On the influence of horizontal temperature stratification of seawater on the range of underwater sound signals

Original title:
Über den Einfluß horizontaler Temperaturschichtung des Seewassers auf die Reichweite von Unterwasserschallsignalen.

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Physik. Zeitschr., 20. Jahrgang, No. 17, pp. 385–389
1. September 1919

A Note

The present re-translation of Lichte's original paper has been performed under the impression that an earlier, often-cited translation by A.F. Wittenborn had been lost. This spurred a renewed search initiated by Ralph Stephen (WHOI), Carl Wunsch (MIT) and Allan Pierce (BU), and the Wittenborn translation has since been recovered by Bill Carey (BU).

The MBL/WHOI Library is now making the Witternborn translation available through WHOAS (Woods Hole Open Access Server):

`http://hdl.handle.net/1912/3021`

and is planning to catalog a paper copy of this work and house it in the WHOI Miscellaneous series at the Data Library & Archives.

Resumed navigation from and to German sea ports has raised the importance of delineating mine-free corridors. Underwater sound signals are among the most important tools to this end. Consequently, learning more about sound propagation in water is of considerable interest.

Water is commonly regarded as significantly better suited for the transmission of sound signals than air because it is perceived to be much more homogeneous. However, this is not the case. To the contrary, for various reasons water is acoustically inhomogeneous along different horizontal layers. As a consequence, sound rays incur a deviation from a straight path, i.e. they are refracted. In the following, the causes of this refraction shall be investigated. For the sake of clarity, numerical examples will be given for some of the cases considered.

The speed of sound (v) in a given media depends on the density (ρ) and the compressibility (κ) of the given media according to

$$v_0 = \frac{1}{\sqrt{\kappa_0 \rho_0}} \qquad (1)$$

The compressibility of water has been determined to be 0.000049 Atm^{-1}, or, in C.G.S. units, equal to [] [1]

$$\frac{980.66 \cdot 1033.3}{0.000049} \; \frac{\mathrm{cm\ sec}^2}{\mathrm{g}}$$

[1] Kohlrausch, Lehrbuch der praktischen Physik, 2nd edition, page 708, Table 19a

Thus, the speed of sound is equal to

$$v_0 = \sqrt{\frac{1013300}{0.000049}}\ \text{cm sec}^{-1}$$
$$= 1439\ \text{m sec}^{-1}$$

The quantities which determine the speed of sound, compressibility and density, depend on several variables, the main of which are temperature, salinity, and pressure.

Our initial goal is to determine the influence of these variables, and to investigate the paths of sound rays in a heterogeneous layer of water. The most important influence is through temperature, and which is therefore considered first.

Let the water temperature decrease uniformly with depth from surface to bottom. We align the x-axis with the water surface (Fig. 1), and the y-axis perpendicular to it, downward. Considering the temperature to be constant within a very thin layer and the sound ray to be a straight line within that layer, we obtain, using the refraction law

$$\frac{v}{\sin\alpha} = \frac{v_o}{\sin\alpha_o} = \frac{v_o}{\cos\delta} \tag{2}$$

where v denotes the sound speed, and α the incidence angle in an arbitrary layer Δy. Values with suffix $_o$ refer to the water surface. δ is the angle between the ray and the horizontal axis. Assume the ray to originate from the center of the coordinate system. Then, according to Fig. 1,

$$\frac{\Delta x}{\Delta y} = \tan\alpha = \frac{\sin\alpha}{\sqrt{1-\sin^2\alpha}} = \frac{\frac{v}{v_o}\cos\delta}{\sqrt{1-\left(\frac{v}{v_o}\right)^2\cos^2\delta}}$$

Since

$$\frac{v}{v_0} = \sqrt{\frac{\kappa_0\rho_0}{\kappa\rho}}$$

where κ and ρ change with depth according to

$$\kappa = \kappa_0(1+\epsilon y) \tag{3}$$

and

$$\rho = \rho_0(1+\gamma y) \tag{4}$$

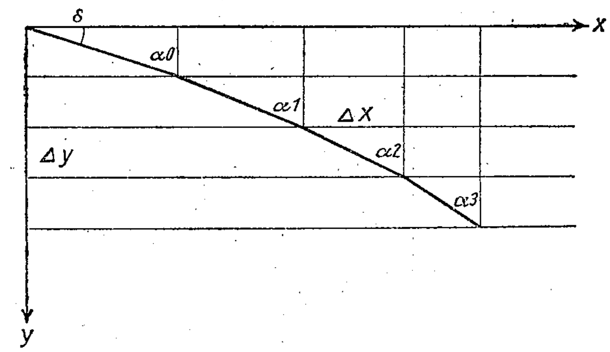
respectively, we obtain

$$\frac{\Delta x}{\Delta y} = \frac{\frac{\cos\delta}{\sqrt{(1+\epsilon\, y)(1+\gamma\, y)}}}{\sqrt{1-\frac{\cos^2\delta}{(1+\epsilon\, y)(1+\gamma y)}}}$$

from which we obtain through integration

$$x = \int\frac{\cos\delta}{\sqrt{\sin^2\delta+\epsilon\, y+\gamma\, y}}$$
$$= \frac{2\cos\delta}{(\epsilon+\gamma)}\cdot\sqrt{\sin^2\delta+\epsilon\, y+\gamma\, y}+C$$

Fig. 1.



The requirement of $y = 0$ for $x = 0$ determines the constant as

$$C = -\frac{2\sin\delta\cos\delta}{\epsilon+\gamma} = -\frac{\sin 2\delta}{\epsilon+\gamma}.$$

Then we obtain

$$\left(x+\frac{\sin 2\delta}{\epsilon+\gamma}\right)^2 = \frac{4\cos^2\delta\sin^2 delta}{(\epsilon+\gamma)^2}+\frac{4\cos^2\delta\cdot y}{\epsilon+\gamma}$$

from which follows

$$y = \frac{\epsilon+\gamma}{4\cos^2\delta}x^2 + \tan\delta\cdot x \tag{5}$$

Eqn. (5) is the equation for our sound ray. For small angles δ, i.e. for rays with an initially horizontal path, we have

$$y = \frac{\epsilon+\gamma}{4}\kappa^2 \tag{6}$$

Eqn. (5) is so far generally valid since no specific assumptions have been made with regard to ϵ and γ. However, if the change in compressibility and density with depth is due to a change in temperature, as we had initially assumed, then we obtain

$$\epsilon = \frac{1}{\kappa_0}\cdot\frac{\partial\kappa}{\partial T}\cdot\frac{\partial T}{\partial y} \tag{7}$$

and

$$\gamma = \frac{1}{\rho_0}\cdot\frac{\partial\rho}{\partial T}\cdot\frac{\partial T}{\partial y} \tag{8}$$

Now, it is [] [2]

$$\kappa = 49\cdot 10^{-6}$$
$$\frac{\partial\kappa}{\partial T} = -2\cdot 10^{-7}$$

thus

$$\epsilon = -4\cdot 10^{-}3\cdot\frac{\partial T}{\partial y}. \tag{9}$$

[2] Kohlrausch, loc. cit., p. 708, Tab. XIXa. Water reaches its minimum compressibility (equal to $42.02\cdot 10^{-6}$) at 62°. Within the limits occurring in practice, $\partial x/\partial T$ may be considered constant.

Furthermore, since $\rho_0 = 1$, and $\frac{\partial\rho}{\partial T}$ of order 10^{-4} [] [3] we have

$$\gamma = -10^{-4}\frac{\partial T}{\partial y}$$

Compared to ϵ we can thus neglect γ without incurring a noticeable error, i.e. the change in the speed of sound in water related to a change in temperature is due to a change in compressibility; the change in density with temperature effectively plays no role.

The equation for a sound ray thus reads, for horizontally stratified temperature,

$$y = -\frac{1}{\cos^2\delta}\cdot 10^{-3}\frac{\partial T}{\partial y}\,x^2 + \tan\delta\cdot x$$

or, for an initially horizontal ray

$$y = -10^{-3}\frac{\partial T}{\partial y}\,x^2 \qquad (10)$$

By way of example, consider a water depth of 30 m and a surface-to-bottom temperature difference of 1°C (with the larger temperature at the surface), thus

$$\frac{\partial T}{\partial y} = -\frac{1}{30}$$

We then obtain

$$y = 3.33\cdot 10^{-5}\cdot x^2$$

An initially horizontal sound ray ($\delta = 0$) which originates at a point near the surface reaches the bottom after having traveled a distance of roughly 1000 m, where the sound energy is essentially absorbed. All other sound rays which initially subtend a non-zero angle δ relative to the surface horizontal, will reach the bottom earlier.

The occurrence, however, in acoustics of sharply confined sound rays, is unrealistic. Sound will be heard also away from the geometric sound ray due to refraction. The ray's curvature will increase with increasing temperature difference between surface and bottom, and the distance over which the sound can be heard will be even smaller than the one inferred in the above example.

If the temperature gradient is reversed, i.e. the temperature at the bottom is larger than at the surface, then the sound rays will be bent upward rather than downward. Since the water surface is essentially a perfect reflector [] [4] the rays will initially be reflected downward at the same angle at which they impinged on the surface, but then bent upward again due to the temperature stratification, unless, as may occur in agitated seas, a significant part of the the sound energy is immediately reflected into the ground and absorbed.

In the same way as a change in temperature with depth, a change in salinity leads to a refraction of sound waves in water and thus a deviation from a straight path. The compressibility diminishes with increasing salinity, as is apparent in the following table, taken from Krümmel's Handbook of Oceanography [] [5]

Table 1

salinity in ‰	0	5	10	15	20
compressibility $10^7\cdot\kappa$	490	484	478	472	466

salinity in ‰	25	30	35	40
compressibility $10^7\cdot\kappa$	461	455	450	422

An absolute change in salinity by 1 ‰ results in a change in compressibility of $1.2\cdot 10^{-7}$. An equal change in compressibility may be achieved by a change in temperature of 0.6°C, and which corresponds to a change in the speed of sound of 1.8 m/sec.

The density dependence on salinity is given, with sufficient accuracy, by the formula [] [6]

$$\rho = 1 + 0.0008\cdot S$$

where S is the salinity given in permille.

Thus, an absolute increase in salinity by 1 permille corresponds to a relative increase in density by 0.8 ‰ and a decrease in the speed of sound by 0.4 ‰, i.e. by 0.58 m/sec. The influence of a change in density on the speed of sound with varying salinity is roughly 1/3 that of a change in compressibility.

A further cause for the refraction of sound waves, but which only plays a role at great depths, is the change of sound speed with varying pressure. This is due to the dependence of density and compressibility, both of which determine the sound speed, on water pressure. With increasing pressure density increases [] [7] , whereas compressibility decreases [] [8]

[3] Kohlrausch, loc. cit., p. 694, Table IV. Density reaches a maximum at 4°. $\partial\rho/\partial T$ is negative only for temperatures above 4°.

[4] Rayleigh, Theorie des Schalls. German edition by Neesen, 1889, II, 98.

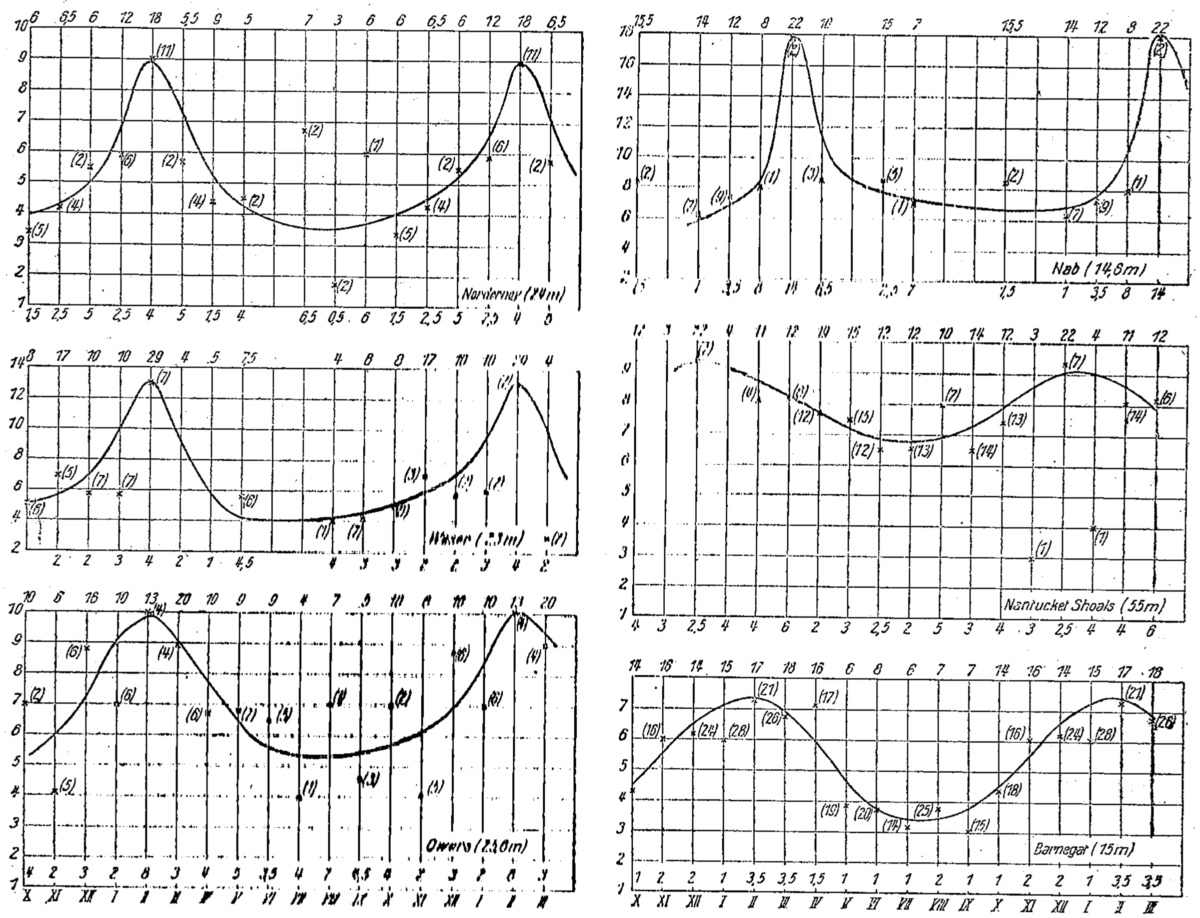
[5] Krümmel, Handbuch der Ozeanographie, Bd. I, 1907, p. 285. See also references therein.

[6] The equation corresponds to the one of Knudsen, reported in Krümmel, loc. cit., p. 237, line 11.

[7] Krümmel, loc. cit., Bd. 1, p. 288.

[8] Kohlrausch, loc. cit., Bd. 1, p. 708, Tab. XIXb.

Fig. 2.



However, the relative change in density with increasing pressure is small compared to the the relative change in compressibility, as can be inferred from the tables. It thus suffices to only consider the influence of compressibility. According to measurements by Amagat [] [9] the change in compressibility of water at 0°C due to an increase in pressure from 1 to 200 Atm is the same as that due to an increase in temperature from 0°C to 20°C. Hence, at 2000 m depth, an "adverse" 20°C temperature difference between the surface and bottom will be fully compensated. At even larger depths a movement of sound waves toward the surface occurs.

Finally, it should be noted that, obviously, water currents play a similar role in the perception of sound as does the wind in the sound propagation in air. It is well known that sound in air is better perceived in the direction of the wind than against it. This may be explained [] [10] by the change in the speed of sound with elevation from the ground, since the wind speed generally increases with distance from the ground. Thus, a sound ray traveling against the wind is gradually bent upward and eventually passes over a listener's head. In contrast, a sound ray traveling with the wind will be gradually bent downward, and leads to a greater range.

Matters in air are exactly analogous to those in water, except that direct observations in water aren't as accessible as in air. What are the implications for the range of underwater sound?

We are mainly interested in the waters of the North Sea and the Baltic Sea. The tables reported by Krümmel's Handbook [] [11] imply an ano-thermic stratification during summer, i.e. water is warmer near the top than near the bottom, and a reversed

[9]

[10] Rayleigh, loc. cit., p. 155.

[11] Krümmel, loc. cit., Bd. 1, p. 468, 480, 349, 350.

(kato-thermic) stratification during winter. Salinity, on the other hand does not change much with depth. Water is either homo-haline, or there is a slight increase in salinity with depth during summer and winter.

In any case, compared to temperature changes, salinity changes have a generally small impact on compressibility. Furthermore, since lightships do not reach great depths, we can limit our considerations to the influence of temperature. The annual cycle in underwater sound range is thus essentially the same as the annual cycle in temperature. For ano-thermic stratification, i.e. during summer, one may expect shorter ranges than for kato-thermic stratification, i.e. during winter.

Observations agree well with this finding. The curves reported in Fig. 2 contain observations of underwater sound ranges which were obtained near various lightships [] [12] Ranges in nautical miles (ordinates) are plotted against months (abscissae, in Roman numbers). Each point has an associated number denoting the number of observations from which a monthly mean value was derived. Numbers below the curves refer to the smallest, numbers above the curves to the largest observed ranges for individual months. Largest ranges during winter and smallest ranges during summer are apparent, in agreement with the above theory. Large differences are found within individual months, which may be explained by random currents (tidal currents), but which may also in part be due to the differences between the receiving devices.

No range observations are available for the abyssal ocean. Nevertheless, significantly larger ranges can be expected than for shallow waters, due to the beneficial impact of pressure on compressibility, and thus on the the propagation speed of sound (compressibility decreasing from top to bottom, leading to increasing speed of sound, and updward directed sound propagation).

The precise attenuation law, i.e. the dependence of the sound intensity on distance, cannot be deduced from the curves presented. Detailed experiments in this regard have been conducted during the years 1915/16, and will be presented elsewhere.

Kiel, April 1919 (received 2 May 1919)

[12] The material has been collected, before the war, by commercial ships which had been equipped with underwater sound receptors, and has been evaluated by me in light of the above-exposed theory. It had been kindly made available to me by the senior engineer Wolf, and for which I am very grateful.