GEOPHYSICAL RESEARCH LETTERS, VOL. 35, L19708, doi:10.1029/2008GL035400, 2008

A nonparametric test for change in variability using a proxy record with an application to ENSO

Andrew R. Solow[1] and Andrew R. Beet[1]

Received 18 July 2008; revised 1 September 2008; accepted 4 September 2008; published 9 October 2008.

[1] A common problem in climate science is determining whether the pattern of variability in a particular process or variable has changed over time. When the modern observational record is short, this problem can be addressed by comparing its variability to that of an historical proxy record. In doing so, it is important to recognize that the statistical properties of the modern and proxy records are different. Here, a nonparametric test for a change in variability in this situation is described that accounts for this difference. The method is illustrated by testing for a change in ENSO variability using a record of an ENSO index over the period 1871–2007 and oxygen isotope records extracted from corals at Palmyra Island that cover four different periods spanning the past millennium. The results are mixed. **Citation:** Solow, A. R., and A. R. Beet (2008), A nonparametric test for change in variability using a proxy record with an application to ENSO, *Geophys. Res. Lett.*, *35*, L19708, doi:10.1029/2008GL035400.

1. Introduction

[2] An important practical problem in climate science is determining whether the pattern of variability in a particular process or variable has changed over time. The ability to address this problem is sometimes limited by the shortness of the modern observational record in relation to the time scale over which the change may have occurred. In some cases, this limitation can be circumvented by extending the modern record back in time using a proxy. In the application described below, the variable of interest is an index of El Niño – Southern Oscillation (ENSO) for which the record extends back to 1871 and the proxy is an oxygen isotope measured in fossil coral extending back nearly 1000 years. As pointed out below, the statistical properties of a variable and its proxy are not the same and this must be taken into account in testing for a change in variability. The purpose of this paper is to describe such a test and to illustrate its use through an application to some data relating to ENSO variability. The test, which is based on the periodograms of the modern and proxy records, is nonparametric in the sense that no particular time series model is assumed.

[3] Related work focusing on changes in ENSO variability in the modern record alone includes *Trenberth and Hoar* [1996, 1997], *Rajagopalan et al.* [1997], *Solow and Huppert* [2003], and *Solow* [2006]. *Cobb et al.* [2003] and J. N. Dorin et al. (Detecting ENSO period change in a proxy record spanning the last millennium, submitted to *Journal of Climate*, 2008) extended the period of analysis through oxygen isotope records in corals. *Timmermann et al.* [1999] discussed the potential effect of global warming on ENSO variability. Results on the spectral analysis of time series are available from many texts including *Priestley* [1981] and *Percival and Walden* [1993] (see also *Diggle and Fisher* [1991] on the nonparametric comparison of spectral densities).

[4] The remainder of this paper is organized in the following way. The basic method is described in Section 2. In Section 3, an illustration aimed at testing for a change in ENSO variability is presented. Section 4 contains some concluding remarks.

2. Method

[5] The situation considered here is the following. Let X_t and Y_t be the values of the variable of interest and its proxy, respectively, at time t. The basic assumption is that:

$$Y_t \cong \beta_o + \beta X_t + \varepsilon_t \quad (1)$$

where β_o and β are unknown and ε_t is a serially uncorrelated error with mean 0 and unknown variance σ^2. This model underlies virtually all of the work on reconstructing climate variables from proxies. Let $X = (X_{t1}, X_{t2},\ldots,X_{tn})$ be the modern time series of length n of the variable of interest and let $Y = (Y_{s1}, Y_{s2},\ldots,Y_{sn})$ be the historical time series of the same length of the proxy. We assume that the modern and historical periods are sufficiently separated that X and Y are independent. The problem is to use these data to test for a change in the variability of the variable of interest.

[6] Details of the statistical results cited in this paragraph are available in most texts on spectral analysis, including those by *Priestley* [1981] and *Percival and Walden* [1993]. Let $f_X(\omega)$ and $f_Y(\omega)$ be the unknown spectral densities of X and Y, respectively. It follows from (1) that, under the null hypothesis H_o of no change in variability:

$$f_Y(\omega) = \beta^2 f_X(\omega) + \sigma^2 \quad (2)$$

for all ω. To our knowledge, no general test of the model in (2) has been proposed. The test of *Diggle and Fisher* [1991] mentioned earlier assumes that $\sigma^2 = 0$. Let $I_X(\omega_j)$ and $I_Y(\omega_j)$, $j = 1, 2, \ldots, k$, be the periodograms of X and Y, respectively, where k is the integer part of $n/2$ and $\omega_j = 2\pi j/n$ is a Fourier frequency. Under H_o, $I_X(\omega_j)$ and $I_Y(\omega_j)$ are approximately exponentially distributed with means $f_X(\omega_j)$ and $\beta^2 f_X(\omega_j) + \sigma^2$, respectively. Moreover, $I_X(\omega_j)$ and $I_X(\omega_k)$ are approximately independent for $j \neq k$ and similarly for $I_Y(\omega_j)$ and $I_Y(\omega_k)$. We will test H_o by testing the goodness

[1]Woods Hole Oceanographic Institution, Woods Hole, Massachusetts, USA.

0094-8276/08/2008GL035400$05.00

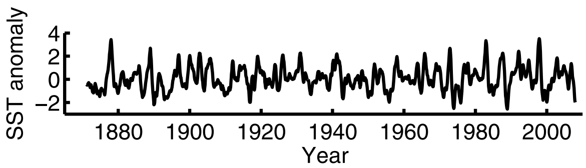

Figure 1. De-seasonalized and smoothed NINO3.4 record, 1871–2007.

of fit of this model for the periodograms. In doing so, no assumption is made about the spectral density $f_X(\omega)$. As a result, the model includes a total of $k + 2$ parameters: β^2, σ^2, and $f_X(\omega_j)$, $j = 1,2,\ldots,k$. The consequences of this for testing goodness of fit are discussed below.

[7] To economize on notation, let $f_j = f_X(\omega_j)$, $I_{Xj} = I_X(\omega_j)$, and $I_{Yj} = I_Y(\omega_j)$. The log likelihood under H_o is given by [*McCullagh and Nelder*, 1989, p. 290]:

$$\log L = -\sum_{j=1}^{k}\left(\log f_j + \frac{I_{Xj}}{f_j} + \log\left(\beta^2 f_j + \sigma^2\right) + \frac{I_{Yj}}{\beta^2 f_j + \sigma^2}\right) \quad (3)$$

The model is fit by maximizing this log likelihood over the unknown parameters. Computational efficiency is gained by noting that, for fixed β^2 and σ^2, the maximum likelihood (ML) estimate of each f_j is the root of the cubic equation:

$$-2\sigma^4 f_j^3 - \left(3\sigma^2\beta - \sigma^4 I_{Xj} - \sigma^2 I_{Yj}\right)f_j^2 - \left(\beta^2 - 2\sigma^2\beta I_{Xj}\right)f_j + \beta^2 I_{Xj} = 0 \quad (4)$$

Let $\hat{\beta}^2$, $\hat{\sigma}^2$, and $\hat{f}_j$, $j = 1,2,\ldots,k$, be the ML estimates of the corresponding parameters. The goodness of the fitted model is measured by the deviance [*McCullagh and Nelder*, 1989, p. 290]:

$$D = -2\sum_{j=1}^{k}\left(\log\frac{I_{Xj}}{\hat{f}_j} - \frac{I_{Xj} - \hat{f}_j}{\hat{f}_j} + \log\frac{I_{Yj}}{\hat{\beta}^2\hat{f}_j + \hat{\sigma}^2} - \frac{I_{Yj} - \left(\hat{\beta}^2\hat{f}_j + \hat{\sigma}^2\right)}{\hat{\beta}^2\hat{f}_j + \hat{\sigma}^2}\right) \quad (5)$$

[8] Because the number of parameters increases with the number of observations, the standard distributional results for D do not hold. The significance of D can instead be assessed through the following parametric bootstrap. For $j = 1, 2, \ldots, k$, simulate periodogram values I_{Xj}* and I_{Yj}* as exponential random variables with means $\hat{f}_j$ and $\hat{\beta}^2\hat{f}_j + \hat{\sigma}^2$, respectively. Re-fit the model in (2) to the simulated periodogram values and find the corresponding deviance D*. Repeat the procedure a large number of times and estimate the significance level by the proportion of values of D* that exceed D.

[9] We conducted a small simulation experiment to assess the performance of this test. In qualitative terms, the results, which are not reported here, show that the test maintains its nominal significance level and that it has good power (~0.8) provided n is not too small (~100) and the departure from H_o is not too slight. Interestingly, unlike the test based on the deviance, a test based on the maximized log likelihood is very conservative.

[10] An advantage of this method is that it can be applied to a subset of the Fourier frequencies that are of particular interest. This is illustrated in the following section where we

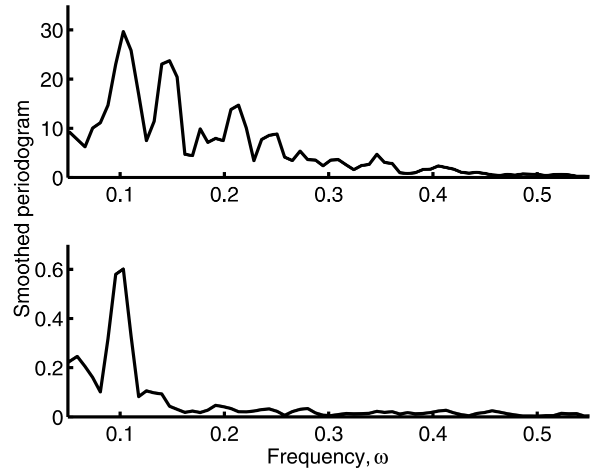

Figure 3. Smoothed periodograms for (top) NINO3.4 and (bottom) Palmyra Island δ^{18}O for the period 1149–1220.

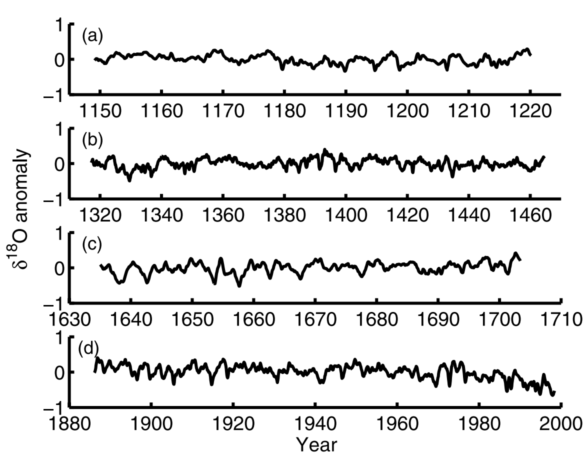

Figure 2. De-seasonalized and smoothed δ^{18}O records from Palmyra Island. (a) 1149–1220, (b) 1317–1464, (c) 1635–1703, and (d) 1886–1998.

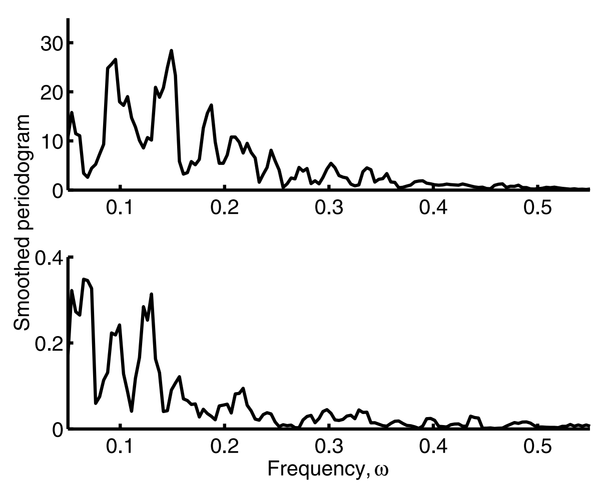

Figure 4. Smoothed periodograms for (top) NINO3.4 and (bottom) Palmyra Island δ^{18}O for the period 1317–1464.

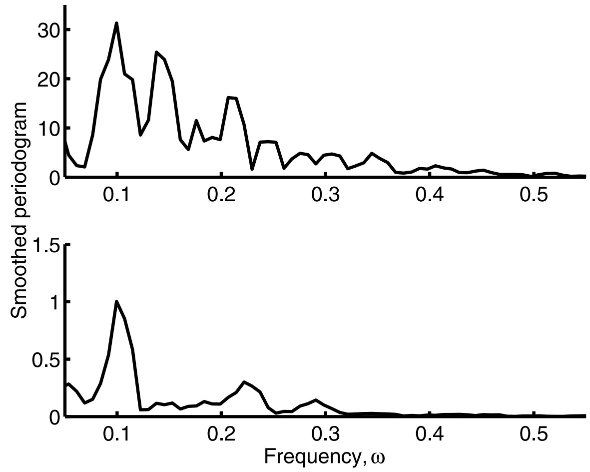

Figure 5. Smoothed periodograms for (top) NINO3.4 and (bottom) Palmyra Island δ^{18}O for the period 1635–1703.

test for a change in ENSO variability over a restricted set of frequencies.

3. Application

[11] Figure 1 shows the monthly record of the NINO3.4 ENSO index over the period 1871–2007. Others have de-seasonalized the original record by subtracting monthly averages and smoothed it by a 5-point moving average. Figure 2 shows monthly records of coral δ^{18}O from Palmyra Island for four separate periods: 1149–1220, 1317–1464, 1635–1703, and 1886–1998 [*Cobb et al.*, 2003]. To maintain comparability, these records have been de-seasonalized and smoothed in the same way as the NINO3.4 record. In this section, these data are used to test for changes in ENSO variability (as reflected in the NINO3.4 record) over the frequency range 0.05 to 0.53 (corresponding to periods between 1 and 10 years). This choice of frequency range is somewhat arbitrary, but is generally consistent with the range of primary interest for ENSO. The recent δ^{18}O record is included in the analysis solely as a check on the method.

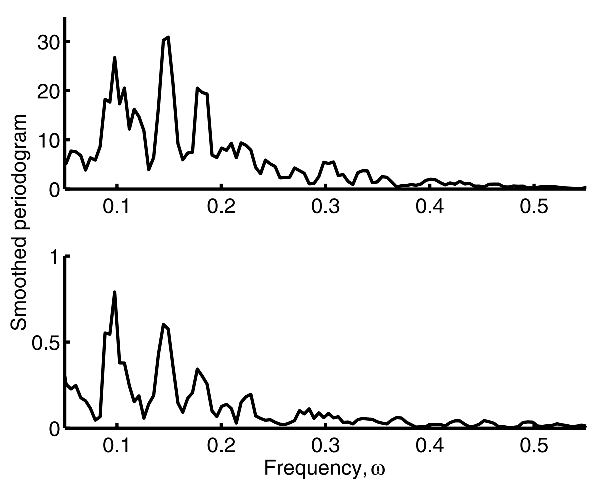

Figure 6. Smoothed periodograms for (top) NINO3.4 and (bottom) Palmyra Island δ^{18}O for the period 1886–1998.

Table 1. Number of Frequencies and Estimated Significance Levels in Testing for Variability Changes Between the Modern NINO3.4 Record and Different Periods of the δ^{18}O Record From Palmyra Island

Period	Number of Frequencies	Significance Level
1149–1200	67	0.052
1317–1464	127	0.116
1635–1703	65	0.176
1886–1998	10	0.984

Indeed, in the applications we envision, no modern version of the proxy record is available.

[12] The records in Figures 1 and 2 vary in length. In the analysis involving the early δ^{18}O records, the most recent NINO3.4 record of equal length was used in the analysis. In the check involving the most recent δ^{18}O record, the overlapping part of the NINO3.4 record was used. Because the records vary in length, the specific frequencies involved in these tests are different. Figures 3–6 show the pair of NINO3.4 and δ^{18}O periodograms used in the test for each period. To facilitate comparison, each periodogram has been smoothed by a simple 3-point moving average.

[13] The results are presented in Table 1. In each case, the number of Fourier frequencies used in the test and the significance level estimated from 10000 bootstrap samples are reported. By conventional standards, only the result for the earliest δ^{18}O record can be viewed as significant. For this period, Figure 3 shows that that both series exhibit a strong spectral peak at around $\omega = 0.10$, but that NINO3.4 also has a strong peak around $\omega = 0.15$ that is not present in the smoothed periodogram for δ^{18}O. The extremely non-significant result for the modern δ^{18}O record is due in part to its overlap with the NINO3.4 record, which results in their non-independence.

4. Discussion

[14] The purpose of this paper has been to describe and apply a nonparametric test for a change in variability between a modern climate record and an historical proxy. From a statistical point of view, the form of the spectral model in (2), which is a result of the proxy relationship in (1), makes it impossible to eliminate the nuisance parameters f_j, $j = 1,2,\ldots,k$, necessitating the brute force approach described here. A central assumption of this test is that the proxy is at least approximately linearly related to the variable in the modern record. As noted, this assumption underlies much of the existing work on climate proxies. In principle, the method could be extended to allow a nonlinear relationship of known form. Another limitation of this approach is that the records must have a common length – or more precisely common Fourier frequencies. We are exploring ways in which this requirement can be relaxed, but this will necessarily involve additional assumptions (e.g., that the underlying spectral densities are smooth).

[15] Turning to the results of the previous section, the situation seems mixed. Only the earliest δ^{18}O record exhibits variability that would be considered significantly different from the modern NINO3.4 record. It is notable that the significance levels in Table 1 decline with the age of the δ^{18}O record. This is consistent with a slow change in ENSO

variability. However, it is also consistent with a slow degradation of the isotope record.

[16] **Acknowledgments.** The very helpful comments of two anonymous reviewers are acknowledged with gratitude. Partial funding for this work was provided by NSF Grant DEB-0515639.

References

Cobb, K. M., C. D. Charles, H. Cheng, and R. L. Edwards (2003), El Niño/Southern Oscillation and tropical Pacific climate during the last millennium, *Nature*, *424*, 271–276.

Diggle, P. J., and N. I. Fisher (1991), Nonparametric comparison of cumulative periodograms, *Appl. Stat.*, *40*, 423–434.

McCullagh, P., and J. Nelder (1989), *Generalized Linear Models*, Chapman and Hall, New York.

Percival, D. B., and A. T. Walden (1993), *Spectral Analysis for Physical Applications*, Cambridge Univ. Press, Cambridge, U. K.

Priestley, M. B. (1981), *Spectral Analysis and Time Series*, Academic, London.

Rajagopalan, B., U. Lall, and M. A. Cane (1997), Anomalous ENSO occurrences: An alternate view, *J. Clim.*, *10*, 2351–2357.

Solow, A. R. (2006), An ENSO shift revisited, *Geophys. Res. Lett.*, *33*, L22602, doi:10.1029/2006GL027632.

Solow, A. R., and A. Huppert (2003), On non-stationarity of ENSO, *Geophys. Res. Lett.*, *30*(17), 1910, doi:10.1029/2003GL018225.

Timmermann, A., J. Oberhuber, A. Bacher, M. Esch, M. Latif, and E. Roeckner (1999), Increased El Niño frequency in a climate model forced by future greenhouse warming, *Nature*, *398*, 694–697.

Trenberth, K. E., and T. J. Hoar (1996), The 1990–1995 El Niño-Southern Oscillation event: Longest on record, *Geophys. Res. Lett.*, *23*, 57–60.

Trenberth, K. E., and T. J. Hoar (1997), El Niño and climate change, *Geophys. Res. Lett.*, *24*, 3057–3060.

A. R. Beet and A. R. Solow, Woods Hole Oceanographic Institution, Woods Hole, MA 02543, USA. (asolow@whoi.edu)