The Deduction Theorem And Ecology

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ABSTRACT

The deduction theorem is based primarily on the formula $[A \supset (B \supset C)] \supset [(A \supset B) \supset (A \supset C)]$. The fisheries of Newfoundland, Iceland, and west Greenland have three aspects, a first aspect of abundance of cod or herring, aspect *A*, a second aspect of overfishing or overfishing plus hydrographic change, aspect *B*, and a third aspect of collapse of the cod-herring fishery or shift to shrimp, aspect *C*. Each aspect implies, connects to the next either in $[A \supset (B \supset C)]$ sequence or in the $[(A \supset B) \supset (A \supset C)]$ double sequence. The Mississippi River system has three parts, the drainage area of the inner U.S.A., part *A*, the plankton-rich, low salinity plume to the west of the Mississippi delta, part *B*, and the oxygen-depleted near-bottom layer of the plume, part *C*. These parts are connected, as shown by the implication connective, $\supset$. There are two indirect applications of the deduction theorem. The first is three aspects of hotspots, each hotspot being a region having a set of many indigenous species and having set of endangered species, the two sets being identical by having all the same species (the principle of extensionality). The second indirect application is three logically valid formulas that describe most of the natural world. *A* is contraposition, an example of which is: a vertebrate is adapted to year-round temperate temperature if it is functional (active) year-round; if and only if a vertebrate is not adapted to year-round temperate temperature only if it is not functional (not active) year-round. *B* is equivalence, an example of which is: if a North American bird species is adapted to its area, then its area is adapted to it, and if its area is adapted to the species, then the species is adapted to its area – equivalent to: species is adapted to area if and only if area is adapted to species. *C* is constructive

dilemma, an example of which is: all insects are in diapause or non-diapause condition; all insects, if in diapause condition, are winter adapted; and all insects, if in non-diapause (winged) condition, are summer adapted: therefore, all insects are winter adapted or summer adapted.

The three logically valid formulas, contraposition, equivalence, and constructive dilemma are thus parts of the larger logically valid formula of the deduction theorem. The empirical data, of considerable value in themselves, become of very great value when inserted into the validity formulas, which seem of limited value without the empirical input. Thus the intent of this several-layered study is just to probe into the underpinnings of nature and to attempt to create some order in our perception of these underpinnings.

INTRODUCTION

There are a number of principles useful in modelling ecology. Principles such as logical validity and relations and functions of set theory have been used (Hulburt, 1992, 1996, 1998, 2001, 2002, 2004). Other principles that could be used are the deduction theorem, the principle of extensionality, the one-many-one and one-many-many basis of the ordered pair, the identity of indiscernibles, and several more. In this article the deduction theorem will be explained and will be shown to be applicable to several biological situations.

The use of the deduction theorem would be a great asset in modelling various biological situations. In the past this theorem has been a well-worked out part of mathematical logic (Church, 1956, pp. 74, 75, 81, 86-91; Nidditch, 1962, pp. 19-40; Kleene, 1964, pp. 90-01; Mendelson, 1964, pp. 32-35; Hamilton, 2000, pp. 32-35; Rautenberg, 2006, p. 17). The theorem has been used in purely axiomatic logic (Thomason, 1970, pp. 68-69; Copi, 1979, pp. 245-246). In all these studies it is part of an axiomatic approach and thus has logical validity as a sine qua non feature.

There are two ways that the deduction theorem will be used in the pages to come. The first will be to convert one of the axioms to words in describing several situations directly and indirectly. The second will be to derive several formulas of logic and then describe several biological patterns with these formulas.

<u>THE DEDUCTION THEOREM</u>

The deduction theorem depends on two logically valid formulas. The first is very simple. The second is more complex and is the one that will be presented next. This formula is of great interest in that it has a deductive and an inductive component. The whole formula when <u>written</u> <u>vertically</u> has parts 1, 2, and 3, three assumptions, where *A*, *B*, and *C* are descriptive statements[1] and ⊃ means *if-then, implies*. There is a final part *C* which is proved by modus ponens, where, for example, step 1 is *if A then (B⊃C)*, step 3 is *given A*, and step 4 is *therefore (B⊃C)*. Next (Rautenberg, p. 17, 231; Nidditch, p. 31):

1.	$[A\supset (B\supset C)]$	Assumption
2.	$(A\supset B)$	Assumption
3.	A	Assumption
4.	$(B\supset C)$	1, 3, modus ponens
5.	B	2, 3, modus ponens
6.	C	4, 5, modus ponens

This structure when *written horizontally* is composed of parts 1, 2, and 3, separated by commas. These three parts yield, ⊢, the final and fourth part, part 6, *C*:

$$[A\supset(B\supset C)],\ (A\supset B),\ A \vdash C$$

[1]The letters A, B, C stand for statements, but never is there given an actual statement in any of the logical accounts.

To go from the group yielding *C* to the last member *A implying C* is an inductive step, meaning that if *C* is proved to be true, then *A* must be true in order that a true *A* precedes a true *C*. Thus a true *A* precedes a true *C*, true implies true, true ⊃ true, *A*⊃*C*. Working from right to left we have:

$$[A \supset (B \supset C)], (A \supset B) \vdash (A \supset C).$$

A second inductive step comes next:

$$[A \supset (B \supset C)] \vdash (A \supset B) \mid (A \supset C).$$

And a final inductive step achieves:

$$[A \supset (B \supset C)] \supset [(A \supset B) \supset (A \supset C)].$$

The three parts are connected by implications. In the logical world the first part implies the second which implies the third – the validity of this sequence is given in the appendix. In the real world the first part is connected to the second which is connected to the third. Three examples from the real world are presented next.

<u>THE DEDUCTION THEOREM AND THE FISHERIES OF NEWFOUNDLAND, ICELAND AND WEST GREENLAND</u>

The *A*, *B*, *C*'s of ecology are the *A*, *B*, *C*'s of the deduction theorem for the data presented here. The *A*, *B*, *C*'s of the deduction theorem involve both human activity and processes of the natural world. In the first case next *B* is human activity. In the second case *B* is partly human and partly hydrographic. And in the third case *B* is more human and less hydrographic.

The downfall of the fisheries of Newfoundland was mostly from overfishing. The downfall of the fisheries in north Iceland was partly from overfishing and partly from hydrographic changes. In West Greenland there was a change in fisheries spurred on by human activity.

At Newfoundland there was a period extending from 1982 to 1987, called "the glory years", that had maximum numbers of cod caught initially by efficient often subsized fleets and that subsequently had an abrupt decline in fish caught until low numbers persisted from 1987 to 1994. In 1992 there was a cod moratorium and after that the population of people decreased considerably (Hamilton et. al., 2004a). In this account one can discern a part *A* of a maximum of fish caught, a part *B* of intense fishing and a part *C* of a minimum of fish caught.

During the first decades of the 20th century a fishery for herring on the north coast of Iceland grew up. In 1890 Siglufjördur was a village of fewer than 100 people. The year-round population went from 144 to 1450 between 1903 and 1924. It doubled from this by the late 1930's. It was added to by summer workers. The photograph of Siglufjördur in 1946 (fig. 1) shows it in its golden years when at peak times the protected harbor might have 400 boats and when 27 salting stations and five fish meal factories were in operation – the salting stations run by young girls, the herring girls (sildarstulkur), who had come from other parts of Iceland to work 14-20 hours/day. No doubt the fish stocks to

[1]Called Norwegian spring-spawning herring

the north of Siglufjördur were probably overfished but additionally slightly colder, lower salinity water intruded southward, shown by the 2°c isotherm and interfered with spawning grounds of the herring (fig. 2). By 1972 the spawning grounds disappeared and though the herring returned at a fraction of their original amount, still the harbor of Siglufjördur had just a single vessel in the 2003 photograph (Hamilton et. al. 2004 b). But here there would seem to be an *A* part of a maximum fishing stocks, a *B* part of intense fishing and hydrographic change and a *C* part of final collapse and denouement, each part implying and connecting to the next sequentially as $[A \supset (B \supset C)]$ or perhaps more perspicuously, as $[A \supset B) \supset (A \supset C)]$.

There was a change in the climate of southwestern Greenland. Between 1950 and 1970 there was a low NAO index with weak westerlies in the North Atlantic and warm southerlies over Greenland and "warmish" water (1.5-2.5 C in June). During this period a cod fishery grew up and exploited the abundant stocks. But between 1970 and 2000 the NAO changed to a higher index with northerly winds over Greenland, "colder" water (0.5 – 2.0 C in June). The cod fishery collapsed from overfishing and lack of spawning in the colder water – total catches exceeded 400,000 metric tons in 1962-63 and in 1967 then dropped below 100,000 metric tons a few years later and never regained high levels. At the same time the shrimp fishery increased from 10,000 metric tons in 1972 to more than 70,000 metric tons in the 1900's. Locally the cod to shrimp change was accomplished well in Sisimiut, where a more varied fish economy existed than in Paamiut to the south, which depended primarily on cod and fell on hard times and had a loss of population. The result was a gradient in shrimp catch locations from the area around Sisimiut to the area of Paamiut, fig. 2. (Hamilton et. al., 2003). Thus there would seem to be an *A* part of abundant cod, a *B* part hydrographic change plus overfishing of cod, and a *C* part of abundant shrimp, each part leading to and connecting to the next, either sequentially as $[A \supset (B \supset C)]$ or more perspicuously as $[(A \supset B) \supset (A \supset C)]$.

It is remarkable how the empirical structure shines through in the simple formula of the deduction theorem.

THE DEDUCTION THEOREM AND THE MISSISSIPPI RIVER

The Mississippi River system has three segments. Segment *A* is the drainage area of the farmland of the interior of the United States. Fig. 3 shows that "the principal sources of nitrate are river basins that drain agricultural land in southern Minnesota, Iowa, Illinois, Indiana, and Ohio, where large amounts of nitrogen are applied to corn and soybean fields" (Nat. Sci. Tech. Council, 2000, p. 23). Nitrate flux from the Mississippi drainage system to the Gulf of Mexico (has almost tripled between the periods 1955-70 and 1980-96." At New Orleans, nitrate was extraordinarily high, 85 ± 3.8 μM in 1975-1980 and 113 ± 8.8 μM in 1981-1987 (Wiseman et. al., 1999, p. 42). Other nutrients, phosphate and silicate were extremely high too. Though silicate decreased between 1955 and 1980, it was still high, 101.7 ± 4.2 μM in 1975-1980 and 108.27 ± 5.0 μM in 1981-1987. So it is clear that Part *A* might be: there is a great amount of nutrient, particularly nitrate nitrogen, from the Mississippi drainage going into the Gulf of Mexico.

Segment *B* of the Mississippi River system is concerned with the phytoplankton growth in a long plume of freshened nitrate-enriched water, which originates from the mouth of the Mississippi River delta and moves westward close along the coast. The composition of the phytoplankton, fig. 4, is predominantly diatoms in the winter (Wiseman et. al., 1999, p. 57). The phytoplankton is composed primarily of picocyanobacteria in the spring and summer. It is this phytoplankton that has a tremendous effect. Segment *B* might be: the nutrient enriched water to the west of the Mississippi delta produces an abundant phytoplankton.

Segment *C* is the tremendous effect that the phytoplankton has as it sinks below the freshened layer into the layer of water just above the bottom. In summer it decomposes and reduces the oxygen to very low amounts, <2.0 mg./1, fig. 4. This layer represents a case of hypoxia (Wiseman et. al., 1999, pp. 81-88). Segment *C* might be: the abundant near-surface phytoplankton produces hypoxia in summer in the near-bottom water.

The three parts of the deduction theorem show that the three segment part, $[A\supset(B\supset C)]$, is connected to the two two-segment parts, $[(A\supset B)$ and $(A\supset C)]$. Thus the desultory, empirical situation could be viewed in this tightly held together manner:

> If there is a great amount of nutrient, particularly nitrate nitrogen, from the Mississippi drainage going into the Gulf of Mexico (*A*),
>
> Then if the nutrient enriched water to the west of the Mississippi delta produces an abundant phytoplankton (*B*),
>
> Then the abundant near-surface phytoplankton produces hypoxia in summer in the near-bottom water (*C*).
>
> But you could go from *A* to *B* and then from *A* to *C* just as coherently – perhaps more coherently.

INDIRECT USE OF THE DEDUCTION THEOREM

The deduction theorem has three descriptive statements, *A*, *B*, and *C*. These are grouped to the left. They are regrouped to the right, to achieve validity. They are connected by implication, to achieve validity.

Suppose that *A*, *B*, and *C* are not simple, but instead are each a complex structure. Suppose that there are three ways to express the principle of extensionality, the principle that two differently defined sets are identical by having all the same members. This principle is illustrated by three ways in which the hotspots of the world present themselves.

A. There are 25 areas in the world called hotspots, fig. 4 (Myers et. al., 2000). "To qualify as a hotspot an area must contain at least 0.5% or 1500 of the world's 300,000 plant species as endemics" and these species must be threatened. Thus there are two derivable sets: (1) the set of areas having a great many plant endemics; and (2) the set of areas having species threatened by extinction. These two sets of areas are identical because each one of the areas belongs to the set of *all 25* which are rich in endemics and to the set of *all 25* which have threatened species.

B. "The 25 hotspots contain the remaining habitats of 133,149 plant species (49% of all the plant species worldwide) and 9,645 vertebrate species" (35% of all land vertebrate species worldwide). (1) Thus there is a set of plant-vertebrate species-rich habitats. (2) There is a set of habitats which formerly occupied 11.8% of the Earth's land surface but now occupy only 1.4% of the Earth's land surface. These two differently defined sets are identical by having all the same habitats and species.

C. There are five principal hotspots, the tropical Andes, Sunaland, Madagascar, Brazil's Atlantic Forest, and the Caribbean. "Together they have 20% and 16%, respectively, of all plants and vertebrates." "At the same time, they feature some of the most depleted habitats: the Caribbean retains only 11.3% of its primary vegetation, Madagascar 9.9%, Sunaland 7.8%, Brazil's Atlantic Forest 7.5%. In this case the property of having a high percentage of all plants and animals and the property of having a high depletion of habitats define two different five-membered sets which are identical because they have all the same five regions having all the same species.

Here, then, are three cases illustrating the principle of extensionality, wherein any pair of identical sets is $F = G$ and is determined (Copi, 1979, p. 178) by the formula:

$$(x)\ (x \in F \equiv x \in G)$$

which is: each region x belongs to ($\in$) the set F of species – rich regions (habitats) if and only if each region x belongs to ($\in$) the set G of species – endangered regions (habitats). That is, for any of the 25 regions x or 5 regions x (5 principal hotspots), if x belongs to the species-rich set F, then x belongs to the species-threatened set G; and for any of the 25 regions x or 5 regions x (5 principal hotspots), if x belongs to the species-threatened set G, then x belongs to the species-rich set F – the reversal being captured by the *if and only if* structure, ≡ (Lipschschutz, 1998, p. 14):

$$(x)[(x \in F) \supset (x \in G)] \bullet (x)\ [(x \in G) \supset (x \in F)]$$

These two formulas are very close to the formula of equivalence that will be considered later.

FURTHER INDIRECT USE OF THE DEDUCTION THEOREM

The deduction theorem, it was said, has three descriptions *A*, *B*, and *C*, which are grouped to the left and are regrouped to the right, to achieve validity. Everything is connected by implication, to achieve validity.

Again suppose that *A*, *B*, and *C* are not simple, but instead are each a complex structure. Suppose that *A* is contraposition, *B* is equivalence, and *C* is constructive dilemma. What are these? The thing to do is to find out what these structures are. To introduce them are brief descriptions: contraposition tells of change, equivalence tells of equilibrium, and constructive dilemma tells of alternative pathways – features of the external, real world of nature.

Briefly contraposition, *A*, is: if this then that; if not that then not this. Equivalence, *B*, is: if this then that and if that then this – equivalent to: this equivalent to that. Constructive dilemma, *C*, is: first or third, if first then second and if third then fourth, therefore second or fourth.

An example of contraposition is: if a vertebrate is active year-round, then it is adapted to year-round temperature; equivalent to: if a vertebrate is not adapted to year-round temperature, then it is not active year-round. An example of equivalence is: if a plant is adapted to its habitat, then its habitat is adapted to it, and conversely; equivalent to: a plant is adapted to its habitat if and only if its habitat is adapted to it. An example of constructive dilemma is: there is leaflessness or there is leafiness; if there is leaflessness then there is winter adaptedness, and if there is leafiness then there is summer adaptedness; therefore, there is winter adaptedness, or there is summer adaptedness.

The three formulas, contraposition, *A*, equivalence, *B*, and constructive dilemma, *C*, describe most of the natural world and each is logically valid in itself.

AXIOMATIC DEVELOPMENT

We will pause for a moment to elaborate simply how the structures *A*, *B*, and *C* are derived axiomatically. There are a number of axiom systems. Rosser (1954) presented a very complete elaboration from three axioms. His system was elaborated in Hulburt (2002). An altered and abbreviated version is presented next.

Thence *if*-then is ⊃, *not* is ~, *and* is • , *or* is *v*, and *P*, *Q*, and *R* are clauses linguistically, but considered structures of nature here. Next we have definitions, one rule of inference, and axioms. There are two definitions.

P ⊃ *Q* defined as ~ (*P*~*Q*), for *P* ⊃ *Q* means getting *Q* if you get *P* and so not to get *Q* must be barred (denied) initially in ~ (*P*~*Q).*

P v *Q* defined as ~ (~*P*~*Q*), for barring not getting both amounts to getting one or the other or both.

There is one rule of inference modus ponens, which is: if this then that, given this, therefore that (*P*⊃*Q*), *P* ∴ *Q*. There are three axioms:

Axiom 1 *P* ⊃ (*PP*),

Axiom 2 (*PQ*) ⊃*P*,

Axiom 3 (P⊃*Q*) ⊃ [~(*QR*) ⊃~ (*RP*)].

One term leads to the next by implying the next, by implication, shown by ⊃. Axiom 2 will be used later, while Axiom 3 will be used first, where there are three terms. (*P*⊃*Q*) is the first term, ~ (*QR*) is the second term, and ~ (*RP*) is the third term. *P* comes first and *Q* second in the first term, but *Q* comes first and *P* second in the second and third terms. The second and third terms are negated and have the liaison *R* in both.

CONTRAPOSITION, *A*, AND EQUIVALENCE, *B*

Axiom 3 will be used in sections 1. – 6. next. These short sections will develop contraposition through substitutions.

1. (*P*⊃*Q*) ⊃ [~ (*QR*) ⊃ ~ (*RP*)] leads by implication from the first to the second to the third terms.

2. The third term is changed to ~ (~ *RP*) by substituting ~*R* for *R* in the second and third terms.

3. Thence ~ (~*PP*) goes to ~ (~ ~ *P*~*P*), to (~ ~*P*⊃*P*), to (~ ~ *Q*⊃*Q*), where in the first step *P* is substituted for *R* in ~*P*. In the second step ~*P* is substituted for P. In the second and third steps the structures are definitionally the same. In the third and fourth steps there is the same structure. The fourth step, (~ ~ *Q* ⊃*Q*), implies the bracket in Axiom 3 next.

4. Axiom 3, with $(\sim\sim Q \supset Q)$ substituted for $(P \supset Q)$, is now $(\sim\sim Q \supset Q) \supset [\sim(Q \sim P) \supset \sim(\sim P \sim \sim Q)]$, in which the first term implies the bracket. The bracket is proved, isolated, by modus ponens: if the parenthesis then the bracket, given the parenthesis, therefore the bracket. $\sim(Q\sim P)$ is definitionally $(Q \supset P)$ and $\sim(\sim P \sim \sim Q)$ is definitionally $(\sim P \supset \sim Q)$, and $\sim P$ replaces the liaison R in the second and third terms of the axiom.

5. The outcome of 4. is $(\sim P \supset \sim Q)$. This can be part of Axiom 3 as the initial term: $(\sim P \supset \sim Q) \supset [\sim(\sim Q Q) \supset \sim (Q \sim P)]$. The first term's $\sim Q$ is the second term's $\sim Q$, and the first term's $\sim P$ is the third term's $\sim P$. There are two uses of modus ponens, one to isolate the bracket, the other to isolate the last term. The first term can be linked to the third term, because the third term has been proved by modus ponens from the first and so it is implied by the first (the deduction theorem).

6. From 4. and 5. we have: $(Q \supset P) \supset (\sim P \supset \sim Q)$,

$(\sim P \supset \sim Q) \supset (Q \supset P)$,

each being the reverse of the other. The first says: reverse and negate. The second says: reverse and unnegate. What has to be done is to put the two together by conjunction (see appendix). Two implications, one the reverse of the other, are put together by conjunction:

$$[(Q \supset P) \supset (\sim P \supset \sim Q)] \bullet [(\sim P \supset \sim Q) \supset (Q \supset P)],$$

This is what equivalence is. And the equivalence of contraposition looks like this – corresponding to *A* of the deduction theorem:

$$(Q \supset P) \equiv (\sim P \supset \sim Q).$$

Where ≡ means *equivalent to* or *if and only if*. A simpler equivalence, without negation, and called officially Equivalence is:

$$[(P \supset Q) \bullet (Q \supset P)] \equiv (P \equiv Q).$$

This corresponds to *B* of the deduction theorem.

CONSTRUCTIVE DILEMMA, *C*

Constructive dilemma has (PvR) initially. Then it has two implications, which are ($P \supset Q$) and ($R \supset S$), which are reversed and negated and surrounded by $\sim Q$ and $\sim R$ in the following way:

$$(\sim Q \sim S) \supset (\sim R \sim Q),$$

$$(\sim R \sim Q) \supset (\sim P \sim R),$$

which are by a) (see appendix):

$$(\sim Q \sim S) \supset (\sim P \sim R),$$

which under reversal and denial is, as in 6. first part:

$$\sim(\sim P \sim R) \supset \sim(\sim Q \sim S),$$

which definitionally is:

$$(P\text{v}R) \supset (Q\text{v}S),$$

which with the initial (PvR) by modus ponens gives us (QvS). We have:

$$(P \vee R),\ [(P \vee R) \supset (Q \vee S)],\ \therefore\ (Q \vee S),$$

and further, since $[(P \vee R) \supset (Q \vee S)]$ is derived from $[(P \supset Q) \bullet (R \supset S)]$, the whole must be:

$$(P \vee R),\ [(P \supset Q) \bullet (R \supset S)]\ \therefore\ (Q \vee S).$$

A. Some Features of Nature: Contraposition

Adaptedness and non-adaptedness occur in temperate land vertebrates and higher plants.

Let us consider the property of being year-round functional (functionality) in the sense that the organism is metabolically and behaviorally active year-round. This single property would be in a non-hibernating mammal, would be exemplified by each such animal. Additionally, this property would dictate having the property of year-round adaptedness in each non-hibernating mammal. Then let us consider the property of not being year-round functional (non-functionality) – getting through the winter in a moribund or inert state and only coming to life, so to speak, with the return of spring and summer. Cold-blooded vertebrates become active and gymnosperm plants grow only with the return of spring and summer. So these organisms are examples, exemplifications of the single property of being year-round nonfunctional. Additionally, this property dictates the property of not being year-round adapted, of having the property of year-round non-adaptedness in each cold-blooded vertebrate and each gymnosperm plant. But though considerable integration of the biological situations is achieved by seeing the structure of nature via properties and their exemplifications, the portrayal is still basically inadequate and disconnected without being reset in a logical framework. And so summarizing we get by contraposition:

> If year-round functionality is exemplified by non-hibernating mammals, then year-round adaptedness is dictated by year-round functionality – equivalent to: if year-round non-adaptedness is dictated by year-round non-functionality, then year-round non-functionality is exemplified by cold-blooded animals and gymnosperms.

This detailed presentation of contraposition can be simplified:

> If there is year-round functionality, then there is year-round adaptedness – equivalent to: if there isn't year-round adaptedness, then there isn't year-round functionality –

$$(P \supset Q) \equiv (\sim Q \supset \sim P).$$

The connectedness of the two aspects of nature is achieved here and this feature is simply a requirement for coherent science.

A specific case of contraposition (*A*) will now be given. The common mud snail *Nassarius obsoletus* hibernates in effect in winter, whereas the worm *Capitella capitata* does not. *Nassarius* grows rapidly near Woods Hole (northeastern United States) during summer months, any population showing at least three size classes of animals from this year, last year, and the year before. During late autumn, winter, and early spring there is essentially no growth. This is related to the cessation of feeding, followed by a migration into deeper water, and a subsequent period of quiescence (Scheltema, 1964, p. 164). But *Capitella*, when the environment is disturbed, may then become overwhelmingly abundant. A period of such abundance extended from October 1969 to September 1970, subsequent to an oil spill near Woods Hole (Grassle and Grassle, 1974). *Capitella* has a short life span, taking 30-40 days to mature, so that a number of generations would be expected to maintain the year-long abundance. In support of this were observations of settlement of larvae in winter and summer, though with the greatest settlement from May to October.

This description can be brought together in this way:

> If *Capitella* is adapted year-round, then it is functional year-round – equivalent to: if *Nassarius* is not functional year-round, then it is not adapted year-round. $(P \supset Q) \equiv (\sim Q \supset \sim P)$.

Thus contraposition organizes the desultory empirical details, transforming crude science into coherent science. There is the structure of coherence that lies deeply beneath the logico-empirical detail just presented.

B. Further Features of Nature: Equivalence

Each flowered plant and spot that bears it go forth hand in hand, for each is adapted to the other. Similarly, each of the 900 species of North American birds is attuned, is adapted to its area of occurrence as its area of occurrence is attuned, is adapted to it (*National Geographic Society*, 1999). Then further, the species of bird that stays in the north throughout the year, is adapted when breeding to its area, which is adapted to it, and when not breeding has the same reciprocal adaptation. And further still, the species of bird that flies south for the winter has its non-breeding area and reciprocal adaptation there and then when it flies north for spring and summer has its breeding area and reciprocal adaptation there.

And so, when species *x* is adapted to its area and its area is adapted to it, we can tighten this linkage by *if species x is adapted to area, then* area is adapted to species. But why should the asymmetry go from species to area; so let's have the asymmetry go the other way: *If its area is adapted to species x, then* species *x* is adapted to its area. Let's just settle for both, and cap the sequence in the following way for all North American bird species.

> If species *x* is adapted to area *y*, *Axy*, then area *y* is adapted to species *x*, *Ayx*, and if area *y* is adapted to species *x*, *Ayx*, then species *x* is adapted to area *y*, *Axy* – equivalent to: species *x* is adapted to area *y* if and only if area *y* is adapted to species *x*;
>
> $$[(Axy \supset Ayx) \bullet (Ayx \supset Axy)] \equiv (Axy \equiv Ayx).$$

This is a complete distributional summarization of the birds of North America. Any other summarization would be incomplete and wrong, in the sense that any other summarization would lack the coherence of this logically valid one. But we are tapping into deeper structure, the deeper structure of coherence.

C. Some Major Features of Nature, Constructive Dilemma

The case of the non-hibernating mammal, which is active year-round and so adapted to year-round temperatures, and the case of the cold-blooded vertebrate and the gymnosperm plant (pine or spruce) which are not active or functional year-round and so are not adapted to year-round temperature are cases of large features of nature that involve contraposition. The next cases are large feature too, but they will involve constructive dilemma.

1.	Leafless limbs	Leafy limbs
2.	Seeds of annuals	Plant forms of annuals
3.	Underground parts of perennials	Above ground parts of perennials
4.	Hibernating individuals of hibernation type mammals	Non-hibernating individuals of hibernation type mammals
5.	Diapause insects	Non-diapause insects

The pairs of the five cases are concrete physical objects, unlike the example in the beginning of this essay where the first pair is two abstract entities. The plural form refers to all entities. In order from the top we have: all limbs are leafless or leafy, all annuals are seeds or plants, all perennials have underground or above-ground parts, all hibernation type mammals are hibernating or not hibernating. The last pair can be put into constructive dilemma form as follows:

1. All insects are in diapause or non-diapause condition.
2. All insects, if in diapause condition, are winter adapted; all insects, if in non-diapause condition (winged condition) are summer adapted.
3. Therefore, all insects are winter adapted or summer adapted.

In more detail:

1. For any insect *x*, *x* is in diapause condition or *x* is in non-diapause condition.

2. For any insect *x*, if *x* is in diapause condition, then *x* is winter adapted; and for any insect *x*, if *x* is in non-diapause condition, then *x* is summer adapted.
3. ∴ For any insect *x*, *x* is winter adapted or *x* is summer adapted.

    a. $(x)\ (Dx \vee Nx)$

    b. $(x)\ (Dx \supset Wx) \bullet (x)\ (Nx \supset Sx)$

    c. $\therefore (x)\ (Wx \vee Sx)$

    a. $(P \vee R)$

    b. $[(P \supset Q) \bullet (R \supset S)]$

    d. $\therefore (Q \vee S)$

All pairs can be gotten into this structure, delineating a large portion of nature explicitly and coherently.

A more specific case of constructive dilemma is the following.

Steele and Koprowski (2001, pp. 66-69) describe the red or pine squirrel (*Tamiasciurus hudsonicus*) and the Douglas squirrel (*T. douglasii*) as perfect examples of larder-hoarders. The pine squirrel in particular has the physical traits of being smaller than other squirrels, of being reddish. It has the behavioral trait of being aggressive in defending its middens of spruce and pine cones that it collects for future eating. It has the trait of making collections of cones that will last at least a year. These collections are close to where it lives. In addition to all these traits, these properties, it has the property of being distributed across northern U.S.A. and southern Canada, and thus we can see its larder-hoarding is carried out where cones are easily gotten. And thus the pine squirrel has the property of being adapted to the environment that has the property of supporting it. And additionally, the pine squirrel has the property of being adapted to an environment having the property of being adapted to it.

Steele and Koprowski (2001, pp. 68-82, 143-147) describe the eastern grey squirrel (*Sciurus carolinensis*) and the fox squirrel (*Sciurus niger*) as good examples of scatter-hoarders. Both have the property of being larger than the pine squirrel and Douglas squirrel, of being grey, of being non-aggressive and having widespread small caches of various foods, quite often acorns. The two species overlap each other in their distributions, though the fox squirrel may occupy a more open, more upland habitat than the grey squirrel. The two species are distributed throughout the eastern U.S.A. So they are adapted to this deciduous environment that supports and is adapted to them.

Let us pause and reflect for a moment. When we look about, we are predominantly surrounded by friendly people. Do we wish to think of them as bearers of friendliness? Yes or no, there are three steps in the following sequence: 1) a friendly person, 2) a person is friendly, 3) a person has the property of friendliness. The austere nominalist, who doesn't believe in properties, chooses 1) and maintains that 2) reduces to 1) because 2) is only a linguistic formula (Loux, 2003, pp. 60-83). The metaphysical realist, on the other hand, believes in properties and endorses 3) and says that 2) is just an ill-formed version of 3) (Loux, 2003, pp. 20-35). 3) is vastly superior, in the opinion of the author. Properties are in the members of the group, in each friendly person. But the group groups its members by taking and marshalling their property. There can be a set or group of friendly people only when the group has the property of friendliness to group them by. There can be a set or group of larder-hoarding squirrels only when the group has the property of larder-hoarding to group them by; this is A_1. There can be a set or group of scatter-hoarders only when the group has the property of scatter-hoarding to group them by; this is set A_2. We can have thence these sets and their properties in the concise and correct format of constructive dilemma:

1. A_1 has the property of larder-hoarding or A_2 has the property of scatter-hoarding.
2. If A_1 has the first property then A_1 has the property of being adapted to the northern spruce forest, and if A_2 has the second property then A_2 has the property of being adapted to the eastern deciduous forest.
3. Therefore, A_1 has the property of northern spruce adaptedness or A_2 has the property of eastern deciduous adaptedness.

DISCUSSION

The deduction theorem, in the form $[(A \supset (B \supset C)] \supset [(A \supset B) \supset (A \supset C)]$, assembles the detail of the North Atlantic fisheries and the Mississippi River system. The theorem has the interesting property of connecting the three parts sequentially. The $[A \supset (B \supset C)]$ part is the orderly sequence but the other two parts, $(A \supset B)$ and $(A \supset C)$, make sensible connections. In the indirect applications of the deduction theorem order makes no difference. But the left part and the two right parts are all required for logical validity.

The hotspots do not have logically valid *A* or *B* or *C*. But the ecological presentation of contraposition (*A*), equivalence (*B*), and constructive dilemma (*C*) have each of these parts as logical validities, validities that emerge and consolidate the desultory empiricalities of the biological situations that are described and refashion them into some semblance of value.

There are three ways we can have: if this then that, or symbolically $P \supset Q$. The three ways can be put down as follows: (1) if this then that, (2) that if this, (3) this only if that. $P \supset Q$ is any one of these (Quine, 1972, p. 47). Same for $\sim Q \supset \sim P$. (1) if not that then not this, (2) not this if not that, (3) not that only if not this. Choosing (2) from the first three and (3) from the second three, we have corresponding to contraposition:

the form

$$(P \supset Q) \equiv (\sim Q \supset \sim P),$$

$$(2) \equiv (3),$$

That if this – equivalent to: not that only if not this.

Here are some examples of this form of contraposition.

> Pure water freezes, if it's below 32°F – equivalent to: pure water does not freeze, only if it's not below 32°F.
>
> Wind blows, if there's a pressure gradient from high to low – equivalent to: wind does not blow, only if there's no pressure gradient (no high and low).[1]
>
> A vertebrate is adapted to year-round temperature, if it is metabolically and behaviorally active under all year-round temperate temperatures – equivalent to: a vertebrate is not adapted to year-round temperature, only if it is not metabolically and behaviorally active under all year-round temperate temperatures (this is true for all non-hibernating land vertebrates).

[1]The Coriolis force makes the wind blow 90° to the right of the pressure gradient, so that the wind goes clockwise around a high.

The first two are factually correct, logically valid structures and cannot be falsified. The interpreted structure of temperature adapted and unadapted vertebrates cannot be falsified. If the four-part as interpreted structure were falsifiable, then it would not be logically valid. But it is logically valid, so it cannot be falsifiable as interpreted.

Equivalence and constructive dilemma have a similar interpreted unfalsifiability. The formulation of the deductive theorem is also valid. But it is true always, is a tautology. Thus the structure that the content of this paper is based on forces one to accept this structure as unfalsifiable and true always.

Being true always is what a tautology guarantees. A tautology is a formula which is true by truth value analysis. There are several methods for accomplishing this; one is given in the appendix. When a deductive vertical proof is reorganized by inductive steps into a horizontal sequence of implications, as was done initially for the deduction theorem, then a truth value analysis can be done on the sequence. Contraposition and equivalence can have a truth-value analysis done on them just as they are. But constructive dilemma must be transformed from vertical deductive proof format to horizontal inductive implicative format. The axioms of the axiomatic proofs are themselves tautologies established by truth-value analysis. Downward deductive proofs from axioms are insufficient without axioms, which can only be established by truth-value analysis. All these cases of truth-value analysis are presented in the appendix, but a large truth-value analysis is the centerpiece in Hulburt (Ecological Modelling, 1996).

Since every aspect of this study is founded on the truth-value analyses presented next, the truth of the basic structure of this study is forced upon us overwhelmingly.

More than this, there is deep structure, the structure of logic itself that penetrates deep in to the facets of reality that have been presented, as in this simple expressive metaphor: afternoon deepens into evening.

APPENDIX

TRUTH VALUE ANALYSIS

There are three ways that two-part structures can be put together, implication (if-then), conjunction (and) and disjunction (or, v). True = ⊤. False = ⊥. For implication ⊤⊃⊤ seems obvious for being true as a whole. And ⊤⊃⊥ being the flat denial of ⊤⊃⊤ should get a rating of false as a whole. But the other two, ⊥⊃⊤, ⊥⊃⊥, with denial of the antecedent, leave the case uncertain what to expect in the consequent and what to make of the whole. What has been settled on is just to lump these two with ⊤⊃⊤ as being true as wholes.

The conjunction of ⊤•⊤ is sensibly true as a whole; ⊤•⊥, ⊥•⊤ , ⊥•⊥ are all false as wholes. The disjunction of an or – connected compound is sensibly true when at least one part is true, otherwise false: ⊤v⊤, ⊤v⊥, ⊥v⊤ are true as wholes; ⊥v⊥ is false as a whole. ⊤•⊤•⊤… is true;⊥ •⊤•⊤… is false. ⊤v⊥v⊥ is true…; ⊥v⊥v⊥… is false. ⊤•⊤ is true as a whole in the sense that it takes both gasoline and electricity to make an engine go. $\top \vee \bot$, $\bot \vee \top$, $\top \vee \top$ are true as wholes in the sense that one can make a million by plan A or plan B and certainly both plans together.

This discussion of truth structure can be summarized in the following tables:

For implication:

$\top \supset \top$	$\top$
$\top \supset \bot$	$\bot$
$\bot \supset \top$	$\top$
$\bot \supset \bot$	$\top$

For conjunction:

$\top \bullet \top$	$\top$
$\top \bullet \bot$	$\bot$
$\bot \bullet \top$	$\bot$
$\bot \bullet \bot$	$\bot$

For disjunction:

$\top \vee \top$	$\top$
$\top \vee \bot$	$\top$
$\bot \vee \top$	$\top$
$\bot \vee \bot$	$\bot$

These presentations of truth structure underpin all logic, and since logic underpins all that there is (in the author's opinion), the truth structure underpins the world. They require the following rules (from Quine, 1972, pp. 28-33).

Rule 1 Reduce an if-then structure (an implication)with $\top$ as a consequent (then part) or $\bot$ as an antecedent (if part) to $\top$. (An implication with $\top\supset\top$, $\bot\supset\top$, or $\bot\supset\bot$ is true as a whole).

Rule 2 Delete $\top$ as an antecedent in an implication (the implication is true or false as the rest is true or false; $\top\supset\top$ is true as a whole, $\top\supset\bot$ is false as a whole).

Rule 3 Delete $\top$ from a conjunction (an *and* compound) (a conjunction with a true component is true or false as the rest of it is true or false).

Rule 4 Reduce a conjunction with $\bot$ as a component to $\bot$.

Rule 5 $\top\equiv\top$ and $\bot\equiv\bot$ reduce to $\top$. $\top\equiv\bot$ and $\bot\equiv\top$ reduce to $\bot$.

The method of operation is to put down the formula to be analyzed in the center of the page. Then next below pick one letter and replace it with true, $\top$, throughout and put all this to the left – for this is how the world really is. To the right replace the same letter with false, $\bot$, for this is how the world isn't but can be imagined to be. By reductions and dichotomies the ultimate end of true, $\top$, everywhere is attained and the hallmark of a tautology of true always is had.

An extensive truth-value analysis is given in Hulburt (Ecological Modelling, 1996).

DEDUCTION THEORUM

$[A\supset(B\supset C)] \supset [(A\supset B) \supset (A\supset C)]$

$[\top \supset (B\supset C)] \supset [(\top\supset B) \supset (\top\supset C)]$

$(B\supset C) \supset (B\supset C)$

$\top$

$[\bot\supset(B\supset C)] \supset [(\bot\supset B) \supset (\bot\supset C)]$

$(\top\supset(\top\supset\top)$

$\top \supset \top$

$\top$

CONTRAPOSITION

$(P\supset Q) \equiv (\sim Q\supset\sim P)$

$(\top\supset Q) \equiv (\sim Q\supset\bot)$

$Q \equiv (\sim Q\supset\bot)$

$(\top\equiv(\bot\supset\bot)$ $\quad \bot\equiv(\top\supset\bot)$

$\top\equiv\top$ $\quad \bot\equiv\bot$

$\top$ $\quad \top$

$(\bot\supset Q) \equiv (\sim Q\supset\top)$

$\top \equiv \top$

$\top$

EQUIVALENCE

$[P\supset Q) \bullet (Q\supset P)] \equiv (P\equiv Q)$

$[(\top\supset Q) \bullet (Q\supset\top)] \equiv (\top \equiv Q)]$

$(Q \bullet\top) \equiv (\top\equiv Q)$

$Q \equiv (\top\equiv Q)$

$\top\equiv (\top\equiv\top)$ $\quad \bot\equiv (\top\equiv\bot)$

$\top\equiv\top$ $\quad \bot\equiv\bot$

$\top$ $\quad \top$

$[(\bot\supset Q) \bullet (Q\supset\bot)] \equiv (\bot\equiv Q)$

$[\top\bullet (Q\supset\bot)] \equiv (\bot\equiv Q)$

$(Q\supset\bot) \equiv (\bot\equiv Q)$

$(\top\supset\bot) \equiv (\bot\equiv\top)$ $\quad (\bot\supset\bot) \equiv (\bot\equiv\bot)$

$\bot\equiv\bot$ $\quad \top\equiv\top$

$\top$ $\quad \top$

CONSTRUCTIVE DILEMMA

$(P \lor R) \supset \{[P \supset Q) \bullet (R \supset S)] \supset (Q \lor S)\}$

$(\top \lor R) \supset \{[(\top \supset Q) \bullet (R \supset S) \supset (Q \lor S)]\}$

$\top \supset \{[Q \bullet (R \supset S)] \supset (Q \lor S)]\}$

$[Q \bullet (R \supset S)] \supset (Q \lor S)$

$(\top \bullet (R \supset S)] \supset (\top \supset S)$ $\quad$ $[\bot \bullet (R \supset S)] \supset (\bot \lor S)$

$(R \supset S) \supset \top$ $\quad$ $\bot \supset (\bot \lor S)$

$\top$ $\quad$ $\top$

$(\bot \lor R) \supset \{[(\bot \supset Q) \bullet (R \supset S)] \supset (Q \lor S)\}$

$R \supset \{[\top \bullet (R \supset S)] \supset (Q \lor S)\}$

$R \supset [(R \supset S) \supset (Q \lor S)]$

$\top \supset [(\top \supset S) \supset (Q \lor S)]$ $\quad$ $\bot \supset [(\bot \supset S) \supset (Q \lor S)]$

$S \supset (Q \lor S)$ $\quad$ $\top$

$\top \supset (Q \lor \top)$ $\quad$ $\bot \supset (Q \lor \bot)$

$\top \supset \top$ $\quad$ $\top$

$\top$

AXIOMS OF THE AXIOMATIC PROOF

$P \supset (PP)$

$\top \supset (\top\top)$ $\quad$ $\bot \supset (\bot\bot)$

$\top$ $\quad$ $\top$

$(PQ) \supset P$

$(\top Q) \supset \top$ $\quad$ $(\bot Q) \supset \bot$

$Q \supset \top$ $\quad$ $\bot \supset \bot$

$\top \supset \top$ $\quad$ $\bot \supset \top$ $\quad$ $\top$

$\top$ $\quad$ $\top$

$(P \supset Q) \supset [\sim(QR) \supset \sim(RP)]$

$(\top \supset Q) \supset [\sim(QR) \supset \sim(R\top)]$ $\quad$ $(\bot \supset Q) \supset [\sim(QR) \supset \sim(R\bot)]$

$Q \supset [\sim(QR) \supset \sim R]$ $\quad$ $\top \supset [\sim(QR) \supset \top]$

$\top \supset [\sim(\top R) \supset \sim R]$ $\quad$ $\bot \supset [\sim(\bot R) \supset \sim R]$ $\quad$ $\sim(QR) \supset \top$

$\sim R \supset \sim R$ $\quad$ $\top$ $\quad$ $\top$

$\top$

CONJUNCTION AND EQUIVALENCE

7. Conjunction can be carried out through the following steps.

Starting with the bracket part of Axiom 3 we get:

$[\sim(QR) \supset \sim(RP)] \supset (RP \supset QR)$

This is like the second part of 6. Next, $(RP \supset QR)$ goes to $(RP \supset PR)$ by putting P for Q. This last structure provides a rule of thumb: surround an implication with the same letter.

8. Next we need the following:

a) $(P \supset Q)$ to $(Q \supset R)$ yields $(P \supset R)$, where Q cancels out to arrive at $(P \supset R)$

b) $(R \supset S)$ leads to $[(PR) \supset (PS)]$

c) $P\,(QR) \supset (PQ)\,R$

In order to get to **b)** we have to have **a)**, and in order to apply **b)** we have to achieve **c)** – see note at end for **c)**.

9. **a)** is developed in this way. Convert $(Q \supset R)$ to $(\sim R \supset \sim Q)$ like the first part of 6. Then put this last structure into Axiom 3:

$(\sim R \supset \sim Q) \supset\sim(\sim QP) \supset \sim (P\sim R)]$

The last term, $\sim(P\sim R)$ or $(P\supset R)$, is achieved by liaison P. Two applications of modus ponens are also required to isolate the single term $(P \supset R)$ – if the first term then the bracket, given the first, therefore the bracket; if the second term then the third, given the second therefore the third, $(P \supset R)$.

b) Once **a)** is established, we can develop **b)** as follows:

surround $(R \supset S)$ with P, $(PR \supset SP)$; surround $(P \supset P)$ with S, $(SP \supset PS)$; with SP canceling and $(PR \supset PS)$ resulting by **a)**. A grand substitution for R and for S in $(PR \supset PS)$ is then:

$$[P \sim\sim (Q \sim R)] \supset [P\,(Q \sim R)].$$

By **c)** we get:

$$[P\,(Q \sim R)] \supset [(PQ) \sim R)]$$

By **a)** we get: $[P\,(Q \sim R)]$ canceling

$$[P \sim\sim (Q \sim R)] \supset [(PQ) \sim R]$$

Reversing and negating, as in 6, we get:

$$\{[(P \sim\sim (Q \sim R)] \supset [(PQ) \sim R]\} \supset \{\sim[PQ) \sim R] \supset \sim[P \sim\sim (Q \sim R)]\}$$

Which is, by modus ponens from the last two steps:

$$\sim[(PQ) \sim R] \supset \sim [P \sim\sim (Q \sim R)]$$

Which is, definitionally:

$$[(PQ) \supset R] \supset [P \supset (Q \supset R)]$$

And substituting (PQ) for R we get:

$$[(PQ) \supset (PQ)] \supset [P \supset (Q \supset (PQ))]$$

Since $[(PQ \supset (PQ)]$ is obvious, like $(P \supset P)$ above, it can be used with the last step in modus ponens to give:

$$[P \supset (Q \supset (PQ))]$$

Which is: if this, then if that, then this and that – if Jane is happy, then if Jane is a child, then Jane is a happy child.

Note on c)

In **c)** Axiom 2 is used. The method: chop off last letter.

Getting P:

$(PQ)\ R \supset (PQ)$	Ax. 2
$(PQ) \supset P$	Ax. 2
$(PQ)\ R \supset P$	a), (PQ) canceling

Getting Q:

$(PQ)\ R \supset (PQ)$	Ax. 2
$(PQ) \supset (QP)$	surround $(Q \supset Q)$ with P
$(PQ)\ R \supset (QP)$	a)
$(QP) \supset Q$	Ax. 2.
$(PQ)\ R \supset Q$	a), (QP) canceling

Getting R:

$(PQ)\ R \supset R\ (PQ)$	surround $(R \supset R)$ with (PQ)
$R\ (PQ) \supset R$	Ax. 2
$(PQ)\ R \supset R$	a), $R\ (PQ)$ canceling

Arriving at (QR):

$(PQ)\ R \supset (QR)$	If $(PQ)\ R$ implies Q and R successively as above, then it implies both in the above order.

Arriving at $P\ (QR)$:

$(PQ)\ R \supset P\ (QR)$	If $(PQ)\ R$ implies P and (QR) successively as above, then it implies both in the above order.

So, the outer pairs are parenthesized in the foregoing manner. The next sequence is to parenthesize the inner pairs, done as follows:

$P\ (QR) \supset (QR)\ P$	surround (QR) with P
$(QR)\ P \supset Q\ (RP)$	parenthesizing outers
$Q\ (RP) \supset (RP)\ Q$	surround (RP) with Q
$(RP)\ Q \supset R\ (PQ)$	parenthesizing outers

$$R\,(PQ) \supset (PQ)\,R \qquad \text{surround } (PQ) \text{ with } R$$

So, *P* (*QR*) top left, by step-wise changes, goes to (*PQ*) *R* bottom right which is **c)**. The stepwise changes are done by taking the right part of one line and putting it to the left in the line below – taking (*QR*) *P* to the right in the first line and putting it to the left in the second line. And so on successively downward.

Thus **c)** is achieved.

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