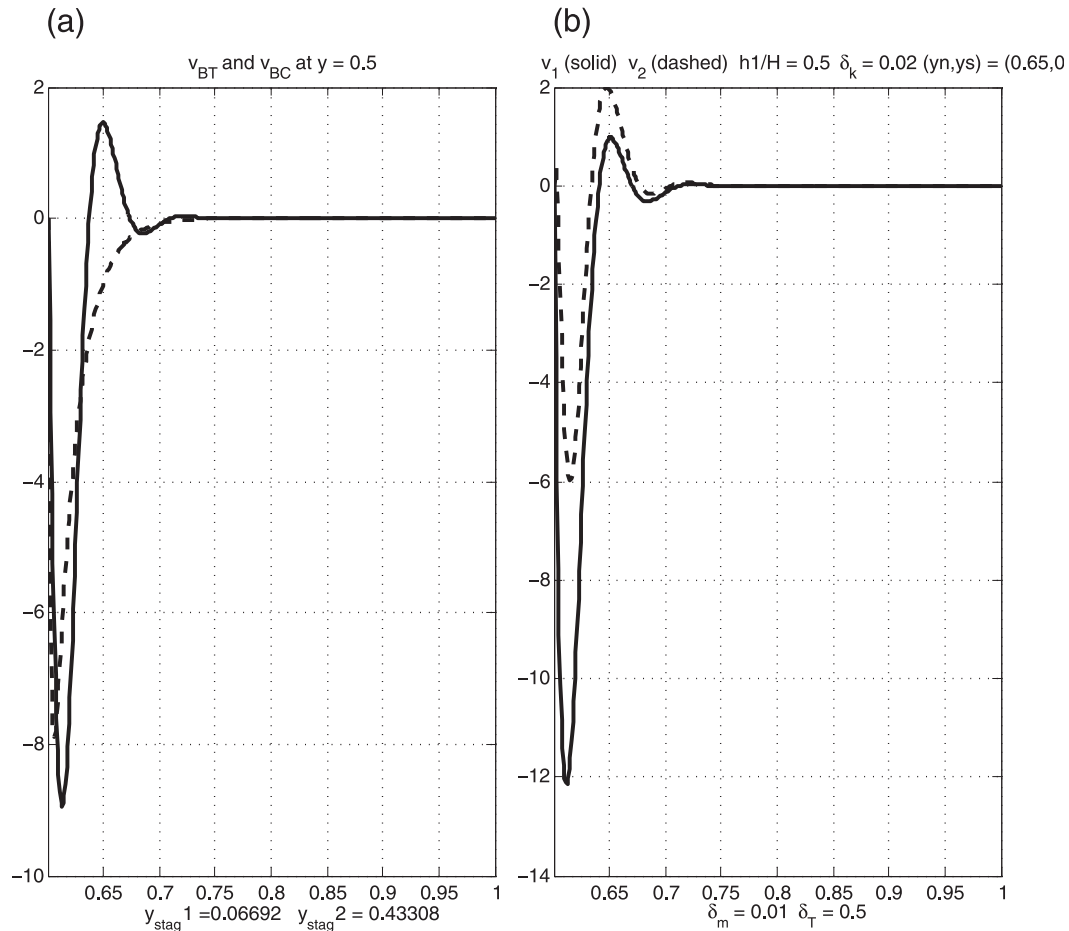


FIG. 7. The meridional flow in the region east of the island illustrating the flow in the boundary layer at the intergyre boundary,  $y = \frac{1}{2}$ : (a) the barotropic (solid) and baroclinic (dashed line) components of the velocity and the (b) upper-layer (solid) and lower-layer (dashed) velocity.

subtropical gyre. In the wind-driven circulation in which the subpolar gyre is defined as the region of positive Ekman pumping, the flow is cyclonic in both layers, although weaker in the lower layer. In the subtropical gyre the directions are again reversed. This holds true also for the interbasin flows. For the combined problem it follows that the direction of the flow in the lower layer depends on the relative size of the wind to buoyancy forcing. To obtain the more oceanographically realistic pattern of poleward interior flow in the subtropical gyre, such a model would have to assume that the wind forcing is dominant.

The boundary between the subpolar and subtropical gyres would naturally be defined by the latitude where the external forcing, wind or buoyancy, changes sign. However, when the island extends from a region of forcing of one sign into a region of opposite signed forcing, the effect of the island, on both the interior flow to the west of the island and to its east, becomes striking. As we

have seen in the case of buoyancy-driven flow, fluid west of the island in the subtropical gyre flows northward as a direct consequence of the westward propagation, by a damped long baroclinic Rossby wave, of the value of the island constant. That flow, while cooled in the subpolar gyre, circulates around the northern tip of the island and flows in a boundary current along the eastern side of the island back into the subtropical gyre where it becomes heated. It flows around a closed region of recirculation to the east of the island. The wind-driven problem exhibits the same intergyre exchange. In each case, the direction of the intergyre flow is completely determined by the sign of the island constant and that sign is determined by the relative contributions of the double-signed forcing of each gyre. Typically, the cross-gyre flow to the east of the island is directed into the gyre that has the greater length of the island contained in it.

This new mechanism for intergyre communication is distinct from the nonlinear fluxes caused by the inertial

penetration of strong currents from their gyre of origin or from the mechanism of topographic steering that distorts the effective gyre boundaries. In considering the dynamics of abyssal flows, it points to a simple way in which ridge systems, broken by fault zones, can lead to cross-gyre flow.

Naturally, it will be of interest to examine the nonlinear extension of the present problem as well as the effects of the addition of topographic elements of the island like a skirted topography around the island in the lower and possibly a portion of the upper layer. The effect of nonlinearity will distort the potential vorticity contours (Rhines and Young 1982) and can lead to fundamental changes in the circulation pattern, including additional possibilities for intergyre flow (Pedlosky 1984). In both of these extensions (nonlinearity and topography), the barotropic and baroclinic responses will no longer be separable.

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#### REFERENCES

- Godfrey, J. S., 1989: A Sverdrup model of the depth-integrated flow from the world ocean allowing for island circulations. *Geophys. Astrophys. Fluid Dyn.*, **45**, 89–112.
- Pedlosky, J., 1984: Cross-gyre ventilation of the subtropical gyre: An internal mode in the ventilated thermocline. *J. Phys. Oceanogr.*, **14**, 1438–1467.
- , 2001: Steady baroclinic flow through ridges with narrow gaps. *J. Phys. Oceanogr.*, **31**, 2418–2440.
- , and M. A. Spall, 2005: Boundary intensification of vertical velocity in a  $\beta$ -plane basin. *J. Phys. Oceanogr.*, **35**, 2487–2500.
- , L. J. Pratt, M. A. Spall, and K. Helfrich, 1997: Circulation around islands and ridges. *J. Mar. Res.*, **55**, 1199–1251.
- , R. Iacono, E. Napolitano, and K. Helfrich, 2009: The skirted island: The effect of topography on the flow around planetary scale islands. *J. Mar. Res.*, **67**, 435–478.
- Rhines, P. B., and W. R. Young, 1982: A theory of the wind-driven circulation. I. Mid-ocean gyres. *J. Mar. Res.*, **40** (Suppl.), 559–596.